University of Rochester

Laboratory IV
Moment of Inertia and Oscillations

DEPARTMENT OF PHYSICS & ASTRONOMY

PHYSICS 113 - 121 - 181

GENERAL PHYSICS I AND MECHANICS

Name: ______________________ **Date:** ______________________

Collaborators: ______________________ **Lab Section:** ______________________

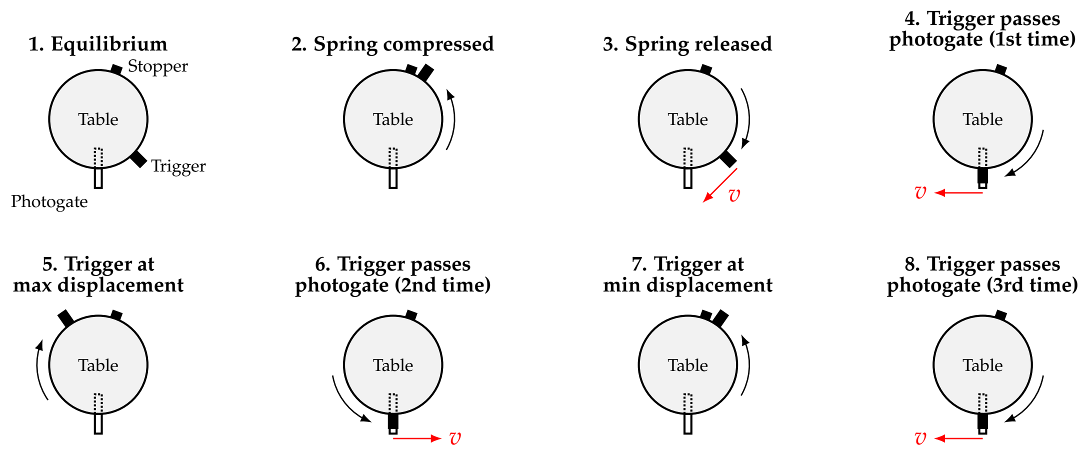
PRELAB EXERCISES (2 points)

This prelab must be completed and handed in to the lab TA at the start of the lab.

Question 1 **1 points**

(a) (1/2 point) In your study of the moment of inertia and the parallel axis theorem, briefly describe the steps you must take to ensure your rotary table is set up correctly, and to ensure the timer is working properly.

(b) (1/2 point) The diagram below shows how the rotary table oscillates.



Which number in the picture corresponds to when the photogate timer measures one period of oscillation, and why?

Question 2 **1 points**

In the second part of the lab, you will observe the oscillations of a spring loaded with a specific mass. However, you may notice that the spring will oscillate even when no mass is attached ($m = 0$).

(a) (1/2 point) What is one reason why this can happen?

(b) (1/2 point) What is one effect this could have on the experiment?

Objective

Measure the moment of inertia of three objects about a specified rotational axis using a photogate timer, and verify the parallel axis theorem. In the second part, investigate Hooke's Law by measuring the oscillation period of a spring-mass system as a function of the attached mass.

Theory

Moment of Inertia and the Parallel Axis Theorem

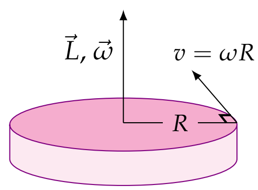


Figure 1: *A rotating solid disk.*

Consider a rigid body rotating about an axis, as in Fig. 1. If the angular velocity is ω, a point in the body at a perpendicular distance r from the axis of rotation will move with linear speed $v = \omega r$. The total angular momentum L of the rotating body points along the axis of rotation and is equal to

$$L = \int_{\text{body}} rv\, dm = \int_{\text{body}} r^2 \omega\, dm = \omega \int_{\text{body}} r^2\, dm = I\omega, \quad (1)$$

where

$$I = \int_{\text{body}} r^2\, dm \quad (2)$$

is called the **moment of inertia** of the body about the axis of rotation. In the SI system of units, I is expressed in units of kg m^2.

