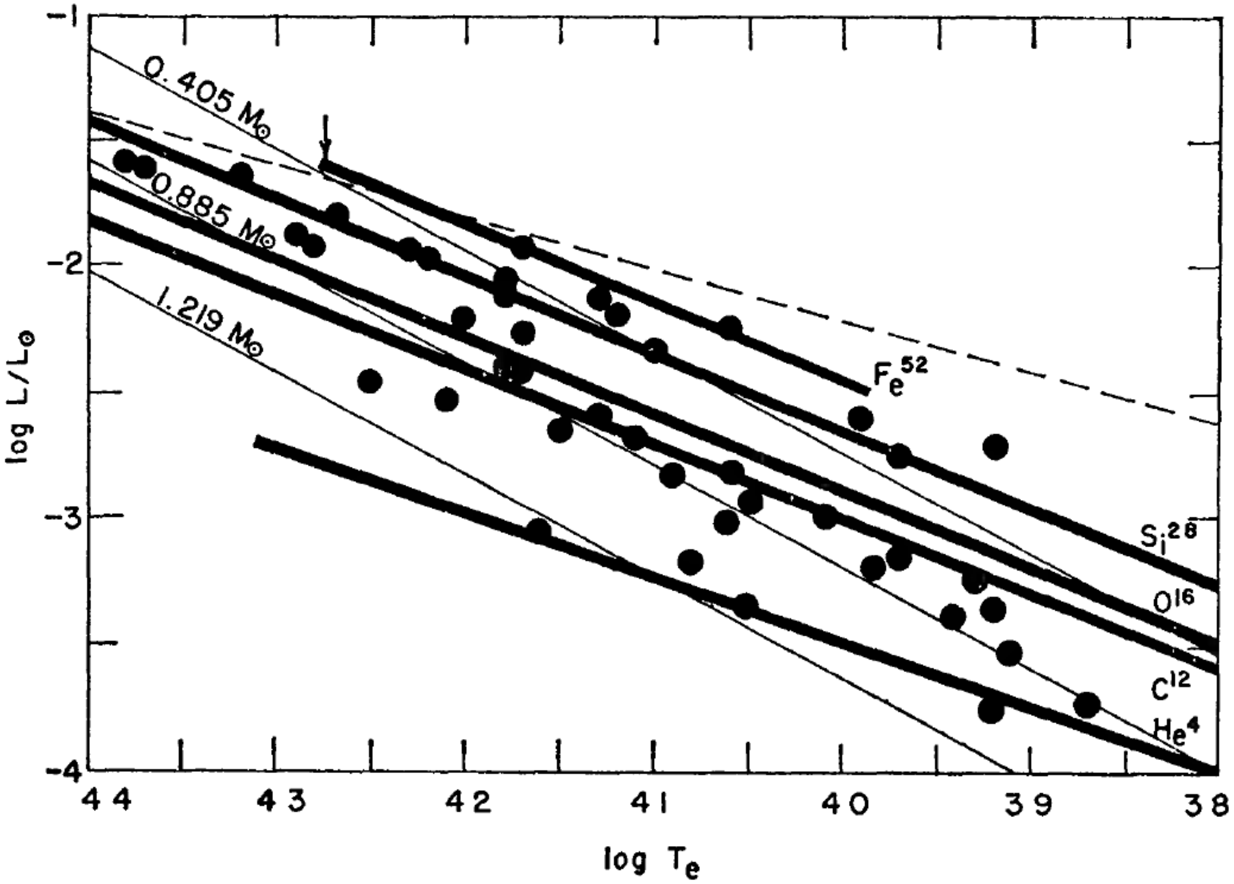
Today in Astronomy 241: degenerate stars III

- White dwarf cooling
- The ages of white dwarfs, the Galaxy, and the Universe
- Very few words about neutron stars
- **Reading**: C&O chapter 16, pp 572-586



Crystallization sequences in the cooling curves for pure Fe, Si, O, C, and He white dwarfs, top to bottom, plotted with field white dwarfs. From Van Horn 1968.

White dwarf cooling

Energy transport in white dwarf interiors is dominated by **conduction** by the degenerate electrons, instead of radiation.

