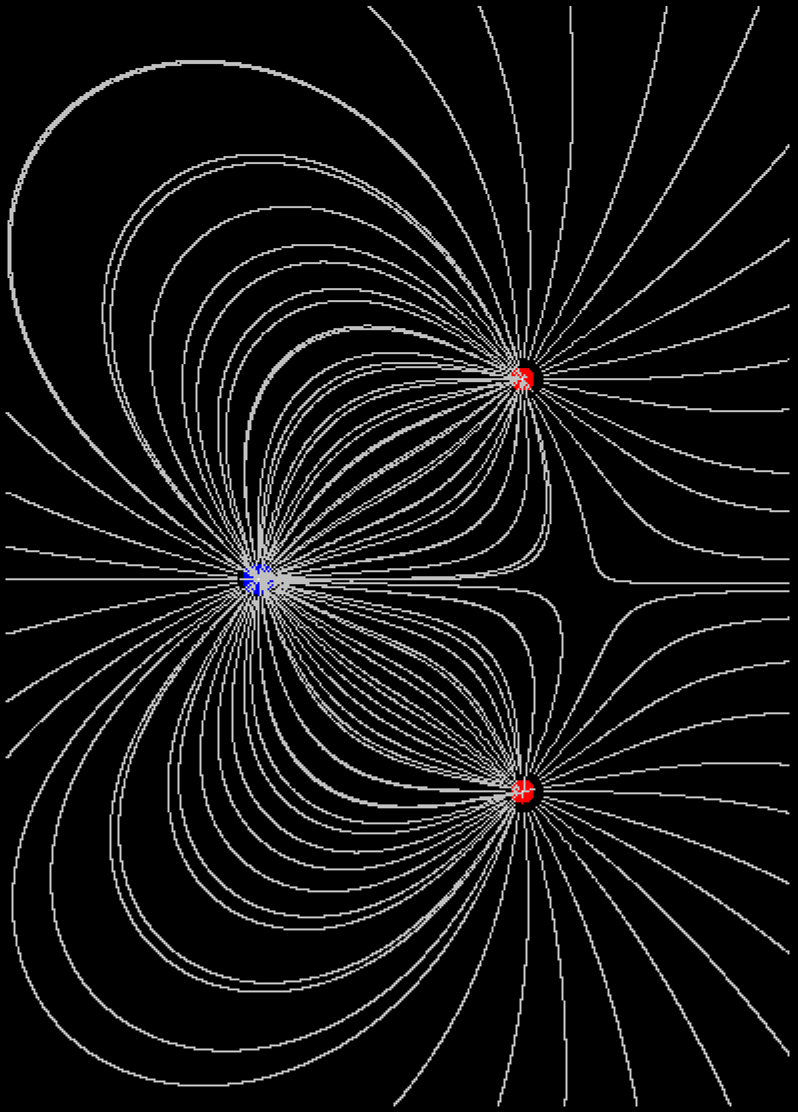
Today in Physics 122: electric charge, electrostatic force, electrostatic field

- What is charge?
- How to solve electrostatic force problems with point charges.
- What is the electrostatic force?
- The electric field ***E***: what charge does to space, and how space in turn tells charge to move.
- Lines of ***E*** and the visualization of electric fields.

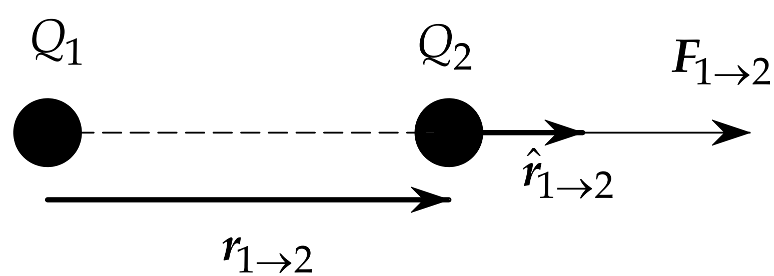


Choose the correct statement.

