University of Rochester

Laboratory III
Conservation of Momentum and Energy

Department of Physics & Astronomy

PHYSICS 113 - 121 - 181

General Physics I and Mechanics

Name: ______________________ **Date:** ______________________

Collaborators: ______________________ **Lab Section:** ______________

PRELAB EXERCISES (2 points)

This prelab must be completed and handed in to the lab TA at the start of the lab.

Question 1 **1 point**

In measurements using the linear track, what is the purpose of measuring the **average fraction lost** for momentum and energy?

Question 2 **1 point**

(a) (1/2 point) Briefly describe both methods used to measure v_b in this lab.

(b) (1/2 point) Why might we want to use two independent methods to estimate v_b?

Objective

Verify the conservation of energy and momentum through two independent experiments: elastic and inelastic collisions on a linear track, and the launch of a projectile captured by a ballistic pendulum. Both methods provide an independent estimate of the projectile velocity, allowing direct comparison of results.

Theory

The laws of conservation of momentum and energy are among the most fundamental and useful in physics. They appear in the solution of many mechanics problems and arise frequently across all fields of science.

Newton's Laws, Conservation Laws, and Symmetries

Conservation of momentum states that if no net force acts on an object of mass m, its momentum $\mathbf{p}$[1] remains constant:

$$\mathbf{p} = m\mathbf{v}, \tag{1}$$

where $\mathbf{v}$ is the object's velocity. This follows directly from Newton's second law:

$$\mathbf{F}_{\text{tot}} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt} = \frac{d(m\mathbf{v})}{dt} = \frac{d\mathbf{p}}{dt}. \tag{2}$$

If there is no net force ($\mathbf{F}_{\text{tot}} = 0$), then

$$\frac{d\mathbf{p}}{dt} = 0 \implies \mathbf{p} = \text{constant}, \tag{3}$$

and momentum is conserved. Conservation of momentum is a restatement of Newton's first law (the **law of inertia**): an object at rest or moving at constant velocity remains in that state unless acted upon by a net external force.

Conservation of energy states that the total energy E of an isolated system is neither created nor destroyed, only transformed between types. In this experiment we are concerned with the conservation (or non-conservation) of kinetic energy,

$$K = \frac{1}{2}mv^2. \tag{4}$$

Conservation laws are intimately related to fundamental **symmetries**[2] in physical systems. Here, *symmetry* refers to a property of a system that can be changed without affecting the underlying laws of physics. Conservation of momentum corresponds to **translational symmetry**: moving the system to a different location in space does not change the outcome of any experiment performed on it. Conservation of energy corresponds to **time-translation symmetry**: performing the experiment at a different time does not change the outcome.

Despite their fundamental nature, the conservation laws are often difficult to observe in everyday life, primarily because of friction. Friction introduces external forces, so the conservation laws no longer strictly apply. To observe them experimentally, friction must be reduced as much as possible.

Conservation of Momentum and Energy in Collisions

When friction is negligible, collisions between objects provide excellent tests of the conservation laws. Collisions are classified as elastic or inelastic. Momentum is conserved in both types, but kinetic energy is conserved only in elastic collisions.

[1] Momentum is a vector quantity with both magnitude and direction.

[2] The relationship between conservation laws and symmetries is formalized in Noether's Theorem, which you may encounter in a more advanced course.

Elastic Collisions

Figure 1 shows a perfectly elastic collision between two masses m_1 and m_2 moving in one dimension. Before the collision, m_1 moves in the $+x$ direction at speed u_1 and m_2 moves in the $-x$ direction at speed u_2. After the collision, m_1 moves in the $-x$ direction at speed v_1 and m_2 moves in the $+x$ direction at speed v_2.