- Conduction is effective enough to render the interior nearly isothermal.
- A thin layer at the surface is usually nondegenerate and radiative or convective, though:
    - Energy leaks slowly to the surface.
    - A steep temperature gradient exists in the surface layer.
    - The photosphere cools more slowly than the isothermal interior.
- Because of this, **luminosity indicates age of the white dwarf**, precisely and without many assumptions.
- And because white dwarfs sit there and cool off forever, the number of white dwarfs per unit luminosity should increase with decreasing luminosity, if the formation rate of white dwarfs is constant.

White dwarf cooling (continued)

- If a WD's energy supply were just its internal energy at the onset of electron degeneracy, its luminosity as a function of time comes out simply (in-class problem 1):

$$L = L_0\left(1+\frac{5}{2}\frac{t}{\tau_0}\right)^{-7/5} \quad \text{, and} \qquad T_e = \left(\frac{L}{4\pi\sigma R^2}\right)^{1/4} \quad ,$$

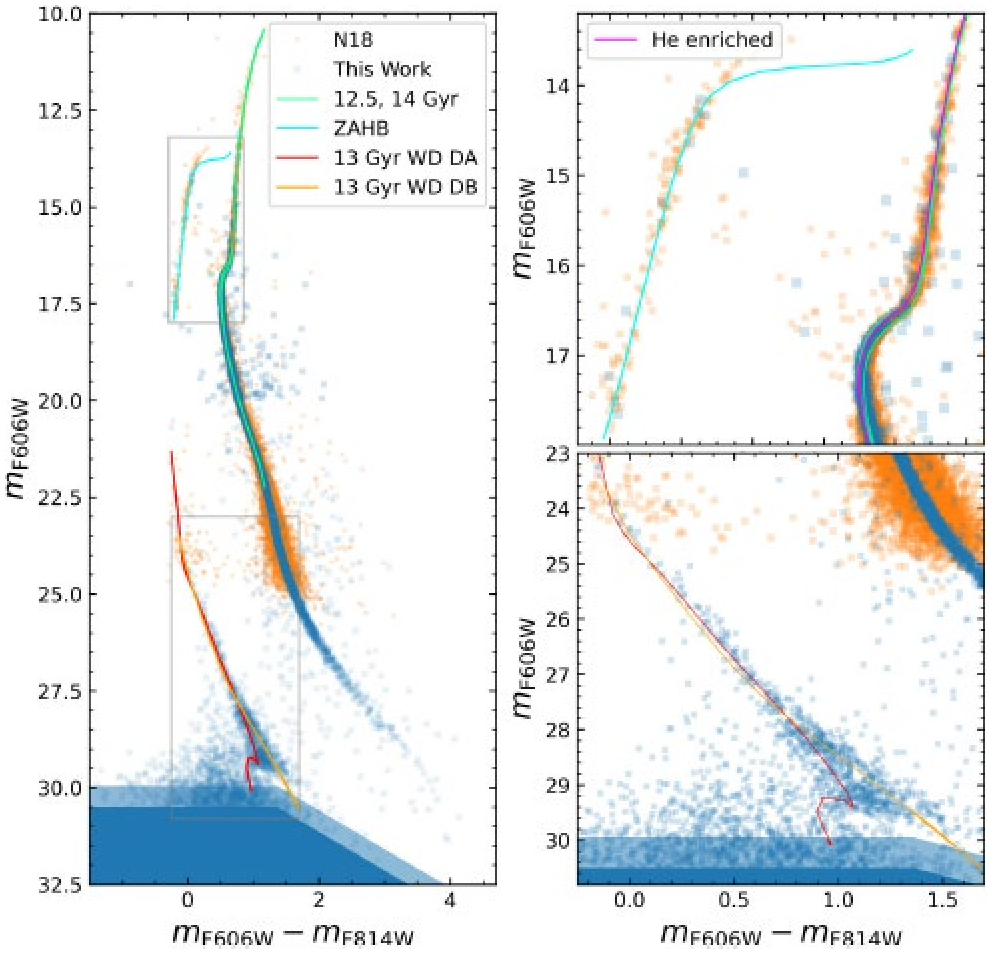
$$\text{where } L_0 = CT_C^{7/2} \quad , \quad \tau_0 = \frac{3}{2}\frac{Mk}{Am_H C^{2/7} L_0^{5/7}} \quad ,$$

$$\text{and } C = \frac{4}{17}\left(\frac{\hbar^2}{3m_e k}\right)^3 \left(\frac{3\pi^2}{m_H}\frac{Z}{A}\right)^2 \frac{64\pi\sigma}{3}\frac{G\mu m_H M}{\kappa_0 k} \quad .$$

- Accounting for the latent heat of crystallization of, and phase separation among, the positive ions, additional energy appears during cooling. See C&O problem 16.8, on Homework #10.

