University of Rochester

Laboratory VI
Coulomb's Law

DEPARTMENT OF PHYSICS & ASTRONOMY

PHYSICS 114 - 122 - 182
GENERAL PHYSICS II AND ELECTRICITY & MAGNETISM

Name: ______________________________ **Date:** ______________________

Collaborators: ____________________________ **Lab Section:** ____________

Prelab Exercises (3 points)

This prelab must be completed and handed in to the lab TA at the start of the lab.

Question 1 **1 point**

Why is it important to recharge the spheres before each measurement?

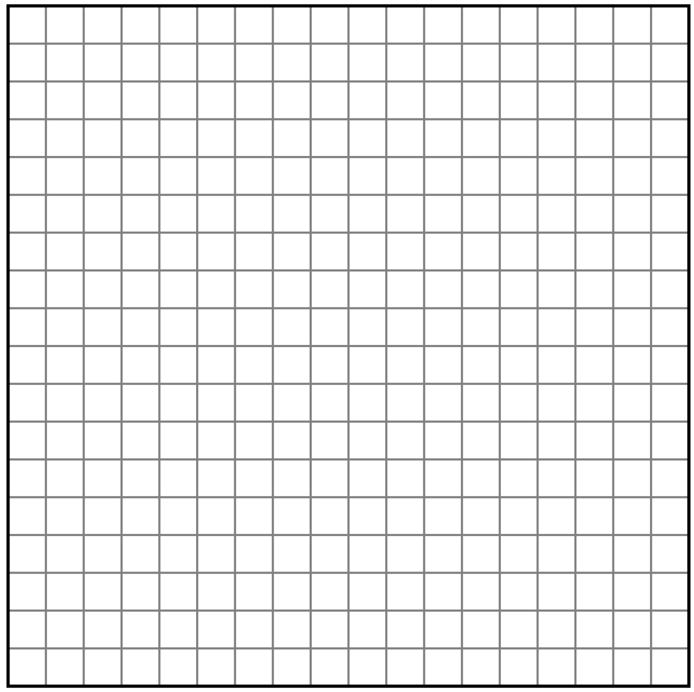
Question 2 **1 point**

Compute the force between one gram of protons and one gram of electrons that are 10 km apart. Show your work and explain your steps.

Question 3 **1 point**

Consider the function $y = x^n$ when $n = -2$. Fill out the table on the left for the given values of x. Plot $\ln x$ vs. $\ln y$ on the right. Make sure you draw your coordinate axes in a smart way. Explain how to obtain n from the graph. (Hint: can you fit a straight line to your plotted points?)

x	y	$\ln x$	$\ln y$
1			
2			
3			
4			
5			
6			



Objective

To verify that the electric force between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.

Theory

Physics in the 18^{th} century included the study of electric phenomena: how charged objects attract and repel one another. In 1785, Charles-Augustin de Coulomb published the relation we now call Coulomb's Law. Given two charges q_1 and q_2 separated by a distance r, Coulomb's Law states that the electric force between them is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}. \tag{1}$$

When we are children, we are told the prefactor is called k, but now that you're starting to grow up, we can let you in on the secret. The 4π makes lots of other formulas simpler, and is there because three-dimensional space subtends that solid angle (remember the formula for the surface area of a sphere?), while the *permittivity of free space*, or *vacuum permittivity*, is in SI units

$$\varepsilon_0 = 8.854\,187\,818\,8(14) \times 10^{-12}\,\frac{\mathrm{C^2\,s^2}}{\mathrm{kg\,m^3}}.$$

When substituting numbers, you can still use the approximation from high school,

$$\frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9\,\frac{\mathrm{N\,m^2}}{\mathrm{C^2}}.$$

The electric force is very powerful compared to gravity. In your prelab, you should find that the force in Question 2 is on the order of 1×10^{15} N! If you could create a force that large to counteract gravity on the Earth's surface, $F = mg$, you could lift Mt. Everest[1]!

Verifying Coulomb's law requires some ingenuity, since we cannot simply observe two charged objects moving and read off the force between them. We need something to measure — what we call an *observable* — that can be related to the force in a way that allows us to test eq. (1). To this end, we will use the torsion balance pictured in Fig. 1.

