Module 9: induction and Faraday's Law

Introduction

In previous units you have studied the magnetic field produced by an electric current in various geometrical arrangements. In this unit you will study the generation of electric current through the use of *time-variable* magnetic flux. The physical law which governs the production in this manner of an electromotive force (emf), and resulting electric current, is called Faraday's Law; the physical process the law describes is called induction. Because of induction, it is possible to generate electrical energy from mechanical work. This process lies therefore at the very root of technology: it makes motors and electric-power generation possible, and, as we will see, helps explain light as an electromagnetic phenomenon.

Along with Faraday's Law, two others we now know well -- Gauss's law and Ampère's law -- and a magnetic equivalent of Gauss's law constitute the four basic equations of electromagnetism, called Maxwell's Equations. As we know them now, these relations lack only one piece of the complete Maxwell equations; we'll add this last bit in at the end of PHY 122.

Objectives

1. Learn to compute the emf produced in a closed loop by the time variation of the magnetic flux linking the loop using Faraday's Law, $\mathcal{E} = \oint E \bullet d\ell = -d\Phi_B/dt$. The time variation can result from change in shape of the loop, motion of the loop relative to the source of flux, or variation of the magnitude of the magnetic field.

2. Learn to use Lenz's Law – the minus sign in Faraday's Law – to deduce the polarity of induced emf, or the direction of electric field, produced by a time-variable magnetic flux. Apply this concept specifically to the production of "eddy currents" in solid metallic objects.

3. Learn Gauss's law for magnetic fields: the magnetic flux through a *closed* surface is zero.

Reading

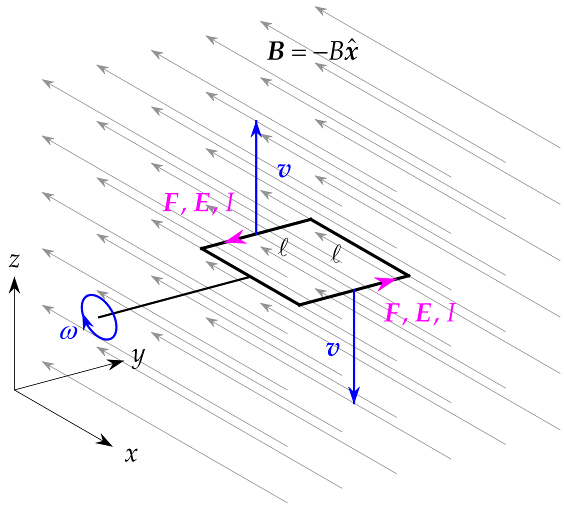
1. Chapter 30 and chapter 32, section 2. (Fundamentals of Physics 10th edition)

Study Guide

Magnetic forces and induced emf

We began our discussion of magnetism in module 7 by noting that a magnetic field is said to be present when there is a force on a moving charge. The quantitative definition is embodied in the Lorentz force law, $\boldsymbol{F} = Q\boldsymbol{v} \times \boldsymbol{B}$, and in the Biot-Savart force law, $\boldsymbol{dF} = I\boldsymbol{d\ell} \times \boldsymbol{B}$. In each case we observe the force and infer the field.

It is clear from these force laws that an observer could say that they were in the presence of either an electric *or* a magnetic field, or a combination of the two, depending on their frame of reference. The force per unit charge to an observer at rest with respect to the charge would be called an electric field, as in HRW chapter 22. At zero velocity there can be no force from a magnetic field, so if a (nongravitational) force is observed it must be due to an electric field. However, an observer moving at velocity $\boldsymbol{v}$ with respect to the charge (so that the charge now looks like a current), such that the total force is given by the Lorentz or Biot-Savart force laws, would say that a magnetic field is present.