$\boldsymbol{F}_{1\to2}$ is the force charge 1 exerts on charge 2. Of course if we know this force, we also know $\boldsymbol{F}_{2\to1}$: it's given by …

A. $\boldsymbol{F}_{2\to1} = \boldsymbol{F}_{1\to2}$, according to Newton's second law.
B. $\boldsymbol{F}_{2\to1} = -\boldsymbol{F}_{1\to2}$, according to Newton's second law.
C. $\boldsymbol{F}_{2\to1} = \boldsymbol{F}_{1\to2}$, according to Newton's third law.
D. $\boldsymbol{F}_{2\to1} = -\boldsymbol{F}_{1\to2}$, according to Newton's third law.

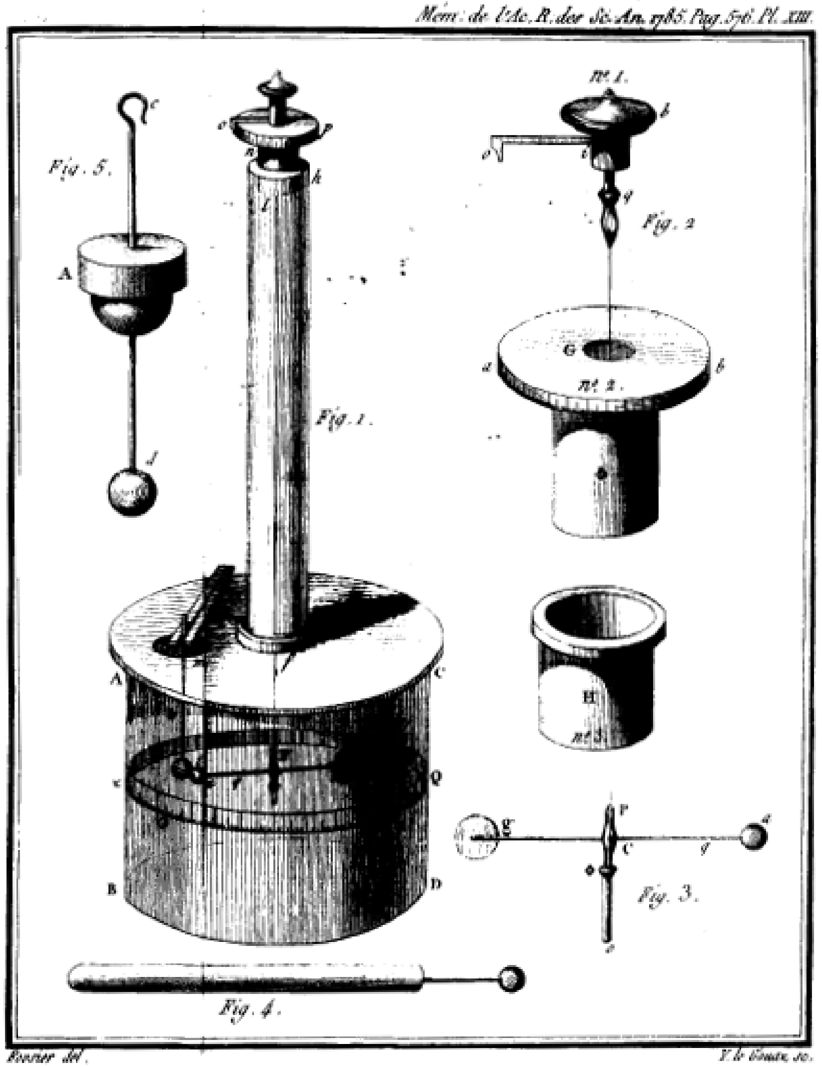
$$\boldsymbol{F}_{1\to2} = k\frac{Q_1 Q_2}{\left|\boldsymbol{r}_{1\to2}\right|^2}\hat{\boldsymbol{r}}_{1\to2}$$



How Coulomb's law is tested

Coulomb's experiment involved an accurate force measurement, using a torsional balance, between two small, charged metal spheres. He adjusted the distance between the spheres and measured the force with the balance.

- ❑ Coulomb probably invented this sort of balance, which has been used ever since in measurement of very small forces.



Coulomb's experimental apparatus (Wikimedia Commons).

