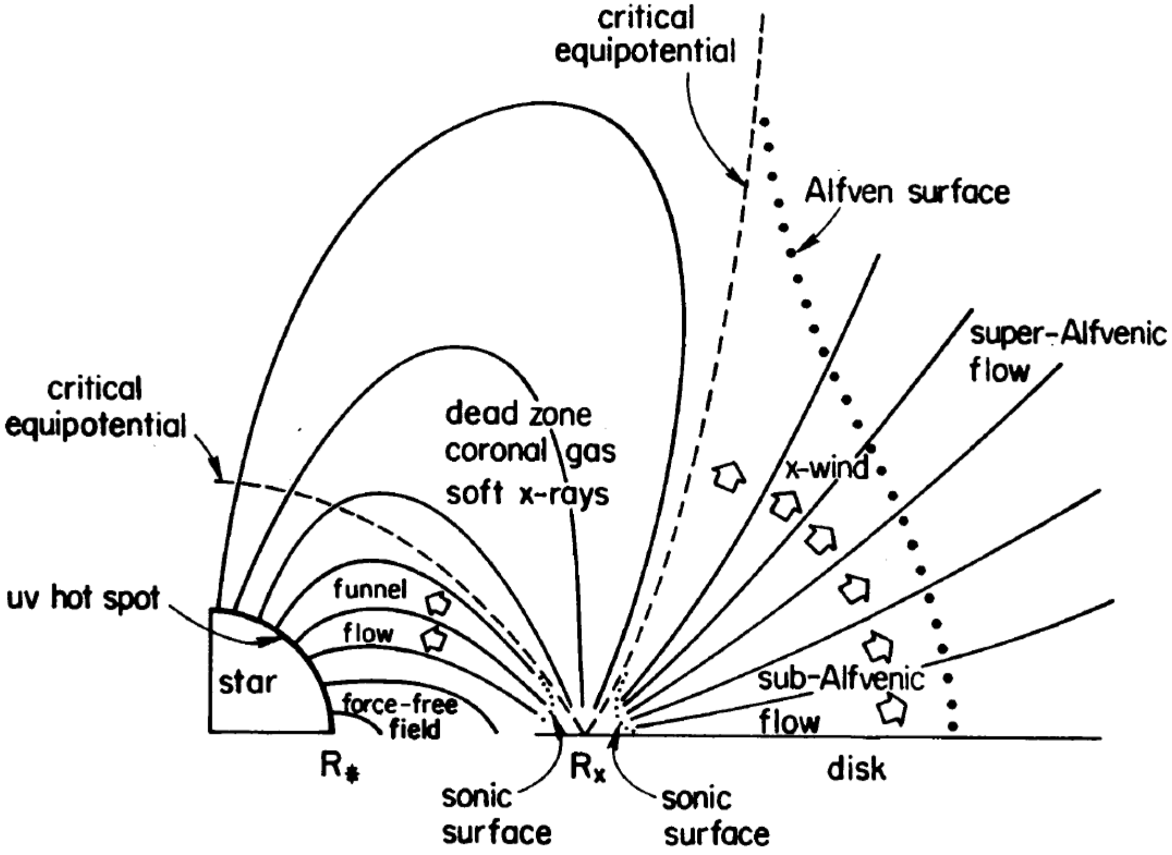
Today in Astronomy 241: accretion disks II