A good example of this unity of electricity and magnetism is provided by a square conducting loop with side ℓ, immersed in a uniform magnetic field $\boldsymbol{B}$ (along $-x$), which rotates at angular speed ω about an axis lying in its plane (along y), as in the diagram at right. At the instant illustrated ($t = 0$), the loop lies in an x-y plane. The velocities of the wires – and thus of the current-carrying charges they contain – are perpendicular to the magnetic field and the loop at this instant. Thus a charge lying a distance r from the rotation axis experiences a force with magnitude $F = QvB = Qr\omega B$, perpendicular both to the field and the direction of the wire's motion. For two sides of the loop, this force is perpendicular to the wire, so there's nowhere for the force to push the charges. For the other two – those parallel to the rotation axis, a distance $r = \ell/2$ from that axis – the force is along the wire, and can produce a current. Each mobile charge along those legs experiences an electric field with magnitude $E = F/Q = \ell\omega B/2$, so the work per unit charge done in traversing one of the legs from end to end is $V = W/Q = F\ell/Q = \ell^2\omega B/2$. The directions of the forces and fields are oppositely directed in these two legs, so they drive current in the same direction. Thus the magnetically induced forces are equivalent to an emf $\mathcal{E} = 2V = \ell^2\omega B$, with polarity such that the current circulates as indicated in the diagram.

Compare this to the result for the same situation according to Faraday's Law. The rotating loop presents a time-variable projected area $A(t) = \ell^2 \sin\omega t$ in the plane perpendicular to $\boldsymbol{B}$ (i.e. y-z; the loop lies in the x-y plane at $t = 0$), so the magnetic flux is $\Phi_B = BA = B\ell^2 \sin\omega t$,[1] and the induced emf is

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} = -B\ell^2 \frac{d}{dt}\sin\omega t = -B\ell^2\omega\cos\omega t \\ &= -\ell^2\omega B \quad \text{at } t = 0,\end{aligned}$$

same as before. The minus sign (Lenz's Law) indicates that the emf has polarity such that its induced currents generate changes in magnetic flux that oppose the flux change just calculated. With rotation of the loop as in the diagram, more flux from magnetic field in the $-x$ direction threads the loop, so current must flow to make the loop's own magnetic field, at locations within the loop, have a component in the $+x$ direction. According to what we learned about loop currents in module 8, this requires a current flowing counterclockwise as seen in the diagram – the same direction as was inferred above from magnetic-force considerations.

The primacy of magnetic flux

But changes in magnetic flux lead to induced emfs even in parts of space where there are no magnetic fields or electric charges: the unity of the relations between electricity and magnetism, expressed by

[1] Strictly speaking, we should note that the area vector $\boldsymbol{A}$ points downward – along $-z$ – at $t = 0$, and write

$$\boldsymbol{A}(t) = -\ell^2\left(\hat{\boldsymbol{x}}\sin\omega t + \hat{\boldsymbol{z}}\cos\omega t\right) \quad ,$$

$$\Phi_B = \boldsymbol{B}\bullet\boldsymbol{A}(t) = -B\hat{\boldsymbol{x}}\ell^2\bullet\left(-\hat{\boldsymbol{x}}\sin\omega t + -\hat{\boldsymbol{z}}\cos\omega t\right) = B\ell^2\sin\omega t \quad .$$

Faraday's Law, go even deeper. For example: a very long $(\rightarrow \infty)$ solenoid which carries a current has a finite magnetic field inside, but zero magnetic field outside. As you will see demonstrated, though, a conducting loop wrapped completely outside of a long solenoid will receive an induced current if the current in the solenoid changes. (See Question 4 and Problem 1, below.) This induction can't be tracked with magnetic forces, as in the previous example, since the magnetic field is zero at the outside coil; it's really the magnetic flux change that induces the emf.

That this is true indicates that induced emf is just the electric-circuit expression of a more general phenomenon: Faraday's Law indicates that time-variable magnetic fields induce electric fields, even in free space.