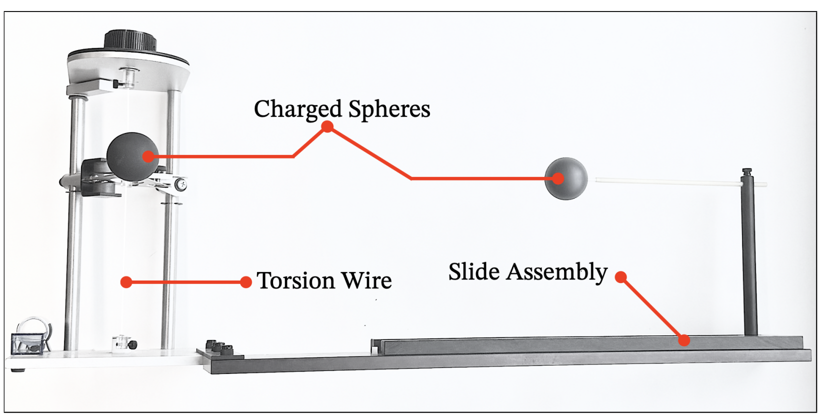


Figure 1: *The PASCO Model ES-9070 Coulomb Balance.*

A conductive sphere is mounted on a rod, counterbalanced, and suspended from a thin torsion wire. An identical sphere is mounted on a slide assembly so it can be positioned at various distances from the

[1]No, seriously: look up the estimated mass of Mount Everest and calculate its weight.

suspended sphere. When the spheres have no net charge, the suspended sphere settles to an equilibrium position and remains there. Once both spheres are charged, there is an electric force F_e on each one.

To charge the spheres, you will use a power supply set to a particular voltage relative to ground, ΔV. Touching a conductive sphere with the charging probe causes electrons to flow until the sphere and probe are at the same voltage. The total charge on the sphere is related to its voltage by

$$q = C\Delta V, \tag{2}$$

where C is the *capacitance* of the conducting sphere, a constant that depends on the sphere's size and material composition, which you will learn more about later in the semester.

The fixed sphere is held in place by the slide assembly, but the suspended sphere is free to move. Its displacement creates a *torque* on the torsion wire[2]. The wire resists being twisted and exerts a counter-torque; the stronger the twist, the stronger the counter-torque, until the two cancel and the system settles into a new equilibrium. See Fig. 2 for details.

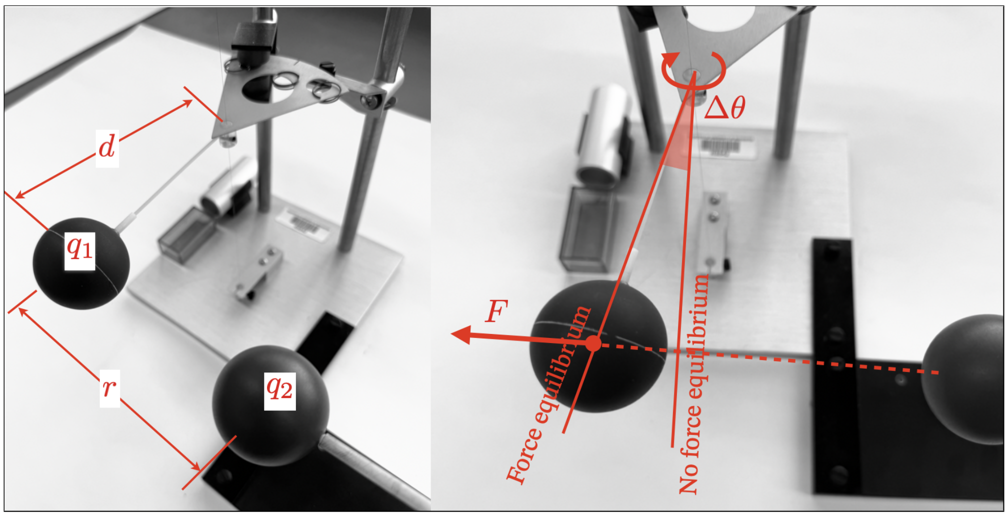


Figure 2: *(Left) The spheres are charged and separated by a fixed distance r, where d is the lever arm for the torque on the wire. (Right) After charging, the suspended sphere moves due to the electric force, twisting the wire.*

