University of Rochester

# Laboratory II
## Acceleration due to Gravity

DEPARTMENT OF PHYSICS & ASTRONOMY

PHYSICS 113 - 121 - 181
GENERAL PHYSICS I AND MECHANICS

**Name:** ______________________ **Date:** ______________________

**Collaborators:** ______________________ **Lab Section:** ____________

## Prelab Exercises (2 points)

*This prelab must be completed and handed in to the lab TA at the start of the lab.*

**Question 1** **1 points**

You have just completed the first part of this lab and have five time values for a particular height: 1.8 s, 1.7 s, 1.9 s, 0.8 s, and 1.9 s.

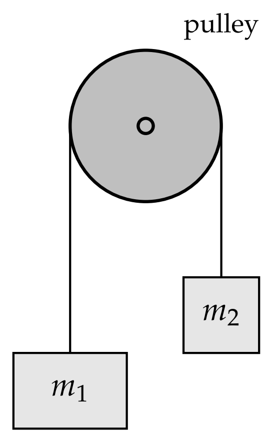
(a) ($^1/_2$ point) Give one quantitative reason why 0.8 s is or is not consistent with the other measurements.

(b) ($^1/_2$ point) If 0.8 s is an outlier, what is one explanation for what could have gone wrong in that trial? Use details of the experiment in your answer.

**Question 2** **1 point**

In this lab you will use Atwood's machine to measure the acceleration due to gravity, $g$. The machine works by hanging two masses on a pulley, with each mass acted upon by gravity. Since the masses are on opposite sides of the pulley, their weights oppose each other and the net acceleration is less than $g$. To see this, draw all relevant forces in the diagram below. Be sure to indicate the direction of friction in the pulley. Assume $m_1 > m_2$.

Figure 1: *A cartoon of Atwood's machine.*



## Objective

*Use an Atwood machine to measure the acceleration due to gravity, accounting for systematic effects such as pulley friction and rotational inertia that prevent a direct free-fall measurement. Analyze how these apparatus imperfections propagate into the final result and compare your measurement to the accepted value of $g$.*

## Theory

Atwood's machine was originally designed by George Atwood in 1784 to demonstrate the effects of uniform acceleration. It reduces the acceleration of the masses to a fraction of $g$, allowing that smaller acceleration to be measured with greater precision than free fall using the same timing device. The acceleration is:

$$a = \frac{M_1 - M_2}{M_1 + M_2} g \tag{1}$$

Inverting this expression gives $g$:

$$g = \frac{M_1 + M_2}{M_1 - M_2} a \tag{2}$$

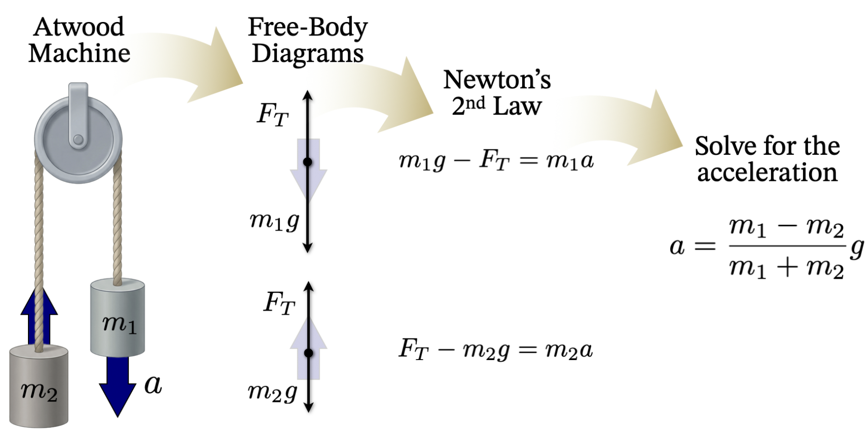



Figure 2: *Derivation of eq. (1) for Atwood's machine.*

In the next section, we show how to calculate the acceleration $a$ of the masses. Then, using eq. (2), you will estimate the acceleration due to gravity at the Earth's surface (or at least Rochester's surface, which is almost as good).

