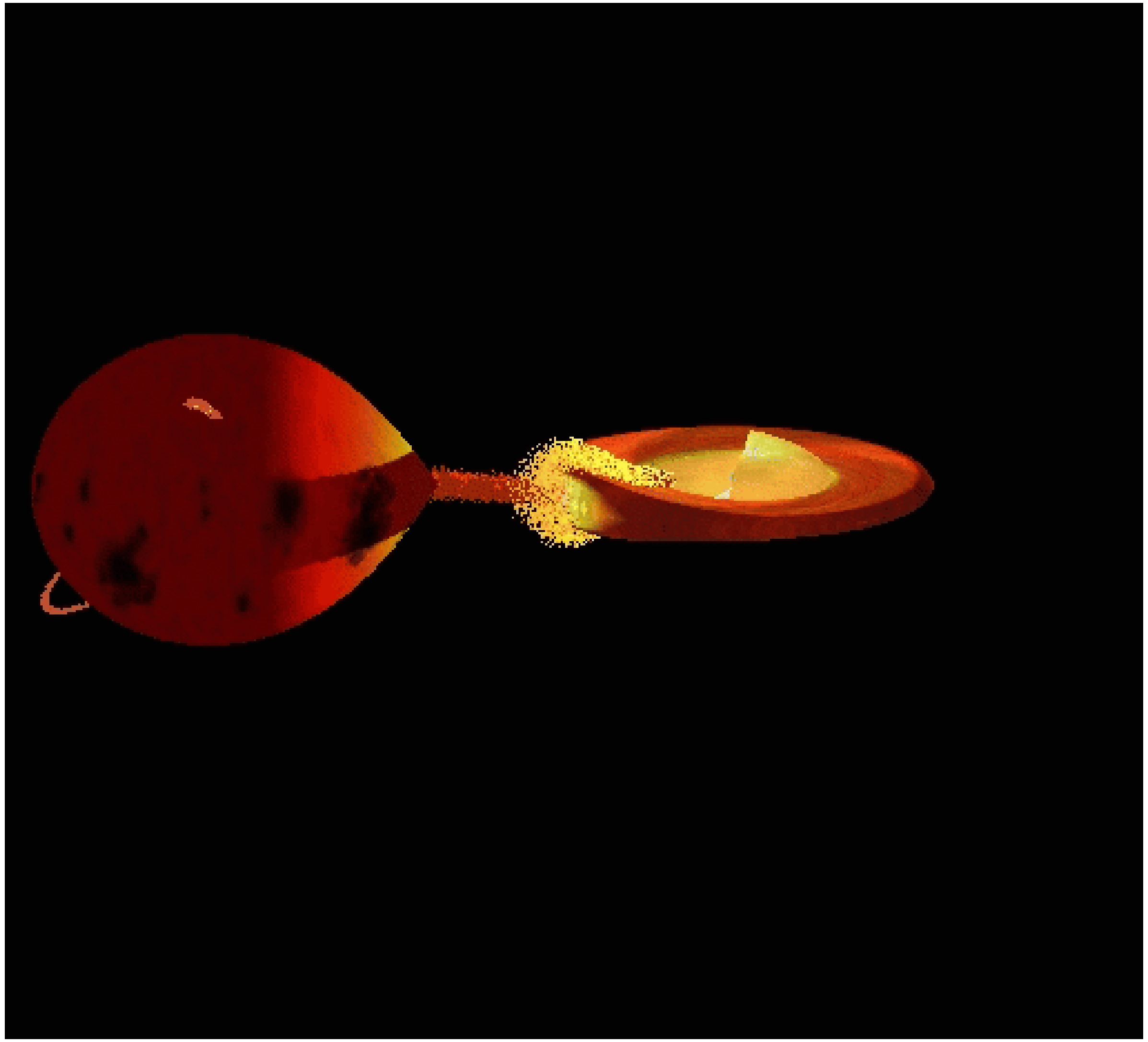
Today in Astronomy 241: accretion disks I

- The restricted three-body problem – review from ASTR 111
- Roche-lobe overflow
- Accretion disk formation
- **Reading**: C&O chapter chapter 18, pp 653-661, 668-673



Realistic animation of a cataclysmic variable system – an intermediate polar, to be precise – by Andy Beardmore, Leicester U.