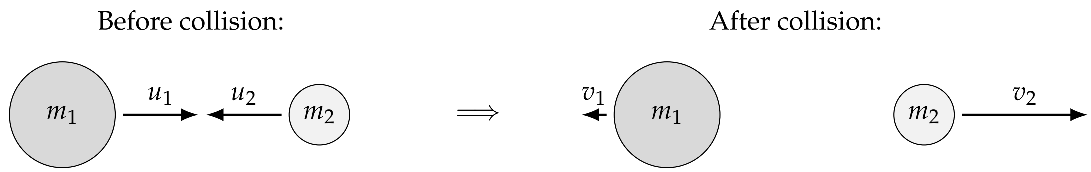

Figure 1: *A perfectly **elastic** 1D collision between masses m_1 and $m_2 = m_1/2$, with initial velocities $u_2 = -u_1$. Both linear momentum and kinetic energy are conserved, giving final velocities $v_1 = -u_1/3$ and $v_2 = -5u_2/3$.*

Since both momentum and kinetic energy are conserved,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2, \tag{5}$$

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2. \tag{6}$$

Combining eqs. (5) and (6), the final velocities are

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2, \tag{7}$$

$$v_2 = \frac{2m_1}{m_1 + m_2} u_1 + \frac{m_2 - m_1}{m_1 + m_2} u_2. \tag{8}$$

Inelastic Collisions

Figure 2 shows a perfectly inelastic collision in 1D. The initial conditions are the same as in Fig. 1, but after the collision the two masses merge into a single object of mass $m_1 + m_2$ moving at velocity v.

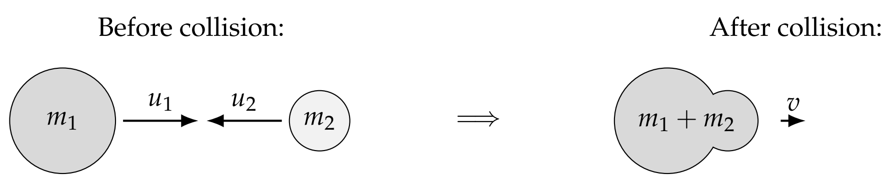

Figure 2: *A perfectly **inelastic** 1D collision between masses m_1 and $m_2 = m_1/2$, with initial velocities $u_2 = -u_1$. The masses stick together and form a single object of mass $m_1 + m_2$. Momentum is conserved but kinetic energy is not. The final velocity of the combined mass is $v = u_1/3$.*

Momentum conservation gives

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v. \tag{9}$$

Kinetic energy is not conserved: some of it is converted to internal heat through the deformation that occurs when the masses merge. Solving eq. (9) for the final velocity,

$$v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}. \tag{10}$$

Measuring Velocity with a Ballistic Pendulum

A ballistic pendulum uses conservation of momentum and energy to estimate the velocity of a small, fast-moving projectile.

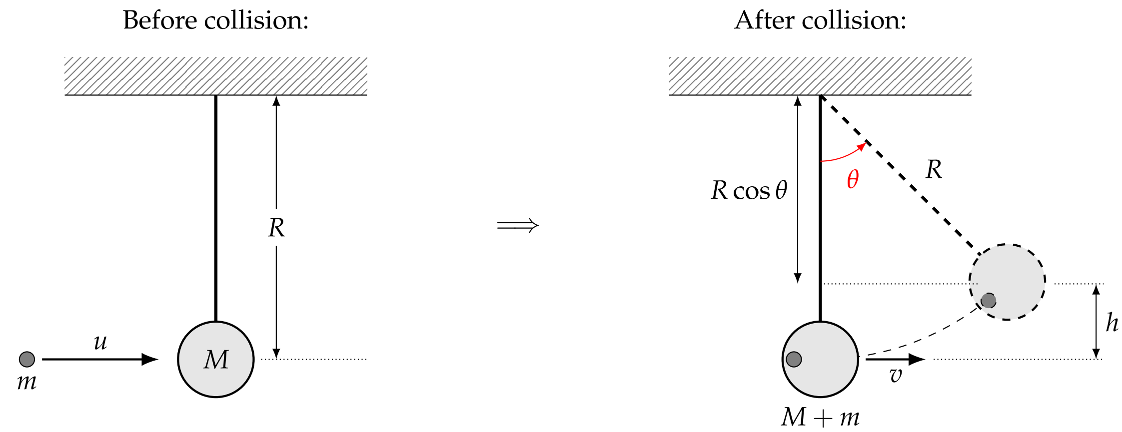

Figure 3: *The ballistic pendulum. Left: a small mass m is fired at velocity u into the pendulum bob of mass M. Right: after the collision, the pendulum swings through an angle θ due to the momentum transferred by m.*

