University of Rochester

Laboratory VIII
Electron Beams and the Electron Charge-to-Mass Ratio

Department of Physics & Astronomy

PHYSICS 122 - 142 - 182

Electricity and Magnetism

Name: ________________________________ **Date:** ________________________

Collaborators: ______________________________ **Lab Section:** ______________

Prelab Exercises (2 points)

This prelab must be completed and handed in to the lab TA at the start of the lab.

Question 1 **1 point**

Name the two forces that are equated in order to derive the charge-to-mass ratio of a charged particle.

Question 2 **1 point**

Describe or make a rough sketch of the path of an electron traveling into a region with a constant B field. Assume the B field is out of the page and the initial velocity of the electron is perpendicular to the field. In this experiment, what provides the B field?

Objective

Inject an electron beam into a uniform magnetic field produced by a pair of Helmholtz coils and observe the resulting circular trajectory due to the Lorentz force. Use the measured radius of curvature and the known field strength to determine e/m_e, the charge-to-mass ratio of the electron.

Theory

A particle with charge q moving with velocity $\mathbf{v}$ through a region with electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$ will experience a force

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}). \tag{1}$$

The force in eq. (1), known as the **Lorentz force**, is parallel to $\mathbf{E}$ and perpendicular to both $\mathbf{v}$ and $\mathbf{B}$.

Suppose that we have a beam of electrons, so that $q = -e \approx -1.602 \times 10^{-19}$ C, and we inject the beam into a region with no electric field and a uniform magnetic field oriented perpendicular to the beam. In this case, eq. (1) will reduce to

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}, \qquad |\mathbf{F}| = F = evB. \tag{2}$$

Due to the vector product, the force will always be perpendicular to $\mathbf{v}$, causing the beam to bend into a circular path of radius r. This makes F a centripetal force:

$$m_e \frac{v^2}{r} = evB. \tag{3}$$

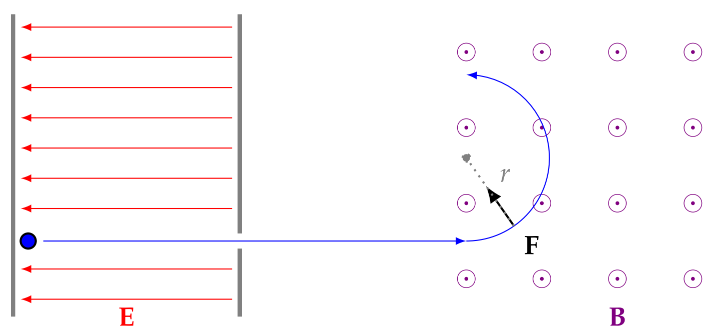

Figure 1: *An electron is accelerated from rest by a constant electric field* **E** *and passes into a region with a uniform magnetic field* **B***, which produces a centripetal force given by eq. (3).*

To precisely control v, imagine two conducting plates with a potential difference (or voltage) V between them. An electron starting at rest at one plate is accelerated to energy eV by the uniform $\mathbf{E}$ field between the plates. Thus, the electron's kinetic energy when it leaves the region of the $\mathbf{E}$ field is

$$\frac{1}{2} m_e v^2 = eV, \tag{4}$$

and therefore its velocity once it leaves the region between the plates is

$$v = \sqrt{\frac{2eV}{m_e}}. \tag{5}$$

Combining eqs. (3) and (5) gives the charge-to-mass ratio

$$\frac{e}{m_e} = \frac{2V}{(rB)^2}. \tag{6}$$

Thus, by measuring V, B, and the radius of curvature r, eq. (6) gives e/m_e entirely from measurable quantities.

Experiment

In this experiment, we will use the apparatus shown in Figure 2 to carry out the measurement sketched out in Figure 1.

