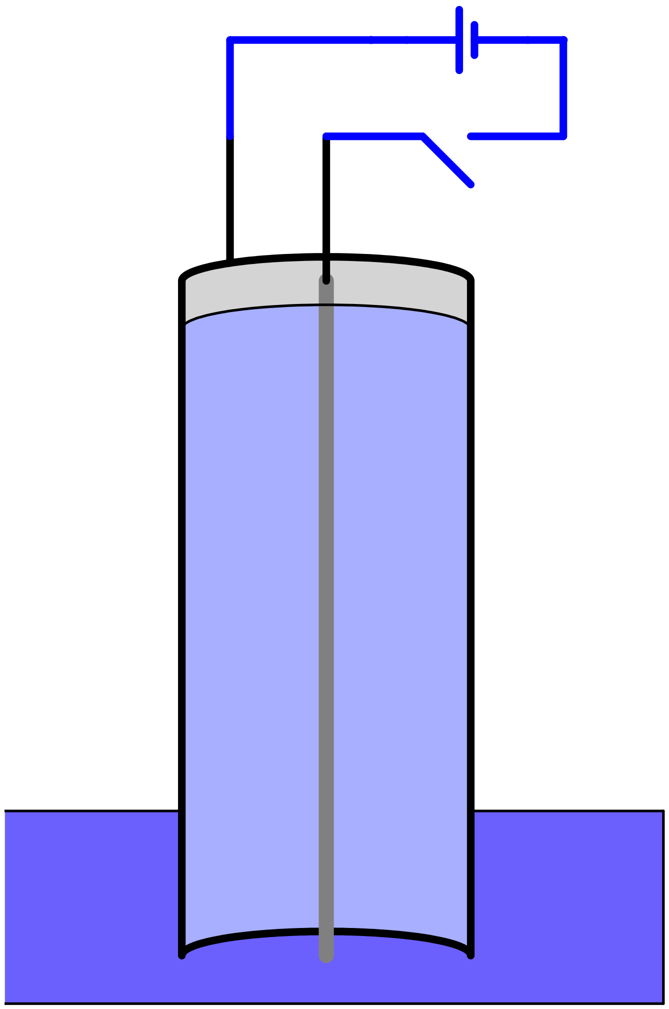
Today in Physics 122: capacitors

- Parallel-plate and cylindrical capacitors: calculation of capacitance as a review in the calculation of field and potential
- Dielectrics in capacitors
- Capacitors, dielectrics and energy
- Capacitors as elements of electrical circuits



Recipe for calculation of the capacitance of arrangements of conductors

Other materials besides conductors have capacitance, but arrangements of conductors lend themselves to straightforward calculation of C. Usually this goes as follows:

- Presume electric charge to be present; say, Q if there is only one conductor, or $\pm Q$ if there are two.
- Either:
  - Calculate the electric field from the charges, and integrate it to find the potential difference V between the conductors, or
  - Solve for the potential difference directly, using

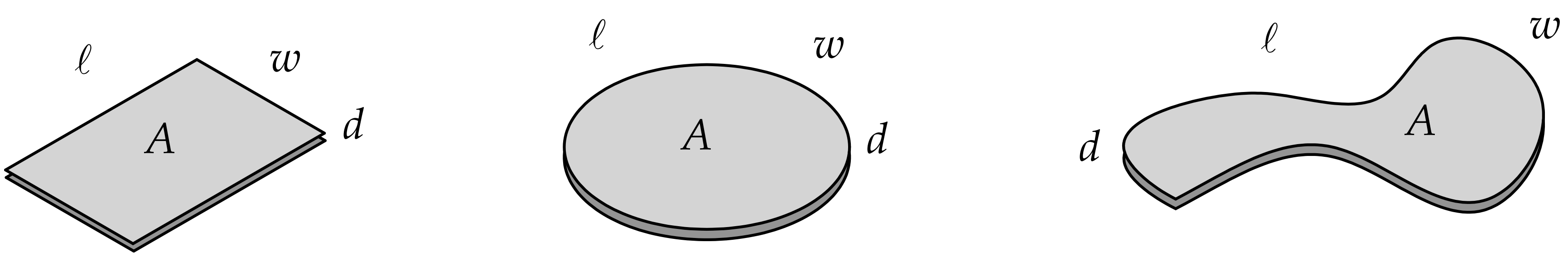
$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dQ}{|\boldsymbol{r} - \boldsymbol{r}'|} \quad .$$

- Then $C = Q/V$.

Parallel-plate capacitor

Consider two parallel conducting plates, separated by a distance d that is very small compared to their extent in other dimensions. Suppose each plate has area A.

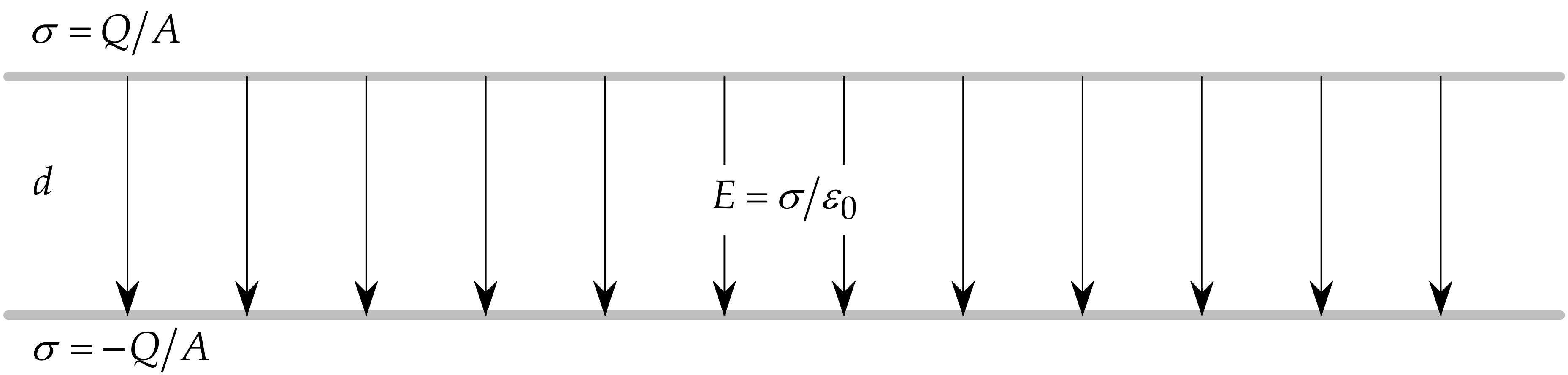
- It doesn't matter what the shape of the flat plates are, as long as they are parallel and very close together.
  - How close? See homework problem set #2, problem 1.
- With charges $\pm Q$ on the plates, the charge densities are uniform and have values $\sigma = \pm Q/A$.