As shown in Fig. 3, a small mass m is fired at velocity u into a stationary pendulum bob of mass M. The collision is inelastic. Using eq. (10), the velocity of the combined mass $M+m$ immediately after impact is

$$v = \frac{m}{M+m}u. \tag{11}$$

The combined mass then swings through an angle θ, and the center of mass rises by a height

$$h = R - R\cos\theta = R(1-\cos\theta). \tag{12}$$

At the top of the swing, all kinetic energy has been converted to gravitational potential energy:

$$K = U$$
$$\frac{1}{2}(m+M)v^2 = (m+M)gh. \tag{13}$$

Combining eqs. (11), (12), and (13), the initial velocity u of the projectile is

$$u = \left(1+\frac{m}{M}\right)\sqrt{2gR(1-\cos\theta)}. \tag{14}$$

Measuring Velocity using Projectile Motion

A second method for measuring the projectile's velocity is to fire it horizontally and observe its trajectory. Suppose the ball travels a horizontal distance L, as shown in Fig. 4. Since there is no horizontal force, the flight time t satisfies

$$L = v_b t \implies t = \frac{L}{v_b}. \tag{15}$$

In the vertical direction, gravity acts on the ball, so

$$H = \frac{1}{2}gt^2 = \frac{1}{2}g\left(\frac{L}{v_b}\right)^2. \tag{16}$$

Solving for v_b,

$$v_b = L\sqrt{\frac{g}{2H}}. \tag{17}$$

Conversely, if v_b and H are known, the expected horizontal distance is

$$L = v_b\sqrt{\frac{2H}{g}}. \tag{18}$$

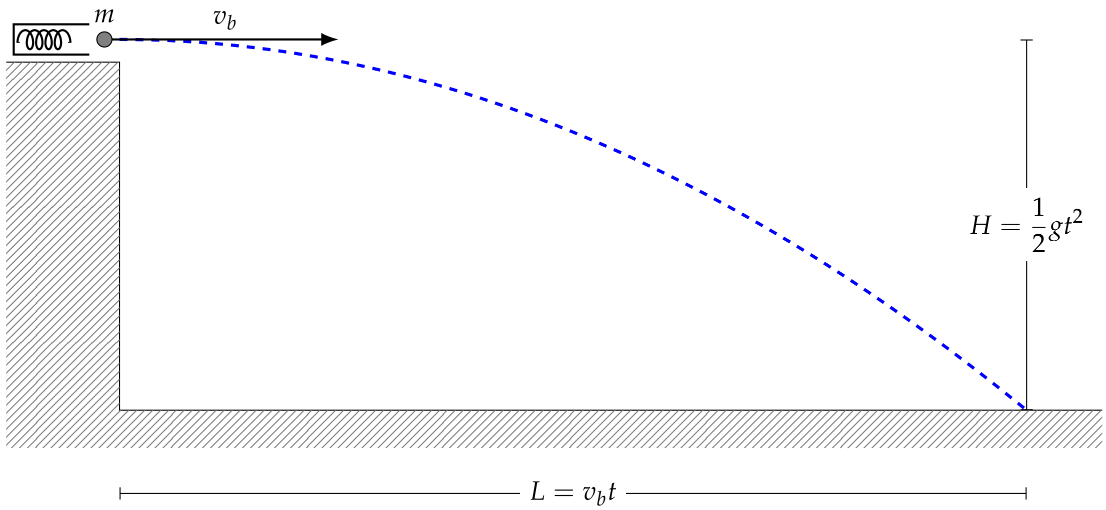

Figure 4: *A ball fired horizontally from a spring-loaded muzzle at velocity v_b travels a horizontal distance L and falls a vertical distance H.*

Experiment

The Low-Friction Track

For these experiments you will use a track with very low-friction cart wheels. The setup is shown in Fig. 5.

