University of Rochester

Laboratory IX
Ohm's Law and Superconductivity

Department of Physics & Astronomy

PHYSICS 114 - 122 - 142 - 182
Electricity and Magnetism

Name: ______________________________ **Date:** ______________________

Collaborators: __________________________ **Lab Section:** ____________

Prelab Exercises (2 points)

This prelab must be completed and handed in to the lab TA at the start of the lab.

Question 1 **1 point**

Define the *critical temperature* for a superconducting material. Explain in words any method for measuring the critical temperature in a superconductor.

Question 2 **1 point**

What is an ammeter? a voltmeter? What is the major difference in the way one uses each to measure electrical quantities in a simple electronic circuit? Be specific.

Objective

> *Observe the Meissner Effect by levitating a magnet above a high-temperature superconductor cooled in liquid nitrogen, demonstrating the expulsion of magnetic flux below the critical temperature. In the second part, build and measure simple DC resistive circuits to verify Ohm's Law and explore series and parallel resistance combinations.*

Theory

Ohm's Law and Equivalent Resistance

In resistive circuits, Ohm's law governs the relationship between voltage, current, and resistance. Ohm's law states that if one applies a driving voltage V to a circuit, the electrical current I inside the circuit will be linearly proportional to V:

$$V = IR. \tag{1}$$

The constant of proportionality R in eq. (1) is the resistance of the circuit.

Simple electronic circuits can be made using a direct-current (DC) voltage supply, such as a battery or a bench supply, which provides a constant potential difference between two junctions in a circuit. The DC voltage supply is connected to a combination of low-resistance wires, individual resistive elements called **resistors**, and (perhaps) measurement devices such as voltmeters and ammeters to measure voltage and current, respectively.

Commonly-used resistors are carbon-based and come color-coded to enable easy visual identification of the resistance. As shown in Figure 1, resistors can be combined in **parallel** combinations that divide the current into several branches, as well as **series** combinations in a single branch. (Other combinations are possible, of course, but we will not investigate them in this experiment.)

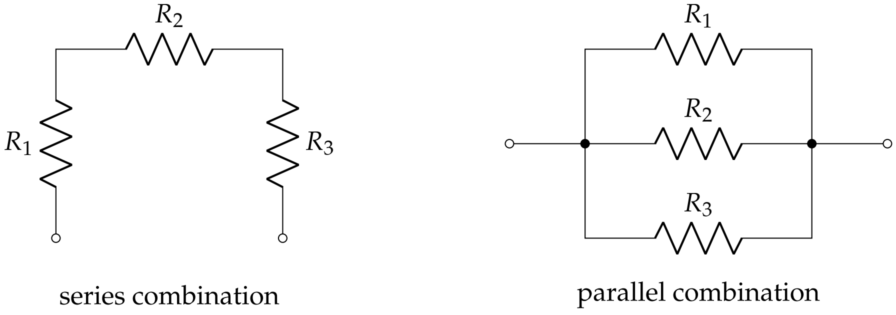


Figure 1: *A set of three resistors combined in series (left) and in parallel (right).*

Resistors combined in this manner can be expressed as a single combined or equivalent resistance R_{eq}. Resistors combined in series have an equivalent resistance that is just the sum of the individual resistances. For resistors combined in parallel, the inverse of the equivalent resistance is equal to the sum of the individual inverse resistances.

$$\text{Series} \qquad R_{\text{eq}} = R_1 + R_2 + R_3 + \ldots \tag{2}$$

$$\text{Parallel} \qquad \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \tag{3}$$

Resistors in Series: A Voltage Divider

A common application of resistors in series is in a device called a **voltage divider**. The divider does exactly what its name suggests: it takes a voltage input and successively steps down (or divides) the voltage by putting the input voltage across a set of resistors in series. This can be used to do useful things like adjust the level of a signal in a circuit, provide fixed bias voltages to output devices, etc.

An example voltage divider is shown in Figure 2. A power supply with voltage V_s is connected across two resistors in series, R_1 and R_2. Let's do a simple analysis of the circuit.

1. Notice that the current through R_1 and R_2 must be the same, because there is only one path through the circuit. Since the only current is I_s from the power supply, we must have
$$I_s = I_1 = I_2.$$
2. The voltages around the loop must sum to zero (Kirchhoff's loop rule), so we must have
$$V_s = V_1 + V_2.$$
3. We can treat the two series resistance as the single equivalent resistance $R_{\text{eq}} = R_1 + R_2$. Thus, by Ohm's law,
$$V_s = I_s R_{\text{eq}} = I_s(R_1 + R_2).$$

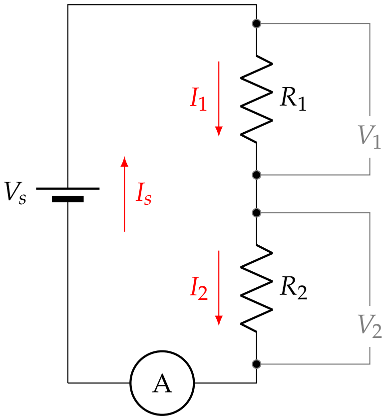


Figure 2: *A voltage divider.*

Putting these elements together, we find that the fraction of the voltage going through each resistor, in terms of the supply voltage V_s, is

$$V_1 = V_s \frac{R_1}{R_1 + R_2}, \qquad V_2 = V_s \frac{R_2}{R_1 + R_2}. \tag{4}$$

That is, the voltage drop across each resistor is proportional to its resistance divided by the equivalent series resistance, and the series resistors divide the supply voltage along the chain.

