Physics Letters A 374 (2010) 4164–4168

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Topology of Earth's climate indices and phase-locked states

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ARTICLE INFO

Article history:
Received 23 July 2010
Received in revised form 10 August 2010
Accepted 11 August 2010
Available online 14 August 2010
Communicated by V.M. Agranovich

ABSTRACT

Phase-locked states in Earth's climate system were identified in a study of a set of climate indices by Swanson and Tsonis (2009) [1] (ST). They reported five climate shift events since 1900 based upon features in a phase-locking parameter *S*. The present study finds that sets of climate indices have important topological properties such as a metric diameter *D* that describes the magnitude of phase locking among the indices. Minima in *D* as a function of time are shown to be associated with climate shifts. Eighteen strong events since 1870 are identified, including the five reported by ST. Ten of these minima correspond to reported events such as the well documented "climate shift of the mid-1970s" and the more recent climate shift of 2001–2002. Most climate shifts tend toward radiative equilibrium.

1. Introduction

Many climate index time series such as the El Niño/La Niña index (Nino3.4), the Southern Oscillation Index (SOI), the Pacific Decadal Oscillation (PDO), the North Pacific Index (NPI), the North Atlantic Oscillation (NAO), the Atlantic Multidecadal Oscillation (AMO) and the South and North Pacific PC1 indices (SPC1, NPC1) have been created to study and characterize various aspects of Earth's climate system. Correlation among these indices is of particular interest. One of the early accounts of correlation was by Bjerknes [2] who reported that

> "… *the temperature variations at the Pacific Equator are associated with Sir Gilbert Walker's Southern Oscillation*".

Bjerknes had found correlation between Nino and SOI. This correlation or phase locking was later made quantitative by Rasmusson and Wallace [3], who determined that the correlation coefficient between equatorial sea surface temperature (SST) and pressure anomalies (SOI) was greater than 0.8. Recently, Swanson and Tsonis [1] considered a set of four northern hemisphere climate indices and defined a synchronization or phase locking parameter *S* based upon the Pearson correlation coefficient between indices (described below). They report five maxima in *S* since 1900 that they identify with synchronization or phase locking in the climate system.

Related phenomena are the changes in the magnitude of various individual climate indices which are interpreted as "climate shifts" in the atmosphere/ocean climate system. For example, a climate shift in the mid-1970s has been observed in a variety of parameters (Ebbesmeyer et al. [4]; Trenberth [5]; Bratcher and Giese [6]; Tsonis, Swanson and Kravtsov [7]). In various studies where a change in the magnitude of a single climate parameter is observed the term "climate shift" is frequently invoked implying correlation to other [unspecified] climate phenomena.

This Letter considers the topological properties of sets of climate indices and proposes that a topological diameter *D* describes the magnitude of phase locking among them. Section 2 describes data sources. In Section 3, minima of the topological diameter *D* are shown to correspond to climate shifts. In Section 4, eighteen climate shifts are enumerated. A summary is in Section 5.

2. Data, methods and definitions

2.1. Data

All data sets consist of monthly values beginning with 1870.

Nino3.4

This is the El Niño/La Niña index based upon anomalies in the sea surface temperature (SST) in the equatorial Pacific region SST3.4 (Kaplan [8]). Region SST3.4 is defined by latitude: 5S–5N, longitude: 120W–170W.

North and South Pacific indices

Shakun and Shaman [9] have calculated the leading principal component (PC1) of detrended Pacific HadISST temperature anomalies poleward of 20N and of 20S. The North Pacific PC1 (NPC1) is very close to the PDO index as the authors expected. (Swanson and Tsonis [1] used the PDO index but NPC1 is used here because it begins in 1870.) The South Pacific PC1 (SPC1) is a