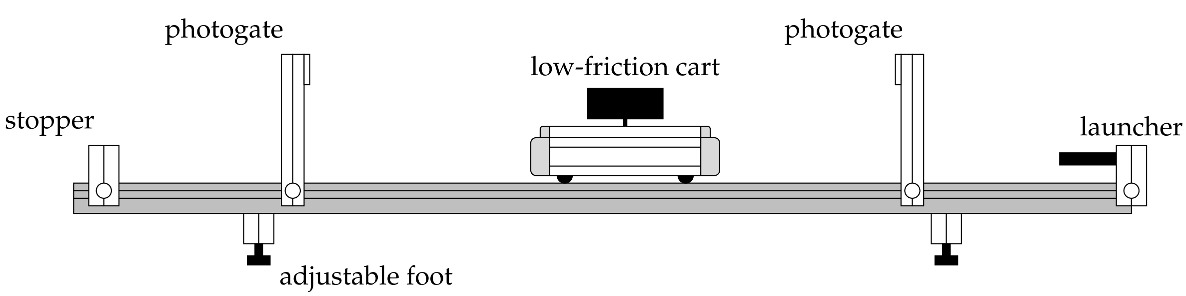

Figure 5: *The linear track used in this experiment.*

Cart velocity is measured using a photogate: an electronic timer triggered by the interruption of an infrared light beam. If an object of length ℓ blocks the beam for a time Δt, its average speed is

$$v = \ell / \Delta t. \tag{19}$$

By measuring velocities before and after various collisions, you will calculate momenta and kinetic energies to test the conservation laws.

Notes on the Use and Setup of the Linear Track

Photogate Timers

The two timers operate independently. To find the speed at a given point on the track, use only the measurement from the timer at that position.

Track

Before starting, check that the track is level. Place one of the bubble levels lengthwise along the track above a supporting foot and observe the bubble. Adjust the foot by turning the screw at its base (see Fig. 5) until the bubble is centered. Repeat at the opposite end of the track, then repeat both checks with the level oriented across the width of the track.

Next, make sure there is an end stop at one end of the track and a launcher at the other. To reposition the end stop or launcher, loosen the screw on the side and slide it into place (see Fig. 6). Finally, position the two photogates about 2.5 cart lengths apart.

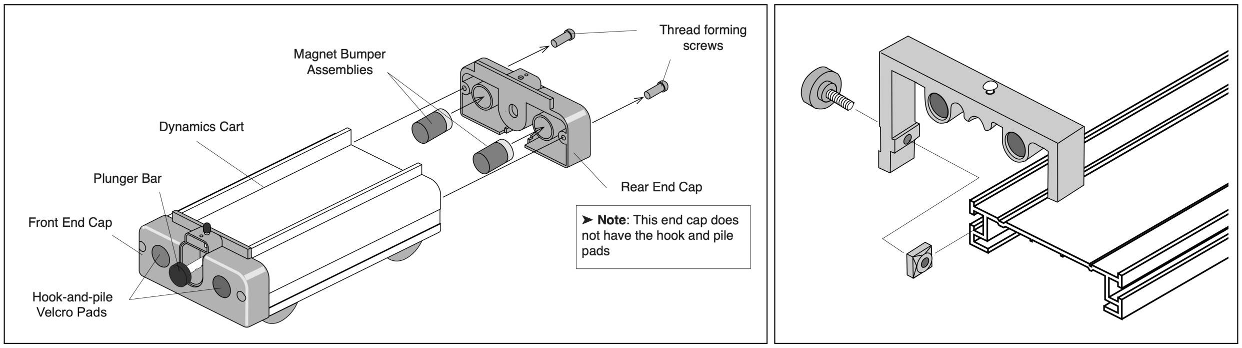

Figure 6: *Left: magnetic and Velcro bumpers on the low-friction carts for elastic and inelastic collisions. Right: the end stop screwed into the track. Image credits: PASCO Scientific.*

Cart

The cart is described in Fig. 6. One end contains a magnet; the other has a plunger and Velcro pads. Use the magnetic bumpers for elastic collisions and the Velcro for inelastic collisions. If the Velcro does not stick well, roughen it by repeatedly pressing the carts together and pulling them apart, or use the tape provided.