### Standard Deviation of a Set of Measurements

Throughout this lab, and in future ones, you will be asked to estimate the uncertainty in your measurements. Unless given further instructions, compute the standard deviation of a data set using

$$\bar{s}_x^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2, \qquad \bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i, \tag{3}$$

where $x_i$ are the data points, $\bar{x}$ is the mean, and $N$ is the number of data points. For instance, if you timed an object dropping to the floor in 10 trials, you would report the fall time as the mean $\bar{t}$ with uncertainty equal to the standard deviation $\bar{s}_t$.

You may then be asked whether a particular data point is consistent with the others, or whether an accepted value is consistent with your results. One way to test this is to check whether the value in question lies within one (or two) standard deviations of the mean. For a Gaussian distribution, about 68% of data points fall within one standard deviation of the mean, and about 95% fall within two. A value outside this range is likely an outlier.

### The Equation of Motion for Atwood's Machine

To find the equation of motion for Atwood's machine, we sum the forces acting on the system. There are three forces: gravity on each mass $M_1$ and $M_2$, and friction in the pulley. The friction force depends on the tension in the strings on either side of the pulley, $T_1$ and $T_2$, and the weight of the pulley $M_p g$, all multiplied by $\mu$, the coefficient of friction.

$$\sum F = M_1 g - M_2 g - \mu(T_1 + T_2 + M_p g) = M_T a \tag{4}$$

Here we define the direction of motion of the heavier mass $M_1$ as positive, $M_T$ is the total mass of the system, and $a$ is the acceleration. The total mass consists of $M_1$, $M_2$, and the pulley mass $M_p$. Substituting for $M_T$ on the right-hand side of eq. (4) gives

$$M_1 g - M_2 g - \mu(T_1 + T_2 + M_p g) = \left(M_1 + M_2 + \frac{1}{2}M_p\right) a. \tag{5}$$

$M_p$ is multiplied by $1/2$ because the pulley undergoes angular acceleration[1] rather than linear acceleration, unlike $M_1$ and $M_2$. The origin of this factor will be covered later in the course.

### Measuring the Acceleration $a$ with Digital Timers

In the first part of the lab, you will measure the acceleration $a$ and then compute $g$ using digital timers. The mass $M_1$ is placed in a plastic tube that triggers the timers as it passes through them. The setup is shown in Fig. 3.

Four variables must be measured: $D$, the length of the plastic tube; $L$, the distance between the two photogates; and $t_1$ and $t_2$, the times recorded by each timer. The timers measure how long it takes an object to pass through them, so $t_1$ and $t_2$ are the transit times of $M_1$ through the first and second timers, respectively. Since $M_1$ has length $D$, the velocity at each timer is found by dividing the distance traveled by the transit time:

$$v_1 = \frac{D}{t_1} \tag{6}$$

The velocity at the second timer is

$$v_2 = \frac{D}{t_2} \tag{7}$$

[1]This means the pulley is being rotated rather than translated.

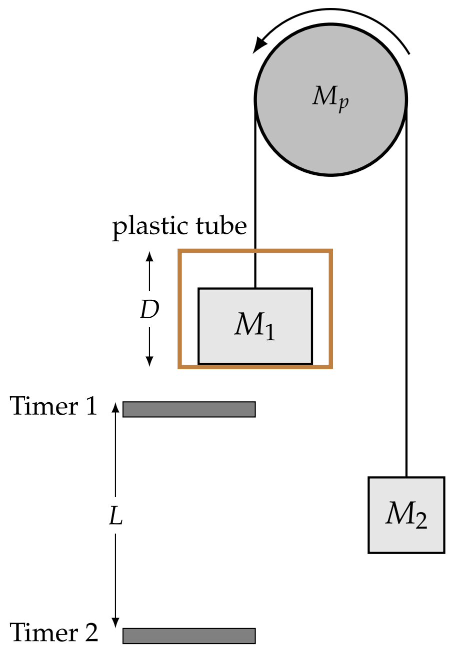



Figure 3: *Setup with two photogate timers.*

The change in velocity between the two timers is due to acceleration $a$. From kinematics,