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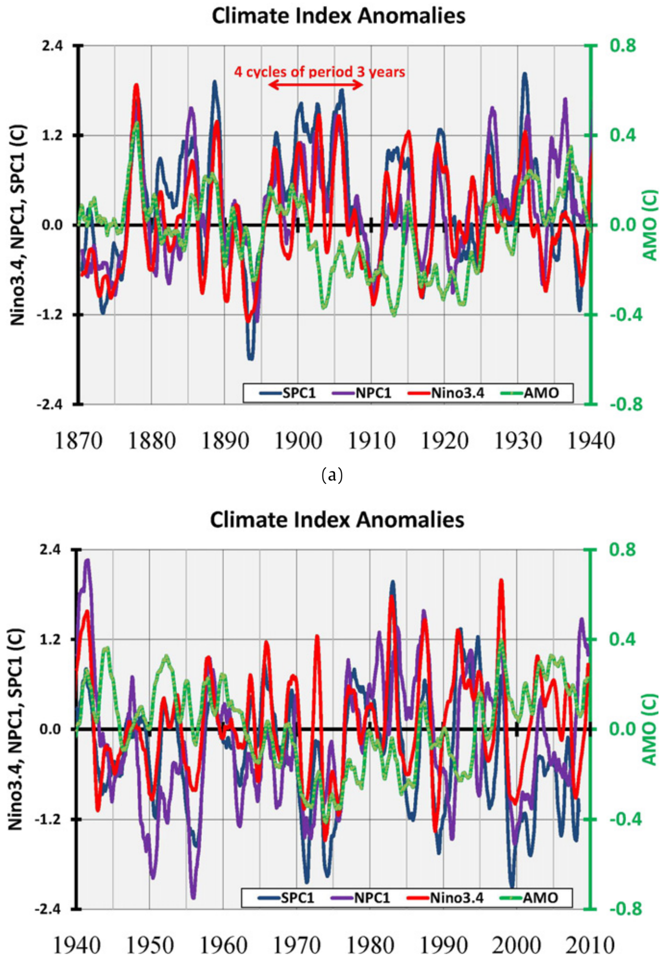
doi:10.1016/j.physleta.2010.08.025



(a)

(b)

Fig. 1. Climate indices Nino3.4, SPC1, NPC1 and AMO. These have been filtered as described in the text. The signs of NPC1 and SPC1 have been changed so that changes common with Nino3.4 will be in the same direction. Note the segment from 1997–1998 showing an El Niño and the segment from 1896–1908 showing four oscillations of period 3-years. (a) 1970–1940; (b) 1940–2010.

new index and is used in this study. It is the only index from the southern hemisphere.

Atlantic Multidecadal Oscillation (AMO)

AMO are the monthly temperature anomalies of the Northern Atlantic Ocean (latitudes: 0 to 70N). These are calculated from the Kaplan SST data set from 1856 to present. See Enfield et al. [10].

As a set these four indices can be considered a proxy for the "global" climate system and Nino3.4, NPC1 and SPC1 can be considered a proxy for the Pacific climate system. Swanson and Tsonis [1] also consider four climate indices: PDO, NPI, ENSO, and NAO. Their index set spans only the northern hemisphere and for some members of the set there are no data before 1900.

The Nino3.4, SPC1, NPC1 and AMO climate index time series from 1870 to 2010 are shown in Fig. 1(a)–(b).

2.2. Methods

In many geophysical data sets an interfering annual effect is a recognized problem. A frequent first step in such studies is the "removal" of the annual effect. Douglass [11] has shown that in some of the commonly used climate indices a small annual component remains. He showed that the digital filter, $\mathcal{F}$, a 12-point symmetric moving average "box" digital filter, reduces the annual effect while preserving the monthly resolution of the original time series. All of the indices used in this study have been filtered with $\mathcal{F}$.

2.3. Definitions of phase locking

The Pearson correlation coefficient ρ_{ij} between two climate index time segments will be used in the definitions below. (The case of $i = j$ corresponds to the autocorrelation function and is not used here.) To calculate the coefficient, the elements in each time segment are considered as components of a vector. Unit vectors are then constructed. The Pearson coefficient is the inner product of the two unit vectors. Geometrically, the inner product is the cosine of the angle between the unit vectors. The angle is found by taking the inverse cosine of ρ_{ij}.

Tsonis, Swanson and Kravtsov [7] and Swanson and Tsonis [1] (ST) have considered correlation in a set of climate indices and have defined a synchronization/phase-locking parameter S that is used for evidence of climate shifts. Their definition of S involves (1) using the Pearson correlation coefficient ρ_{ij} to define a "distance" d_{ij} between two climate indices (i and j) and (2) defining a relation for combining many d_{ij}s.