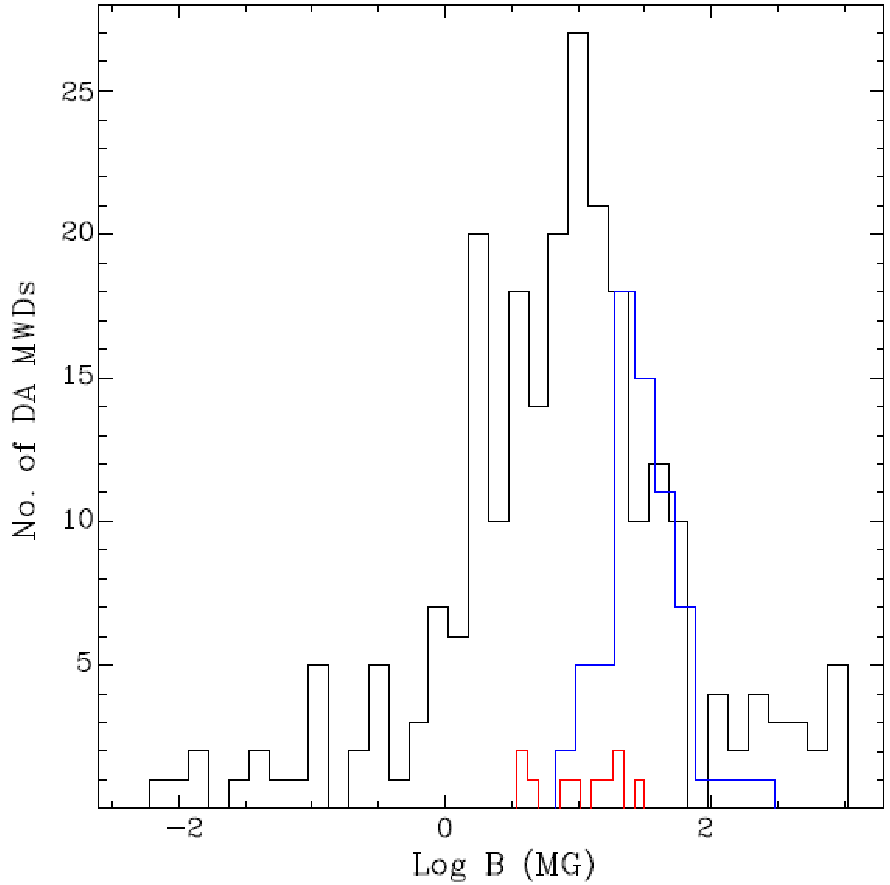
- Magnetic truncation of accretion disks, and magnetic accretion funnels.
- Vertical structure of accretion disks
- **Reading**: C&O chapter 18, pp 661-668



A magnetically truncated disk; Shu+1994.

White dwarfs, *B*, and accretion disks

- Some white dwarfs have large magnetic moments, including large numbers of those in cataclysmic variables.
  - *B* >> 1 megagauss at the poles.
  - Measurable directly by Zeeman effect, and by several indirect means.
- Their *B* is dynamically important, and the situation is simple enough to provide a better introduction to the dynamics than other magnetized disks.
  - Protoplanetary disks, for example, which are quite messy.
- We will focus on inner truncation as a particularly important dynamical effect of *B*.



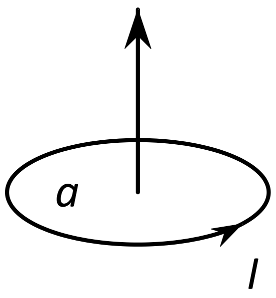
Surface *B* at the magnetic poles. Black: all magnetic DAs; blue: polars; red: intermediate polars. From Ferraio+2015.

Field and moment

- B is easiest to measure directly at the poles, as for neutron stars.
- B is dynamically more important where the disk is, in the equatorial plane.
- Thus white dwarf magnetism is often catalogued by magnetic dipole moment m_B (cgs units: gauss cm^3), so remember the spherical-coordinate result from PHYS 217,

$$\boldsymbol{B} = \frac{2m_B \cos\vartheta}{r^3}\hat{\boldsymbol{r}} + \frac{m_B \sin\vartheta}{r^3}\hat{\boldsymbol{\vartheta}} \quad .$$

$\boldsymbol{m}_B = Ia\hat{\boldsymbol{z}}$



- Also helps to remember the current-loop paradigm for magnetic dipoles, though they don't look much like magnetized stars.

Magnetic disk truncation

- As viscous torques (*vide* next class) rearrange an accretion disk's angular momentum, disk material flows toward the center – this is the accretion flow.