The torsion dial on the Coulomb Balance is then used to twist the wire back and return the suspended sphere to its original position. Once the system is motionless, the reading on the dial gives the total twist $\Delta\theta$, which is our desired *observable*. Here $\Delta\theta$ serves as a proxy for the electric force F_e, while the applied voltages ΔV_1 and ΔV_2 serve as proxies for the charges q_1 and q_2. The restoring torque from the wire obeys Hooke's Law,

$$\tau_+ = -\kappa\Delta\theta, \tag{3}$$

where κ is the *torsion constant* of the wire. The torque due to the electric force is

$$\tau_- = F_e d, \tag{4}$$

where d is the *lever arm*, the distance from the wire to the point where the electric force is applied. The torques cancel in equilibrium ($\tau_+ + \tau_- = 0$), so combining eqs. (1), (2), (3), and (4), the net twist is

$$\Delta\theta = \frac{d}{4\pi\varepsilon_0\kappa}\frac{q_1q_2}{r^2} = \frac{C_1C_2d}{4\pi\varepsilon_0\kappa}\frac{\Delta V_1\Delta V_2}{r^2}. \tag{5}$$

The prefactor on the right side of eq. (5) is a constant, while the second factor contains the quantities you control: the independent variables ΔV_1, ΔV_2, and r. The dependent variable, the observable $\Delta\theta$, is on the left. By connecting the electric force to controllable quantities and a measurable observable, we can test Coulomb's Law experimentally.

[2]Don't worry if you don't understand what a torque is yet. You will learn more about torque later in the semester.

In **Experiment 1**, every variable in eq. (5) is held constant except r. This tests the proportionality

$$\Delta\theta \propto \frac{1}{r^2}.$$

In **Experiment 2**, every variable is held constant except $q_2 = C_2 \Delta V_2$, testing

$$\Delta\theta \propto q_2 \propto \Delta V_2.$$

In both experiments, note that charges on your body, the surrounding air, and nearby objects can interfere with measurements or cause the sphere charges to leak into the environment.

Equipment

- PASCO Model ES-9070 Torsion Balance (see Fig. 1)
- Two conducting spheres of radius $a = 1.9\,\text{cm}$
- Power supply with a power cord
- **Red** and **Black** wires with probes

Experiment

Tips for Accurate Results

- To minimize the influence of static charge on your body, stand directly behind the balance and as far away as possible.
- Avoid synthetic fabrics, which tend to acquire large static charges. Short-sleeve cotton shirts are best. Attaching a grounding wire to yourself further reduces the impact of static charge.
- Turn the power supply on, charge the spheres, and immediately turn the power supply off. The large voltages can affect the torsion balance.
- While charging, hold the probe near the end of the handle so that your hand is as far from the sphere as possible, to minimize bioelectric effects.
- Perform the measurements as quickly as possible after charging to minimize charge leakage.
- Recharge the spheres before each measurement.

Experiment 1: Varying r

1. Touch the spheres with the grounding probe to ensure they are fully discharged. Move the sliding sphere as far as possible from the suspended sphere. Set the torsion dial to $0°$. Zero the torsion balance by rotating the bottom torsion wire retainer until the pendulum is at the zero displacement position.
2. Ground both spheres again with the grounding probe. Position the slide arm to $2a = 3.8\,\text{cm}$, the diameter of a sphere. Loosen the thumbscrew on the rod supporting the sliding sphere and move it until the sliding sphere just touches the suspended sphere; do not push the suspended sphere far enough to cause it to rotate. Tighten the thumbscrew.
3. Move the sliding sphere as far away as possible from the suspended sphere. Use the charging probe to charge each sphere to a potential of 6 kV.
4. Move the sliding arm to $r = 20\,\text{cm}$. Adjust the torsion dial to bring the pendulum back to the zero position. Record this measurement of $\Delta\theta$ in the first row of Table 1.
5. Repeat steps 3–4 until successive results fall within $\pm 1°$ of each other, recording every measurement of $\Delta\theta$ in Table 1.
6. Repeat steps 3–5 for $r = 14\,\text{cm}$, $10\,\text{cm}$, $9\,\text{cm}$, $7\,\text{cm}$, and $5\,\text{cm}$.