$$v_2 = v_1 + at, \tag{8}$$

where $t$ is the time for $M_1$ to travel from the first timer to the second. Although $t$ is difficult to measure directly, the distance $L$ is easily measured. These are related by another kinematic equation,

$$L = v_1 t + \frac{1}{2}at^2. \tag{9}$$

Solving eq. (8) for $t$ gives

$$t = \frac{v_2 - v_1}{a}. \tag{10}$$

Substituting eq. (10) into eq. (9) to eliminate $t$,

$$\frac{v_1(v_2 - v_1)}{a} + \frac{(v_2 - v_1)^2}{2a} = L. \tag{11}$$

Solving for $a$ gives

$$a = \frac{D^2}{2L}\left(\frac{1}{t_2^2} - \frac{1}{t_1^2}\right). \tag{12}$$

By measuring two lengths and using the photogate timers, we can determine the acceleration of the masses.

### Analog Hand Timer Setup

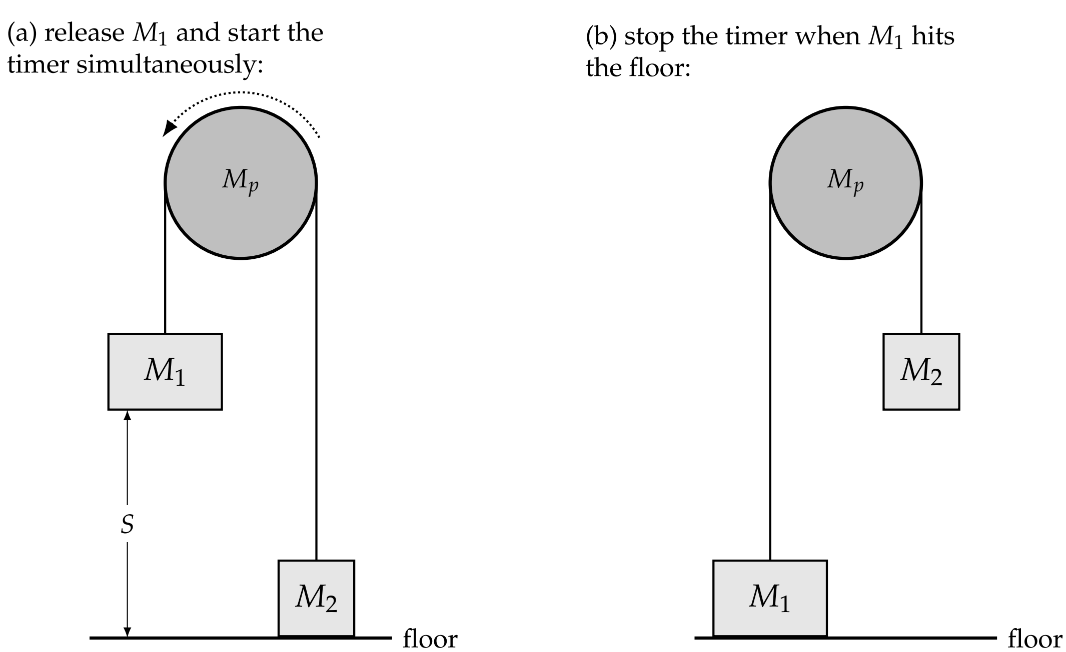



Figure 4: *The "analog" setup where you manually time the fall of $M_1$. (a) Start with $M_2$ on the floor and release $M_1$, starting the timer simultaneously. (b) Stop the timer the moment $M_1$ hits the floor.*

In this part of the experiment, shown in Fig. 4, $M_1$ starts from rest, which simplifies the kinematics. Only two variables need to be measured:

1. The distance from the bottom of $M_1$ to the floor, denoted $S$.

2. The time $t$ it takes for $M_1$ to reach the floor.

You will measure $t$ with the hand timer.