Restricted three-body problem

In corotating frame of reference, neglecting Coriolis force:

$$r_1 + r_2 = a \qquad M_1 r_1 = M_2 r_2$$

$$s_1^2 = r_1^2 + r^2 + 2 r_1 r \cos\vartheta$$

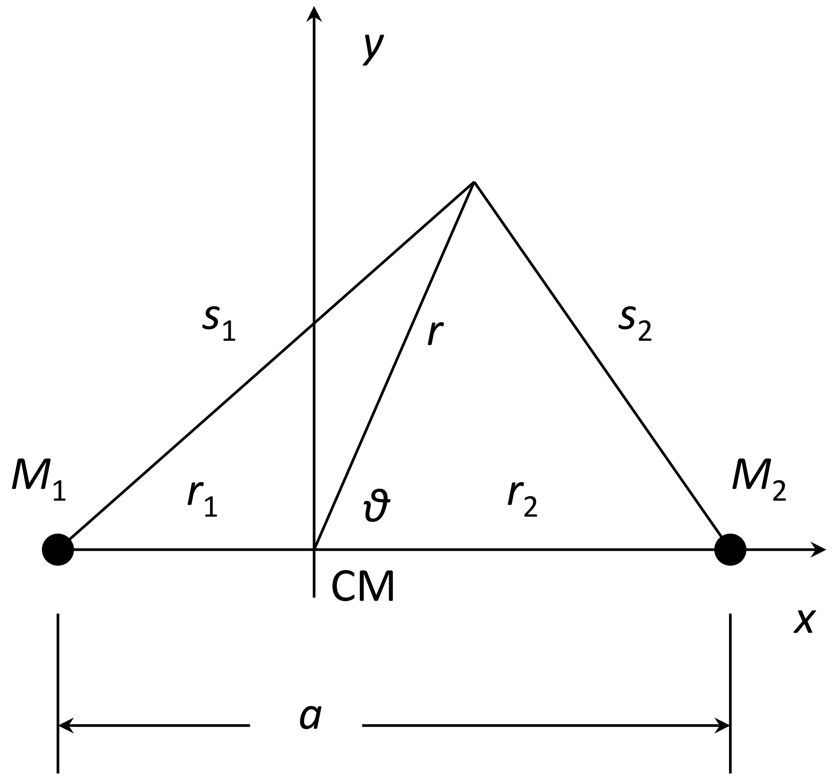
$$s_2^2 = r_2^2 + r^2 - 2 r_2 r \cos\vartheta$$

Gravitational potential:

$$\Phi = -G\left(\frac{M_1}{s_1} + \frac{M_2}{s_2}\right) - \frac{1}{2}\omega^2 r^2$$

Kepler's third law:

$$\omega^2 = \frac{G\left(M_1 + M_2\right)}{a^3}$$



Restricted three-body problem (continued)

Suppose mass is transferred at rate $\dot{M}_1$ from star 2 to star 1. Mass M and angular momentum L are conserved, so

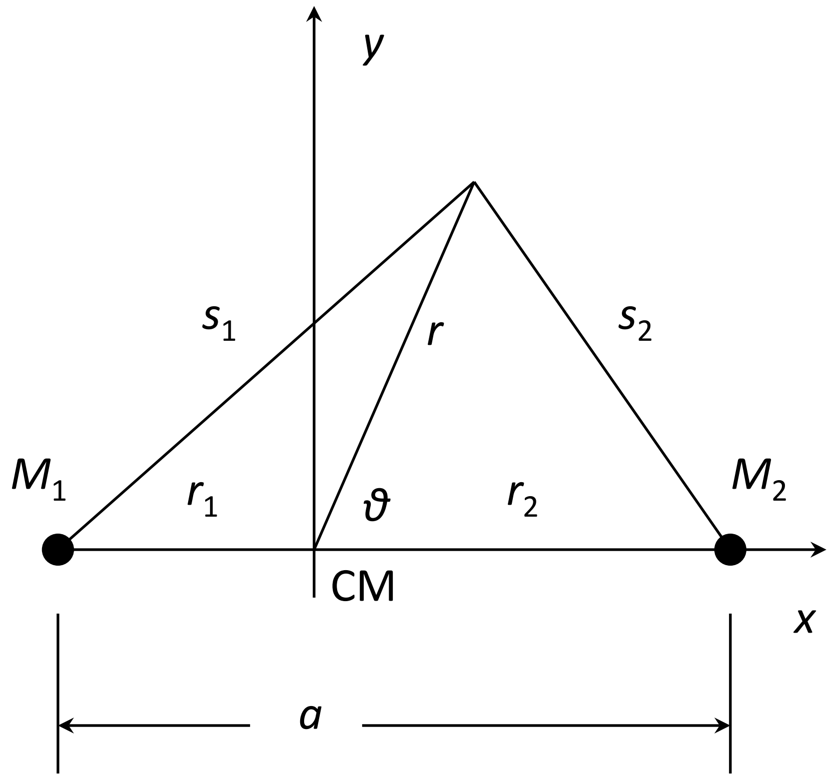
$$L = \frac{M_1 M_2}{M_1 + M_2}\sqrt{G(M_1+M_2)a} \equiv \mu\sqrt{GMa}$$