Before starting, test each cart on the track. Place it on the track with the wheels in the grooves and give it a small push. There should be minimal speed loss as it rolls. If you notice a significant loss, recheck the level; if the level is fine, inspect the wheels and ask your TA for help.

The track will be used to create collisions between the carts. A moving cart passes through a photogate before the collision, recording its initial velocity. After the collision, one or both carts pass through the photogates, recording the final velocities. With these velocities and the measured masses, you can calculate and compare momenta and kinetic energies before and after each collision.

To reduce systematic uncertainties, the effects of friction will be estimated first, and a launcher will be used to ensure reproducible initial conditions.

Newton's First Law

In this part, one cart at a time is launched down the track past both timers.

In the absence of friction, the cart's velocity should remain constant since no net external force acts on it. In practice, a small amount of rolling friction reduces the velocity. This section does not test the conservation laws; its purpose is to quantify the systematic loss due to friction, which you will use to interpret the collision data in the following sections.

With friction present, the momentum and energy ratios satisfy

$$\frac{p_f}{p_i} < 1, \qquad \frac{K_f}{K_i} < 1.$$

You will measure the average final momentum $\overline{p}_f$ and average final kinetic energy $\overline{K}_f$, and use them to estimate the average fractional losses due to friction:

$$\frac{\overline{\Delta p}}{p} = 1 - \left(\frac{\overline{p}_f}{p_i}\right), \tag{20}$$

$$\frac{\overline{\Delta K}}{K} = 1 - \left(\frac{\overline{K}_f}{K_i}\right). \tag{21}$$

Key Idea: The average fractional losses give rough estimates of the momentum and energy lost to friction. If the fractional losses measured during a collision differ from the friction-only losses by **less than a factor of 3**, we will conclude that no additional momentum or energy was lost in the collision, and that the conservation laws hold.

Procedure

1. Answer question 3 in the Postlab exercises.
2. Check that the track is level along both its length and width. Test at several locations along the track.
3. Place a wooden block on top of one of the carts and insert a plastic flag firmly into the block. This flag is what the photogates will detect.
4. Adjust the photogate heights so that the flag cuts the beam; a red LED on the side of the photogate lights up when the beam is interrupted. Loosen the screw on the post to move the bracket up or down. Separate the two photogates by about 2.5 cart lengths.
5. Adjust the cart launcher so that the scale reads about 1 cm to 2 cm when cocked. Adjust the compression by loosening the screw on the latching clamp and sliding it into position.
6. Place the cart in front of the launcher with the wheels in the grooves and the launcher aimed at the center of the cart.
7. Launch the cart by pulling the string. Once it has passed through both photogates, stop it and record the two times in Data Table 1 in the Postlab exercises.
8. Repeat steps 6 and 7 for a total of two trials.

Elastic Collisions

Place two carts on the track with their magnetic ends facing each other. Before starting, note the following:

- The launcher setting and photogate positions are recommendations. You may adjust them, but keep them consistent across all trials.
- Position the timers so that you can measure the speed of each cart both before and after the collision.
- Since friction continuously removes energy and momentum, place the timers as close to the collision point as practical.

You will perform the following two experiments:

Carts of equal mass : Place equal wooden masses on each cart. Set the launcher scale to about 1 cm and separate the photogates by about 2.5 cart lengths.

Carts of unequal mass : Place a wooden mass on the launched cart and heavier metal masses on the stationary cart. Set the launcher scale to about 1 cm.

Procedure

1. Answer question 5 in the Postlab exercises.
2. Set up the launcher and photogates for the **Carts of equal mass** experiment.
3. Launch one cart into the stationary one, arranged so the collision occurs between the two photogates. The launched cart passes through the first photogate before the collision; record this time as $t_{1,i}$ in Data Table 2. After the collision, read the final times of both carts from the photogates and record them in Data Table 2.