Gauss's Law for magnetic fields

In Section 2 of Chapter 32 it is pointed out that the total amount of magnetic flux entering a closed volume in space must be the same as the amount leaving it; that is,

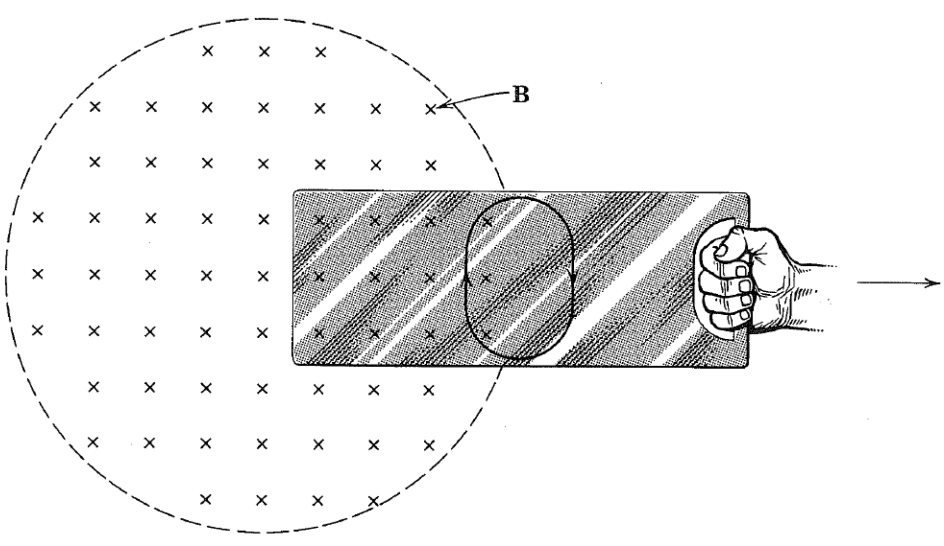
$$\oint \boldsymbol{B} \bullet d\boldsymbol{A} = 0$$

Another way to describe this phenomenon is to say that all magnetic flux lines form closed loops. No lines of $\boldsymbol{B}$ can come to an end, as lines of $\boldsymbol{E}$ do on an electric charge, since there is no such thing as a magnetic monopole—the equivalent of charge for magnetic fields.

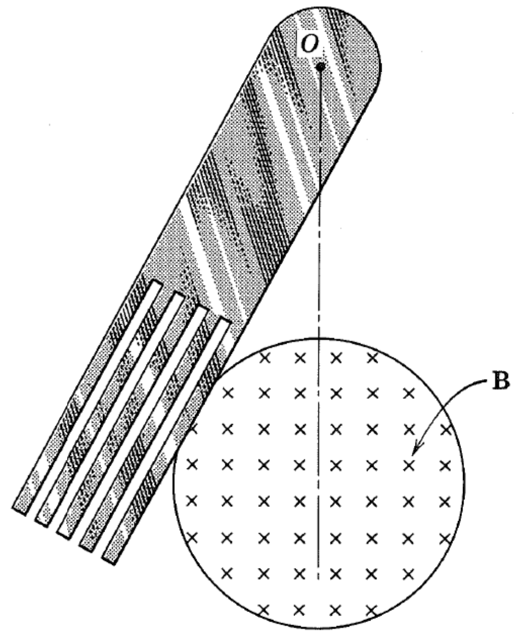
An electric field which is induced by a changing magnetic flux also have this property of having zero flux through a closed surface: the field lines are closed paths, not ending on charges.