$$\frac{1}{a}\frac{da}{dt} = -\frac{2}{\mu}\frac{d\mu}{dt}$$

$$\frac{d\mu}{dt} = \frac{\dot{M}_1}{M}\left(M_2 - M_1\right)$$

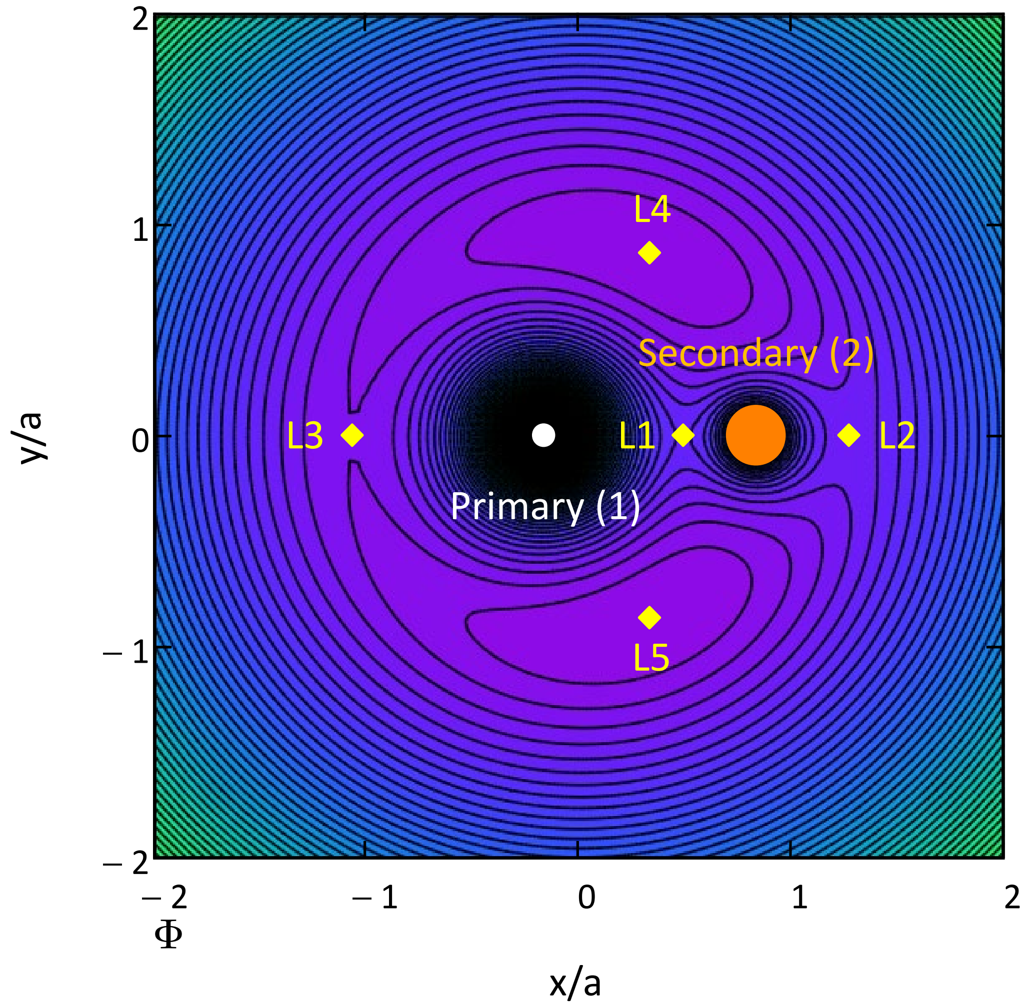
$$\frac{da}{dt} = 2a\dot{M}_1\frac{M_1 - M_2}{M_1 M_2}$$

$$\frac{1}{\omega}\frac{d\omega}{dt} = -\frac{3}{2}\frac{1}{a}\frac{da}{dt}$$



Restricted three-body problem (continued)