4. Both timers should be set to gate mode. If a timer displays two times, the first displayed value is t_1. To find t_2, flip the memory switch to read the total time $t_1 + t_2$, then subtract t_1.

5. Replace the wooden mass on the stationary cart with a metal mass so that $m_1 < m_2$. If the lighter cart derails after the collision, reduce the launcher power. You do not need to record times for this configuration; instead, observe the collision qualitatively and answer question 10 in the Postlab exercises.

Inelastic Collisions

In an inelastic collision the two carts stick together, so orient their Velcro ends toward each other. Use the same setup as before, with the launched cart (m_1) in front of the launcher and the stationary cart (m_2) on the track. The goal is to determine which mass combination results in the smallest kinetic energy loss.

Procedure

1. Set $m_1 < m_2$ by placing the wooden mass on the launched cart and the metal mass on the stationary cart. Set the launcher scale to 2 cm.

2. Place the carts on the track with their Velcro ends facing each other. If they do not stick after the collision, add tape to the end of the stationary cart.

3. Launch the cart, record t_1 for the first cart and t_2 for the combined system, and enter the results in Data Table 3 in the Postlab exercises.

4. Repeat for a second trial and record the results in Data Table 3.

5. Set up the carts with equal mass ($m_1 = m_2$) and repeat steps 3 and 4.

6. Set up the carts so the stationary cart is lighter ($m_1 > m_2$) and repeat steps 3 and 4.

Velocity of a Projectile using a Ballistic Pendulum

In this part you will measure the velocity of a metal ball fired from a spring gun using a ballistic pendulum (see Fig. 3).

> **CAUTION**: Never look into the barrel of the launcher or put your fingers into the barrel.

Procedure

1. Level the base of the ballistic pendulum apparatus and adjust if necessary.

2. Measure the mass m of the metal ball and record it in Data Table 4.

3. Mount the ball on the spring gun's push rod via the hole through its diameter. Confirm that the ball loads smoothly and that the ball catcher is aligned with the barrel.

4. Measure the pendulum mass M by unscrewing the pivot pin (the small knob at the top). Slide out the pin, remove the pendulum, and weigh it with the ball catcher attached. Keep the pin in a safe place while weighing. Record the pendulum + ball catcher mass in Data Table 4.

5. Find the distance R from the pivot pin to the center of mass of the ball-pendulum system. Place the metal ball in the catcher and balance the entire assembly on the table edge, as shown in Fig. 7.

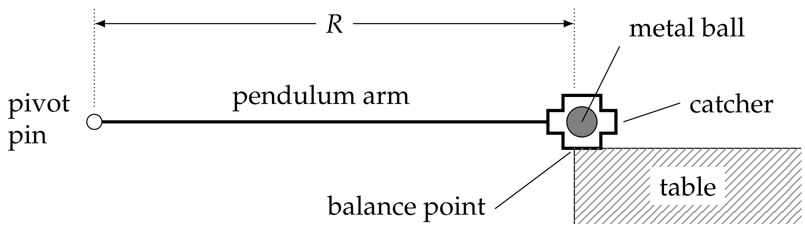

Figure 7: *Finding the center of mass of the pendulum and ball by balancing on the table edge.*

Slide the assembly outward until it is just barely balanced; any further and it would fall. The balance point lies directly below the center of mass. Measure R as the distance from the pivot pin to this point and record it in Data Table 4.

6. Reassemble the pendulum by reversing the steps in step 4.
7. Cock the gun by placing the ball in the barrel and pushing it down with the ramrod until the trigger catches at the desired setting.
8. Three range settings are available: short, medium, and long. We recommend the long-range setting, but any setting is acceptable provided you use the same one for every trial.
9. With the pendulum at rest and the angle indicator at zero, fire the ball into the catcher five times. Record the maximum swing angle for each trial in Data Table 4. **Keep your hand clear of the spring gun mechanism.**

Velocity of a Projectile using Projectile Motion

In this final part, you will measure the ball's velocity by firing it horizontally from the same spring gun and analyzing its trajectory, as shown in Fig. 4.