Study questions

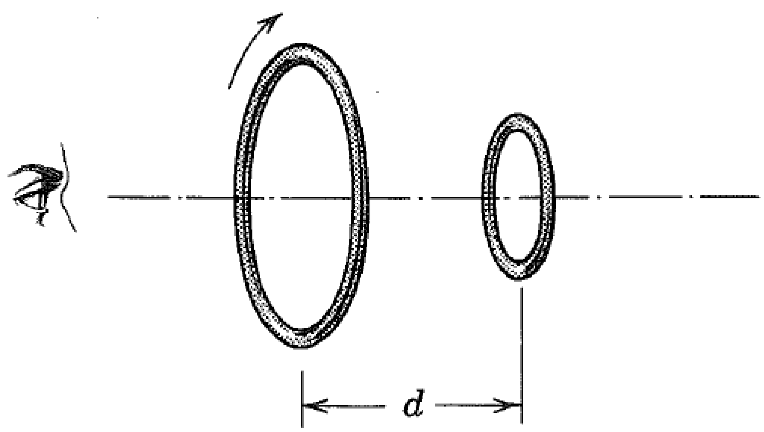
1. A sheet of copper is placed in a magnetic field as shown below. If we attempt to pull it out of the field or push it further in, an automatic resisting force appears. Explain its origin. Use Lenz's Law to determine the direction of the (eddy) currents for when the sheet is being pulled out or pushed in. With the direction of the eddy currents determined, explain why there is resistance to the movement of the sheet.



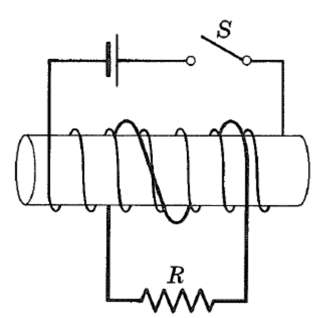
2. *Magnetic damping*. A strip of copper is mounted as a pendulum about point O in the figure at right. It is free to swing through a magnetic field normal to the page. If the strip has slots cut in it as shown, it can swing freely through the field. If a strip without slots is substituted, the vibratory motion is strongly damped. Explain. (*Hint*: use Lenz's law; consider the paths that the charge carriers in the strip must follow if they are to oppose the motion.)



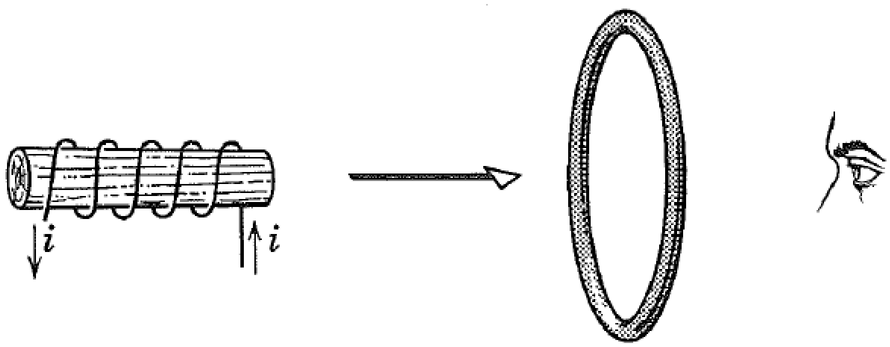
3. Two conducting loops face each other a distance d apart (see below). An observer sights along their common axis in the direction shown. If a clockwise current i is suddenly established in the larger loop, what is the direction of the induced current in the smaller loop? What is the direction of the force (if any) that acts on the smaller loop?



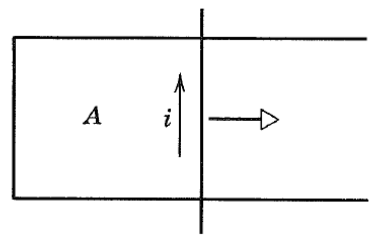
4. What is the direction, if any, of the current through resistor R in the circuit shown at right (a) immediately after switch S is closed, (b) some time after switch S was closed, and (c) immediately after switch S is opened? When switch S is held closed, which end of the coil acts as a north pole?



5. A current-carrying solenoid is moved toward a conducting loop, as shown below. What is the direction of circulation of current in the loop as we sight toward it as shown?