- Extrema of the gravitational potential, of which there are five besides the stars, are the familiar Lagrange points.
    - Gravitational force vanishes there; an object placed at one will stay fixed there, and orbit the CM along with the stars…
    - … unless they suffer perturbations: they are maxima or saddle points of Φ, in general. So these are not stable equilibria.
    - … except if one adds the Coriolis force, and considers $M_1 > 25M_2$; then L4 and L5 are stable equilibria.
- The **inner Lagrange point,** L1, is the only one of interest to us this semester.

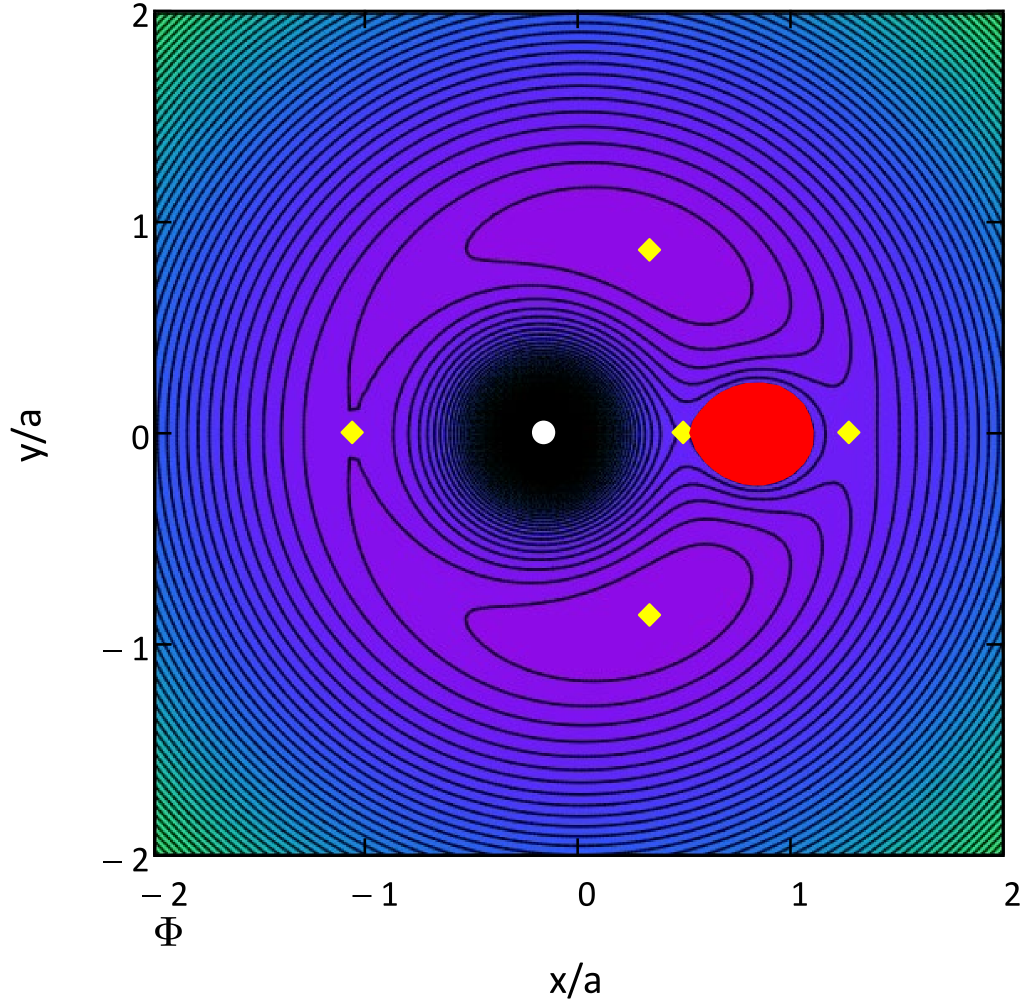


$M_1/M_2 = 5,\ a = 1$ AU

Roche lobes

The space enclosed by the equipotential surface containing L1 is called the **Roche lobe** of the star within that space.