If the axis of rotation passes through the center of mass of the object, then I is called I_{CM}. For example, for the solid disk of mass M and radius R shown in Fig. 1,

$$I_{\text{CM}} = \frac{1}{2} M R^2.$$

Table 1 shows I_{CM} for several common mass distributions.

Table 1: *Moments of inertia of several mass distributions.*

Object	**Rotational Axis**	$\mathbf{I_{CM}}$
Thin Ring	Symmetry Axis	MR^2
Thick Ring	Symmetry Axis	$\frac{1}{2}M\left(R_1^2 + R_2^2\right)$
Solid Disk	Symmetry Axis	$\frac{1}{2}MR^2$
Thin Spherical Shell	About a Diameter	$\frac{2}{3}MR^2$
Solid Sphere	About a Diameter	$\frac{2}{5}MR^2$

The **parallel axis theorem**[1] relates I_{CM} to the moment of inertia I about any parallel axis displaced from the center of mass. The theorem states

$$I = I_{\text{CM}} + Md^2, \quad (3)$$

where M is the body's mass and d is the perpendicular distance between the two axes. This implies that I_{CM} is the minimum moment of inertia among all parallel axes.

[1] Also called the Huygens–Steiner theorem, after Christiaan Huygens (1629–1695) and Jakob Steiner (1796–1863).

Analogies between Rotational and Linear Motion

When working with rotational motion for rigid bodies, many of the equations are analogous to those of linear kinematics. For example, angular velocity plays the role of linear velocity, and moment of inertia plays the role of mass. Table 2 summarizes the correspondence.

Table 2: *Analogies between linear and rotational motion.*

Linear Kinematics	**Rotational Kinematics**
Linear Velocity $= v$	Angular Velocity $= \omega$
Mass $= M$	Moment of Inertia $= I$
Linear Momentum $= p = Mv$	Angular Momentum $= L = I\omega$
Kinetic Energy $= K = \frac{1}{2}Mv^2$	Kinetic Energy $= K = \frac{1}{2}I\omega^2$

Determining the Moment of Inertia of an Object

The apparatus in Fig. 2 consists of a rotary table on which you can mount an object in order to measure its moment of inertia. A torsion spring restricts the motion of the table and provides a restoring torque τ. If the table is rotated by an angle θ, then the torque acting on it is

$$\tau = -\kappa\theta, \tag{4}$$

where κ is the torsion spring constant, analogous to the spring constant k for compressed and stretched springs. The value of κ must be measured.

If the table has moment of inertia I_{tab} and an object with moment of inertia I_{obj} is mounted on it, the combined system performs rotary oscillations with angular frequency

$$\omega = \sqrt{\frac{\kappa}{I_{\text{obj}} + I_{\text{tab}}}}, \tag{5}$$

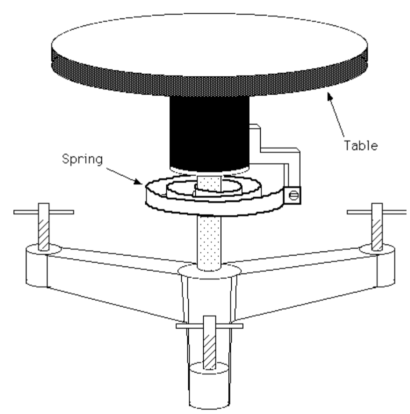


Figure 2: *Rotary table with torsion spring.*

which corresponds to a period of oscillation

$$T = 2\pi\sqrt{\frac{I_{\text{obj}} + I_{\text{tab}}}{\kappa}}. \tag{6}$$

The two unknown parameters are I_{tab} and κ. To determine their values, you will measure the oscillation period of the table alone (T_{tab}) and the period of the table together with an object of known moment of inertia I_{obj} ($T_{\text{tab+obj}}$). From eq. (6), one finds that

$$T_{\text{tab}} = 2\pi\sqrt{\frac{I_{\text{tab}}}{\kappa}}, \tag{7}$$

$$T_{\text{tab+obj}} = 2\pi\sqrt{\frac{I_{\text{obj}} + I_{\text{tab}}}{\kappa}}. \tag{8}$$