In the definition of the distance d_{ij} between two indices one wants it to be small when the correlation is high. Onnela, Kasti and Kertesz [12] (OKK) in their studies state that any distance d_{ij} based upon ρ_{ij} that is positive and monotonically decreasing is suitable. They use $d_{ij}(t) = (2(1-\rho_{ij}(t)))^{1/2}$, which maps the correlation coefficient from values $[-1$ to $1]$ onto d_{ij} values [0 to 2]. This is undesirable because one wants the equivalent correlation maximal values -1 and $+1$ to be mapped onto the single d_{ij} value 0. ST [1] achieve this by defining $d_{ij}(t) = (2(1-|\rho_{ij}(t)|))^{1/2}$. These definitions of distance do not satisfy the usual mathematical criteria for a metric distance.

If the distance between indices satisfies the following three properties:

$$\begin{aligned} &\text{(a)} \quad d_{ij} \geqslant 0 && \text{(non-negativity)}, \\ &\text{(b)} \quad d_{ij} = d_{ji} && \text{(symmetry)}, \\ &\text{(c)} \quad d_{ik} + d_{kj} \geqslant d_{ij} && \text{(triangle inequality)} \\ &\quad (i \neq k \neq j), && \end{aligned} \tag{1}$$

then the set I_0 of indices together with the pairwise distances d_{ij} between these indices form a metric space, (I_0, d_{ij}). Having a metric space allows the topological diameter D to be used to describe the "phase locking" among the members of the set I_0. The distances defined by OKK and ST do not satisfy these conditions.

The distance used in this Letter is

$$d_{ij}(t) = \cos^{-1}\left(\left|\rho_{ij}(t)\right|\right), \tag{2}$$

where the multivalued inverse cosine function is restricted to the principal branch that goes from 0 to 90 degrees. This distance (spherical angle) satisfies properties (1) and is thus a metric. Metric (2) also has the desired mapping: the equivalent maximal correlation values -1 and $+1$ are mapped to 0 degrees while correlation 0 is mapped to 90 degrees.

The diameter D from topology can be used as a measure of phase locking among the indices. From Munkres [13], the topological diameter $D(t)$ of the metric space (I_0, d_{ij}) is defined as

$$D_{I_0}(t) = \max\left[d_{ij}(t) \mid i, j \in I_0;\ i \neq j\right], \tag{3}$$

where i and j are any of the indices in the set I_0. Geometrically, D selects the largest angle (d_{ij}) among the members of the set. The diameter D may be considered a "dissimilarity" index because large D means weak correlation. There are six independent d_{ij}s among the four indices considered in this Letter.

Table 1
Sets of index pairs shown in Fig. 2.

Set	Indices involved	# of pairs	Number of minima for D less than 53 degrees
A	Pacific only (NPC1, Nino3.4, SPC1)	3	18
B	Pacific plus the North Atlantic (NPC1, Nino3.4, SPC1, AMO)	6	8

Tsonis, Swanson and Kravtsov [7] define a phase-locking parameter S as the average of their non-topological d_{ij}s. In a later paper, Swanson and Tsonis [1] change the definition of S to be the [also non-topological] square root of the mean square of the Pearson coefficients.

The diameter D is clearly quite sensitive to the distance value of a single index pair. For example, consider the case of six pairs consisting of five perfectly correlated pairs and one uncorrelated pair. The value of D is 90 degrees for the six pairs but becomes 0 degrees when the uncorrelated pair is removed. For this same example, the value of the S of Swanson and Tsonis [1] goes from 0.83 to 1.0. Thus, D is more sensitive to an uncorrelated pair than is S.

In Section 4 below it is proposed that narrow minima in D are indicators of climate shifts. Furthermore, the geographical range of the climate shift is determined by the range of the set of d_{ij}s.

