Astronomy 241 Problem Set #4

Due 27 February 2024, in Box

Please submit your work in PDF form, for which the filename includes your name(s) and the number of the assignment, e.g. payne_hw1_solo.pdf or baade-zwicky_hw2_team.pdf.

Solo problems: C&O problems 10.3, 10.4, 10.6, and 10.10.

Team problems: problems H and I below. Team **Chapman** is Amii and Lara; team **Cleese** is Nora and Rafe; team **Gilliam** is Avi and Conor; team **Idle** is Annie and Ethan; team **Jones** is Joey and Waly; and team **Palin** is Angel and Rianna.

H. In hydrogen, the ionization energy for level n, and the corresponding frequency of light, is

$$E_n = \frac{2\pi^2 m_e q_e^4}{h^2}\frac{1}{n^2} \quad \text{, or} \quad \nu_n = \frac{2\pi^2 m_e q_e^4}{h^3}\frac{1}{n^2} .$$

(It will be easier to use frequencies instead of wavelengths in this problem and its sequel.) The cross section for ionization can be calculated quantum-mechanically; for an ensemble of hydrogen atoms with their energy levels populated according to thermal equilibrium at temperature T, the effective ionization cross section for level n is

$$\begin{aligned}\sigma_{\nu n}(\nu,n,T) &= \frac{64\pi^4}{3\sqrt{3}}\frac{m_e q_e^{10}}{h^6 c}\frac{1}{\nu^3 n^3}\exp\left(-\frac{2\pi^2 m_e q_e^4}{h^2 kT}\left[1-\frac{1}{n^2}\right]\right) \quad \text{for } \nu \ge \nu_n , \\ &= 0 \quad \text{for } \nu < \nu_n .\end{aligned}$$

The total cross section at any given frequency ν is the sum over all n of these cross sections:

$$\sigma_\nu(\nu,T) = \sum_{n=1}^{\infty} \sigma_{\nu n}(\nu,n,T) \quad .$$

Thus the bound-free opacity of stellar material, with solar abundances but with hydrogen considered artificially the only absorber, is $\kappa_\nu = \sigma_\nu / \mu m_{\mathrm{H}}$.

a. Write a function in your programming tool – Matlab, Mathematica, python -- for $\sigma_\nu(\nu,T)$, that can be used for temperatures near that of the Sun's photosphere (5800 K). You will find the **`if`** function useful; you will also need to estimate a safe upper cutoff for the sum. Plot your result for frequencies between 10^{14} and 10^{16} Hz.

b. Calculate the Rosseland mean opacity, using the Planck function as the flux-weighting function:

$$\frac{1}{\kappa} = \frac{\int_0^\infty \frac{B_\nu(T)}{\kappa_\nu}d\nu}{\int_0^\infty B_\nu(T)\,d\nu} \quad , \quad \text{where } B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \quad .$$

What is the value of this Rosseland mean opacity for Solar photospheric conditions (T = 5800 K)?

Note in this problem that one of the integrals must be carried out numerically. Note also that it is easier on your programming tools if the integrand is made dimensionless by substitution of variables.

I. a. With the bound-free opacity for hydrogen that you constructed in problem H, calculate the Rosseland mean opacity, as derived in class with a flux-weighting function for a non-gray atmosphere:

$$\frac{1}{\bar{\kappa}} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{dB_\nu}{dT} d\nu}{\int_0^\infty \frac{dB_\nu}{dT} d\nu} \quad .$$

What is the value of this Rosseland mean opacity for Solar photospheric conditions?

It is easier numerically if the integrand is made dimensionless by substitution of variables.

b. Then calculate the flux produced by the photosphere with solar-photospheric conditions and hydrogen bound-free opacity, using the expression derived in class:

$$F_\nu(0) \cong \pi B_\nu(T_e) + \frac{\pi T_e}{8}\left(\frac{\bar{\kappa}}{\kappa_\nu} - 1\right)\frac{dB_\nu}{dT}(T_e) \quad .$$

Plot this in the frequency range $10^{14} - 10^{16}$ Hz, and plot on the same graph the flux from a gray atmosphere of the same effective temperature. Comment on the differences.