Units for electric charge

In magnitude, $F = kQ_1Q_2/r^2$

- The MKS unit of electric charge is the **coulomb.** (abbreviated coul or C) Thus the units of the constant k have to be

$$N = [k]\frac{\text{coul}^2}{\text{m}^2} \quad \Rightarrow \quad [k] = \frac{\text{Nm}^2}{\text{coul}^2}$$

  and its value is measured to be $k = 9.0 \times 10^9\,\text{Nm}^2\text{coul}^{-2}$.

- For reasons to be made clearer later this semester, we usually define another constant to use in place of k:

$$k = \frac{1}{4\pi\varepsilon_0} \quad \Rightarrow \quad \varepsilon_0 = 8.85 \times 10^{-12}\,\frac{\text{coul}^2}{\text{Nm}^2}.$$

Permittivity of vacuum (free space)

What is electric charge?

1. Is charge a continuous, fluid-like property of matter, in the sense that it can be made as small as one wants?

2. Is charge completely independent of other properties of matter, something that can be "poured into" matter, as Franklin (among others) thought?

These questions were hotly debated in the 1700s-1800s, at about the same time that the final arguments took place about whether matter it self was made of continuous fluid or "atoms." The answers are

1. No
2. No

What is electric charge? (continued)

So what is it, then?

Charge turns out to be **an intrinsic property of the subatomic constituents of matter**, the elementary particles.

- There is a smallest finite amount that electric charge comes in. This was theorized to be true in the mid-1800s (by Faraday) and finally demonstrated experimentally in 1909 by Millikan and Fletcher.
- The *magnitude* of that smallest amount, called the **quantum of electric charge** or **elementary charge**, is

$$e = 1.6022 \times 10^{-19}\ \text{coul.}$$

  That is to say, one coulomb is a lot of charge.

More on the elementary particles and charge in PHY 123.

What is electric charge? (continued)

As you're no doubt aware, ordinary matter is made of atoms, which in turn are made of protons, neutrons and electrons.

- Atoms in their stable states contain equal numbers of electrons and protons.
- The charges of the electron and proton are likely to be **exactly** equal and opposite: the electron has $-e$ and the proton $+e$, following Franklin's definition.
  - Experimentally, $$Q_{\text{electron}} + Q_{\text{proton}} = 0 \pm 10^{-21}\, e$$ $$Q_{\text{neutron}} =$$ (Bressi et al. 2011).
- So ordinary matter is **electrically neutral**, to very high precision.

“Triboelectricity”

Rubbing amber and fur together results in a charge on both. The sign of the charge on the amber is negative.

Explain physically what must be going on in this process, which is called **triboelectricity**.

Electrostatic forces

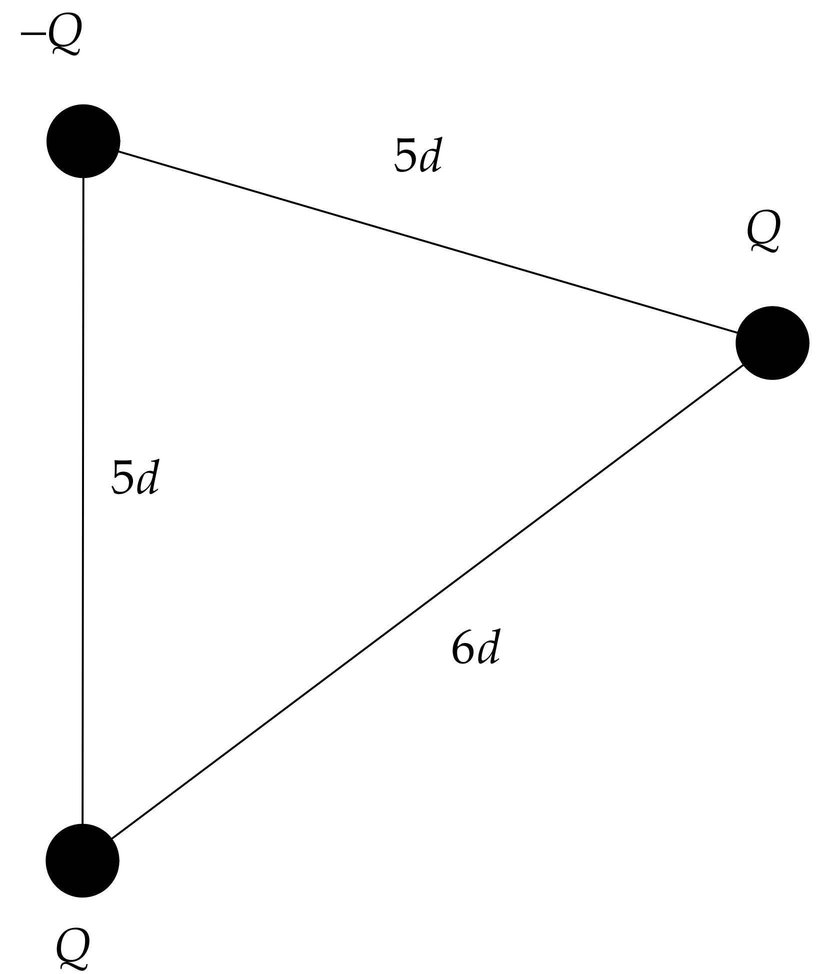
Upshot from the homework and exam perspective: we now have a new force, the electrostatic force, to treat in mechanics problem by use of Newton's laws.