The kinematic equations for this setup are:

$$v_{\text{final}} = at \qquad \text{[final velocity of } M_1\text{]} \tag{13}$$

$$S = \frac{1}{2}at^2 \qquad \text{[distance fallen by } M_1\text{]} \tag{14}$$

These are eqs. (8) and (9) with the initial velocity set to zero ($v_1 = 0$). Solving for $a$ gives

$$a = \frac{2S}{t^2}. \tag{15}$$

The purpose of this part is to determine whether timing by hand, as George Atwood had to do in the 1780s, produces different or similar measurements of $g$ compared to the photogate timers.

## Experiment

### Measuring Acceleration $a$ and $g$ using Photogates

In this section, photogates are used to time the falling mass. You will apply the kinematic equations from the previous section along with statistical techniques to estimate $g$ and its uncertainty. Before starting, **read the operating instructions on the back of the photogate**. You will use the timer in gate mode. A list of equipment is given below:

- ☐ Pulley Wheel
- ☐ Pole with photogates
- ☐ Double Pulley
- ☐ Photogate
- ☐ Photogate Timer
- ☐ Power cord
- ☐ PVC Mass Holder
- ☐ 3 Ring Masses
- ☐ Mass Holder
- ☐ Mass Set:
  - $1 \times 200$ g
  - $2 \times 100$ g
  - $1 \times 50$ g
  - $4 \times 20$ g
  - $1 \times 10$ g
  - $1 \times 5$ g

#### Procedure

1. Measure the length $D$ and mass $M_D$ of the plastic tube. Enter your measurements in Table 1.

2. Position the photogates successively along the path of the mass. Make sure the plastic tube will not hit them on its way down.

3. Measure the distance $L$ from the top of one photogate to the top of the other (see Fig. 3). Enter your measurement in Table 1.

4. Add mass to the plastic tube until the total mass (including $M_D$) equals $M_1 = 260$ g. Enter the amount added in Table 1.

5. Add masses to one of the mass hooks until the total equals $M_2 = 240$ g. You may use tape to secure the masses on the hook.

6. Begin the experiment with the lighter mass ($M_2$) on the floor. After ensuring the photogate timers are reset, release the masses. Do not push them; the measurement requires that they start from rest. Hold the pulley, not the masses, to keep them steady before release.

7. Record the time from the upper timer as $t_1$ and the time from the lower timer as $t_2$ in Table 2. Note that $t_1$ is the value initially displayed. To find $t_2$, flip the `MEMORY` switch to `READ`; the display will show $t_1 + t_2$. Subtract $t_1$ to obtain $t_2$. As a sanity check, confirm that $t_2 < t_1$ (the mass moves faster through the lower gate).

8. Repeat 10 times to obtain a reliable estimate of $g$ and its standard deviation. Enter all values in Table 2.

Table 1: *Lengths and masses used in the Atwood machine. In column 4, indicate the mass added to the tube to bring $M_1$ to* 260 g.

| $D$ [cm] | $L$ [cm] | $M_D$ [g] | $M_1 - M_D$ [g] | $M_1$ [g] | $M_2$ [g] |
|---|---|---|---|---|---|
| | | | | 260 | 240 |

Table 2: *Atwood machine $M_1$ fall times using photogates.*

| Trial | $t_1$ [s] | $t_2$ [s] | Trial | $t_1$ [s] | $t_2$ [s] |
|---|---|---|---|---|---|
| 1 | | | 6 | | |
| 2 | | | 7 | | |
| 3 | | | 8 | | |
| 4 | | | 9 | | |
| 5 | | | 10 | | |

### Measuring Acceleration $a$ and $g$ with Manual Timers

This setup is the same as before, without the photogate timers. Move the photogates out of the way so the falling mass does not hit them.

#### Procedure

1. With $M_1$ (the plastic tube) on the floor, measure the distance $S$ from the floor to the bottom of $M_2$. Do not hold the masses or string while measuring, as this will affect the result.