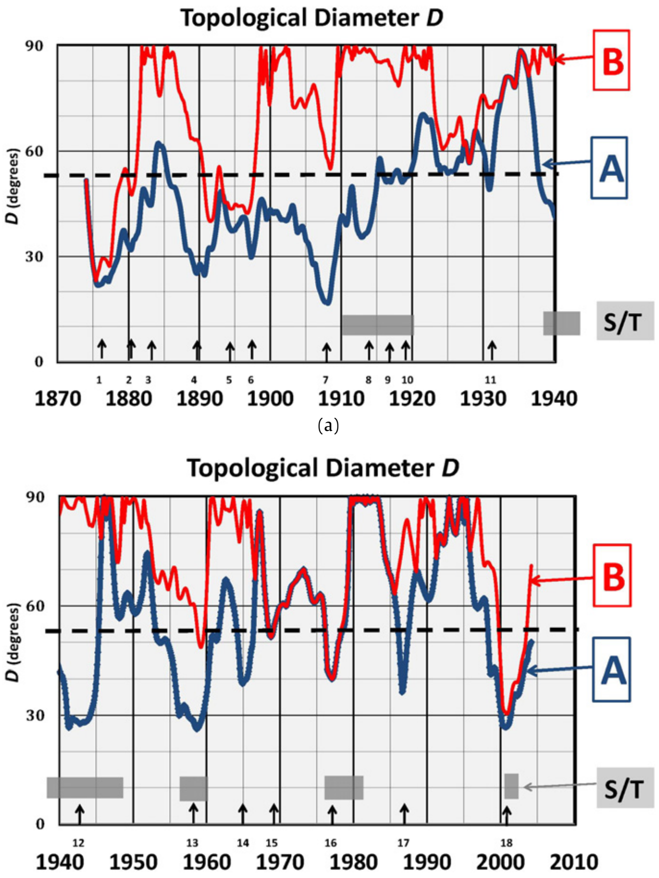
3. Analysis of data

The subject of this study is coherent behavior among the various climate regions of Earth. Evidence for such behavior is in correlation between climate indices from those regions. The filtered indices Nino3.4, SPC1, NPC1 and AMO are shown in Fig. 1(a)–(b). In general, these indices are bounded oscillations of period 2 to 7 years. From inspection, one sees many time segments showing correlation among the indices. These regions of coherence will be found and enumerated in the analysis below. However, before quantitative correlation studies can be undertaken the question of lag times must be considered. The Pearson correlation coefficient between index pairs was calculated as a function of lag time for various ten-year segments since 1870. It was found that the maximum correlation between pairs of Pacific indices (Nino3.4, SPC1 and NPC1) occurred at lags of not more than several months among them with Nino3.4 almost always leading. The maximum correlation of Atlantic Multidecadal Oscillation (AMO) with any of the Pacific indices occurred at AMO lags of 7 to 9 months. Accordingly, for all calculations in the next section, the AMO index was adjusted to lag Nino3.4 by 8 months. No lag adjustments were made in the other indices.

The six independent correlation coefficients $\rho_{ij}(t)$ among the four indices were computed for each date t over the interval $t - \Delta t/2$ to $t + \Delta t/2$ where, following Swanson and Tsonis [1], $\Delta t = 84$ month s was chosen. Next, the topological diameter $D(t)$ was computed for the two sets of indices listed below (see Table 1):

A) Pacific indices (Nino3.4, SPC1 and NPC1): 3 pairs.
B) Pacific indices plus Northern Atlantic (AMO): 6 pairs.

Fig. 2(a)–(b) shows $D(t)$ for these two cases, where it is seen that $D_A(t) \leqslant D_B(t)$ as expected. The important features of the D time series are the narrow minima that are identified below with climate shifts. The smallest minimum in D (strongest correlation) occurred during 1908–1909 and had a value of 18 degrees (set A). The number of minima below threshold D increases with threshold as shown in Table 2. In this study only the eighteen strongest minima in set A corresponding to $D < 53$ degrees will be enumerated (see Tables 2 and 3).



