University of Rochester

Laboratory VII

Voltage & Electrostatic Potential

Department of Physics & Astronomy

PHYSICS 122 - 142 - 182

Electricity and Magnetism

Name: ______________________________ **Date:** ______________________

Collaborators: ___________________________ **Lab Section:** ____________

PRELAB EXERCISES (2 points)

This prelab must be completed and handed in to the lab TA at the start of the lab.

Question 1 **1 point**

In the first part of this lab, you will be using a mechanically-balanced, axially-symmetric cylindrical capacitor to measure an absolute voltage. Each time you take a measurement, but before you read the microscope scale, you should perform a critical procedural technique. What is it, why is it critical, and how many times should you perform it?

Question 2 **1 point**

How do you determine the electric field lines for the second part of the experiment? Electric fields are vector quantities. How will you determine the direction that they should point?

Objective

> *The purpose is to calculate an absolute voltage measurement by mechanical means and to illustrate the concepts of the electric field by experimental demonstration.*

Theory

Capacitance and the Principle of Virtual Work

A force is exerted on any electric charge by every other charge in its vicinity. The total electrical force on a test charge varies as the charge is moved through space; this spatial variation of force per unit charge defines the *electric field*. Since the electrostatic force (Coulomb's Law) is conservative, the work done moving between two points is independent of the path taken. This motivates the concept of *electric potential*: the potential difference between two points is the work per unit charge done by the electric field in moving a test charge between them.

The charge Q on a conductor and the potential V it creates are linearly related through the *capacitance* C,

$$C = \frac{Q}{V}. \tag{1}$$

Capacitance depends only on the geometry of the conductor system. The stored electrical energy of a capacitor held at constant potential V is

$$U = \frac{1}{2} C V^2. \tag{2}$$

Since this energy changes as the geometry changes, work is required to move the conductors. By the *principle of virtual work*, the force between the electrodes in the direction of a coordinate z (at constant V) is

$$F_e = \frac{1}{2} V^2 \frac{dC}{dz}. \tag{3}$$

The Cylindrical Capacitor

In this experiment the capacitor consists of two coaxial cylinders. For a long cylindrical capacitor the capacitance per unit length is

$$\frac{C}{\ell} = \frac{2\pi\varepsilon_0}{\ln(r_2/r_1)}, \tag{4}$$

where r_1 is the radius of the inner cylinder (the beverage can) and r_2 is the radius of the outer cylinder. When the inner cylinder is displaced axially by a small amount $\Delta z = s$, the change in capacitance is

$$\frac{dC}{dz} = \frac{C}{\ell} = \frac{2\pi\varepsilon_0}{\ln(r_2/r_1)}, \tag{5}$$

so the electrical force on the inner cylinder is

$$F_e = \frac{\pi\varepsilon_0 V^2}{\ln(r_2/r_1)}. \tag{6}$$

Measuring the Voltage

The inner cylinder floats on a glass stem partially submerged in water; changes in the electrical force shift the equilibrium position of the float. The displacement s of the optical target on the shaft is observed through a low-power microscope. Balancing the electrical force against the change in buoyant force due to the displaced float yields the applied voltage in terms of purely mechanical quantities:

$$V = \sqrt{\frac{\rho g \pi d^2 s \ln(r_2/r_1)}{2\pi\varepsilon_0 \left(1 - \frac{d^2}{D^2}\right)}}, \tag{7}$$

where $\rho = 1000\,\mathrm{kg\,m^{-3}}$ is the density of water, d is the diameter of the float stem, D is the diameter of the beaker, and s is the measured displacement. The right-hand side depends only on mechanical and geometric quantities.

Electric Field Mapping

The *electric field* $\mathbf{E}$ is everywhere perpendicular to surfaces of constant potential (equipotentials) and points in the direction of decreasing potential. In this experiment, electrodes are placed on a sheet of resistive graphite paper and connected to a battery. A voltmeter and two probes are used to trace out equipotential contours by finding points at which the potential difference between the hand probe and a fixed reference probe is zero. Once the equipotentials are mapped, the electric field lines can be drawn perpendicular to them, with arrows pointing from higher to lower potential.