- The flow follows the gravitational force until the energy density in B from the star's magnetic field begins to match the kinetic energy density of the flowing material:

$$\frac{1}{2}\rho v^2 = \frac{B^2}{8\pi} \qquad \text{in cgs units}$$

- By the means you will demonstrate today in C&O 18.18, the equality defines the **Alfven radius**, r_A :

$$r_A = \left(\frac{m_B^4}{2GM_1\dot{M}^2}\right)^{1/7} \qquad \text{in cgs units}$$

Magnetic disk truncation (continued)

- By about $r_d = r_A / 2$, the (ionized) material is following the poloidal lines of *B*, and rather than the gravitational field, which is perpendicular to *B* and would subject the ions to Lorentz force.

- The disk is truncated for smaller radii.

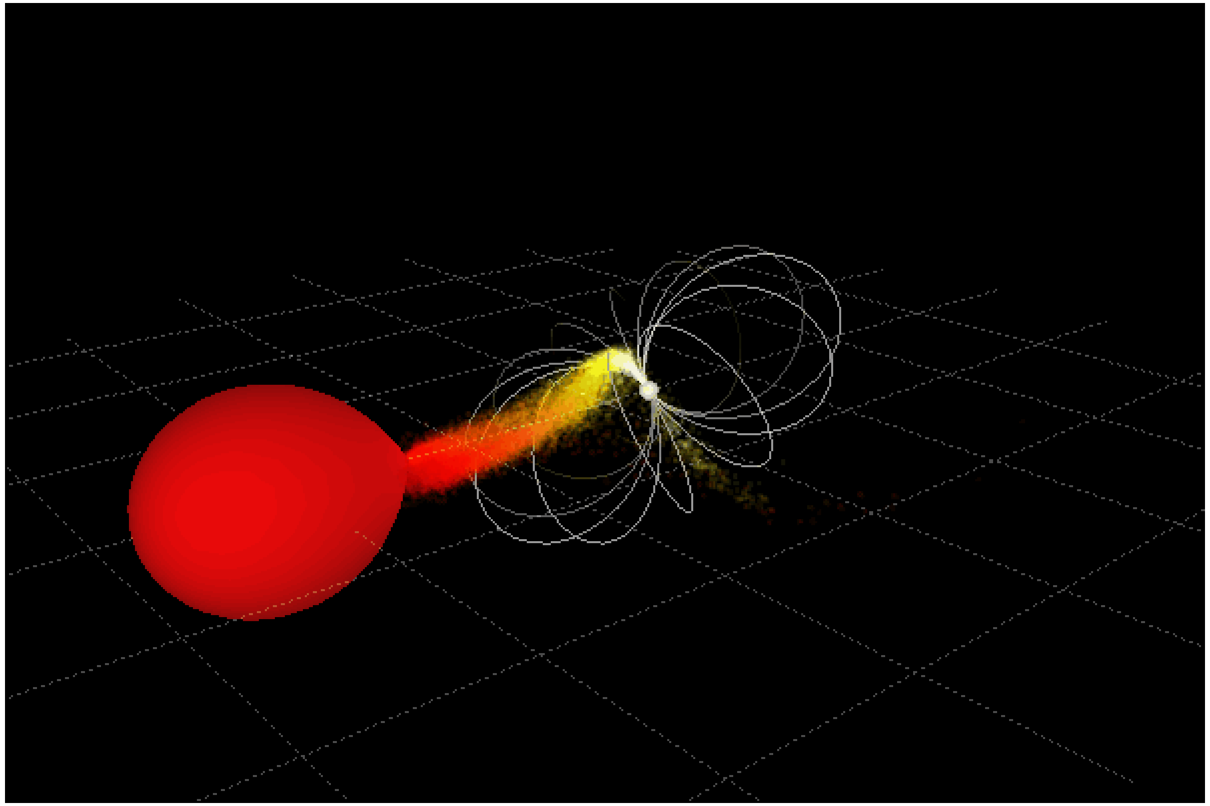
- The resulting field-guided flow is called a funnel flow, as the flow's cross-sectional area gets smaller as it approaches the stellar surface.