Procedure

1. Secure the apparatus on a block on the table and level it. Latch the pendulum bob out of the way. Practice firing the ball a few times to check for wild shots.
2. Measure the height H from the floor to the bottom of the ball when it is seated in the launcher. The easiest approach is to measure from the floor to the tabletop, then from the tabletop to the bottom of the ball, and add the two. Record H in Data Table 5.
3. With the ball in the gun and the spring uncompressed (ball at the front of the barrel), mark the position on the floor directly below the center of the ball with a piece of tape.
4. Fire one shot at the chosen range setting and note where the ball lands. Tape a sheet of white paper over that area and cover it with carbon paper, carbon side down. Do not tape the carbon paper.
5. Reload and fire five more shots at the same range setting. The ball will leave carbon marks on the paper. Measure the horizontal distance L from the tape mark to each impact point and record each value in Data Table 5.

Name: ______________________ **Date:** ______________________

Collaborators: ______________________ **Lab Section:** __________

Postlab Exercises (20 points)

Submit the postlab to the TA at the end of the lab.

Newton's First Law (2 points)

Question 3 **1 points**

(a) (1/2 point) What values do you expect for the fractions of momentum and kinetic energy lost,

$$\frac{\Delta p}{p} = ? \qquad \frac{\Delta K}{K} = ?$$

when friction is infinitely large? Explain.

(b) (1/2 point) What values do you expect when there is no friction? Explain.

Question 4 **1 point**

Fill in Data Table 1, where t_i is the transit time through the first photogate and t_f is the transit time through the second.

Data table 1: *Measurements of fractional momentum and energy loss due to friction.*

Trial	t_i	t_f	p_f/p_i	K_f/K_i
1				
2				

For a cart of length ℓ, the velocity at each photogate is $v_i = \ell/t_i$ and $v_f = \ell/t_f$, so the momentum and energy ratios simplify to

$$\frac{p_f}{p_i} = \frac{mv_f}{mv_i} = \frac{t_i}{t_f}, \qquad \frac{K_f}{K_i} = \frac{1/2\, mv_f^2}{1/2\, mv_i^2} = \frac{t_i^2}{t_f^2}.$$

Compute the average ratios and calculate the fractional losses using eqs. (20) and (21). Show your work.

$$\overline{\frac{\Delta p}{p}} = \qquad \overline{\frac{\Delta K}{K}} =$$

Elastic Collisions of Equal and Unequal Masses (6 points)

Question 5 **1 point**

(a) ($^1\!/_2$ point) For a perfectly elastic collision, what fractional losses do you expect, given the friction you measured?

$$\frac{\overline{\Delta p}}{p} = \qquad\qquad \frac{\overline{\Delta K}}{K} =$$

(b) ($^1\!/_2$ point) Do you expect these values to be the same as or different from those in question 4? Explain.

Question 6 **1 point**

(a) ($^1\!/_2$ point) If the two carts have equal length and mass, do the simplified ratio equations from question 4 still apply? Explain.

(b) ($^1\!/_2$ point) Based on your answer, which equation will you use to compute $\overline{\Delta p}/p$ and $\overline{\Delta K}/K$ for the elastic collision?

Question 7 **1 point**

Fill in Data Table 2. The launched cart has mass m_1 and the stationary cart has mass m_2. $t_{1,i}$ is the time m_1 takes to pass the first photogate before the collision; $t_{2,f}$ is the time m_2 takes to pass the second photogate after the collision.

Data table 2: *Measurements of elastic collisions between equal masses.*

Trial	m_1	m_2	$t_{1,i}$	$t_{1,f}$	$t_{2,i}$	$t_{2,f}$
1				N/A	N/A	

Question 8 **1 point**

Using your answer to question 6 and the data in Data Table 2, find the fractions of momentum and kinetic energy lost.