Parallel-plate capacitor (continued)

- At points well inside the gap, the plates can be regarded as infinite, to good approximation.
- As we found on 9 September, the electric field between two oppositely-charged infinite parallel plates is uniform, with magnitude

$$E = \frac{\sigma}{\varepsilon_0} \quad .$$

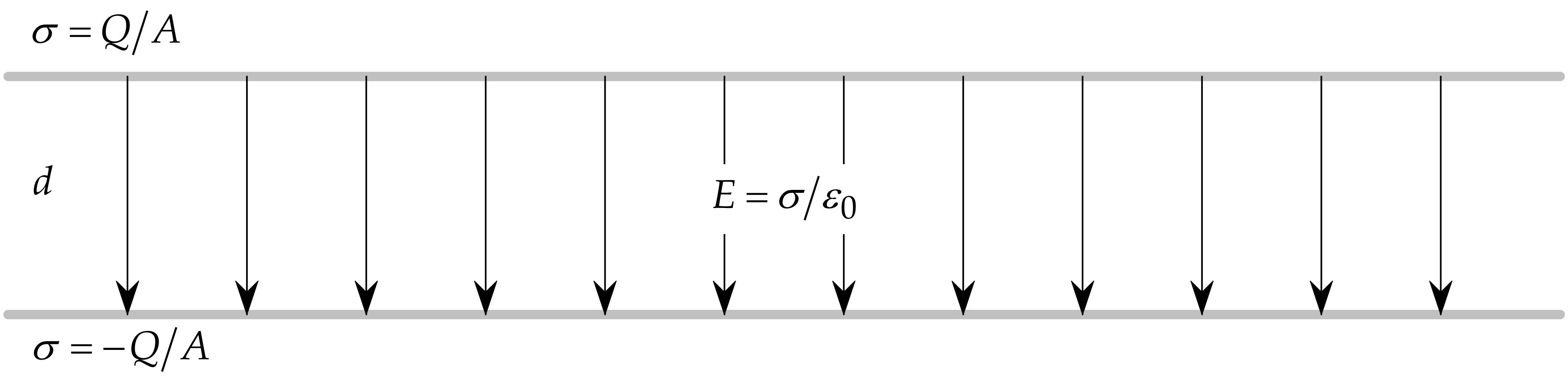


Parallel-plate capacitor (continued)

- The uniformity of $\boldsymbol{E}$ makes the integration trivial:

$$\Delta V = -\int_{-}^{+} \boldsymbol{E} \bullet d\boldsymbol{\ell} = Ed = \frac{\sigma d}{\varepsilon_0} \equiv \frac{Q}{C} = \frac{\sigma A}{C};$$

$$\boxed{C = \frac{\varepsilon_0 A}{d}.}$$

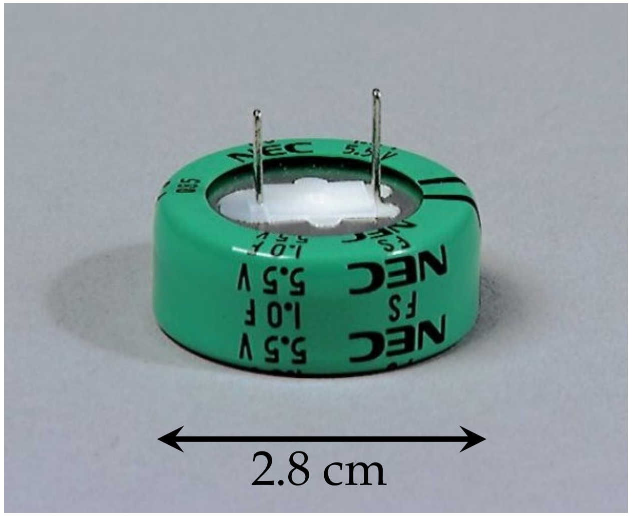


Units of capacitance

In MKS: the unit of capacitance, named in honor of Michael Faraday, is the **farad** :