Equipment

- Absolute voltage apparatus (cylindrical capacitor with float and microscope)
- High-voltage power supply (use the 3 kV terminal only)
- Beaker with distilled water and a few drops of Eastman Kodak Photo-Flo 200 wetting agent
- Machine oil (for bearings)
- Small set of masses and weight pan
- Table probe and hand probe
- Battery and multimeter (voltmeter)
- Board with conducting graphite paper
- Box of metal electrode pieces

Experiment

Part 1: Absolute Measurement of Voltage

Caution: Keep the high-voltage supply at 3 kV. Do **not** use the 6 kV setting — it will spark. Do **not** touch the inner cylinder; potentials of several thousand volts are involved.

1. Check that the water level is approximately halfway up the glass stem of the float. Add or remove small masses from the weight pan as needed so that the can clears the upper bearing by about 1 cm.

2. Familiarize yourself with the apparatus. Gently spin the shaft using clean, dry thumb and forefinger. Once started, the spinning should persist for five to ten seconds and die out gradually. If it stops abruptly, the bearings need cleaning — ask your TA for help. **Important:** spin the shaft at least twice after every change to overcome bearing friction and confirm a reproducible reading.

3. Ensure the microscope is powered on and aimed at the ruler attached to the shaft. Adjust the height, focus, and zoom until you can clearly see both the ruler markings (spaced 0.5 mm apart) and the target mark where two cylinder sections meet.

4. **Calibrate displacement vs. voltage.** Apply five different voltages to the apparatus (suggested values: 0, 1000, 2000, 2500, and 3000 V). Spin the shaft at least twice after each voltage change. Measure the displacement of the optical target at each voltage and record the readings in Data Table 1.

5. **Measure an unknown voltage (displacement method).** Set the power supply to an intermediate voltage (e.g. 2200 V) and record this value. Switch the supply off and note the microscope reading. Switch it back on, spin the shaft twice, and record the new reading. Subtract to obtain the displacement s. Repeat several times and record all values in Data Table 2.

6. **Measure the unknown voltage (null method).** With the same unknown voltage applied, note the "zero" position on the microscope scale with the supply off. Turn the supply on. Carefully add masses to the weight pan until the zero point is just passed, then interpolate to find the mass m_{null}

corresponding to the zero-point. Record the balancing force $F_{\text{null}} = m_{\text{null}}g$ in the space provided in the Postlab Exercises.

7. Switch off all power when finished.

Part 2: Electric Field Mapping

1. Place the conducting graphite paper on the board and clip the electrodes into place. Connect the battery to the binding posts and the voltmeter to the two probes.
2. Place the stationary probe at a reference position between the two electrodes and mark its position on your data sheet.
3. Move the hand probe across the paper to find points where the voltmeter reads zero (within $\pm 0.05\,\text{V}$). If the reading is positive, move in the direction that gives a negative reading — the null point lies between. Mark each null point on the data sheet. Gently agitate the probe tips if necessary to ensure good contact.
4. Without moving the stationary probe, find enough null points to draw a smooth equipotential line through them.
5. Move the stationary probe to a new position (not on the previous contour) and repeat to map another equipotential. Map **at least four** equipotential lines for each electrode configuration.
6. Map both electrode configurations: the **two-point** arrangement and the **parallel-plate** arrangement. For the parallel plates, probe beyond the edges of the plates to observe fringe effects.
7. Disconnect the battery and turn off the voltmeter when finished.

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Postlab Exercises (18 points)

Submit the postlab to the TA at the end of the lab.

Part 1: Absolute Voltage Measurement (10 points)

Question 3 **$^1/_2$ point**

Record your displacement measurements for each applied voltage in Data Table 1.

Data table 1: *Displacement vs. applied voltage.*

Voltage [V]	Displacement s [mm]
0	
1000	
2000	
2500	
3000	

Question 4 **2 points**

Make a plot of displacement s versus V^2 using the data from Data Table 1. Plot V^2 on the horizontal axis and s on the vertical axis. Draw a best-fit straight line, measure its slope, and show your calculation. Include axis labels with units.

Question 5 **½ point**

Record your repeated displacement measurements for the unknown voltage in Data Table 2, and compute the average displacement s_{avg}.