- A star approaching the size of its Roche lobe begins to assume the lobe's teardrop shape, since the star's surface is also, perforce, an equipotential.
- A star that fills or overfills its Roche lobe can donate mass to its companion, since **no work need be done to move a mass through L1**.



$M_1/M_2 = 5,\ a = 1$ AU

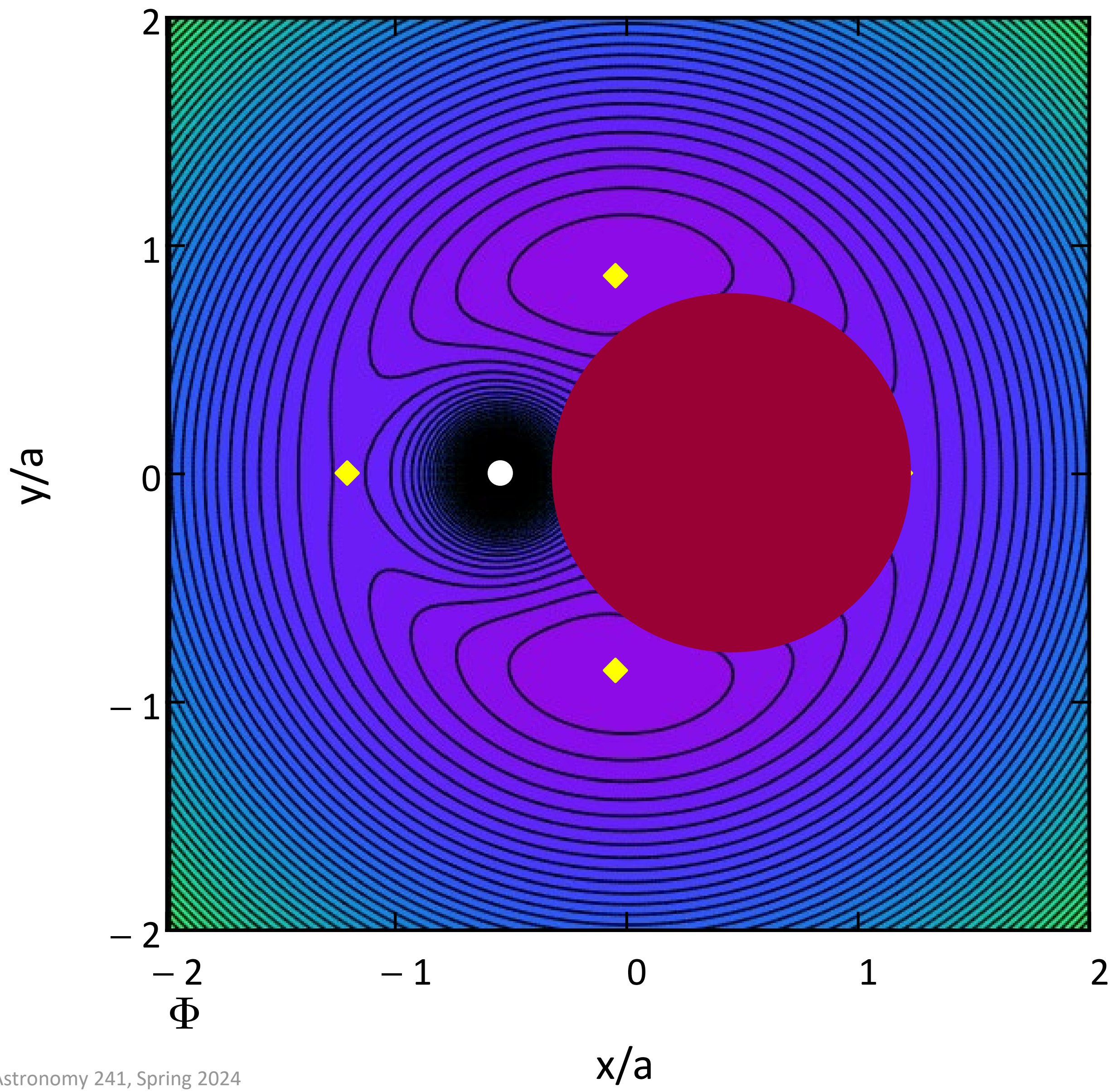
Restricted three-body problem (continued)

Gravitational potential Φ for the most common mass of white dwarf and a mass near the main-sequence turnoff,

$$M_1/M_2 = 0.58M_\odot/0.72_\odot\,,$$

separated by *a* = 1 AU.

