Astronomy 241 Problem Set #9

Due 16 April 2024, in Box

Please submit your work in PDF form, for which the filename includes your name(s) and the number of the assignment, e.g. payne_hw1_solo.pdf or baade-zwicky_hw2_team.pdf.

If it's being submitted for a regrade, prepend Regrade_ to the file name.

Solo problems: U (10 points) and V (20 points).

Team problems: C&O 16.8 and W, each worth 10 points. Team **Higgs** is Avi and Ethan; Team **Kibble** is Conor and Joey; Team **Brout** is Angel and Rianna; Team **Englert** is Amii and Waly; Team **Guralnik** is Nora and Rafe; Team **Hagen** is Lara and Annie.

U. In the in-class problems on 4 April 2024, you showed that the equation of state for degenerate electrons of arbitrary velocity distribution can be expressed parametrically as

$$P = \frac{\pi}{3}\frac{m_e^4 c^5}{h^3} f(x) \equiv af(x) \quad ,$$

$$\rho = \frac{m_H A}{Z}\frac{8\pi m_e^3 c^3}{3h^3} x^3 \equiv bx^3 \quad ,$$

where $$f(x) = x\left(x^2+1\right)^{1/2}\left(2x^2-3\right) + 3\,\mathrm{arcsinh}(x) \quad ,$$

and $x = p_F / m_e c$. Use this form for the EOS, define new dimensionless variables,

$$r_0 = \left(\frac{a}{Gb^2}\right)^{1/2} \quad , \quad r = r_0\eta \quad , \quad M_r = br_0^3\theta \quad ,$$

and show thereby that the equations of hydrostatic equilibrium and mass conservation can be expressed as

$$\frac{dx}{d\eta} = -\frac{\left(1+x^2\right)^{1/2}}{8x}\frac{\theta}{\eta^2}$$

$$\frac{d\theta}{d\eta} = 4\pi\eta^2 x^3 \quad .$$

Show also that the boundary conditions are split:

$$x = 0 \text{ at } \eta = \frac{R}{r_0} \quad ,$$

$$\theta = 0 \text{ at } \eta = 0 \quad .$$

V. **Exact white-dwarf structure.** Solve the system of equations derived in problem U, to obtain thereby the radius and the run of x (and therefore density and pressure) inside a white dwarf for arbitrary electron velocity distribution, for a range of masses. Do this backwards compared to problem L, homework #5: starting at the stellar surface for a white dwarf with dimensionless mass $\theta = M/br_0^3$, guess a value for the radius, and then integrate the dimensionless hydrostatic-equilibrium and mass-conservation equation from the outside in, checking the mass in the central zone, and iteratively refining the guess for the radius until the mass contained within the central zone is a very small fraction of the total mass. Repeat the calculations and obtain the radius as a function of mass for masses in the range 0.001-1.44 $M_{\odot}$, for helium-carbon-oxygen $(Z/A = 0.5)$ white dwarfs. Plot your results with the data on radii and masses for white dwarfs in visual or eclipsing binary systems, available on the course website.

Hint: you might do better to use an adaptive-stepsize Runga-Kutta solver this time, rather than the fixed-stepsize ones recommended in Homework #5.

W. **The Stoner-Anderson-Chandrasekhar mass.** Show that one can also express the system of equations as

$$\frac{dx}{d\theta} = -\frac{\left(1+x^2\right)^{1/2}}{32\pi x^4}\frac{\theta}{\eta^4}$$
$$\frac{d\eta}{d\theta} = \frac{1}{4\pi\eta^2 x^3} \quad .$$

Integrate this system from the inside out, as in problem L, homework #5 – starting at the center with very large density, and adjusting the dimensionless mass steps so that the resulting dimensionless density is very small in the outermost zone – to determine the maximum mass, in $M_{\odot}$, of helium-carbon-oxygen $(Z/A = 0.5)$ white dwarfs, more accurately than can be done with the setup of problem V. Report your answer to at least three significant figures past the decimal point.

Hint: again, it might work better to use an adaptive-stepsize R-K solver.