(a)

(b)

Fig. 2. Topological D for the following sets of pairs: A) The 3 pairs that span the Pacific (in blue); B) The 6 pairs that span the Pacific plus the North Atlantic (in red). Note that $D_A(t) \leqslant D_B(t)$. The equality occurs if the ds in set A are larger than any of those added to make set B. There are 18 minima in D whose value is less than 53 degrees associated with set A. For set B, there are 8 minima for which $D < 53$ degrees. The five events reported by Swanson and Tsonis [1] are indicted by grey rectangles. (a) 1870–1940; (b) 1940–2010. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

Table 2
Variation of the D threshold (set A).

D threshold (degrees)	30	40	50	53	60
Number of minima below threshold	7	12	14	18	22?

Eight minima in set B have $D < 53$ degrees and are consistent with those found in set A. By this criterion these minima can be characterized as “global”. Optimization of lag times could increase this number.

4. Discussion

Comparison with observations of climate events (Section 4.2) strongly suggests a correspondence between the minima in D and “climate shifts”. This observation is elevated to hypothesis (H)

> A minimum in the topological diameter D of a set of distance metrics of climate indices correlates with a shift in Earth’s ocean/atmospheric climate system within the geographic region of the set. (H)

The threshold value of D in (H) is arbitrary. The choice $D < 53$ degrees yields eighteen minima. Choosing a lower threshold would

Table 3
Eighteen minima of the topological diameter D (changes in phase-locked states of the climate system of the Earth) since 1870. [D values less than 53 degrees.]

Diameter D for set A			Swanson/Tsonis [1] maxima	Douglass/Knox [14] and Overland et al. [16] change in state	Comment
#	Date	Min (degrees)			
1	1875–1876	21			
2	1880–1881	32			
3	1882–1884	45			
4	1889–1891	25			
5	1894–1895	38			
6	1897–1898	30			Begin segment of 3 year period [Note 1]
7	1908–1909	18			End segment of 3 year period [Note 1]
8	1912–1913	36	1910–1920		
9	1916–1917	51			
10	1919	51			
11	1931	49			
12	1941–1945	28	1938–1948		
13	1956–1959	28	1956–1960	Early 1960s	
14	1964–1966	39			Two minima?
15	1969	51			
16	1976–1977	40	1976–1981	Early 1970s [Note 2]	Climate "shift" of 1970s.
17	1986–1987	37		[Note 2]	
18	2001–2002	26	2001–2002	Early 2000s [Note 2]	

Note 1. In Douglass [11] and here, a time-segment in Nino3.4 from 1896 to 1908 shows a strong oscillation of period 3 years. This implies changes in state at the beginning and at the end.
Note 2. Overland et al. [16] report climate shifts at 1976–1978, 1988–1990 and 1998–1999.

exclude some minima that seem to correspond to known climate shifts. Obtaining more minima to explain can be achieved by choosing a higher threshold.

This identification of a climate shift is quite different from that of Swanson and Tsonis [1]. Theirs involves two necessary conditions. The first is the occurrence of a maximum in their phase-locking parameter S, which is close to hypothesis (H), and the second independent condition is that there be no "increase in Coupling between modes". (See their paper for the definition of Coupling.) The 1956–1960 event, clearly identified here, is eliminated by ST's Coupling condition, which disagrees with the results of this Letter.

4.1. Robustness

Hypothesis (H) is robust. A "climate shift" is not indicated unless all index pairs of a set have a metric distance less than a certain value. Weaker minima may be missed by this criterion. For example, a missed minimum may exist around 1990. If any of the pairs of the set had had a larger metric d, there would be no identification of a climate shift. Gershunov et al. [14] have shown that correlation between a climate time-series pair can sometimes arise from stochastic noise. This possibility seems unlikely in this study because of the larger number of pairs.

4.2. Comparison of minima in D to climate shifts in prior studies

Eighteen minima in D in set A are identified in Fig. 1 and are listed in Table 3. The twelve that occurred since 1900 include the five events (1910–1920, 1938–1945, 1956–1960, 1976–1981 and 2001–2002) identified by Swanson and Tsonis [1] (ST) and are indicated in Fig. 2 by grey rectangles. The correspondence of the ST events is one-to-one with events identified in this Letter except that their 1910–1920 event has been resolved into three.

Three climate shifts since 1950 were reported by Douglass and Knox [15] in a study of ocean heat content and "top of the atmosphere" data. The shifts were described as occurring during the 1960s, mid-1970s and early 2000s, consistent with the results of the present study.