$$1\text{F} = \frac{1\text{ coul}}{1\text{ volt}} \quad .$$

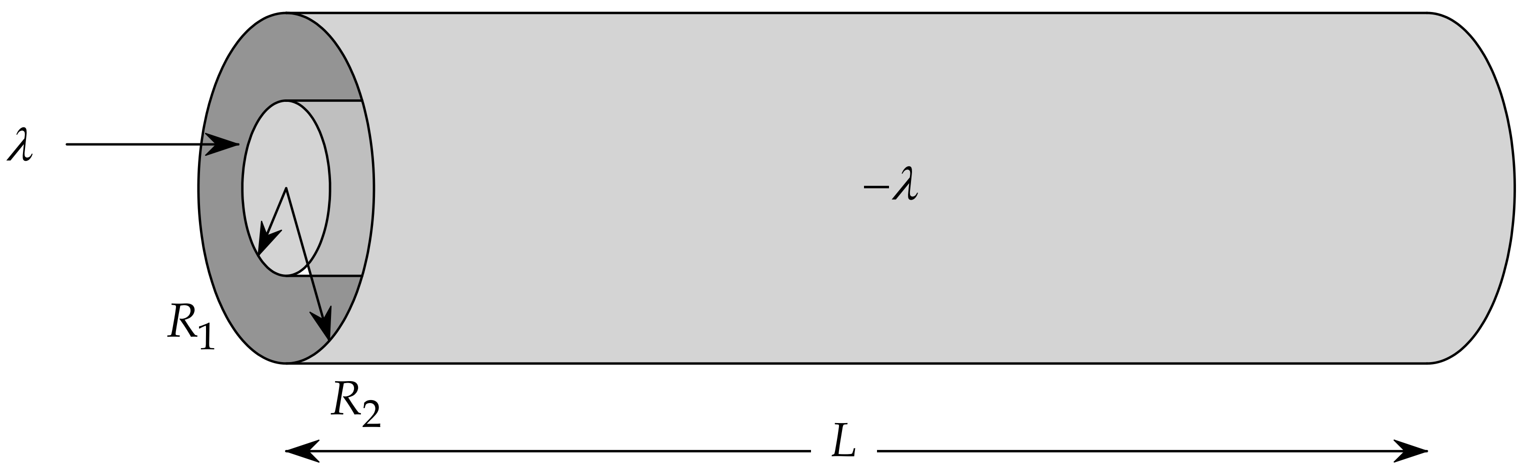
- ❑ Note that $\varepsilon_0 = 8.85\times 10^{-12}\ \text{coul}^2\ \text{Nt}^{-1}\ \text{m}^{-2} = 8.85\times 10^{-12}\ \text{F m}^{-1} = 8.85\ \text{pF m}^{-1}$.
- ❑ One farad is a Huge capacitance.
  - Those found around the lab and in circuits are usually in the pF-μF range (10^{-12}-10^{-6} F).
  - A 1F parallel-plate capacitor with d = 25 μm (0.001 in) has A = 2.8 km^2: 1.7 km on a side, if square.
  - Modern supercapacitors are made with *much* smaller d: < 1 nm.



Cylindrical capacitor

Consider two, coaxial, conducting cylinders with radii R_1 and $R_2 > R_1$. Their length is $L \gg R_2$ and they carry opposite charges $\pm Q$ (charge per unit length $\lambda = \pm Q/L$).

- At points well inside the gap, the cylinders can be regarded as infinite, to good approximation.

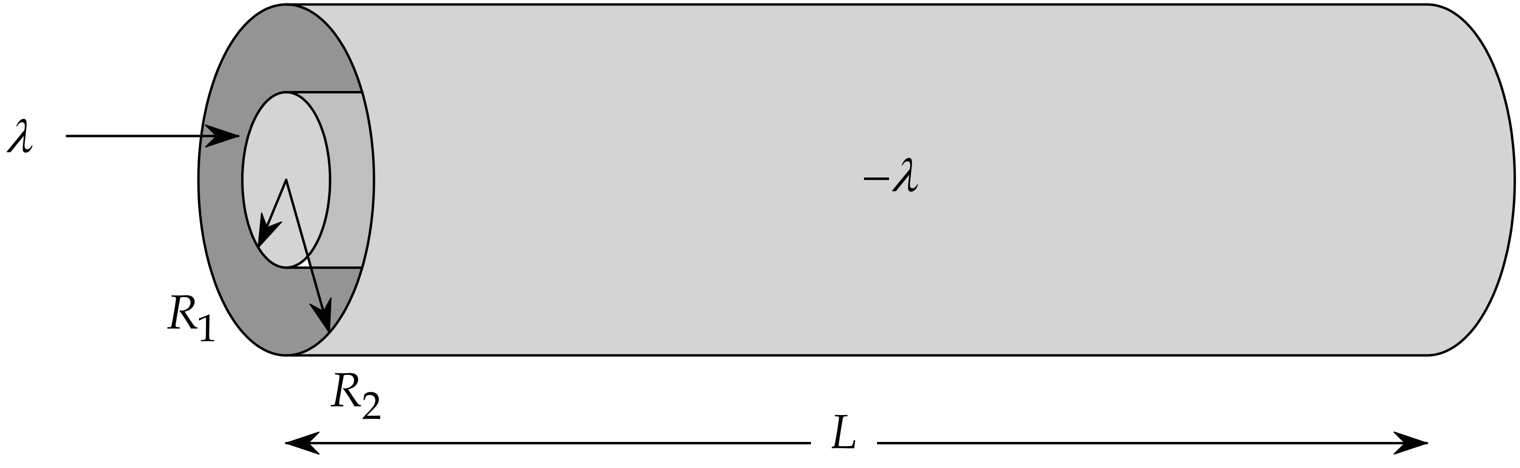


Cylindrical capacitor (continued)

- We have shown by use of Gauss's law (13 September) that

$$\boldsymbol{E} = \begin{cases} \lambda\hat{\boldsymbol{r}}/2\pi\varepsilon_0 r\,, & R_1 < r < R_2 \\ 0, & r \geq R_2\,, \leq R_1 \end{cases}$$

for infinite, oppositely-charged coaxial cylinders.