Data table 2: *Displacement measurements for the unknown voltage.*

Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	s_{avg} [mm]

Unknown voltage (from power supply): $V_{\text{known}} =$ ____________

Balancing force (null method): $F_{\text{null}} =$ ____________

Question 6 **1 point**

Using the slope found in the graph above (in mm V^{-2}) and s_{avg} from Data Table 2, calculate the unknown voltage. Compute the percent error using the known value.

Question 7 **1 point**

Using eq. (6) and the balancing force F_{null}, calculate the unknown voltage a second time.

Question 8 **4 points**

What is the limiting experimental parameter for measuring an unknown voltage in each method? That is, what variable has the greatest uncertainty?

(a) (1 point) Displacement method:

(b) (1 point) Null method:

Question 9 **1 point**

Which method of measurement is more reliable? Why? Be specific.

Part 2: Electric Field Mapping (8 points)

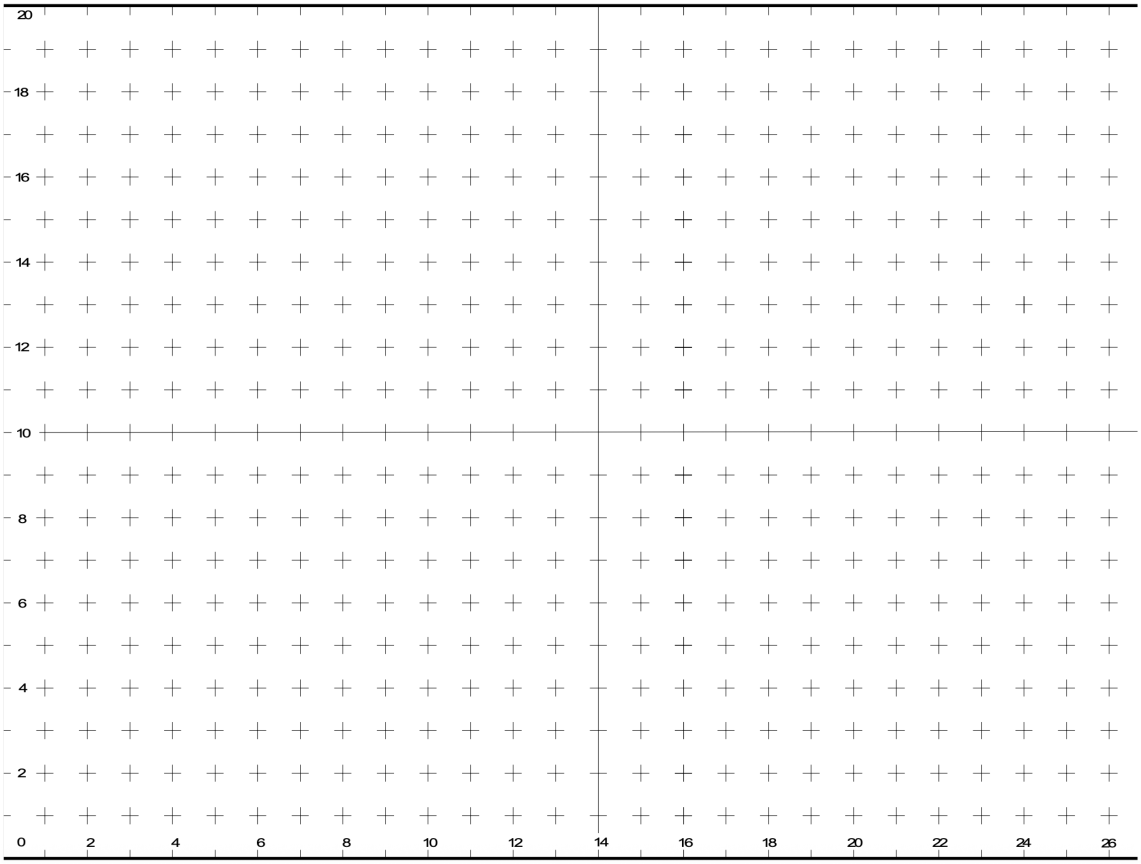
Question 10 **1 point**

Draw the electric field lines you would expect for an isolated positive point charge $+q$ below. Identify two arbitrary regions and label which has the stronger field. Explain how you can determine this from the field line pattern.

Question 11 **3 points**

In Graph 1, sketch the electric field lines according to your measured mapping of equipotential surfaces for the two-point electrode arrangement. Include arrows showing the direction of the field lines.