- And it will work a lot like gravity, since it's inverse-square.
- Good news: you already have mastered how to do such problems, in PHY 121.
- Bad news: you now have to remember how to do such problems.
  - So shake the rust off by looking at the example electrostatic force-balance problems in the textbook (chapter 22), and the different one which follows.

Example: three charges

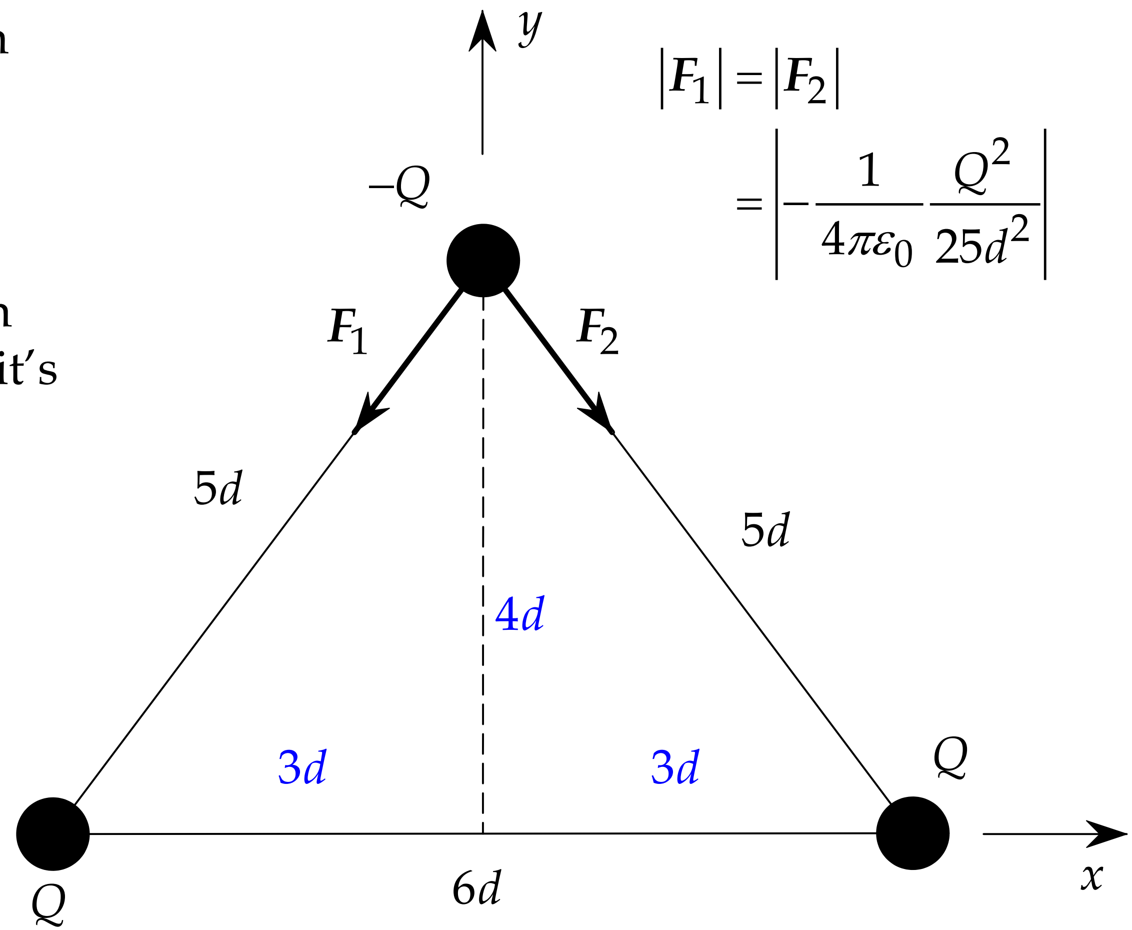
Three electric charges, Q, Q, and $-Q$, are arranged as shown. Find a formula for the force on $-Q$.