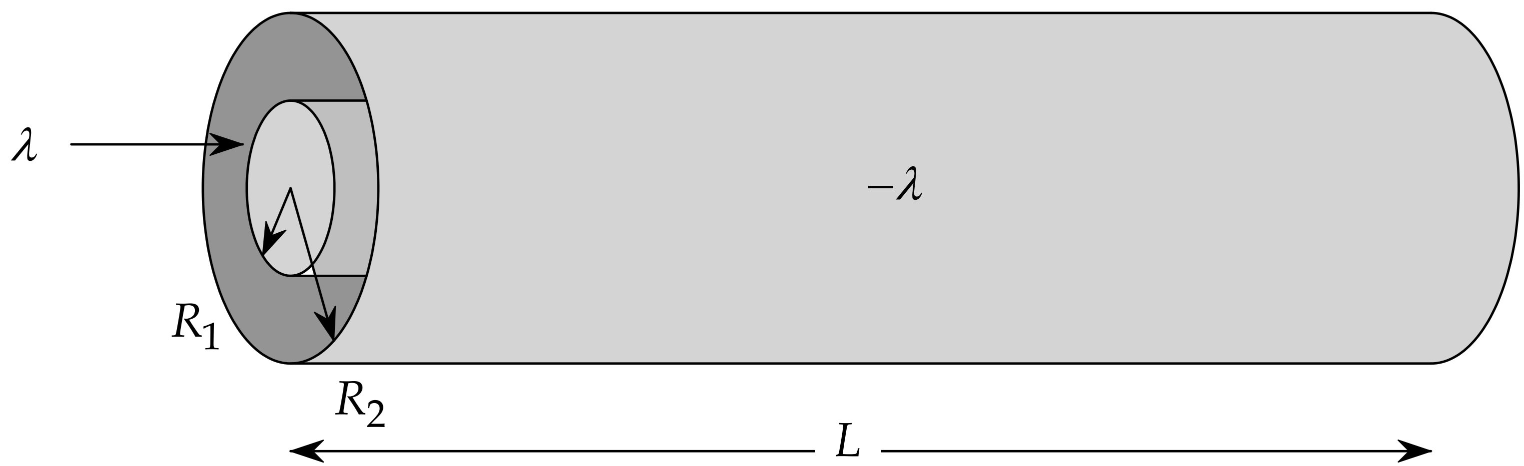


Cylindrical capacitor (continued)

- Thus

$$\Delta V = -\int_{-}^{+} \boldsymbol{E} \bullet d\boldsymbol{\ell} = \frac{\lambda}{2\pi\varepsilon_0} \int_{R_1}^{R_2} \frac{dr}{r} = \frac{1}{2\pi\varepsilon_0} \frac{Q}{L} \ln\left(\frac{R_2}{R_1}\right) \equiv \frac{Q}{C};$$

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(R_2/R_1\right)}.$$

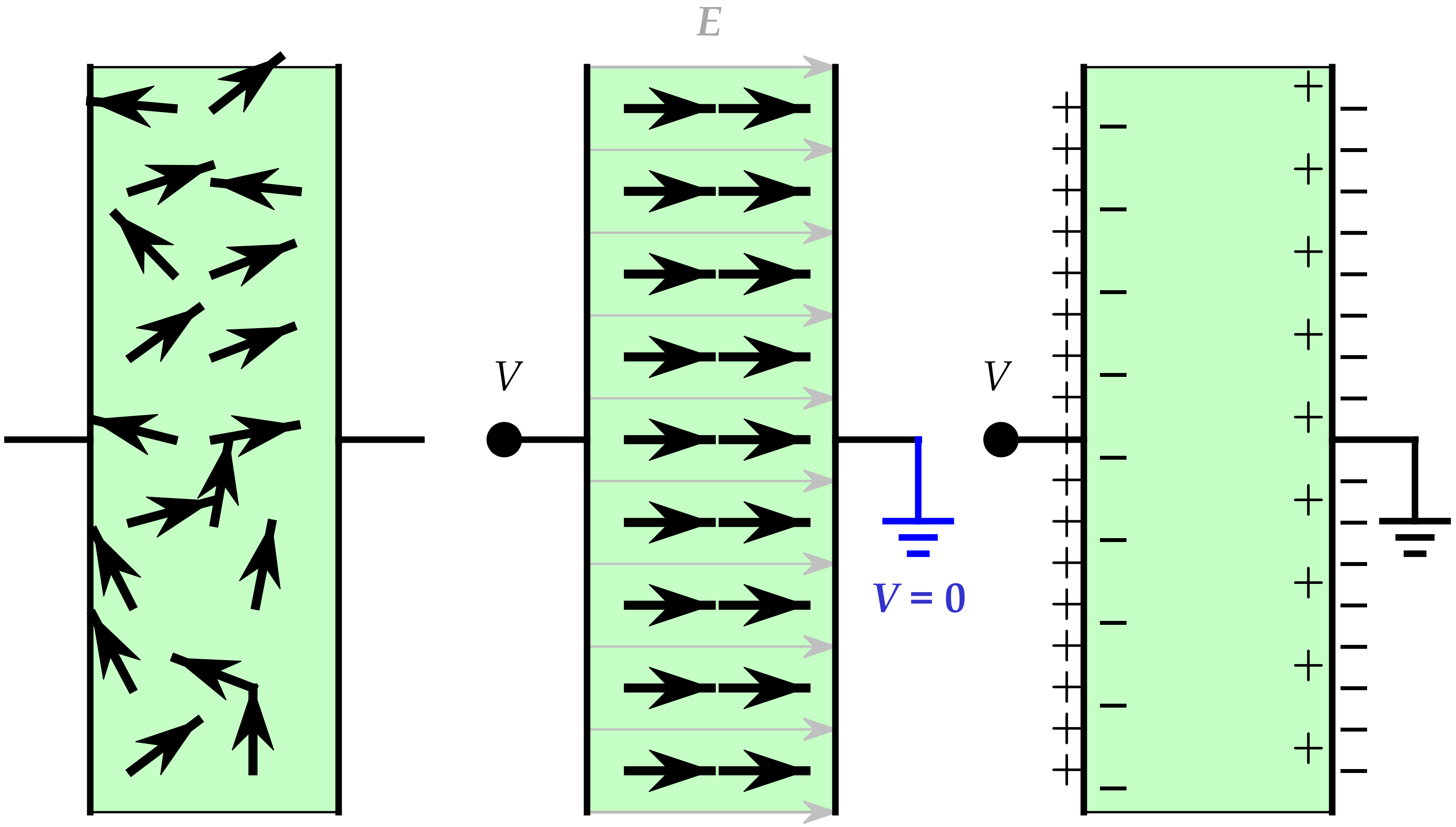


Dielectric insulators in capacitors

It would be asking too much for the electrodes of these vacuum-gapped capacitors to stay put.