White dwarf cooling and the age of the Galaxy

- The WD formation rate is not constant, as the Universe hasn't lasted forever.
- Indeed, the observed distribution of WDs in globular clusters has a sharp cutoff, **indicating a maximum age to their WDs**.
- Thus is obtained an age estimate for globular clusters, which potentially is more precise than that from isochrone fitting.
  - The latter is easier to observe, though.
- For example: Bedin+2023 get an age for NGC 6752 of 12.7-13.5 Gyr, consistent between isochrone fitting and maximum WD age.
- The Galaxy's disk and the Universe must therefore be at least this old: a lower limit to their ages which is independent of Universal expansion, the cosmic background, and the standard model of cosmology.



NGC 6752 from HST, by Bedin+2023.

Neutron stars

- If a stellar remnant is more massive than the Stoner-Anderson-Chandrasekhar limit, it cannot be in hydrostatic equilibrium, and thus collapses past white dwarf size.
- As it does so, it heats up dramatically, allowing endothermic nuclear reactions such as inverse beta decay and nuclear dissolution to take place.
- This results in conversion of all of the protons and electrons into neutrons.
- Neutrons are fermions, so eventually the collapse may be stopped by neutron degeneracy pressure.
  - As for electrons, neutron degeneracy pressure sets in when neutron separations are similar to their deBroglie wavelengths.
- Thus there are **neutron stars**, directly analogous to white dwarfs.
  - Which, according to their deBroglie wavelengths, have radii about 1/1836 (= m_e/m_n) the size of a white dwarf. Regard a neutron star as a gigantic atomic nucleus, except without net electric charge.

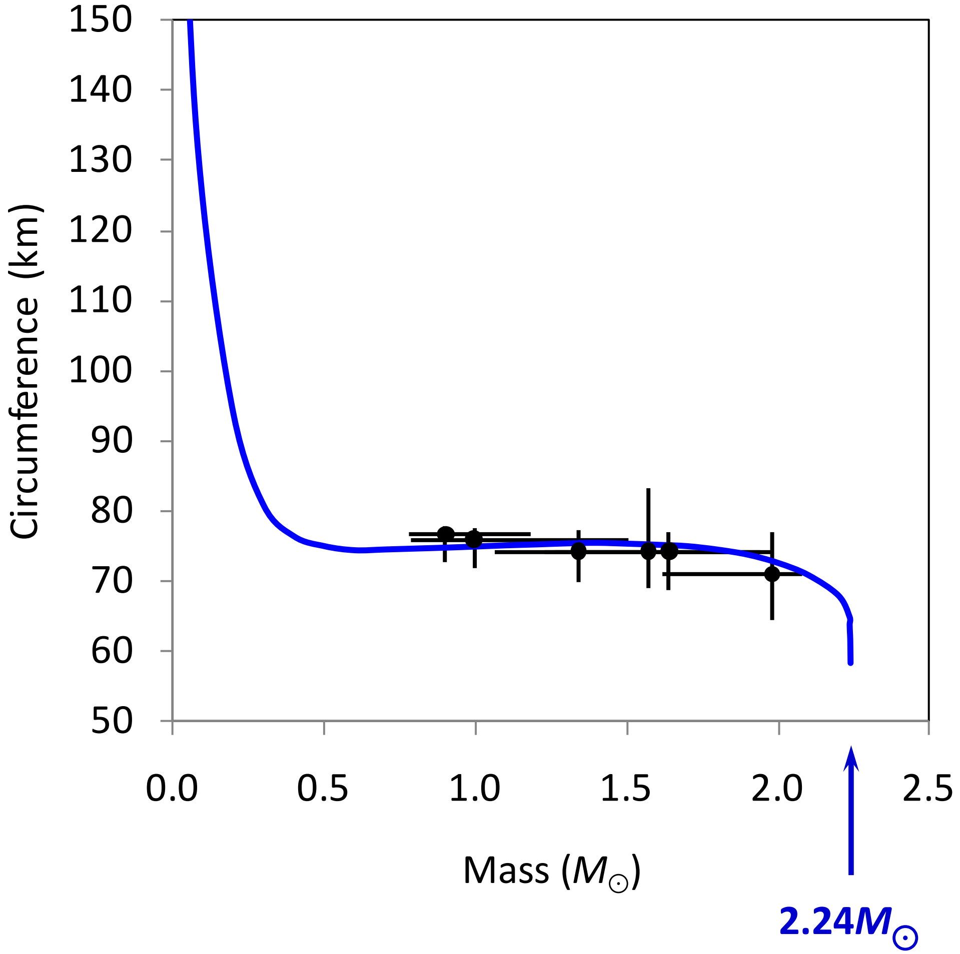
Neutron stars (continued)