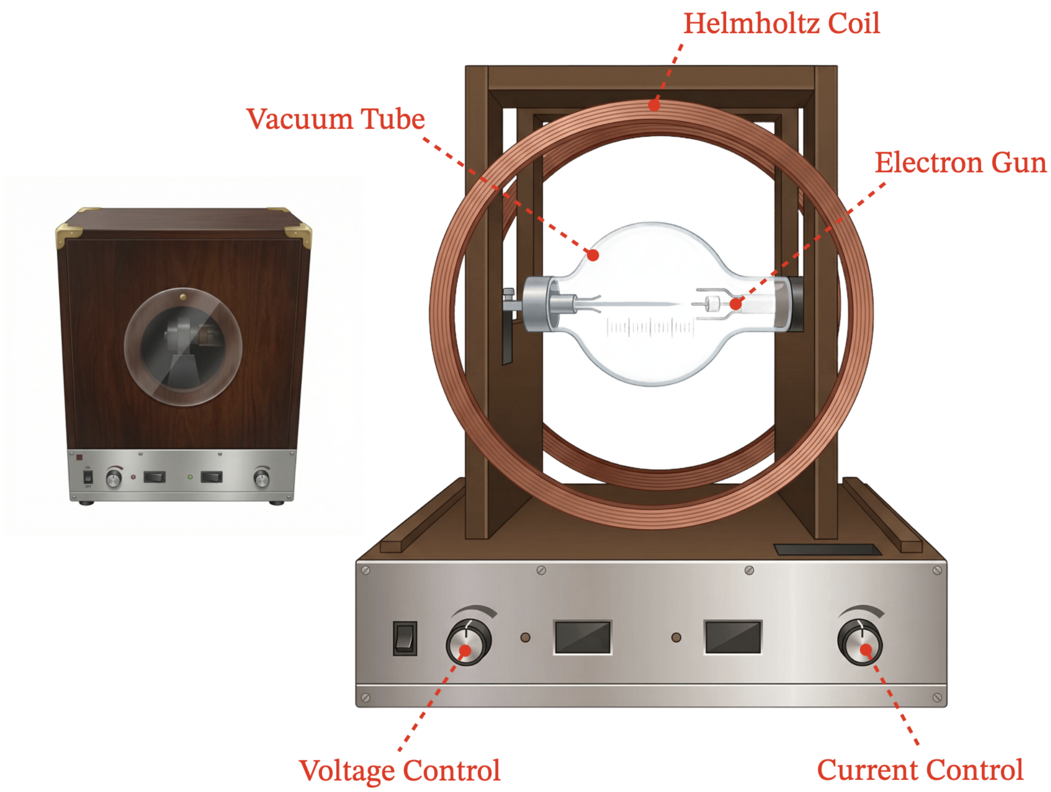

Figure 2: *The vacuum tube and Helmholtz coil used to measure e/m_e in this experiment.*

The apparatus consists of two major components. First, a **vacuum tube** containing a heated filament at one end provides a source of electrons. The electrons are accelerated by an applied voltage V which sets up an $\mathbf{E}$ field, and the resulting electron beam is collected on an electrode at the other end of the vacuum tube.

Second, outside the vacuum tube is a **Helmholtz coil**, which establishes a uniform $\mathbf{B}$ field in the tube. The Helmholtz coil is made of two coaxial circular electromagnets of radius R separated by a length $\ell = R$. If each electromagnet has N windings of magnet wire carrying current I, then the magnitude of the field inside the coil is given by

$$B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{R}. \tag{7}$$

Given $N = 130$ windings per coil, the permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m $\approx 1.26 \times 10^{-6}$ H/m, and assuming $R = 15\,\text{cm} = 0.15\,\text{m}$, the field magnitude can be expressed in terms of coil current I as

$$B = 0.77 \cdot I\ \text{mT} = (7.7 \times 10^{-4}) \cdot I\ \text{T} \tag{8}$$

The apparatus allows you to adjust the applied voltage V and the current I, from which you may compute B using eq. (8). You will measure the coil radius R directly and recompute this coefficient later in the Postlab. Referring to eq. (6), the final measurement needed to estimate e/m_e is the electron beam's radius of curvature r. The tube is filled with a trace amount of He gas, and when the electron beam moves through it, the gas fluoresces with green light, allowing you to see its circular path. A scale inside the tube lets you read off the diameter of the circular path; dividing by 2 gives the radius of curvature.