- This means that there's no turbulent boundary layer at the inner edge of the disk, since it does not contact the star. The disk's temperature therefore lacks its original second term:

$$T(r) = \left(\frac{3GM_1\dot{M}}{8\pi\sigma R_1^3} \right)^{1/4} \left(\frac{R_1}{r} \right)^{3/4} \quad .$$

Polars

- Between strong magnetic fields and rotation, some cataclysmic variables avoid having disks altogether.
- Polars – distinct from intermediate polars – like EX Hydrae have such strong *B* that transferred mass simply free-falls down the field lines all the way to the primary…
- …never forming a disk.
    - The accreting material goes through a shock just before arrival at the primary, slightly complicating description of its emission.



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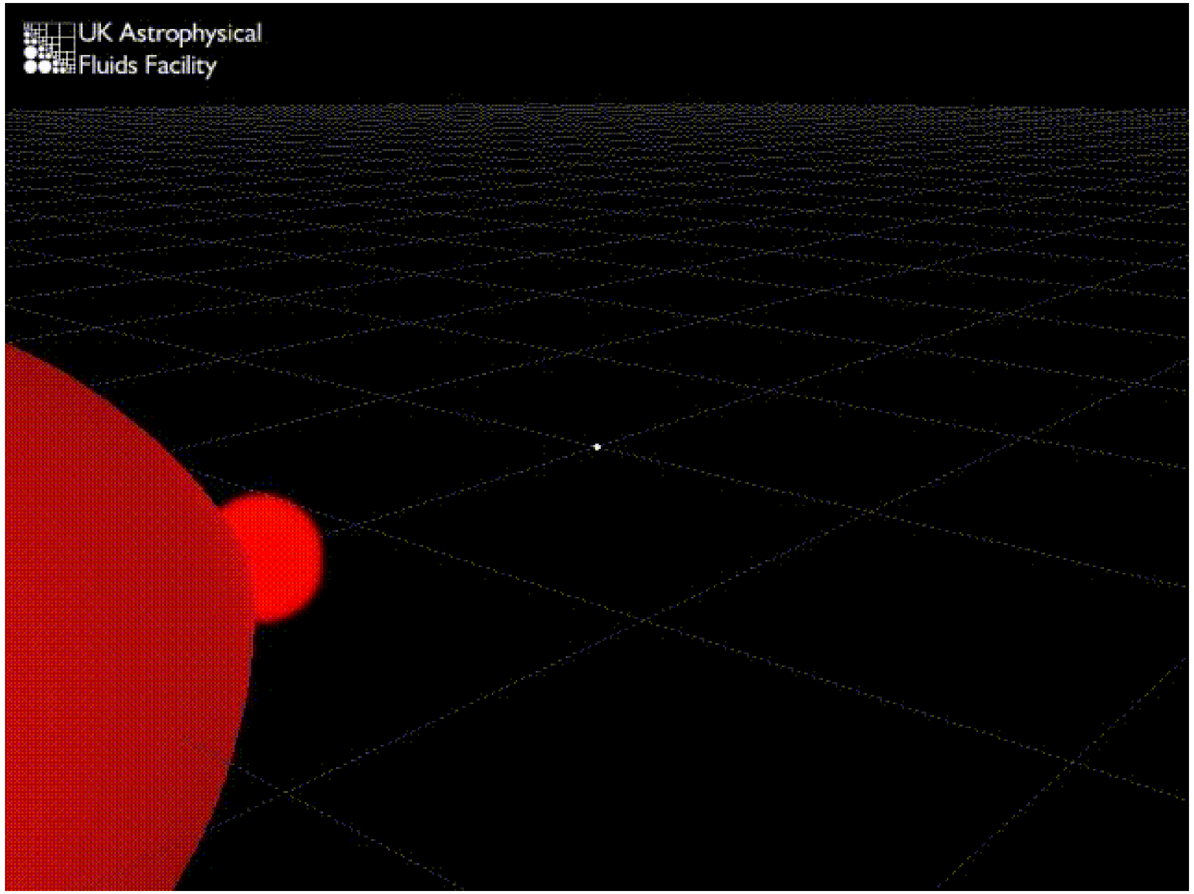
Propellers