- Unfortunately for us, the resulting dimensions are such that there are necessary ingredients to the equation of state, the combination of which is beyond the scope of an undergraduate education:
  - the strong interaction – which, for neutron-star conditions is inaccessible to lab measurements.
  - general relativity.
- But no matter what, there is still a maximum speed for neutrons (c) just as for electrons, so there is a maximum mass for neutron stars …
- … past which nothing can stop the collapse. Such an object would be a black hole.
- The maximum mass of neutron stars is thought to lie in the range $2.2 - 2.4 M_{\odot}$.

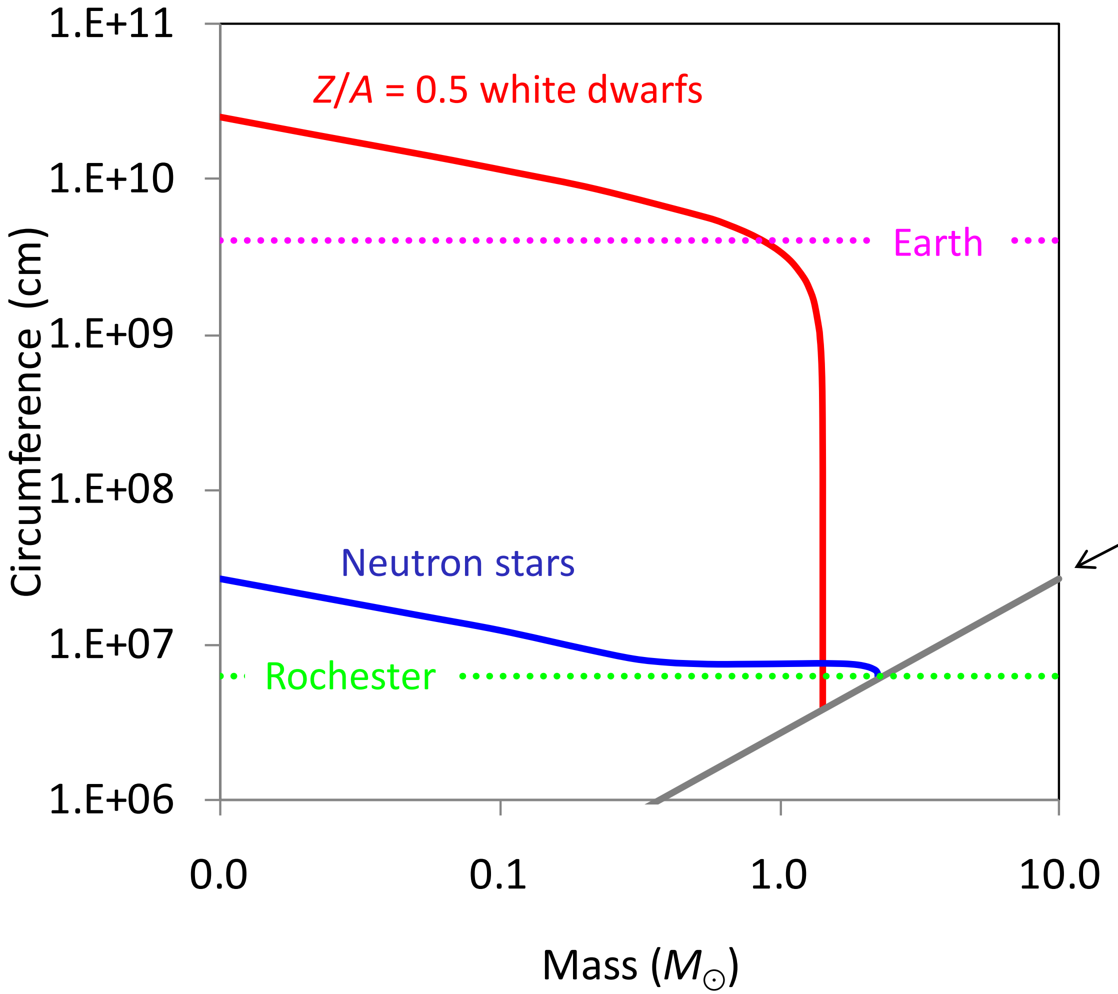
Neutron stars (continued)

Theoretical calculations (— , Lattimer & Prakash 2007; see also Hebeler+2013) currently give a good account of the data on neutron stars in binaries (●; Steiner+2010; see also Demorest+2010).