To solve for κ, we square and subtract eqs. (7) and (8):

$$(T_{\text{tab+obj}})^2 + (T_{\text{tab}})^2 = 4\pi^2\frac{2I_{\text{tab}} + I_{\text{obj}}}{\kappa}, \tag{9}$$

$$(T_{\text{tab+obj}})^2 - (T_{\text{tab}})^2 = 4\pi^2\frac{I_{\text{obj}}}{\kappa}. \tag{10}$$

Eq. (10) lets us isolate the torsion spring constant κ in terms of known and measured quantities:

$$\boxed{\kappa = 4\pi^2 \frac{I_{\text{obj}}}{(T_{\text{tab+obj}})^2 - (T_{\text{tab}})^2}} \tag{11}$$

To find the unknown I_{tab}, divide eq. (9) by eq. (10):

$$\frac{(T_{\text{tab}})^2 + (T_{\text{tab+obj}})^2}{(T_{\text{tab+obj}})^2 - (T_{\text{tab}})^2} = \frac{2I_{\text{tab}}}{I_{\text{obj}}} + 1, \tag{12}$$

which simplifies to

$$\boxed{I_{\text{tab}} = I_{\text{obj}} \frac{(T_{\text{tab}})^2}{(T_{\text{tab+obj}})^2 - (T_{\text{tab}})^2}} \tag{13}$$

Once κ and I_{tab} are known, the moment of inertia I_x of any other object x can be found from

$$\boxed{I_x = I_{\text{tab}} \left[\left(\frac{T_{\text{tab}+x}}{T_{\text{tab}}} \right)^2 - 1 \right]}, \tag{14}$$

provided we also measure the oscillation period $T_{\text{tab}+x}$ of the table with the new object mounted.

Hooke's Law and Spiral Spring Oscillations

It is often assumed that a long spiral spring obeys Hooke's law when it is not stretched too far. If the spring is hung vertically from a fixed support and a mass is attached to its free end, the mass can be set into simple harmonic motion by stretching and releasing it. The period of oscillation is the time required for the attached mass to return to its initial position. The period T depends on the attached mass M, the spring force constant k, and the spring mass m, and is given by

$$T = 2\pi \sqrt{\frac{M + bm}{k}}. \tag{15}$$

In eq. (15), b is a dimensionless constant called the spring mass coefficient. You will calculate b in this lab and compare it to the theoretical value

$$b_{\text{theory}} = \frac{1}{3}. \tag{16}$$

During the lab, you will measure T for a variety of values of M. To make it easy to estimate k from T and M, square eq. (15):

$$T^2 = \frac{4\pi^2}{k} M + \frac{4\pi^2}{k} bm. \tag{17}$$

A plot of T^2 versus M should therefore yield a straight line of the form

$$y = (\text{slope})x + (y\text{-intercept}). \tag{18}$$

You will plot the data in this manner and determine whether the expected linear relationship holds.

Experiment

Moment of Inertia and Parallel Axis Theorem

Procedure

1. Set up the rotary table and allow it to come to its equilibrium position. Make sure the tabletop is level by placing a level onto it and adjusting the apparatus' feet as needed.
2. Set the photogate timer to pendulum mode and arrange the photogate so the table's trigger (see Fig. 3) passes through the photogate when the table rotates.