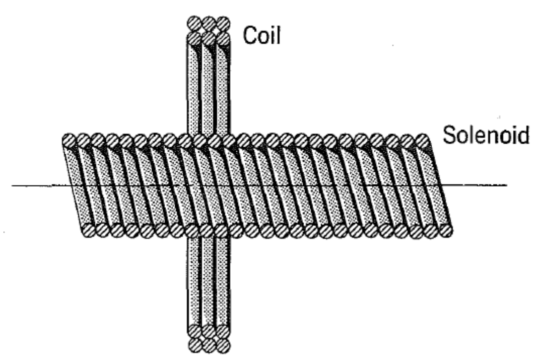
6. In the rectangular loop in the diagram at right, the movable wire is moved to the right, resulting in an induced current as shown. What is the direction of **B** in region A?



Study problems

Whenever you can, express you answers in terms of symbols (variables) first and then plug in numbers at the end.

2. A closed copper coil, with $N_c = 100$ turns and a total resistance $R = 5.0\ \Omega$, is placed outside a solenoid with radius $r_s = 1.5$ cm and $n_s = 200$ turns cm^{-1}, as shown at right. Starting at $I_1 = 1.5$ amp, the current in the solenoid is reduced to zero, and then to $I_2 = 1.5$ amp in the opposite direction, at a steady rate over a time $\Delta t = 0.05$ sec. What current appears in the 100-turn coil?

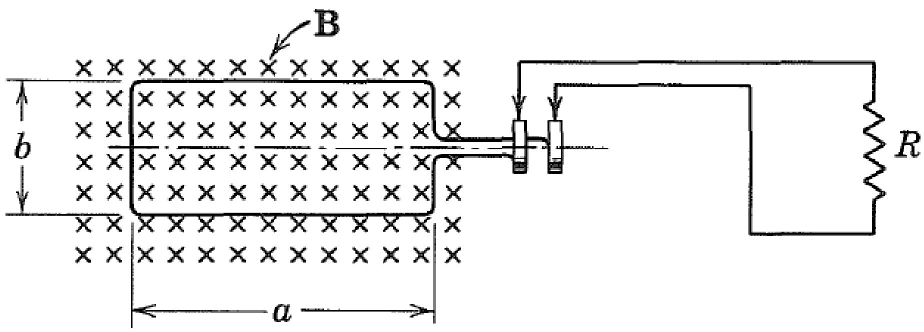


3. A uniform magnetic field **B** is normal to the plane of a circular ring, $D = 10$ cm in diameter, made of #10 copper wire (diameter $d = 0.1$ inch). At what rate must **B** change with time if an induced current of $I = 20$ amp is to appear in the ring? The resistivity of copper is $\rho = 1.68\times10^{-8}\ \Omega$ m.

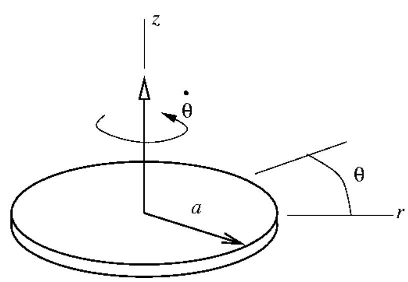
4. *Alternating current generator*. A rectangular loop of N turns, and of length a and width b, is rotated at an angular frequency ω in a uniform magnetic field **B**, as in the schematic diagram below. (a) Show that an induced emf $\mathcal{E}$, given by

$$\mathcal{E} = \omega NbaB \sin\omega t = \mathcal{E}_0 \sin\omega t \quad ,$$

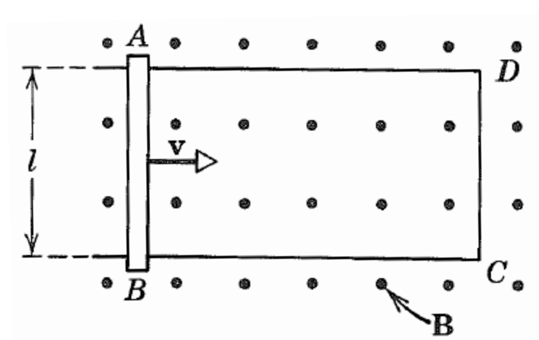
appears in the loop. This is the principle of the commercial alternating-current generators. (b) Design a loop that will produce an emf with $\mathcal{E}_0 = 150$ volts when rotated at frequency $\nu = \omega/2\pi = 60$ revolutions sec^{-1} in a field $B = 5000$ gauss.



