Astronomy 241 Problem Set #5

Due 5 March 2024, in Box

Please submit your work in PDF form, for which the filename includes your name(s) and the number of the assignment, e.g. payne_hw1_solo.pdf or baade-zwicky_hw2_team.pdf.

Team problems: J, below. Team **Aragon** is Lara and Waly; team **Boleyn** is Nora and Angel; team **Seymour** is Avi and Amii; team **Cleves** is Ethan and Rafe; team **Howard** is Joey and Conor; team **Parr** is Rianna and Annie.

Solo problems: K and L, below.

J. Using your programming tool – Matlab, Mathematica, python – solve the Lane-Emden equation for polytrope indices $n = 3/2$ and 3. Plot your results, $D_n(\xi)$ against ξ, and determine to six-place precision the values of ξ for which $D_n(\xi)$ first becomes zero.

Hint: use your programming tool's built-in fourth-order Runge-Kutta solver. Note that R-K solvers work on first-order differential equations, so you will have to turn the second-order Lane-Emden equation into a system of two first-order ODEs, as discussed in class.

K. Integrate the mass-conservation equation for a star with a polytropic equation of state with index *n*, to determine a relationship between mass, radius, and central density. Calculate the central density for a star with solar mass and radius.

Hint: express the integral in dimensionless form, and use the Lane-Emden equation itself to simplify it.

L. From the result of problem K and the equation of state for a polytrope with index *n*, obtain expressions for the central pressure, central temperature, and the constant *K* in the pressure equation of state $P = K\rho^{\gamma} = K\rho^{1+1/n}$, all as functions of stellar mass *M*, radius *R*, and mean molecular mass μ.