- ❑ What kind of problem is this, and what tools do we need?
  - A force-addition problem. We need Coulomb's Law, an appropriate coordinate system, and trigonometry to add the vectors.



Three charges (continued)

- What is the appropriate coordinate system in which to decompose the force vectors and then add the results?
  - Any coordinate system will do, of course, but it's easiest to choose one which fits the symmetry of the problem.
  - Like the one symmetrical about the $-Q$.



$$\left|\mathbf{F}_1\right| = \left|\mathbf{F}_2\right| = \left| -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{25d^2} \right|$$

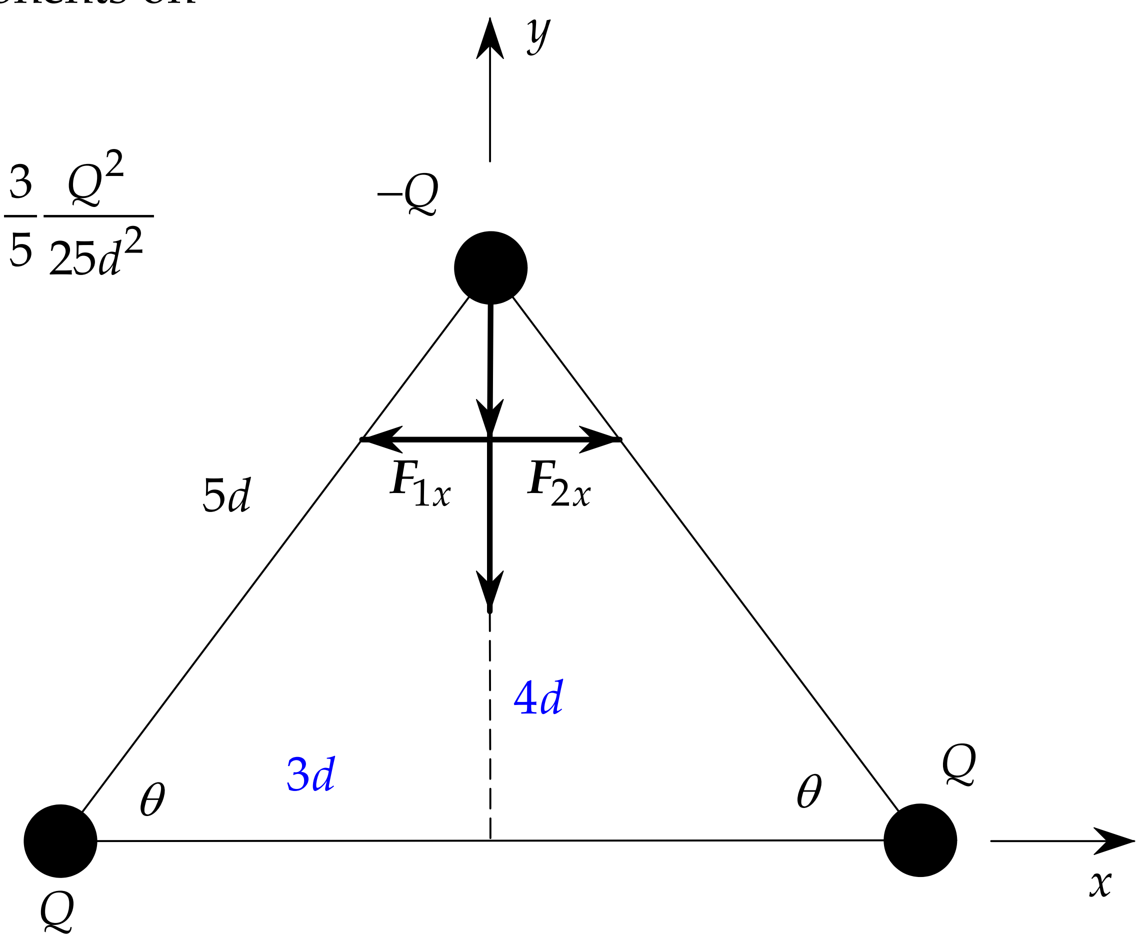
Three charges (continued)