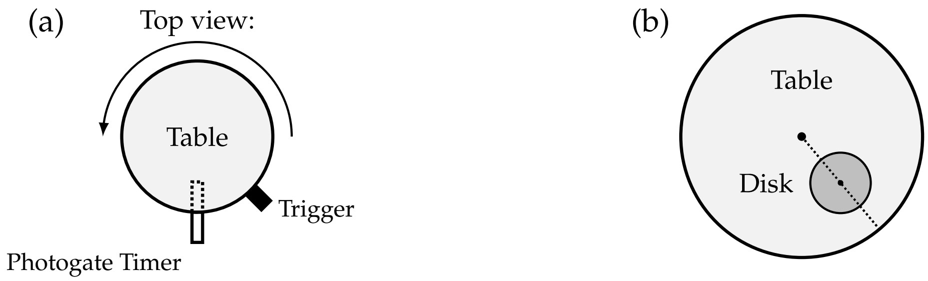


Figure 3: *(a) view of rotary table, photogate timer, and trigger; (b) the table with the disk at an offset from the axis of rotation.*

3. Wind the table either clockwise or counter-clockwise such that *the trigger does not pass the timer while winding*. Stop when the table hits the block and cannot be turned any further.
4. Release the table. Count the number of times the trigger passes through the photogate before the timer stops counting. It should be three passes before the timer stops. The displayed time will be the period of one oscillation.
5. Measure the mass and inner and outer radii of the *brass ring*. Record your values in Data Table 1 in the Postlab Exercises.
6. Measure the period of one oscillation *of the table alone* (T_{tab}) five (5) times. Record your data in Data Table 1 in the Postlab Exercises.
7. After ensuring the brass ring is mounted securely with screws to the tabletop and is centered on the table's axis of rotation, measure the period of oscillation *of the table/ring combination* ($T_{\text{tab+ring}}$) five (5) times. Record the data in Data Table 1.
8. Measure the mass and radius of the solid disk, and record your measurements in Data Table 1.
9. To measure the period of oscillation for the table/disk combination, secure the disk at one of five equally spaced positions along the table radius, a distance d from the table's axis of rotation (see Fig. 3). Start at the center ($d = 0.000\,\text{m}$). The recommended positions are 0.000 m, 0.015 m, 0.030 m, 0.045 m, and 0.060 m, using the spacing holes on the table.
10. For each of the five positions d, measure the period of oscillation five (5) times. Record your data in Data Table 2.

Hooke's Law and Spiral Spring Oscillations

Introduction

According to Hooke's law, the extension x of a stretched spring from its equilibrium position should be proportional to the applied force F,

$$F = -kx, \tag{19}$$

where k is the *force constant* of the spring (also called the *spring constant*). You will test the validity of Hooke's law by measuring x while incrementally increasing the attached mass M. Then, you will measure the period of oscillation for several hanging masses to estimate the spring mass coefficient b needed to account for the spring's own mass m, as described in eqs. (15) and (16).

Procedure

1. Weigh the Slinky Jr.™ and mass holder to determine the mass of the spring/slinky system m. Record it in Data Table 4 in the Postlab Exercises.

2. Mount the spring by clipping one end of it to the flat metal piece mounted to the stand.

3. For each of the attached masses M listed in Data Table 4, calculate the force of gravity $F = Mg$, where $g = 9.8\,\mathrm{m\,s^{-2}}$. Record your calculations in Data Table 4.

4. Let the spring come to equilibrium. If possible, move the ruler/spring vertically so that the ruler's zero is at the bottom of the spring (or the bottom of the mass holder, if you prefer). Record the equilibrium position of the spring's bottom end on the ruler.

5. Measure the change in extension of the spring, x, as you attach different amounts of mass to the end of the spring. The extension is measured relative to the equilibrium length of the unloaded spring. Use masses of 5 g, 10 g, 15 g, 20 g, and 25 g, and attach each mass gently. Record all results in Data Table 4.

6. For each of the five masses M, measure the period of ten (10) oscillations. Set the spring and mass into motion by **gently** stretching the mass 15 cm to 20 cm and then releasing it. The mass should not touch the floor or any other surface as it moves. If you pulled the mass down, then one oscillation is completed each time the mass returns to its lowest point. **Divide the total time by 10 to get the average period of one oscillation.** Record the average period for each value of M in Data Table 4.