Graph 1: *Two-point potential and electric field lines.*



+1+1+1

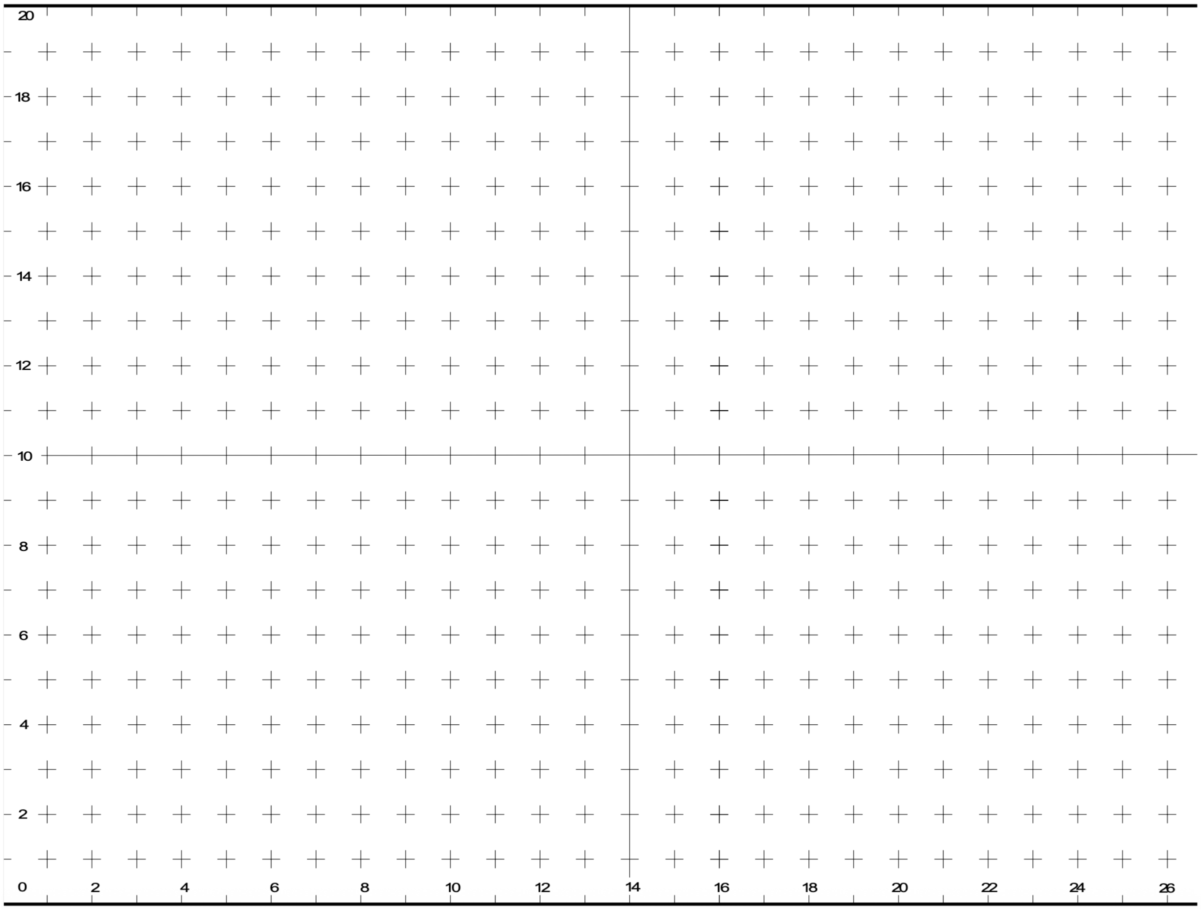
Question 12 **½ points**

In which region(s) of the two-point pattern would the electric force on a test charge be largest? Why?

Question 13 **3 points**

In Graph 2, sketch the electric field lines according to your measured mapping of equipotential surfaces for the parallel-plate electrode arrangement. Include arrows showing the direction of the field lines.

Graph 2: *Parallel-plate potential and electric field lines.*



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Question 14 **$^1/_2$ points**

Describe the trajectory of a positive unit charge introduced midway between the parallel plates with a velocity directed parallel to the plates. Draw a figure if you wish.