Procedure

Note to TAs: If the electron gun in the vacuum tube is not pointed downward, the electron beam will travel on a spiral path. To correct this, loosen the mounting screws of the vacuum tube and rotate the tube a little until the electron gun is in the proper orientation and the beam is circular. This adjustment should be done by a laboratory technician before the laboratory session and should not be left to the student.

1. Set the apparatus on a level table. The room light should not be too bright, or the electron beam will be hard to see. You may wish to use the "box" with the viewing window to reduce glare (see Fig. 2).

2. To minimize the effect of Earth's magnetic field on your measurement, use a compass to locate magnetic North and align the Helmholtz coil axis to be perpendicular to the needle.

   Note that the influence of the geomagnetic field or other sources of magnetic fields can be observed via the deflection of the circular motion of the electron beam while the apparatus is rotated. The magnitude of this deflection is greater when a small current is flowing through the coils.

3. With the power switched off, make sure the apparatus is plugged in. Turn on the power switch. The unit will perform a 30-second self-test, indicated by the digital display changing values rapidly.

   During the self-test, the controls are locked out to allow the cathode to heat to the proper operating temperature. When the self-test is complete, the display will stabilize and show "000." A further 5-10 minute warm-up time is recommended before starting your measurements.

4. After 5–10 minutes have elapsed, turn the **Voltage Adjust** control up to 350 V and observe the bottom of the electron gun. You should see a bluish beam traveling straight down to the envelope of the tube.

   Note: the Voltage and Current outputs are controlled by an onboard microprocessor that automatically locks out the controls at both the minimum and maximum settings. As a result, when the knob reaches the minimum or maximum setting, it will still turn, but the value shown on the corresponding display will not change. This feature prevents excessive loads from being placed on the tube and provides a smooth, jitter-free display.

5. Turn the **Current Adjust** control up and observe the circular deflection of the beam. When the current is high enough, the beam will form a complete circle within the envelope of the vacuum tube. The diameter of the circular beam can be measured using the centimeter scale inside the tube. To help you read the correct diameter, the scale numbers fluoresce when struck by the beam.

6. With the accelerating voltage set to 350 V, measure the beam diameter d for 9 coil current settings and record your results in Data Table 2 in the Postlab exercises.

7. Set the accelerating voltage to 400 V, repeat the procedure in Steps 5 and 6, and measure the beam diameter d for 9 coil current settings. Record your results in Data Table 3.

8. Repeat the measurements in Steps 5 and 6 for one additional accelerating voltage of your choice, recording your data in Data Table 4.

9. If a small ceramic magnet is available, move it close to the tube and observe the deflection of the beam. It is easy to see how properly designed magnets can be used to focus and steer an electron beam.

10. Reset both controls to zero and switch off the apparatus.

11. Measure the internal and external diameter of the Helmholtz coils at several different locations and record your results in Data Table 1. They may not be quite round, and the two coils may not be quite the same. Using eq. (7), with proper units, recalculate the coefficient in eq. (8) using your measured radius R, and use this coefficient in your analysis when you convert I to B.

12. Average your measurements and determine the standard error, so that you will be able to decide whether it has a significant effect on the accuracy of your results.

Data Analysis and Estimation of e/m_e

To estimate e/m_e, you will make nine measurements of the radius of curvature r given a fixed accelerating voltage V and a varying Helmholtz coil current I. From these, you will estimate the product rB that appears in eq. (6). You will need to calculate the mean, the standard deviation, and the uncertainty on the mean value of rB. These three quantities are defined below:

$$\text{Mean:} \qquad \overline{rB} = \frac{1}{9}\sum_{i=1}^{9}(rB)_i, \tag{9}$$

$$\text{Standard deviation:} \qquad \sigma_{rB} = \sqrt{\frac{\sum_{i=1}^{9}\left((rB)_i - \overline{rB}\right)^2}{9-1}}, \tag{10}$$

$$\text{Uncertainty on the mean:} \qquad \sigma_{\overline{rB}} = \frac{\sigma_{rB}}{\sqrt{9}}. \tag{11}$$