- ❑ Real capacitors are made by putting conductive coatings on thin layers of **insulating** (non-conducting) material.
- ❑ In turn, most insulators are **polarizable**:
  - The material contains lots of randomly-oriented molecules with dipole moments.
  - When such a capacitor is charged, these dipoles experience torque (see 4 September) that aligns them in the capacitor's internal $\boldsymbol{E}$.
- ❑ The alignment of the dipoles takes work, which takes up some of the energy put into the charging.

Dielectric insulators in capacitors (continued)



Dielectric insulators in capacitors (continued)

- In turn, work changes the potential energy of the dipole. Consider a dipole that starts off pointed perpendicular to the applied electric field:

$$\boldsymbol{\tau} = \boldsymbol{p} \times \boldsymbol{E} \quad \Rightarrow \quad \tau = pE\sin\theta$$

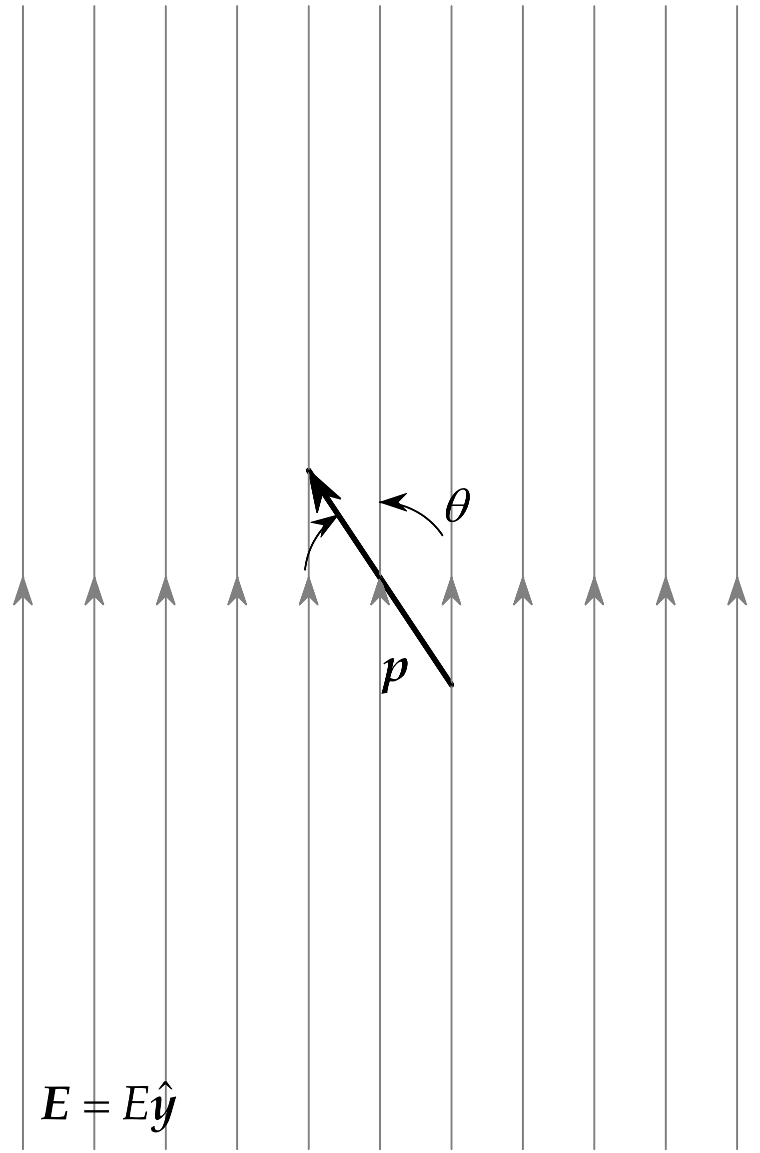
$$W = \int_{\pi/2}^{\theta} \tau(\theta')\,d\theta' = \int_{\pi/2}^{\theta} pE\sin\theta'\,d\theta'$$

$$= -pE\cos\theta'\Big|_{\pi/2}^{\theta} = -pE\cos\theta$$

$$= -\boldsymbol{p}\bullet\boldsymbol{E}$$

$$= U$$

Minimum when $\boldsymbol{p}$ is parallel to $\boldsymbol{E}$.



Dielectric insulators in capacitors (continued)

- Because the ***E*** fields generated by the dipoles themselves points in the opposite direction as the field from the applied voltage,
  - more charge will flow to the electrodes for a given voltage, than would in the absence of the dielectric.
  - Or put it the other way: a smaller applied voltage is required for a given charge, since the attraction to the dipoles's charges can do some of the work of the voltage.
- Thus the capacitance is larger with the dielectric between the plates, than it is with vacuum.