- What are all the x-y components on our test charge –Q?

$$F_{1x} = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{25d^2}\cos\theta = -\frac{1}{4\pi\varepsilon_0}\frac{3}{5}\frac{Q^2}{25d^2}$$

$$F_{2x} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{25d^2}\cos\theta = -F_{1x}$$

So the x components cancel.



Three charges (continued)

$$F_{1y} = -\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{25d^2}\sin\theta = -\frac{4}{5}\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{25d^2}$$

$$F_{2y} = F_{1y}$$

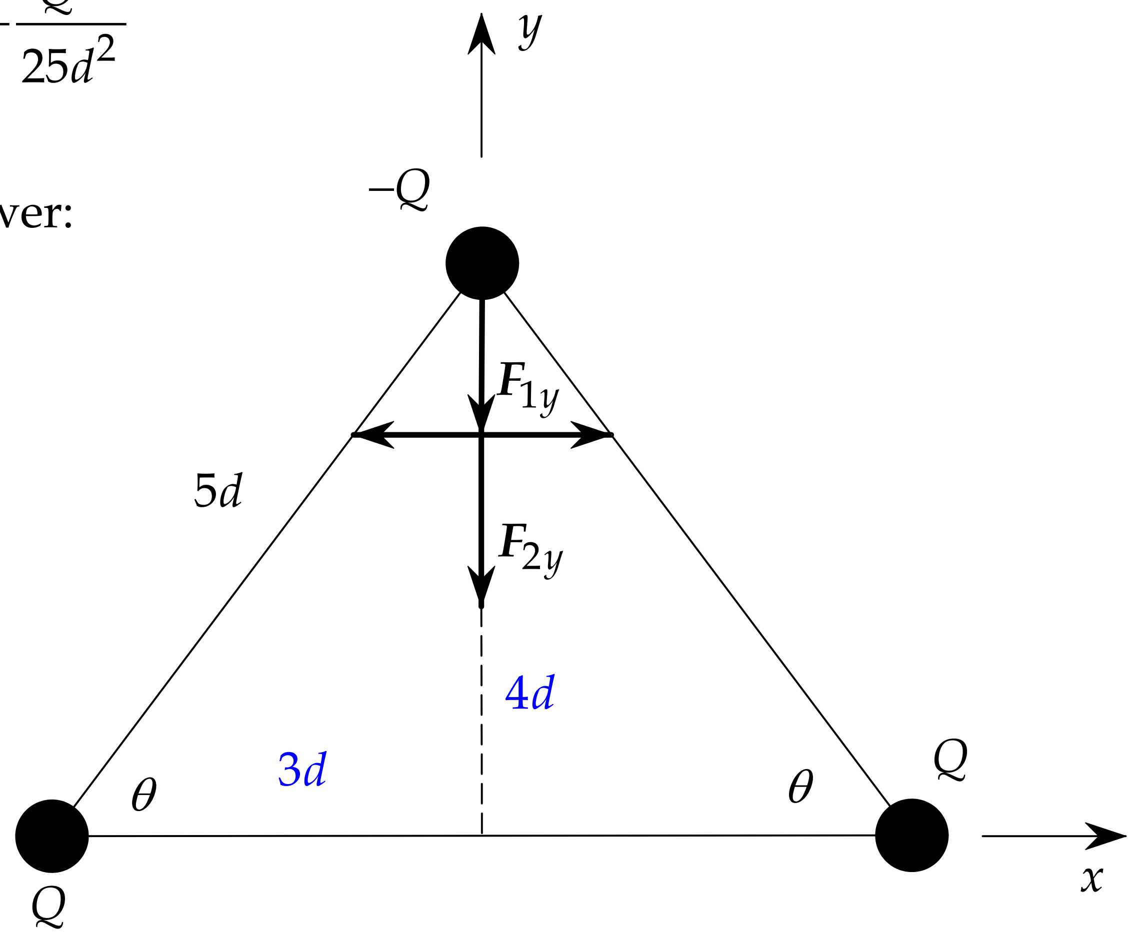
So add them up for the final answer:

$$F_y = F_{1y} + F_{2y} = \boxed{-\frac{2}{125\pi\varepsilon_0}\frac{Q^2}{d^2}}$$

That is: magnitude $\frac{2}{125\pi\varepsilon_0}\frac{Q^2}{d^2}$,

toward the midpoint

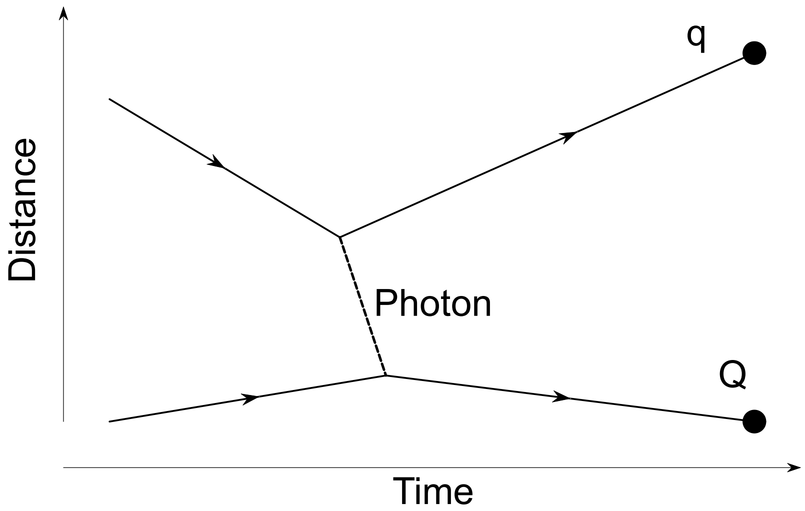
of $Q-Q$.



What is the electrostatic force?

There are two consistent ways to think about the nature of the electrostatic force, which turn out to be equivalent mathematically and physically at a deep level.

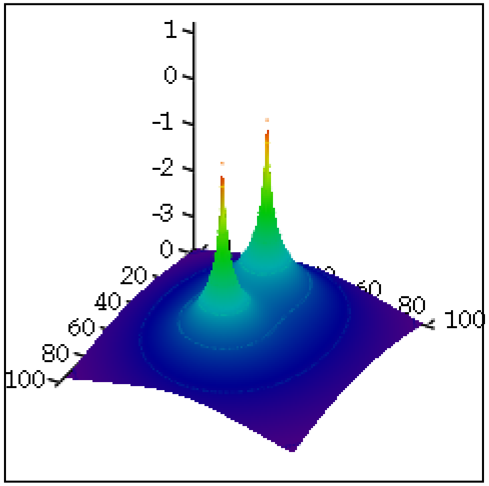
- **The quantum picture**. Electrically-charged objects interact by **exchange** of quanta which carry energy and momentum.
- Thus the force is a local phenomenon: **a property of a charge's state when exchanging quanta**.
- The particle exchanged in the electrical interaction is the **photon**. (A particle of light.)



What is the electrostatic force? (continued)