Note: circumference plotted, instead of radius.



Degenerate stars



Causality cutoff
A star smaller in circumference than

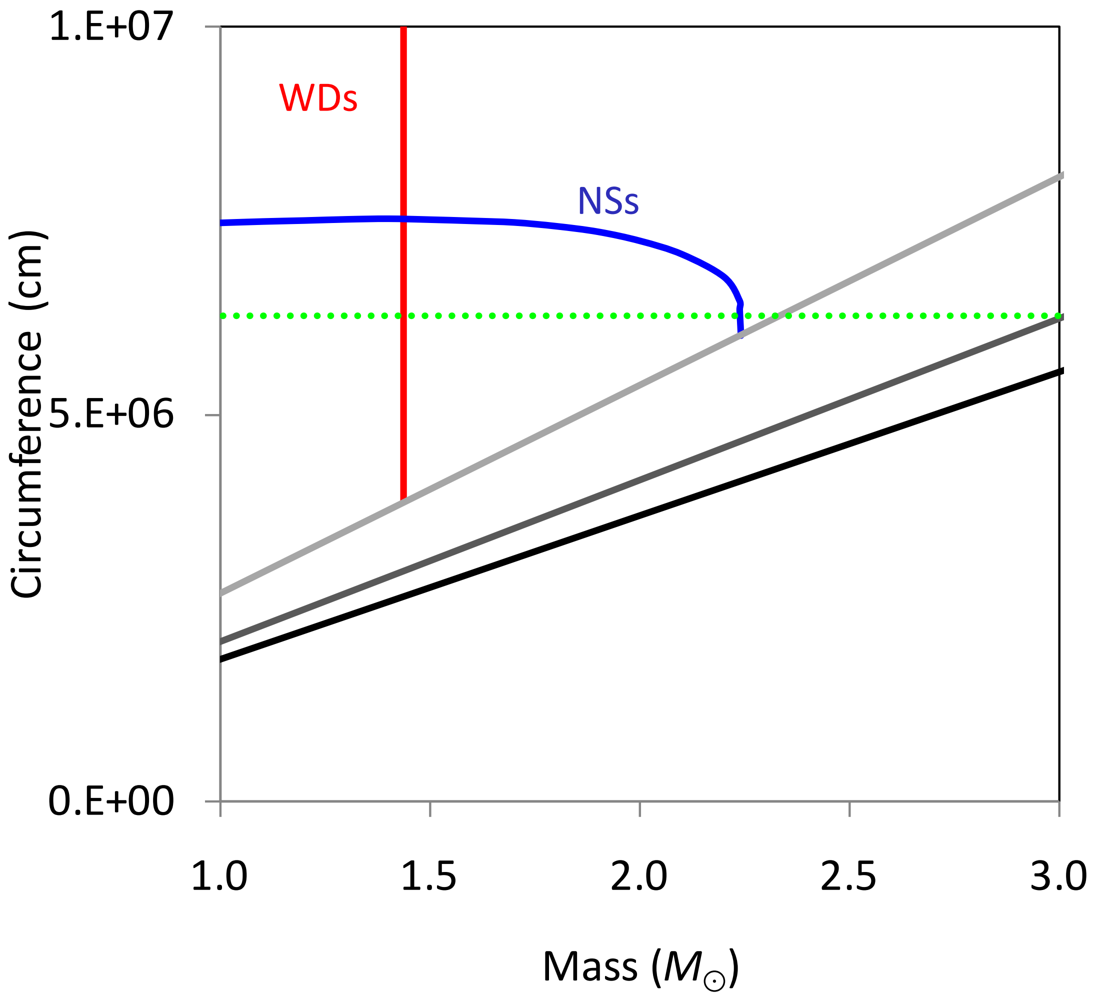
$$C = 5.8\pi GM/c^2$$

turns out to require an EOS with

$$\partial P/\partial\rho > c^2,$$

which in turn implies speed of sound $v_s > c$: violates order of cause and effect.