Name: ____________________ **Date:** ____________________

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Postlab Exercises (20 points)

Submit the postlab to the TA at the end of the lab.

Moment of Inertia and the Parallel Axis Theorem (10 points)

Question 3 **3 points**

Fill in data from your measurements of the moment of inertia in Data Table 1 and test of the parallel axis theorem in Data Table 2.

Data table 1: *Data for the moments of inertia.*

Ring Measurements

Ring mass [kg] = ____________

Inner radius R_1 [m] = ____________

Outer radius R_2 [m] = ____________

Disk Measurements

Disk mass [kg] = ____________

Disk radius R [m] = ____________

Trial	T_{tab} [s]	$T_{\text{tab+ring}}$ [s]
1		
2		
3		
4		
5		
Average [s]		

Data table 2: *Data for parallel axis theorem.*

Distance [m]	$d_{x_1} = 0.000$	$d_{x_2} =$	$d_{x_3} =$	$d_{x_4} =$	$d_{x_5} =$
Trial	T_{x_1} [s]	T_{x_2} [s]	T_{x_3} [s]	T_{x_4} [s]	T_{x_5} [s]
1					
2					
3					
4					
5					
Average [s]					

Next, calculate all values in Data Table 3. For the measured periods, use the average values from Data Tables 1 and 2. Write the formula for each quantity (except for I_{x_2} to I_{x_5}) and the uncertainty. Remember to include units. You do not have to show your calculations.

Data table 3: *Calculation of moments of inertia.*

Formula:	Theoretical value of the moment of inertia of the brass ring (Table 1). Note: we consider it a thick (and heavy) ring.
$I_{\text{ring}} =$	
Formula:	Spring constant of the table, eq. (11), where $I_{\text{obj}} = I_{\text{ring}}$ and $T_{\text{tab+obj}} = T_{\text{tab+ring}}$.
$\kappa =$	
Formula:	Calculated moment of inertia of the table, eq. (13), where $I_{\text{obj}} = I_{\text{ring}}$.
$I_{\text{table}} =$	
Formula:	Theoretical value of the moment of inertia of the brass disk (Table 1).
$I_{\text{disk}} =$	
Formula:	Calculated moment of inertia when the brass disk is placed at d_1. Use eq. (14), where $T_{\text{tab}+x} = T_{x_1}$.
$I_{x_1} =$	
$I_{x_2} =$	Calculated moment of inertia when the brass disk is at d_2.
$I_{x_3} =$	Calculated moment of inertia when the brass disk is at d_3.
$I_{x_4} =$	Calculated moment of inertia when the brass disk is at d_4.
$I_{x_5} =$	Calculated moment of inertia when the brass disk is at d_5.
Uncertainty $= \lvert I_{\text{disk}} - I_{x_1} \rvert =$	Calculated uncertainty in the moment of inertia of the solid disk.