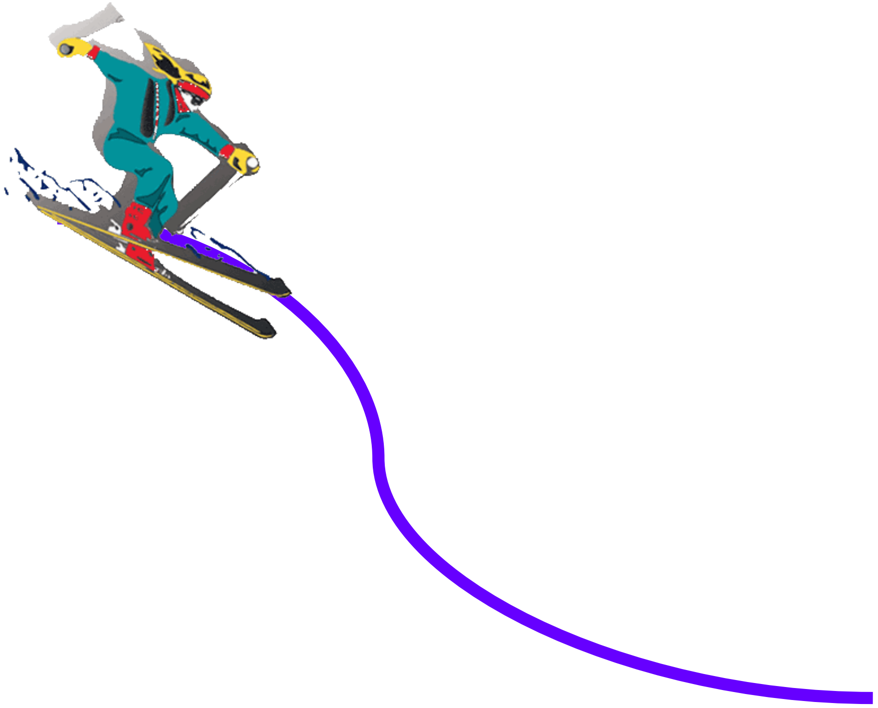
- **The classical picture.** The presence of electric charge changes space itself: it introduces "hills and valleys" that, in turn, tell other charges how to move around. Thus force is a **global property of space**.

  - In this picture, space containing charges is characterized by the **electric field:** the force per unit charge, defined everywhere in space.
  - The magnitude of the field tells a **"test"** charge placed at that point how fast to move.
  - The direction of the field tells the test charge which direction in which to move.



What is the electrostatic force? (continued)

- Or, more prosaically,
  - Electric field is like a ski slope
  - A test charge would be like a skier.
  - The slope is there whether you ski down or not.



The electric field and its calculation

For most purposes involving macroscopic objects moving at everyday speeds, the classical picture works just fine, and is much easier in which to do calculations.

- So we will envision the force as due to the electric field, which turns Coulomb's law for point charges into this:

$$\boldsymbol{F}_{Q\to q} = q\boldsymbol{E}_Q \quad , \quad \boldsymbol{E}_Q = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\boldsymbol{r}} \quad .$$

  where the unit vector $\hat{\boldsymbol{r}}$ points away from the charge Q.

- Like forces, therefore, the electric field obeys the **principle of superposition:** if multiple charges are present, one writes their fields down (**in a single coordinate system**, of course) and adds the vectors to obtain the total field:

$$\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2 + \ldots = k\frac{Q_1}{\left(\boldsymbol{r}-\boldsymbol{r}_1\right)^2}\widehat{\left(\boldsymbol{r}-\boldsymbol{r}_1\right)} + k\frac{Q_2}{\left(\boldsymbol{r}-\boldsymbol{r}_2\right)^2}\widehat{\left(\boldsymbol{r}-\boldsymbol{r}_2\right)} + \ldots$$

Lines of force for *E*

In visualization of electric fields, it is handy to have a pictorial representation which shows the direction of ***E*** and gives an impression of the field strength.

- You now know one way to generate formulas for such things, but pictorially there's something handier, which is easier to calculate in another way which we'll learn later: the **lines of force.**
- Goes as follows:
  - Starting at the positive charges, start radially outward in a number of directions evenly spaced around a circle. The number of directions is bigger, the bigger the charge.

Lines of force for *E* (continued)

- Similarly, draw lines radially inward to negative charges, also evenly spaced around the circle, also more divisions for more-negative charges.
- According to the vector formula for ***E***, follow each of these directions until lines from + join up with lines from -, or until they disappear off to infinity.
- Then you will have a picture full of curves: the **curves** (the lines of ***E***) **will be tangent to *E***, and the density of the curves (per unit area) gives an impression of the magnitude of ***E*.**

❑ **Visualization aid:** try playing with this Java applet:

www.cco.caltech.edu/~phys1/java/phys1/EField/Efield.html

as we will now do.