2. Familiarize yourself with the hand timer and plug it in when ready. You may also use a phone or any other stopwatch.

3. Hold $M_2$ on the floor, release it, and simultaneously start the timer. Stop the timer when $M_1$ hits the floor.

4. Make 10 measurements of the fall time and record them in Table 3.

Table 3: *Atwood machine $M_1$ fall times using manual timers.*

| Trial | $t$ [s] | Trial | $t$ [s] |
|---|---|---|---|
| 1 | | 6 | |
| 2 | | 7 | |
| 3 | | 8 | |
| 4 | | 9 | |
| 5 | | 10 | |

### Accounting for the Rotational Inertia of the Pulley

When pushing an object along a line, a higher mass requires more force to achieve the same acceleration. A similar principle applies to rotating objects: a car wheel is harder to spin than a bicycle wheel because it is heavier. Rotating objects are more complicated, however, because their motion depends not only on their total mass but also on how that mass is distributed around the axis of rotation. This will be treated formally later in the course.

In the previous measurements, we treated the pulley as massless. In reality, the pulley has mass and must be accelerated rotationally as the masses fall. In this section, you will carry out measurements that account for the pulley's mass, and in the final postlab section you will assess how much it affects your estimate of $g$.

**Procedure**

1. Replace the current pulley with the large-disk pulley.
2. Attach two metal mass holders to the pulley and add masses so they match the first combination of $M_1$ and $M_2$ in Table 4.
3. With $M_1$ on the floor, measure the distance $S$ from the floor to the bottom of $M_2$. Record the result in Table 4.
4. For each combination of $M_1$ and $M_2$ in Table 4, take one hand-timed measurement of the fall time. Record the values in the column labeled "$M_{\text{ring}} = 0$".
5. Measure the mass of one metal ring ($M_{\text{ring}}$), add it to the pulley, and record $M_{\text{ring}}$ in Table 4. Repeat the previous step for each mass combination and record the results in the middle column.
6. Measure a second ring, add it to the pulley, and record $2M_{\text{ring}}$ in Table 4. Repeat for each mass combination and record the results in the right column.

Table 4: *Manual timing data with the large pulley and ring masses.*

| $M_1$ | $M_2$ | Fall times [s] | | |
|---|---|---|---|---|
| | | $M_{\text{ring}} = 0$ | $M_{\text{ring}} =$ ________ [g] | $2M_{\text{ring}} =$ ________ [g] |
| 105 g | 100 g | | | |
| 110 g | 100 g | | | |
| 115 g | 100 g | | | |

**Name:** ______________________________ **Date:** ______________________

**Collaborators:** ____________________________ **Lab Section:** ______________

## POSTLAB EXERCISES (20 points)

*Submit the postlab to the TA at the end of the lab.*

### Measurement using Photogate Timers (4 points)

**Question 3** **1 point**

Using your measurements from Table 2, calculate the means $\bar{t}_1$ and $\bar{t}_2$ and the standard deviations $\Delta t_1$ and $\Delta t_2$. Include units.

$\bar{t}_1 =$ ____________________

$\bar{t}_2 =$ ____________________

$\Delta t_1 =$ ____________________

$\Delta t_2 =$ ____________________

**Question 4** **1 point**

Using eq. (12) with $\bar{t}_1$ and $\bar{t}_2$, find the average acceleration $\bar{a}$. Show all work and include units.

$\bar{a} =$ ____________________

**Question 5** **2 point**

(a) (1 point) Estimate your uncertainties in $L$ ($\Delta L$) and $D$ ($\Delta D$) based on how accurately you read the ruler. Use the error propagation formula below to estimate $\Delta a$, the uncertainty in the acceleration:

$$\Delta a = \bar{a} \cdot \sqrt{\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{2\Delta D}{D}\right)^2 + \left(\frac{2\Delta t_1}{\bar{t}_1}\right)^2 + \left(\frac{2\Delta t_2}{\bar{t}_2}\right)^2}.$$

Include units.

$\Delta L =$ ________________

$\Delta D =$ ________________

$\Delta a =$ ________________

(b) ($1/2$ point) Which of the four terms in the square root contributed most to $\Delta a$?

(c) ($1/2$ point) What is one possible reason this term dominates the uncertainty?