Overland et al. [16] in a study of various individual climate time series report climate shifts during 1976–1978, 1988–1990 and 1998–1999 all of which agree with the results here.

In a study of Nino3.4, Douglass [11] showed a time segment from 1896 to 1908 that consisted of four oscillations of period 3 years, implying the existence of a coherent climate state. This same time segment is indicated in Fig. 1(a). Minima #6 and #7 of Fig. 2(a) closely correspond to the beginning and the end of this coherent state (see Table 3).

Thus, ten of the eighteen minima correspond to identified climate shift events.

4.3. Cause of climate shifts?

What is causing the climate shifts? Any proposed mechanism must explain that:

1. The D time-series show minima.
2. Some of the minima are very narrow.
3. The frequency of occurrence of climate shifts is 18 in 150 years.

One possibility is strong short-duration events – *i.e.*, impulses. One thinks of large volcano events. There are several cases where the times of these volcanoes are close to the times of climate shifts and will be discussed in a later paper. However, volcanoes cannot explain most of the shifts since there are many more climate shifts than known large volcano events.

The various geographic regions represented by the climate indices would have to be strongly coupled during the impulse even though they may not have been either before or after. Otherwise, there would be no climate shift. If this possibility is correct the following questions are not answered:

1. What is the nature of the impulse?
2. Do the proposed impulses couple to all indices directly or to only a few and those few transmitting the impulse via "teleconnections"?
3. Is there more than one kind of impulse?

4.4. Toward radiation equilibrium

Three of these eighteen climate shifts are identified with the shifts of Douglass and Knox [14] (DK) who found that the shifts were between states of positive and negative radiation imbalance.

It is likely that these states are close to the state of radiative balance. If one considers many successive changes in state it seems unlikely that they would all be away from radiative equilibrium. It is suggested that even though there are only the three examples from DK that most of the shifts in Earth's climate system are toward radiative equilibrium.

4.5. Future studies

This study opens new lines of inquiry concerning phase-locked states of Earth's climate system. Many follow-up studies are suggested. The claims of dates of eighteen climate shifts are quite specific. Evidence in other climate studies should be sought as to the correctness of each of these identifications. Hypothesis (H) and its implications should be tested in all possible ways. One should consider other sets of climate indices and various subsets. The threshold of D should be varied and the set of associated minima in D compared to observations of climate shifts. The 84-month window in the computation of the correlation coefficients should also be varied.

The topological diameter D as a measure of phase locking among the indices is convenient and simple to calculate but is not unique and is probably not the best measure. The maximum area A of the polygons that the unit vectors of the climate indices define on the unit sphere may be a better measure of phase locking. This is because by definition the magnitude of D gives all weight to the two indices having the largest distance between them. The area A, on the other hand, gives equal weight to all the indices. This possibility should be considered.

That AMO lags the Pacific indices by 7 to 9 months suggests that the climate shifts originate in the Pacific. Additionally, within the Pacific indices Nino3.4 sometimes leads, suggesting that those events originate in the Tropical Pacific. These questions should be studied for each climate shift listed by varying lag times and different geographic sets of indices.

5. Summary

This study finds that sets of Earth's climate indices have important topological properties such as the topological diameter D that measures the magnitude of phase locking among the indices.

Comparison of the D time series with prior studies on climate shifts leads to the hypothesis that a minimum in the topological diameter D of a set of distance metrics of climate indices correlates with a shift in Earth's ocean/atmospheric climate system. Eighteen strong minima since 1870 are found. Ten of these correspond to reported events such as the well documented "climate shift of the mid-1970s" and a more recent climate shift of 2001–2002.

Three of these climate shifts are consistent with the findings of Douglass and Knox that the climate shifts are between states of positive and negative radiation imbalance. This suggests that most changes in Earth's climate system, on average about one every eight years, are toward radiative equilibrium.

Acknowledgements

Data values of NPC1 and SPC1 were supplied by Jeremy Shakun. The author thanks Jonathan Pakianathan for suggesting the metric distance given in Eq. (2). Also many helpful discussions were had with Robert S. Knox.

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