Dielectric insulators in capacitors (continued)

- Experiments show that most dielectric insulators increase the capacitance by a factor κ, the material's **dielectric constant**.
  - κ is different in general for different materials, and usually lies in the range 1-40.
- So if an arrangement of conductive electrodes yields a capacitance of C_0 without dielectric, its capacitance will be $C = \kappa C_0$ when filled with dielectric.
  - Replace ε_0 with $\kappa\varepsilon_0$ in formulas for C_0.

$$C = \frac{\kappa\varepsilon_0 A}{d}$$ **Dielectric-filled parallel- plate capacitor**

$$C = \frac{2\pi\kappa\varepsilon_0 L}{\ln\left(R_2/R_1\right)}$$ **Dielectric-filled cylindrical capacitor**

Energy, capacitors and dielectrics

Recall the expression for energy stored in a capacitor:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

- For a given V, more energy can be stored in a dielectric filled capacitor $(C = \kappa C_0)$ than in a vacuum-filled one $(C = C_0)$, since $\kappa \geq 1$.
- For a given Q, *less* energy can be stored thereby.

Thus the **electrostatic pump**:

- Use a cylindrical capacitor like a straw in a dielectric, nonconducting fluid.
- First, with the capacitor out of the fluid, charge the capacitor up to Q with V.

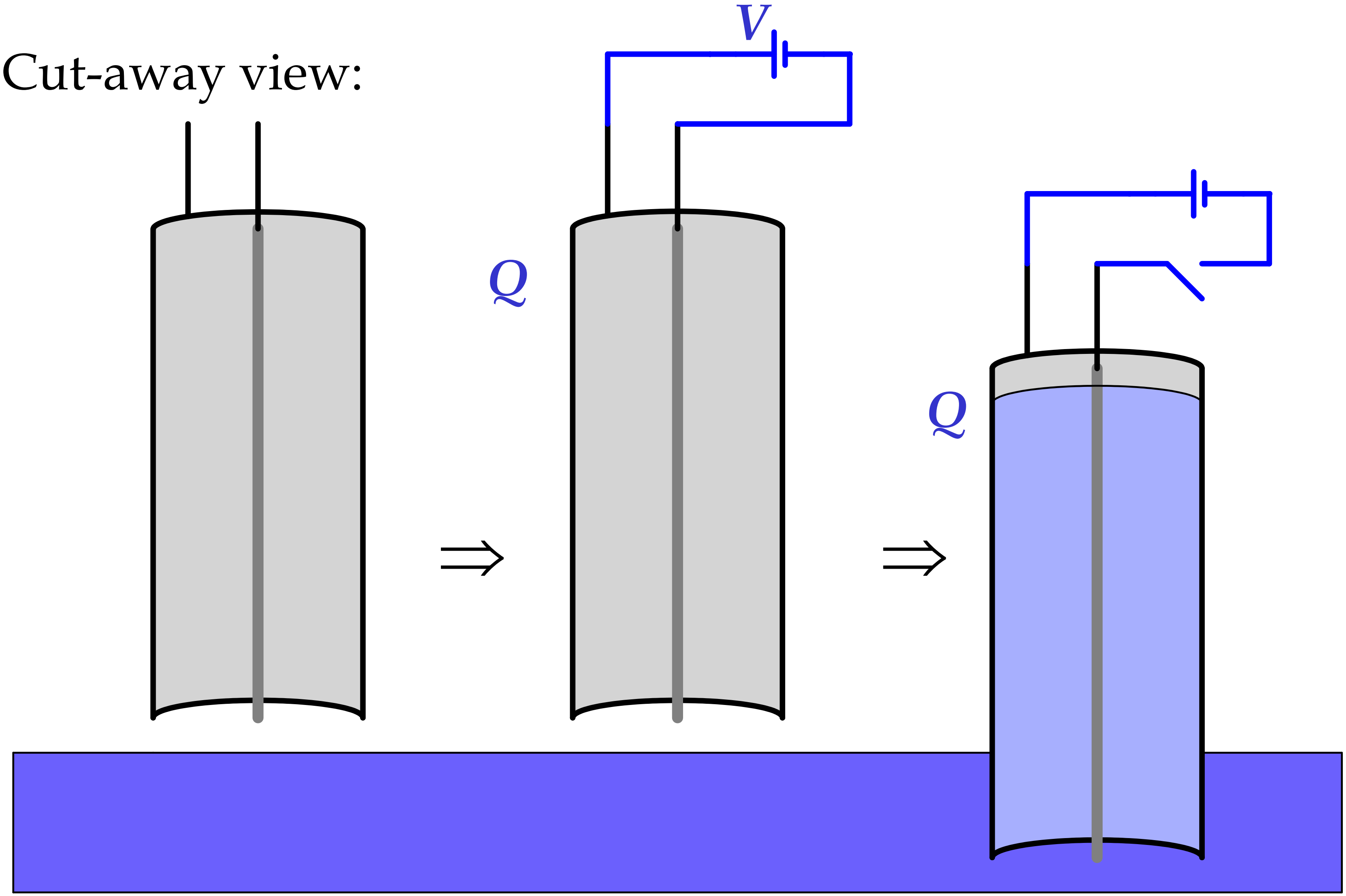
Energy, capacitors and dielectrics (continued)

- Then disconnect V. The capacitor retains the charge Q.
- Now put one end of the capacitor into the fluid.
- Because the (positive!) potential energy U in the capacitor is less with dielectric than without ($Q^2/2C_0 \rightarrow Q^2/2\kappa C_0$), fluid will be drawn into the capacitor.
  - and will rise to the level at which the electrostatic potential energy decrease is balanced by the gravitational potential energy increase.

See diagram on next page.