Propagation of Uncertainties

When you have completed the statistical analysis of the three data sets, you will calculate e/m_e from eq. (6) for each of the three accelerating voltages. You will calculate an uncertainty on e/m_e by propagating the uncertainty on the mean $\sigma_{\overline{rB}}$:

$$\text{Uncertainty on } e/m_e\text{:} \qquad \Delta(e/m_e) = 2\frac{\sigma_{\overline{rB}}}{\overline{rB}}\frac{e}{m_e}. \tag{12}$$

Reporting the Charge to Mass Ratio

At the end of the data analysis, you will have found three different values for e/m_e and its uncertainty for each of the three accelerating voltages. You will want to quote results to **three significant figures**. Below are two proper ways to quote your results:

$$\frac{e}{m} = (1.91 \pm 0.02) \times 10^{11}\ \text{C/kg}, \qquad ✓$$
$$\frac{e}{m} = 1.91 \times 10^{11}\ \pm 0.02 \times 10^{11}\ \text{C/kg}. \qquad ✓$$

The following are examples of *incorrect* ways to report your results. Look carefully and make sure you understand why each is incorrect. If you are unsure, ask your TA or TI.

$$\frac{e}{m} = 1.91 \times 10^{11}\ \pm 0.0205 \times 10^{11}\ \text{C/kg}, \qquad ✗$$
$$\frac{e}{m} = 1.9 \times 10^{11}\ \pm 0.0005 \times 10^{11}\ \text{C/kg}, \qquad ✗$$
$$\frac{e}{m} = 1.9 \times 10^{11}\ \pm 2.05 \times 10^{9}\ \text{C/kg}. \qquad ✗$$

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Postlab Exercises (20 points)

Submit the postlab to the TA at the end of the lab.

Question 3 **2 points**

Record your measurements of the Helmholtz coil diameter D and radius $R = D/2$ and compute an average value in Data Table 1.

Data table 1: *Measurements of Helmholtz coil radius.*

D_{inner} (cm)	D_{outer} (cm)	R_{inner} (cm)	R_{outer} (cm)	$(R_{\text{inner}} + R_{\text{outer}})/2$ (cm)

$R = \overline{R} =$ ______________ m

Next, using the average value of the Helmholtz coil radius R, use eq. (7) to recompute the coefficient in eq. (8), assuming $N = 130$:

$$B = \underbrace{\left[\left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N}{\overline{R}}\right]}_{\text{coefficient (T} \cdot \text{A)}} I.$$

In the experiment, you measured the diameter of the electron beam d, which is related to the radius of curvature by $r = d/2$. The calculation of e/m_e uses the product rB, so write a formula for computing rB given the measured current I and beam diameter d:

$rB =$ ______________ $d \cdot I$ (m · T)

Question 4 **3 points**

Record your measurements from the first voltage setting (350 V). Compute the mean of rB, the standard deviation, and the uncertainty on the mean, and estimate e/m_e.

Data table 2: *Beam measurements using first voltage setting.*

$V =$ 350 V

N	Coil current I (A)	Beam diameter d (m)	rB (m · T)
1			
2			
3			
4			
5			
6			
7			
8			
9			

Mean $\overline{rB}$:

Standard deviation σ_{rB}:

Uncertainty on the mean $\sigma_{\overline{rB}} = \sigma_{rB}/\sqrt{9}$:

Estimated e/m_e:

Uncertainty on e/m_e:

Final result: $e/m_e \pm$ uncertainty, with units:

Question 5 **3 points**

Record your measurements from the second voltage setting (400 V). Compute the mean of rB, the standard deviation, and the uncertainty on the mean, and estimate e/m_e.