Question 4 **2 points**

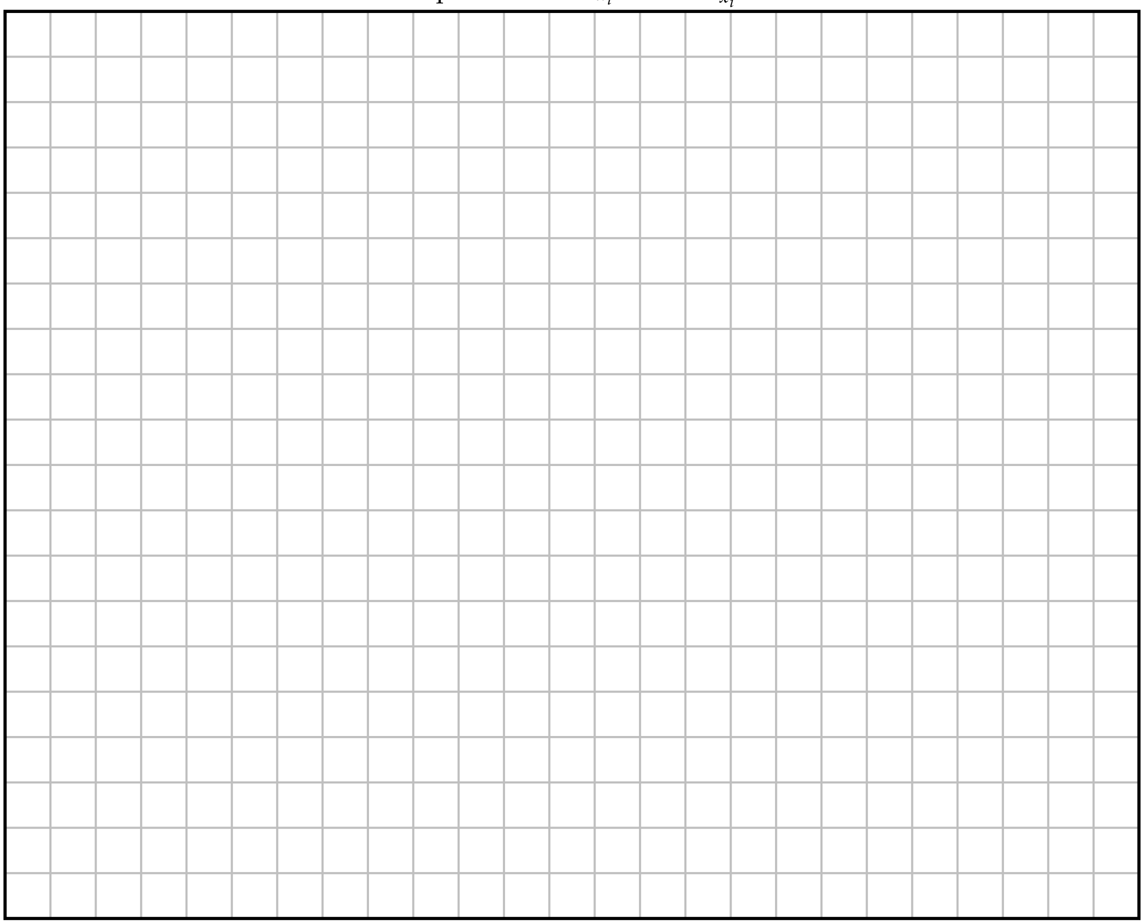
In Graph 1:

(a) (1 point) Plot I_{x_i}, the moment of inertia of the solid disk on the y-axis versus $d_{x_i}^2$, the *square* of the distance between the center of the table and the center of the solid disk, on the x-axis.

(b) ($^1\!/_2$ point) Include error bars on each data point. Each error bar should have the same length $|I_{\text{disk}} - I_{x_1}|$ from Data Table 3.

(c) ($^1\!/_2$ point) Add a title to the plot, label the axes, and include units.

Graph 1: *Plot of I_{x_i} versus $d_{x_i}^2$.*



Question 5 **1 points**

(a) ($^1/_2$ point) In the plot, draw a best-fit straight line for your data. It need not go through the origin.

(b) ($^1/_2$ point) Find the y-intercept of your best-fit line and circle it on the graph. Then circle two points on your best fit line, (x_1, y_1) and (x_2, y_2), and compute the slope. Write both values below:

$$b_{\text{exp}} = y - \text{intercept} =$$

$$m_{\text{exp}} = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} =$$

Question 6 **1 point**

If you take the parallel axis theorem

$$I = Md^2 + I_{\text{CM}}$$

and plot I versus d^2 (y vs. x), the data should be linear. Given that the equation for a line is $y = mx + b$, where m is the slope and b is the y-intercept, what variables in the parallel axis theorem correspond to m and b?

$$m_{\text{theory}} = \text{slope} =$$

$$b_{\text{theory}} = \text{intercept} =$$

Question 7 **2 points**

Using your answers to questions 5 and 6, how much do your measured values for the slope and y-intercept differ from the theoretical values? Remember to include units.

$$|m_{\text{theory}} - m_{\text{exp}}| =$$

$$|b_{\text{theory}} - b_{\text{exp}}| =$$

Question 8 **1 points**

Name two sources of experimental error. Give a reason as to how each source of error would affect your values of the moment of inertia.