Degenerate stars near their size limits



Below these lines, the interior of a **stable** star would have:

$v_s > c$

$P > \infty$

an event horizon (it would be a black hole).

Proximity to the last one is how you can tell we need general relativity to describe degenerate stars in this regime.

Today's in-class problems

1. Armed with nothing but the equations of hydrostatic equilibrium and energy transfer,

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2} \quad , \quad \frac{dT}{dr} = -\frac{3}{16\sigma}\frac{\overline{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2} \quad ,$$

   the ideal gas law and the Kramers bound-free opacity,

$$P = \frac{\rho kT}{\mu m_H} \quad , \quad \overline{\kappa} = \kappa_0 \frac{\rho}{T^{3.5}} \quad , \quad \kappa_0 = 4.34\times 10^{25}\frac{g_{bf}}{t}Z(1+X)\ \mathrm{cm^5\ gm^{-2}\ K^{3.5}} \quad ,$$

   and the idea that the pressure and temperature are related at the **base** of the nondegenerate atmosphere – where it turns degenerate – by

$$\frac{T}{\rho^{2/3}} = \mathcal{D} = \frac{\hbar^2}{3m_e k}\left(\frac{3\pi^2}{m_H}\frac{Z}{A}\right)^{2/3} = 1.3\times 10^5\mathrm{K\ cm^2\ gm^{-2/3}} \quad ,$$

Today's in-class problems (continued)

derive C&O equation 16.19, the relation between the white dwarf's luminosity L and the nearly-uniform temperature T_c of its interior:

$$L = \frac{4\mathcal{D}^3}{17}\frac{64\pi\sigma}{3\kappa_0}\frac{G\mu m_H M}{k}T_c^{7/2} = \frac{4}{17}\left(\frac{\hbar^2}{3m_e k}\right)^3\left(\frac{3\pi^2}{m_H}\frac{Z}{A}\right)^2\frac{64\pi\sigma}{3}\frac{G\mu m_H M}{\kappa_0 k}T_c^{7/2} \quad .$$

2. C&O 16.12.

Answers and/or secrets to in-class problems

1. Combine hydrostatic equilibrium with energy transport and use the chain rule:

$$\frac{dP}{dT} = \frac{dP}{dr}\frac{dr}{dT} = +\frac{GM_r\rho}{r^2}\frac{16\sigma}{3}\frac{T^3}{\overline{\kappa}\rho}\frac{4\pi r^2}{L_r}$$

$$= \frac{64\pi\sigma}{3\overline{\kappa}}\frac{GM}{L}T^3 \quad \text{near the star's surface.}$$

Then throw in the bound-free opacity and the ideal-gas equation of state: $\overline{\kappa} = \kappa_0 \rho T^{-3.5} = \frac{\kappa_0 \mu m_H}{k} P T^{-4.5}$,
and one obtains

$$PdP = \left(\frac{64\pi\sigma}{3\kappa_0}\frac{GM}{L}\frac{k}{\mu m_H}\right)T^{15/2}dT \quad .$$

Integrate this directly using the usual surface boundary conditions, between far far away – where pressure and temperature both approach zero – and the base of the nondegenerate photosphere, where $\rho = \left(T/\mathcal{D}\right)^{3/2}$,
and solve the result for *L*.

Answers and/or secrets to in-class problems (continued)

2. (C&O 16.12) First estimate the speed:

$$v = \frac{p}{m_n} \approx \frac{\hbar}{m_n} n_n^{1/3} = \frac{\hbar}{m_n}\left(\frac{\rho}{m_n}\right)^{1/3} \cong 6\times 10^9 \text{ cm sec}^{-1} = 0.2c.$$

This is small enough that, for a crude estimate, we can use the nonrelativistic expression for degeneracy pressure. Adapt that for electron pressure, using Z/A = 1 and replacing $m_e \rightarrow m_n$:

$$P_n = \frac{\left(3\pi^2\right)^{2/3}}{5}\frac{\hbar^2}{m_n}\left(\frac{\rho}{m_n}\right)^{5/3} = 1.1\times 10^{35} \text{ dyne cm}^{-2} \approx 6\times 10^{11} P_C\left(\text{Sirius B}\right).$$