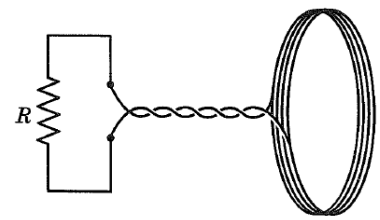
5. A circular copper disk of diameter $D = 10$ cm rotates at a frequency $\nu = \omega/2\pi = 1800$ revolutions minute^{-1} (1800 rpm) about an axis through its center and at right angles to the disk. A uniform magnetic field $B = 10^4$ gauss is perpendicular to the disk. What potential difference $\mathcal{E}$ is induced between the axis of the disk and its rim?



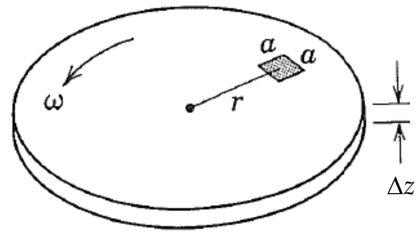
6. In the diagram at right is depicted a rectangular circuit with one movable side (AB), $\ell = 2.0$ m long, which moves at speed $v = 50\ \mathrm{cm\ sec}^{-1}$. $\boldsymbol{B}$ is the earth's magnetic field, directed perpendicularly out of the page and having a magnitude $B = 6.0 \times 10^{-5}$ T at the circuit's location. The resistance of the circuit ADCB, assumed constant (explain how this may be achieved approximately), is $R = 1.2 \times 10^{-5}\ \Omega$. (a) What is the emf $\mathcal{E}$ induced in the circuit? (b) What is the electric field magnitude E in the wire AB? (c) What force F does each electron in the wire experience due to the motion of the wire in the magnetic field? (d) What is the magnitude and direction of the current I in the wire? (e) What force F_w must an external agency exert in order to keep the wire moving with this constant velocity? (f) Compute the rate P_w at which the external agency is doing work. (g) Compute the rate P_J at which electrical energy is being converted into Joule energy.



7. Prove that if the flux of $\boldsymbol{B}$ through the coil of N turns at right changes in any way from Φ_1 to Φ_2, then the absolute value of the charge Q which flows through the circuit of total resistance R is given by the absolute value of $Q = N(\Phi_2 - \Phi_1)/R$. *Hint*: remember that charge is the integral over time of current.



8. An electromagnetic eddy-current brake – a simple example of the principle on which is based the regenerative brakes in hybrid cars and electric trains – consists of a disk with resistivity ρ and thickness Δz, rotating about an axis through its center with a magnetic field $\boldsymbol{B}$ applied perpendicular to the plane of the disk over a small area a^2 (see the diagram at right). Assuming the area a^2 to lie at a distance r from the axis, find an approximate expression for the torque that slows down the disk, evaluated at the instant its angular velocity equals ω.



9. Refresh your memory on Study Problem 14 in Module 8 before proceeding. Got it? OK, now: prove that the electric field $\boldsymbol{E}$ in a charged parallel-plate capacitor cannot drop abruptly to zero as one moves at right angles to it, as suggested by the arrow in the figure below (see point a). In actual capacitors, *fringing* – bowing outward from the gap – of the lines of force always occurs, which means that $\boldsymbol{E}$ approaches zero in a continuous and gradual way.