Astronomy 241 Problem Set #7

Due 30 March 2024, noon, in Box

Please submit your work in PDF form, for which the filename includes your name(s) and the number of the assignment, e.g. payne_hw1_solo.pdf or baade-zwicky_hw2_team.pdf.

If it's being submitted for a regrade, prepend Regrade_ to the file name.

Solo problems: P, Q, and R.

Team problem: S. Team **0.2** is Avi and Nora; Team **0.5** is Rafe and Lara; Team **1** is Conor and Waly; Team **2** is Annie and Angel; Team **5** is Amii and Rianna; Team **10** is Ethan and Joey.

P. (10 points) Continuing problem 1 in the in-class problems on 19 March 2024, for the star with $M = 1M_\odot$ and $R = 1R_\odot$ which is partially supported by gas pressure and radiation pressure.

i. Assume that $\beta \ll 1$, and show that

$$\beta = \left[4 + \frac{3c}{4\sigma(\pi G)^3}\left(\frac{k}{\mu m_{\rm H}}\right)^4 \frac{1}{M^2}\left(-4\pi\Delta_{3m}\right)^2\right]^{-1} ,$$

where $\Delta_{3m} = \xi_1^2 D_3'(\xi_1)$ is the stellar-surface value of the derivative component of the n = 3 Lane-Emden solution, in the nomenclature of homeworks #5 and #6.

ii. Calculate β for $M = 1M_\odot$ and $R = 1R_\odot$, and thereby validate the assumption in part i.

Q. (20 points) As in homework #6 problems M, N, and O, follow up the result of problem P by calculating and plotting the density $\rho(r)$, pressure $P(r)$, temperature $T(r)$, criterion for convective instability

$$C = \frac{1}{n+1}\frac{T(r)}{P(r)}\frac{dP}{dT} \quad ,$$

luminosity gradient dL_r/dr, and luminosity generated within r, L_r. This time include the CNO process along with the p-p chain in the star's energy generation. Include in your plots the same quantities for the MESA model of the present-day Sun. Is this new polytrope a better model than the one considered in homework #6? Why did se add CNO fusion to the mix? Discuss in detail the similarities and differences between your new model and the MESA solar model.

R. (10 points) Calculate the total luminosity L and effective temperature T_e of your new polytrope solar model star.

i. Compare the results to those for the Sun.

ii. What additional information would you need to calculate a theoretical HR diagram using this new polytrope model, as you did in problem O?

S. (30 points) **Eighteen MESA models, all to get 36 numbers**. Each team has a name indicating a stellar mass in Solar-mass units. Generate models of your stellar mass for three different metallicities: Z = 0.004, as characteristic of Population II as in typical Galactic globular clusters like M 3 and M 15; Z = 0.02, typical Population I, similar to the Sun, and Z = 0.03, similar to the metal-richest Population II stars such as Kepler-6 (homework #3).

i. For your team's mass, determine the luminosity L and effective temperature T_e as the star reaches the main sequence, for each value of metallicity. Note that this is not the same age for all the stars and metallicities – you will have to hunt around among the models, guided by MESA's movie output, to find the one corresponding to the **beginning** of main-sequence development. (This is why there are two of you. Share the work.)

ii. Assemble with all the other teams. Using all the results, produce the HR diagram for each metallicity. Plot them in the same graph with the HR diagram of John Southworth's database of detached eclipsing binary stars. And then discuss the comparison of these diagrams in detail, dealing with questions such as: Are the three metallicities distinctly identifiable in the data? What is the likely spread of metallicity among the main-sequence stars represented in the empirical HR diagram?