(a) ($^1/_2$ point) First source of error:

(b) ($^1/_2$ point) Second source of error:

Hooke's Law and Spiral Spring Oscillations (10 points)

Add your measurements into Data Table 4 below:

Data table 4: *Hooke's Law and Spiral Spring measurements.*

m = mass of spring [kg] =				
M = attached mass [kg]	Force = Mg [N]	Extension [m]	Period [s]	Period^2 [s^2]
0.005				
0.010				
0.015				
0.020				
0.025				

Question 9 **2 points**

(a) (1 point) Plot the **force** of gravity versus **extension** (y vs. x) in Graph 2 using the data from Data Table 4.

(b) ($^1/_2$ point) Include a plot title, axis labels, and units.

(c) ($^1/_2$ point) Draw a best-fit straight line through the data points.

Graph 2: *Force of gravity versus spring extension.*

Question 10 **1 point**

Based on the best-fit straight line, does your data agree with Hooke's Law? Explain why.

Question 11 **1 point**

As in question 5, calculate the slope of the best-fit straight line. Circle two points (x_1, y_1) and (x_2, y_2) on the line that you will use to compute the slope, and use it to find the spring constant, k_{Hooke}. Don't forget units!

$$k_{\text{Hooke}} = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} =$$

Question 12 **1 point**

If we were to plot T^2 along the y-axis and M along the x-axis:

(a) ($1/2$ point) What would be an equation for the theoretical value of the slope, m_{theory}, remembering that $y = mx + b$ is the equation for a line?

(b) ($1/2$ point) What would be an equation for the theoretical value of the y-intercept, b_{theory}?

Question 13 **1 point**

Using your data for Period^2 and the attached mass M from Data Table 4:

(a) (½ point) Plot Period^2 versus attached mass, i.e., T^2 vs M, in Graph 3.

(b) (½ point) Draw a best-fit straight line through the data, and include a title, axis labels, and units.

Graph 3: *Period squared versus mass.*

Question 14 **1 point**

Calculate the slope of the best-fit straight line. Circle two points on the line, (x_1, y_1) and (x_2, y_2). Setting the measured slope equal to the theoretical expression you found in question 12, find the spring constant k_{harm}:

$$m_{\text{exp}} = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = m_{\text{theory}}$$

$$k_{\text{harm}} =$$

Question 15 **1 point**

Find and circle the y-intercept of the best-fit straight line. Call this intercept y_0. Setting y_0 equal to the theoretical expression for the y-intercept from question 12, find the spring mass coefficient b. Use the spring constant k_{harm} you computed in question 14.

$y_0 = \text{intercept} =$

$b =$

Question 16 **1 point**

Suppose that if the effective spring mass, bm, is less than 10% of the smallest attached mass, $M = 0.005\,\text{kg}$, then we can say the spring mass is negligible. Based on the value of b found in question 15, is the spring mass negligible? That is, do you find that

$$\frac{bm}{M} \leq 0.1?$$

Question 17 **1 point**

Check whether the spring constants k calculated in questions 11 and 14 are comparable in two steps:

(a) ($^1\!/_2$ point) Find the difference between the two k values, including units:

$|k_{\text{Hooke}} - k_{\text{harm}}| =$

(b) ($^1\!/_2$ point) Suppose we assume the k measurements are consistent if they differ by less than 10% of the smaller of the two values. Is it the case that

$$\frac{|k_{\text{Hooke}} - k_{\text{harm}}|}{k_{\text{smaller}}} \leq 0.1?$$