## Measurement using Manual Timers (8 points)

**Question 6** **1 point**

Calculate the mean and standard deviation of your hand timer data (Table 3). Include units.

$\bar{t} =$ ________________

$\Delta t =$ ________________

**Question 7** **1 point**

Using eq. (15) with $\bar{t}$, find the average acceleration $\bar{a}$. Show all work and include units.

$\bar{a} =$ ________________

**Question 8** **2 point**

(a) (1 point) Estimate your uncertainty in $S$ ($\Delta S$) based on how accurately you read the ruler. Use the error propagation formula below to estimate $\Delta a$:

$$\Delta a = \bar{a} \cdot \sqrt{\left(\frac{\Delta S}{S}\right)^2 + \left(\frac{2\Delta t}{\bar{t}}\right)^2}.$$

Include units.

$\Delta S =$ ______________

$\Delta a =$ ______________

(b) (1 point) Which quantity, $S$ or $t$, contributes more to the uncertainty $\Delta a$?

**Question 9** **1 point**

Use eq. (2) to estimate $g$ from both the photogate and manual timer data. Compute the uncertainty on $g$ using

$$\Delta g = \frac{M_1 + M_2}{M_1 - M_2} \Delta a.$$

Include units.

$g_{\text{photogate}} =$ ______________ $\pm$ ______________

$g_{\text{hand timer}} =$ ______________ $\pm$ ______________

**Question 10** **3 point**

Compare the accuracy and precision of the two methods. The accepted value is $g = 9.8039\,\mathrm{m\,s^{-2}}$.

(a) (1 point) Which method was more precise? Explain based on your estimates for $g$.

(b) (1 point) Which method was more accurate? Explain based on your estimates for $g$.

(c) (1 point) How many standard deviations apart are your two estimates for $g$? Use the formula

$$N_{\mathrm{SD}} = \left| \frac{g_{\text{photogate}} - g_{\text{hand timer}}}{\Delta g_{\text{photogate}} + \Delta g_{\text{hand timer}}} \right| .$$

### Accounting for the Rotational Inertia of the Pulley (8 points)

**Question 11** **1 point**

Using the data in Table 4, calculate $(M_1 - M_2)/(M_1 + M_2)$ for each mass combination. Then compute the acceleration for each trial using eq. (15). Include units.

| $\frac{M_1 - M_2}{M_1 + M_2}$ | Acceleration | | |
|---|---|---|---|
| | 0 rings | 1 ring | 2 rings |
| | | | |
| | | | |
| | | | |

**Question 12** **2 points**

If the pulley is massless, eq. (1) holds and

$$a = \frac{M_1 - M_2}{M_1 + M_2} g.$$

A plot of $a$ versus $(M_1 - M_2)/(M_1 + M_2)$ should then be a straight line with slope $g$. To assess whether the pulley mass affects the results, make this plot and compare the slope to the accepted value $g = 9.8039\,\mathrm{m\,s^{-2}}$.

Plot $a$ versus $(M_1 - M_2)/(M_1 + M_2)$ for the 0-ring, 1-ring, and 2-ring data on the grid below. Draw a best-fit line through each set of points. Include axis labels, units, a title, and a legend.

**Question 13** **1 point**

How does your measured value of $g$ change as the pulley mass increases? *Hint: $g$ is the slope of each best-fit line.*

**Question 14** **1 point**

Given your answer to question 13, can the effect of the pulley's mass explain the discrepancy between your estimates of $g$ in question 9 and the accepted value $g = 9.8039\,\mathrm{m\,s^{-2}}$?

**Question 15** **1 point**

What is another significant source of error in Atwood's machine that is not explored in this lab? How might it affect your results?

**Question 16** **1 point**

(a) ($^1/_2$ point) What is one improvement to Atwood's machine that would increase the accuracy of your results?

(b) ($^1/_2$ point) What is one improvement that would increase the precision?

**Question 17** **1 point**

Atwood's machine has several systematic errors that would not be present in a simpler experiment. What is one reason we might nonetheless expect better results from Atwood's machine than from simply dropping objects off tall buildings, as Galileo did?