$\frac{\overline{\Delta p}}{p} =$ $\frac{\overline{\Delta K}}{K} =$

Question 9 **1 point**

Using the key idea from the Newton's First Law section, determine whether the following inequalities are satisfied:

$$\left| \left(\frac{\overline{\Delta p}}{p} \right)_{\text{Question 8}} - \left(\frac{\overline{\Delta p}}{p} \right)_{\text{Question 4}} \right| > 3 \left(\frac{\overline{\Delta p}}{p} \right)_{\text{Question 4}}, \tag{22}$$

and

$$\left| \left(\frac{\overline{\Delta K}}{K} \right)_{\text{Question 8}} - \left(\frac{\overline{\Delta K}}{K} \right)_{\text{Question 4}} \right| > 3 \left(\frac{\overline{\Delta K}}{K} \right)_{\text{Question 4}}. \tag{23}$$

In other words, is the fractional loss during the collision more than 3 times larger than the loss due to friction alone?

Question 10 **1 point**

With the stationary cart replaced by a heavier metal mass ($m_1 < m_2$), describe what you observe. How does this collision differ from the equal-mass case? Do the simplified ratio equations still apply? Explain.

Inelastic Collisions (3 points)

Question 11 **1 point**

Record your data for three mass combinations in Data Table 3.

Data table 3: *Measurements of inelastic collisions.*

Trial	t_1	t_2
$m_1 < m_2$		
$m_1 = m_2$		
$m_1 > m_2$		

Question 12 **1 point**

(a) (½ point) Which mass combination ($m_1 < m_2$; $m_1 = m_2$; $m_1 > m_2$) produced the smallest change in speed after the collision?

(b) (½ point) Why does this combination perform best? Recall from eq. (10) that momentum conservation gives

$$m_1 u_1 = (m_1 + m_2)v.$$

Question 13 **1 point**

What is one factor that limits the final velocity v of the combined carts?

Velocity Measurements with a Ballistic Pendulum (4 points)

Question 14 **1 point**

Record your ballistic pendulum measurements in Data Table 4.

Data table 4: *Measurements of the ballistic pendulum.*

$R =$ ____________________

Ball mass $m =$ ____________________

Pendulum mass $M =$ ____________________

Trial	1	2	3	4	5	$\overline{h}$
Angle θ						—
$h = R(1 - \cos\theta)$						

Question 15 **1 point**

Calculate the mean height $\overline{h}$ and its standard deviation Δh. Show your work and include units.

Question 16 **1 point**

Using eq. (14) with $v_b = u$, find the initial velocity v_b of the ball. Show your work and include units.

Question 17 **1 point**

Find the uncertainty Δv_b using propagation of uncertainties:

$$\frac{\Delta v_b}{v_b} = \frac{\Delta h}{2\overline{h}}.$$

Assume the mass measurements have no uncertainty. Show your work.

Velocity Measurements from Projectile Motion (5 points)

Question 18 **1 point**

Record your measurement of H, its estimated uncertainty ΔH, and all range measurements L in Data Table 5.

Data table 5: *Measurements of projectile motion.*

$H =$ ______________________

$\Delta H =$ ______________________

Trial	1	2	3	4	5	Average $\overline{L}$
Distance L						

Question 19 **1 point**

Calculate the mean range $\overline{L}$ and its standard deviation ΔL. Show your work.

Question 20 **1 point**

Using eq. (17), calculate v_b from H and $\overline{L}$. Show your work.

Question 21 **1 point**

Calculate the uncertainty Δv_b using propagation of uncertainties:

$$\frac{\Delta v_b}{v_b} = \sqrt{\left(\frac{\Delta L}{\overline{L}}\right)^2 + \left(\frac{\Delta H}{2H}\right)^2}.$$

Show your work and include units.

Question 22 **1 point**

(a) (1/2 point) Are the values of v_b and Δv_b from projectile motion consistent with those from the ballistic pendulum (questions 16 and 17)? That is, do the two results agree within their uncertainties?

(b) (1/2 point) If the results do not agree, what is one possible explanation?