Eligible Proofs, t < 1 December 2026

In your Proof Meeting, which takes place in Dan's office, 418 B&L, you will have 30 minutes to prove one theorem, selected randomly from a list of six by roll of an ordinary, fair six-sided die. You will be graded by two instructors on the basis of correctness and rigor of your steps, and by how far through the proof you get. The maximum score for a correct, complete proof is 10.

You may bring one (1) 3 inch by 5 inch index card of notes to your Proof Meeting. Plan to write your proof on Dan's whiteboard, to keep your board-writing organized, and not to erase anything from the board without instructor agreement.

Here is the current list of six, which will change whenever a new eligible proof appears in class.

1. **Griffiths theorems 1 and 2**, in the form of problems 1.51 and 1.52: prove that

   a. if vector field $\boldsymbol{F}$ and scalar function V satisfy $\boldsymbol{F} = -\boldsymbol{\nabla} V$, then $\boldsymbol{\nabla} \times \boldsymbol{F} = 0$ everywhere; $\int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{F} \cdot d\boldsymbol{\ell}$ is independent of path between any two given end points $\boldsymbol{a}$ and $\boldsymbol{b}$; and $\oint \boldsymbol{F} \cdot d\boldsymbol{\ell} = 0$ for any closed loop.

   b. if vector fields $\boldsymbol{F}$ and $\boldsymbol{A}$ satisfy $\boldsymbol{F} = \boldsymbol{\nabla} \times \boldsymbol{A}$, then $\boldsymbol{\nabla} \cdot \boldsymbol{F} = 0$ everywhere; $\int_{\mathcal{S}} \boldsymbol{F} \cdot d\boldsymbol{a}$ is independent of open surface $\mathcal{S}$ for any given boundary $\mathcal{C}$; and $\oint \boldsymbol{F} \cdot d\boldsymbol{a} = 0$ for any closed surface.

2. **Stokes's theorem**: prove that, for any vector field $\boldsymbol{v}$ on a surface $\mathcal{S}$ bounded by curve $\partial\mathcal{S}$, $\int_{\mathcal{S}} (\boldsymbol{\nabla} \times \boldsymbol{v}) \cdot d\boldsymbol{a} = \oint_{\partial\mathcal{S}} \boldsymbol{v} \cdot d\boldsymbol{\ell}$.

3. **Gauss's Law of Electricity:** if $\boldsymbol{E}$ is an electric field, then $\boldsymbol{\nabla} \cdot \boldsymbol{E} = 4\pi\rho(\boldsymbol{r})$, and $\oint_{\partial\mathcal{V}} \boldsymbol{E} \cdot d\boldsymbol{a} = 4\pi Q_{\mathcal{V}}$, where $Q_{\mathcal{V}}$ is the electric charge contained within surface $\partial\mathcal{V}$ which bounds volume $\mathcal{V}$.

4. **Uniqueness of solutions**: prove that the Poisson equation $\nabla^2 V = 4\pi\rho$, when subjected to boundary conditions on the solution, its normal gradient, or combinations of these, yields unique solutions for $V(\boldsymbol{r})$.

5. **Ampère's Law of Magnetism, magnetostatic version:** If $\boldsymbol{B}$ is a magnetic field generated by a steady current with density $\boldsymbol{J}(\boldsymbol{r})$, then $\boldsymbol{\nabla} \times \boldsymbol{B}(\boldsymbol{r}) = 4\pi \boldsymbol{J}(\boldsymbol{r})/c$, and $\oint_{\partial\mathcal{A}} \boldsymbol{B} \cdot d\boldsymbol{\ell} = 4\pi I_{\mathcal{A}}/c$, where $I_{\mathcal{A}} = \int_{\mathcal{A}} \boldsymbol{J}(\boldsymbol{r}') \cdot d\boldsymbol{a}'$ is the current enclosed by the curve $\partial\mathcal{A}$ which bounds area $\mathcal{A}$.

6. **Helmholtz's field theorem:** If a vector field $\boldsymbol{F}(\boldsymbol{r})$ vanishes as $r \to \infty$; if $\boldsymbol{F}$ has specified divergence and curl $\boldsymbol{\nabla} \cdot \boldsymbol{F} = d(\boldsymbol{r})$ and $\boldsymbol{\nabla} \times \boldsymbol{F} = \boldsymbol{c}(\boldsymbol{r})$; and if $d(\boldsymbol{r})$ and $\boldsymbol{c}(\boldsymbol{r})$ are confined, in the sense that their integrals with $1/r$ over all space converge:

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$$U(\boldsymbol{r}) = \frac{1}{4\pi} \int_{\text{all space}} \frac{d(\boldsymbol{r}')}{\mathscr{r}} d\tau' \quad \text{and} \quad W(\boldsymbol{r}) = \frac{1}{4\pi} \int_{\text{all space}} \frac{c(r')}{\mathscr{r}} d\tau' \quad ,$$

then $\boldsymbol{F}$ is uniquely specified by its curl and divergence through these integrals: $\boldsymbol{F}(\boldsymbol{r}) = -\boldsymbol{\nabla} U(\boldsymbol{r}) + \boldsymbol{\nabla} \times \boldsymbol{W}(\boldsymbol{r})$.

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