Data table 3: *Beam measurements using second voltage setting.*

$V =$ 400 V

N	Coil current I (A)	Beam diameter d (m)	rB (m · T)
1			
2			
3			
4			
5			
6			
7			
8			
9			

Mean $\overline{rB}$:

Standard deviation σ_{rB}:

Uncertainty on the mean $\sigma_{\overline{rB}} = \sigma_{rB}/\sqrt{9}$:

Estimated e/m_e:

Uncertainty on e/m_e:

Final result: $e/m_e \pm$ uncertainty, with units:

Question 6 **3 points**

Record your measurements from the third voltage setting. Compute the mean of rB, the standard deviation, and the uncertainty on the mean, and estimate e/m_e.

Data table 4: *Beam measurements using third voltage setting.*

$V =$ ____________________ V

N	Coil current I (A)	Beam diameter d (m)	rB (m · T)
1			
2			
3			
4			
5			
6			
7			
8			
9			

Mean $\overline{rB}$:

Standard deviation σ_{rB}:

Uncertainty on the mean $\sigma_{\overline{rB}} = \sigma_{rB}/\sqrt{9}$:

Estimated e/m_e:

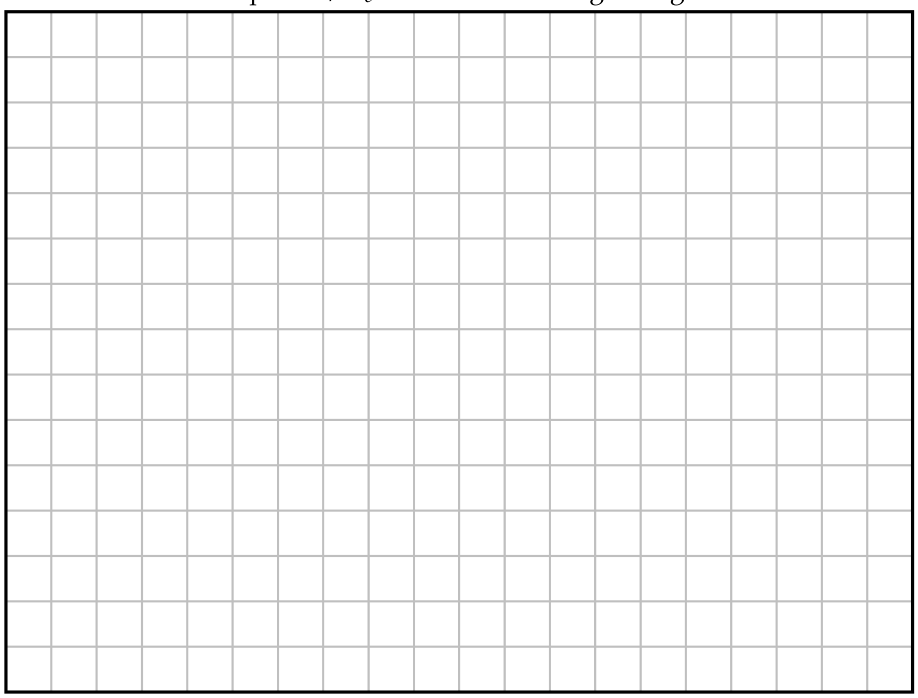
Uncertainty on e/m_e:

Final result: $e/m_e \pm$ uncertainty, with units:

Question 7 **4 points**

In Graph 1, plot e/m_e vs. accelerating voltage V and include error bars determined from your calculation of the uncertainty in e/m_e. Include axes labels with units. Use scientific notation in your y-axis label by stating it in terms of 10^{11} C/kg. Draw a horizontal line at the y-axis value of $e/m_e = 1.759 \times 10^{11}$ C/kg, the accepted value of e/m_e.

Graph 1: e/m_e *versus accelerating voltage.*



Question 8 **3 points**

Taking into account each measured value with its associated error, are your results consistent with each other? With the true value? Explain your answer and discuss why or why not.

Question 9 **2 points**

Name at least two of the major sources of uncertainty in the measurement of e/m_e in this lab. Be specific (e.g., "human error" as a reason is vague and imprecise).