- Rare polars of the **propeller** sort, like AE Aquarii here, have too large a spin and too strong a magnetic field even to allow the transferred mass to climb onto the lines of *B*.
  - Spun up and intensified *B* by earlier accretion eras.
- So these just swat the material away.
- In homework #10 you will deal with an **intermediate polar**, for which the primary is spinning fast and has strong *B*, but still has a disk.



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Vertical structure

- Between the disk's outer and inner edge – at least til the Alfven radius – the accretion disk still behaves like an ideal gas. You will show today that, if it is vertically isothermal, the pressure *P* varies with radius *r* according to

$$P(r,z) = P(0)e^{-z^2/H(r)^2} \quad \text{, where}$$

$$H(r) = \left(\frac{2kT(r)r^3}{G\mu m_{\mathrm{H}}M_1}\right)^{1/2} \quad .$$

Pressure scale height

- At radii past the temperature maximum you found last time, the temperature decreases with increasing radius, so the scale height increases with increasing radius: the disk is **flared**.

Today's in-class problems

1. C&O problem 18.18 (a). It will be better done in cgs than MKS. Hint: C&O page 692.

2. (a) Supposing that the disk is supported centrifugally in the radial direction, determine the vertical component of the gravitational acceleration at radius *r*, and write thereby the vertical, 1-D equation of hydrostatic equilibrium which applies to the disk's vertical structure.

   (b) Eliminate density in favor of pressure, and integrate the resulting expression from the mid-plane of the disk (z = 0) to produce the formula for the pressure scale height H, page 8.

   (c) Eliminate T assuming the optically-thick accretion disk expression on page 5, to obtain

$$H(r) = \left(\frac{2k}{\mu m_{\mathrm{H}}}\right)^{\frac{1}{2}} \left(\frac{3\dot{M}}{8\pi\sigma G^3}\right)^{\frac{1}{8}} M_1^{-3/8} r^{9/8}$$

   – that is, the disk is thin, and flared.

Answers

1. See page 4, and C&O page 692.

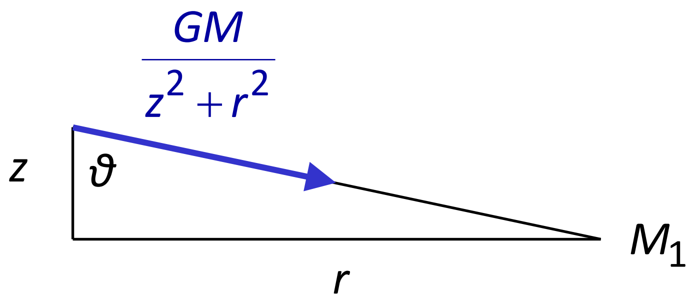
2.
$$\frac{dP}{dz} = -\rho g_z = -\rho \frac{GM_1}{z^2+r^2}\cos\vartheta \qquad \text{Take } z \ll r:$$

$$\cong -\rho \frac{GM_1}{r^2}\frac{z}{r} = -\frac{G\mu m_H M_1}{kTr^3} Pz$$

$$\int_{P_0}^{P} \frac{dP'}{P'} = -\frac{G\mu m_H M_1}{kTr^3}\int_0^z z'dz'$$

$$\ln\left(\frac{P}{P_0}\right) = -\frac{G\mu m_H M_1}{kTr^3}\frac{z^2}{2}$$

$$P = P_0 e^{-z^2/H^2} \quad , \quad H = \left(\frac{2kTr^3}{G\mu m_H M_1}\right)^{1/2} \quad .$$



Vertical component of stellar gravity balanced by pressure; horizontal component supplies centripetal acceleration.