- The symbols for the stars are not to scale, for the WD and the turnoff star, but the symbols for the secondary in the red giant and AGB phases are.



Mass transfer in semidetached binaries

For a star with radius *R* that has slightly over-filled its Roche lobe, mass flows through L1 at a rate

$$\dot{M} \approx \pi \rho R d \sqrt{\frac{3kT}{m_H}}$$

where *d* is the distance of overlap between the star's surface and its Roche lobe. (See example 18.1.)

- The mass transfer rate increases rapidly with *d* for more substantial overfilling.
- Mass transfer can impart **very** large energy to the primary, since it has a long way to fall: especially in the most interesting cases of degenerate primaries.

Accretion disks

- Transferred mass generally does not fall straight down to the primary, because it has angular momentum in that body's reference frame. (See page 3.)
- Instead it falls into orbit around it, assuming the form of a disk.
- Viscous dissipation in the disk results in flow of matter through the disk large radii to small, and eventually onto the primary: this is an **accretion disk**. Its basic properties, crudely:

$$T(r) = \left(\frac{3GM\dot{M}}{8\pi\sigma R^3}\right)^{1/4} \left(\frac{R}{r}\right)^{3/4} \left(1 - \sqrt{\frac{R}{r}}\right)^{1/4}$$

$$L = \frac{GM\dot{M}}{2R}$$

$$R_{\mathrm{disk}} \approx 2a\left[0.5 - 0.227\log\left(\frac{M_1}{M}\right)\right]^4 \left(1 + \frac{M_1}{M}\right)$$

M, R are the mass and radius of the primary star (#1)

Today's in-class problems

1. C&O problem 18.3

2. C&O problem 18.4

3. C&O problem 18.5

Answers

1. (a) All victims of ASTR 111 will remember that mass flux is $f_m = \rho v$, so, assuming the flux to be uniform over an area A, $\dot{M} = \rho v A$.

   (b) Whence it follows from the book's Figure 18.5 – or your own work on transits in homework #3 – that

$$R^2 = \left(R - \frac{d}{2}\right)^2 + x^2 \cong R^2 - Rd + x^2$$

$$x^2 = Rd = A/\pi \quad \Rightarrow \quad \dot{M} = \pi\rho v R d \approx \pi\rho R d\sqrt{\frac{3kT}{m_{\mathrm{H}}}}$$

2. Letting $x = R/r$ and straightforwardly setting $dT/dx = 0$:

$$\frac{dT}{dx} = T_{\mathrm{disk}} x^{3/4}\left(1-\sqrt{x}\right)^{1/4} = T_{\mathrm{disk}}\left[\frac{3}{4}x^{-1/4}\left(1-\sqrt{x}\right)^{1/4} - \frac{1}{4}x^{3/4}\left(1-\sqrt{x}\right)^{-3/4}\left(\frac{1}{2\sqrt{x}}\right)\right] = 0$$

$$1-\sqrt{x} = \frac{1}{6}\sqrt{x} \quad \Rightarrow \quad \frac{7}{6}\sqrt{x} = 1 \quad \Rightarrow \quad x = \frac{36}{49} \quad \Rightarrow \quad T_{\max} = T_{\mathrm{disk}}\left(\frac{36}{49}\right)^{3/4}\left(1-\sqrt{\frac{36}{49}}\right)^{1/4} = 0.488T_{\mathrm{disk}}$$

Answers (continued)

3. Straightforward integration:

$$L_{\text{disk}} = \int_R^\infty 4\pi r\sigma \left(\frac{3GM\dot{M}}{8\pi\sigma R^3}\right)\left(\frac{R}{r}\right)^3 \left(1-\sqrt{\frac{R}{r}}\right)dr = \frac{3GM\dot{M}}{2R^2}\int_R^\infty \left(\frac{R}{r}\right)^2 \left(1-\sqrt{\frac{R}{r}}\right)dr$$

Let $x = R/r$, $dx = -Rdr/r^2$, $x = 1 \rightarrow 0$, and switch the integration limits to cancel the minus sign:

$$L_{\text{disk}} = \frac{3GM\dot{M}}{2R}\int_0^1 \left(1-\sqrt{x}\right)dx = \frac{3GM\dot{M}}{2R}\left[x - \frac{x^{3/2}}{3/2}\right]_0^1 = \frac{GM\dot{M}}{2R}$$, q.e.d.