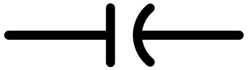
Energy, capacitors and dielectrics (continued)

Cut-away view:



Capacitors in circuits

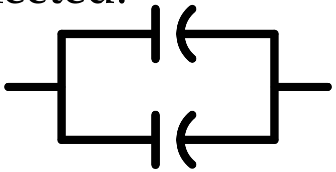
Capacitors are used ubiquitously in electrical circuits as energy-storage reservoirs. The appear in circuit diagrams as



where

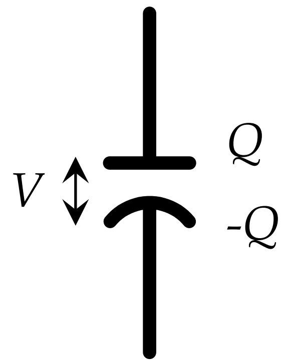
- the two short lines are supposed to remind you of a parallel-plate capacitor,
- the other lines represent wires used to connect the capacitor to other components, and
- *all* of the lines are understood to be perfect conductors.

There are two ways two capacitors can be connected:

series,  and **parallel**. 

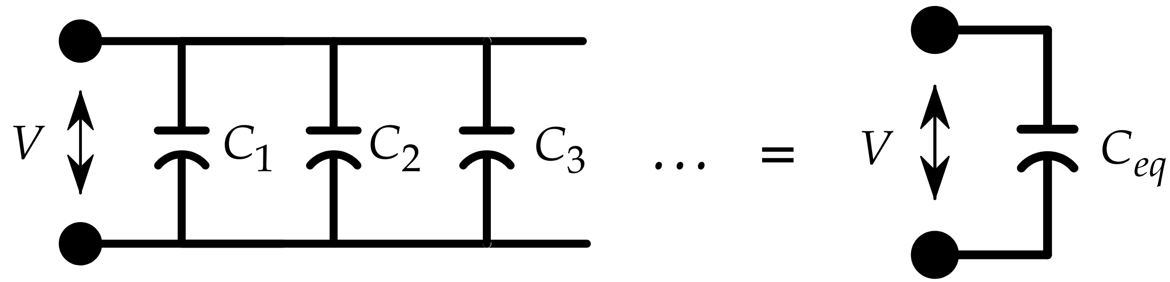
Capacitor abbreviations

- When we say "the charge on the capacitor is Q," we mean there's Q on one conductor and $-Q$ on the other one; the latter is understood to be there.
  - …though sometimes that other charge lies at infinity, as in workshop 4, problem 1.
  - The curved plate in the diagram is conventionally where $-Q$ is.
- When we say "the voltage on the capacitor is V, we really do mean a voltage (potential difference) but abbreviate ΔV by V.



Capacitors in circuits (continued)

By definition, capacitors connected in parallel have the same voltage:



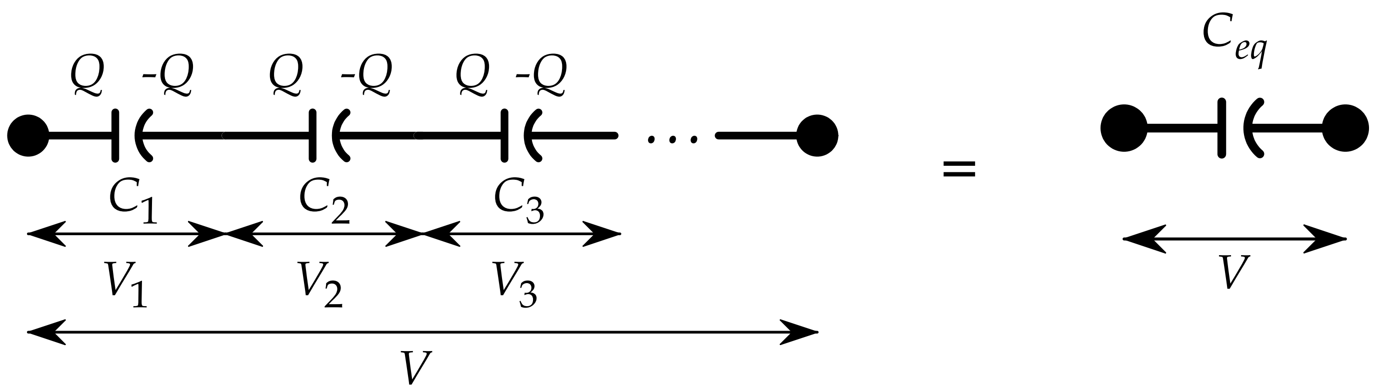
- Their charges are all in the ratio of their capacitances, and the total charge of the combination is

$$
\begin{aligned}
Q &= C_1 V + C_2 V + C_3 V + \dots \\
&= \left(C_1 + C_2 + C_3 + \dots\right) V \equiv C_{eq} V
\end{aligned}
$$

parallel capacitors are **equivalent** to a single capacitor with C equal to the sum of the capacitances.

Capacitors in circuits (continued)

Capacitors in series have different voltages, but all the same charge, since there is no way to get a net charge to the "middle" conductors:



so

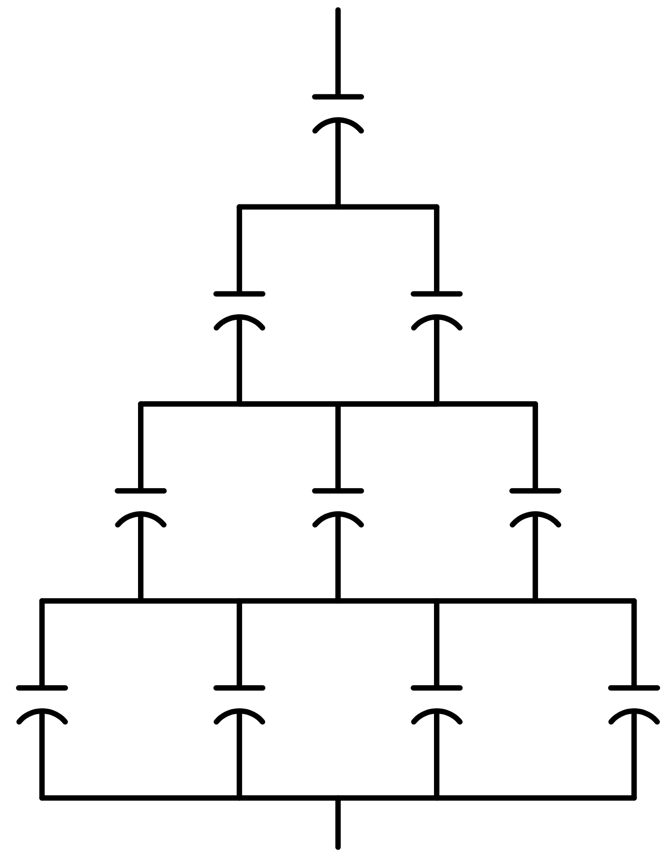
$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \dots$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \right) Q \equiv \frac{1}{C_{eq}} Q$$

Capacitors in circuits (continued)

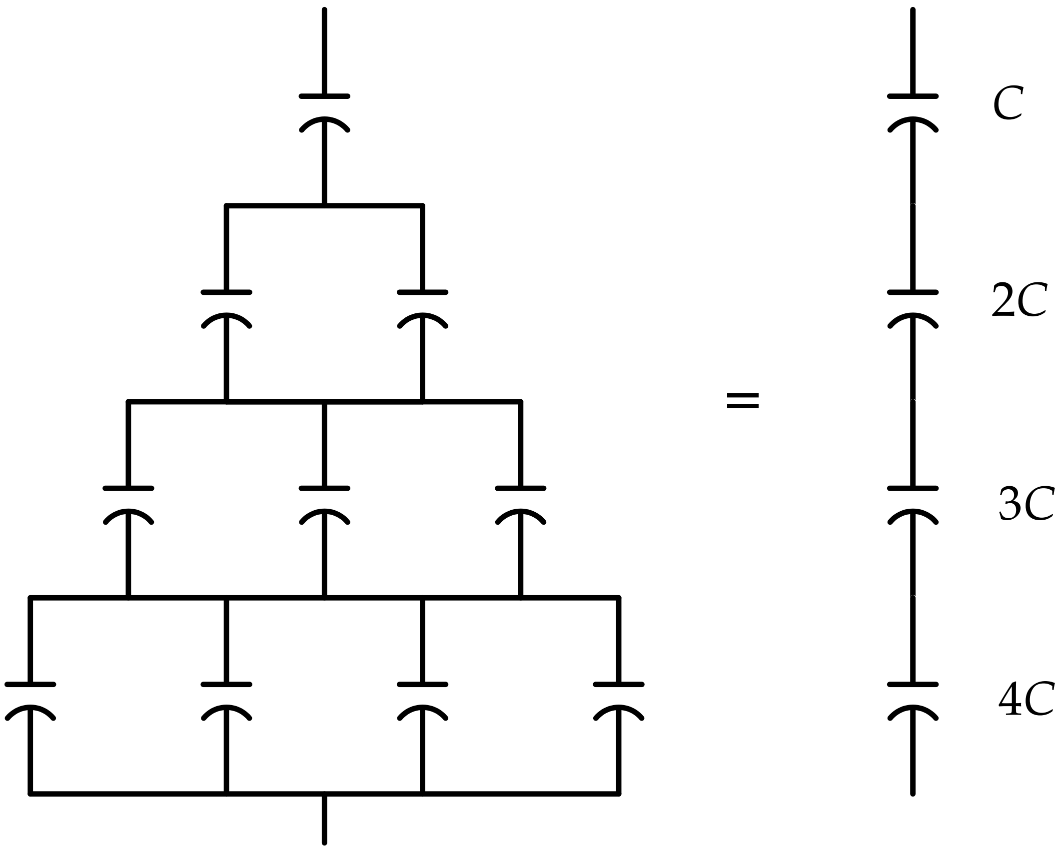
With these rules, one can calculate the single C equivalent to any network of Cs which involve **purely** series or parallel combinations of components.

Example. *What single capacitance is equivalent to this pyramid of ten capacitors, all with the same value C?*



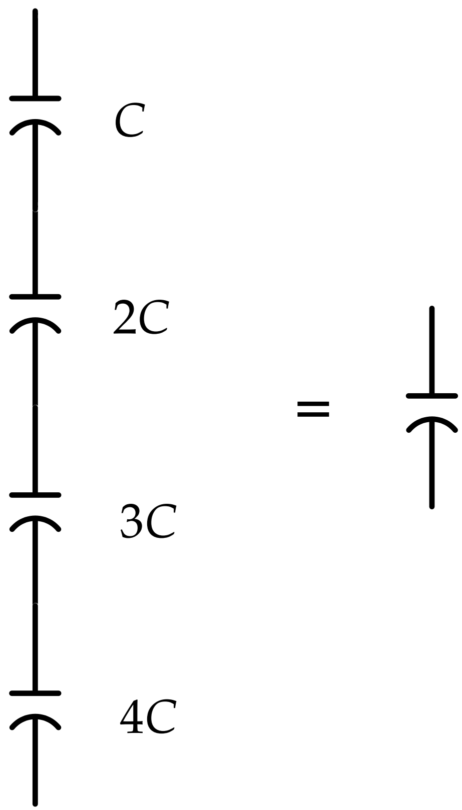
Capacitors in circuits (continued)

- First resolve the parallel combinations:



Capacitors in circuits (continued)

- And then the remaining series combination:



$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C} + \frac{1}{4C} = \frac{25}{12C}$$

$$\boxed{C_{eq} = \frac{12}{25}C}$$