Experiment 2: Varying q_2

1. Same as Experiment 1.
2. Same as Experiment 1.
3. Same as Experiment 1.
4. Move the sliding arm to 8 cm. Adjust the torsion dial to bring the pendulum back to the zero position. Record $\Delta\theta$ in Table 2.
5. Repeat steps 3–4 at least five times, until successive measurements fall within $\pm 1^\circ$ of each other. Record every measurement.
6. Repeat steps 3–5, charging the sliding sphere to 5 kV, 4 kV, 3 kV, 2 kV, and 1 kV in turn, while keeping the suspended sphere at 6 kV.

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Postlab Exercises (13 points)

Submit the postlab to the TA at the end of the lab.

Measurement 1: Force vs. Separation (2 points)

Coulomb's Law applies to point charges, but we use spheres of radius a. When the spheres are far apart ($r \gg a$), they behave like point charges because the charge on each is distributed nearly uniformly. When the spheres are close together ($r \approx a$), the charges bunch toward each other, leading to non-point-like behavior. In a more advanced course you will learn how to calculate this effect, but for now it suffices to know that the corrected observable is

$$\Delta\theta_c = \frac{\Delta\theta}{\beta}, \qquad \text{where} \qquad \beta = 1 - 4\left(\frac{a}{r}\right)^3 \tag{6}$$

is the correction factor. Record your measurements of $\Delta\theta$ for each separation r in Table 1 and compute the mean of your N measurements:

$$\overline{\Delta\theta} = \frac{1}{N}\sum_{i=1}^{N} \Delta\theta_i. \tag{7}$$

Then compute the correction factor β and the corrected mean twist angle $\overline{\Delta\theta}_c$ using eq. (6). Finally, for each separation r, tabulate $1/r^2$, $\ln(r/a)$, and $\ln \overline{\Delta\theta}_c$. You will plot the final two columns on the next page.

Table 1: **Measurement 1 Data**

r [cm]	$\Delta\theta$	$\overline{\Delta\theta}$ [°]	β	$\overline{\Delta\theta}_c$ [°]	$1/r^2$	$\ln(r/a)$	$\ln(\overline{\Delta\theta}_c)$
20							
14							
10							
9							
7							
5							

Question 4 **2 points**

Measurement 2: Force vs. Charge (11 points)

Table 2: **Measurement 2 Data**

ΔV_2 [kV]	$\Delta\theta$	$\overline{\Delta\theta}$ [°]	β	$\overline{\Delta\theta}_c$ [°]	$\ln(\overline{\Delta\theta}_c)$	$\ln(\Delta V_2/1\,\text{kV})$
6						
5						
4						
3						
2						
1						

Question 5 **2 points**

Question 6 **2 points**

Show one complete example of how you corrected your data. Choose any one row from Table 1 and compute β and $\overline{\Delta\theta}_c$ step by step.

Question 7 **2 points**

Using your log-log plot from Question 6, calculate the slope of the best-fit line. Show your calculation by identifying two points on the line.

Question 8 **1 point**

Make a scatter plot of $\ln\left(\overline{\Delta\theta_c}\right)$ versus $\ln\left(r/a\right)$ with the independent variable on the horizontal axis and the dependent variable on the vertical axis. Label your axes.

Question 9 **1 point**

Make a scatter plot of $\ln(\overline{\Delta\theta_c})$ versus $\ln(\Delta V_2/1\,\text{kV})$ with the independent variable on the horizontal axis and the dependent variable on the vertical axis. Label your axes.

Question 10 **1 point**

For both graphs, add a dashed line that fits the data well. Use each plot to estimate the slope m and y-intercept b, and write down the equation of the best-fit line in the form $y = mx + b$.

Question 11 **2 points**

Analyze the best-fit lines from both plots. What mathematical relationship exists between $\ln\left(\overline{\Delta\theta_c}\right)$ and $\ln\left(r/a\right)$? What relationship exists between $\ln\left(\overline{\Delta\theta_c}\right)$ and $\ln(\Delta V_2/1\,\mathrm{kV})$? Do these relationships match what Coulomb's Law predicts? If your measured slopes differ from the expected values, identify possible sources of experimental error.