Resistors in Parallel: A Current Divider

In Figure 3, a voltage supply V_s is connected across two resistors in parallel. We notice the following details:

1. The current splits between the two branches of the circuit. By Kirchhoff's sum rule, the currents must sum to the current from the supply:
$$I_s = I_1 + I_2.$$
2. The voltage across the two resistors must be the same, because their endpoints are connected by a conducting wire with negligible resistance. Thus,
$$V_s = V_1 = V_2.$$
3. Using eq. (3) and Ohm's law, we find that
$$V_s = I_s R_{\text{eq}} = I_s \frac{R_1 R_2}{R_1 + R_2}.$$

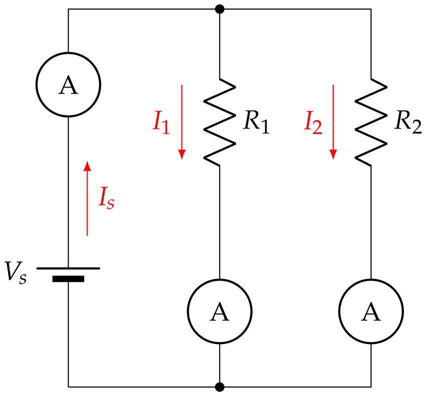


Figure 3: *A current divider.*

Combining these results, it is straightforward to show that

$$I_1 = I_s \frac{R_2}{R_1 + R_2}, \qquad I_2 = I_s \frac{R_1}{R_1 + R_2}. \tag{5}$$

That is, the current in resistor 1 is proportional to R_2, and the current in resistor 2 is proportional to R_1. Note that this makes physical sense: if $R_1 > R_2$, then $I_2 > I_1$ because the lower resistance in the second branch allows more current to flow through it.

Superconductivity

At the microscopic scale, an electrically conducting material can be thought of as an atomic lattice with energy levels that allow electrons to move freely when a voltage is applied across the material. The electrons' collective motion gives rise to the macroscopic current we measure in the lab. As electrons move through the material, they scatter off the atomic lattice, converting their kinetic energy to vibrational energy in the lattice and heating the material. (This is why live wires are hot to the touch!) This "frictional" dissipation of energy is characterized by the resistivity of the material and is the origin of Ohm's law.

However, some materials called **superconductors** lose all measurable electrical resistivity at low temperatures. In a superconductor, even without any driving voltage, an electrical current will flow indefinitely with no discernible decay. This phenomenon was originally observed in 1911, when the Dutch physicist Heike Kamerlingh Onnes used liquid helium to cool solid mercury to 1.5 K[1]. Kamerlingh Onnes observed that the electrical resistance of mercury rapidly drops to zero around 4.2 K.

Critical Temperature

Between 1911 and the mid-1980s, superconductivity was observed in a total of ten metals and alloys. In all cases, it was found that the transition from the normal resistive state to a superconducting non-resistive state occurs over a relatively small change in temperature. The temperature at which the transition occurs is called the **critical temperature** T_C, and its value depends on the material in question. Until the mid-1980s, the highest known critical temperature was $T_C = 23\,\mathrm{K}$, for the niobium-germanium alloy $\mathrm{Nb_3Ge}$. In 1986 and 1987, a class of ceramic superconductors was discovered with $T_C = 90\,\mathrm{K}$ to 100 K. The discovery of such "high-T_C" superconductors[2] is significant because T_C is greater than the boiling temperature of liquid nitrogen (77 K), a common and inexpensive refrigerant.

Our theoretical understanding of superconductivity is incomplete. In 1957, John Bardeen, Leon Cooper, and J. Robert Schrieffer modeled superconducting current as the resistance-free transport of bound pairs of electrons ("Cooper pairs") through the superconductor[3]. This so-called BCS theory, named after the authors, cannot describe the high-T_C superconductors, whose behavior is still not well-understood.

The Meissner Effect

In addition to being **perfect conductors**, superconductors are **perfectly diamagnetic**. That is, when $T < T_C$, they expel magnetic fields from the interior of the material. An external magnetic field that tries to penetrate the superconductor is perfectly canceled by an opposing field produced by superconducting currents inside the material. A diagram of this effect is shown in Figure 4. The expulsion of magnetic fields from superconductors, first observed by Walther Meissner and Robert Ochsenfeld in 1933, is called the Meissner effect.

If the external field comes from a magnet, the expulsion of the field lines produces a repulsive force on the magnet. Thus, a magnet positioned above the superconductor will be levitated, a phenomenon that you will investigate in this experiment.

Practical, real-world examples of the Meissner effect are its use in magnetic suspension systems, where it is desirable to eliminate mechanical friction (in maglev trains, for example) or to provide mechanical stability in applications using strong magnetic fields (e.g., magnetic resonance imaging).

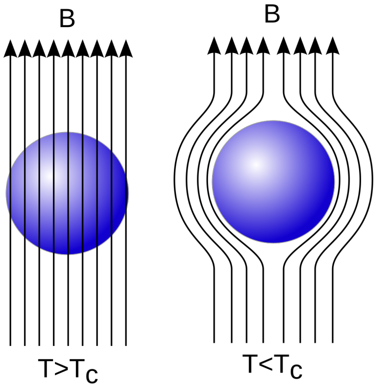


Figure 4: *At high T, a B field penetrates a conductor (left), but the field is expelled below the superconducting transition (right). Credit: P. Jaworski, Wikimedia Commons.*

---

[1] Kamerlingh Onnes was the first person to liquefy helium, and he was awarded a Nobel Prize for this work in 1913.

[2] Georg Bednorz and K. Alex Müller were awarded the Nobel Prize in 1987 for the discovery of high-T_C ceramic superconductors.

[3] Bardeen, Cooper, and Schrieffer were awarded the Nobel Prize for their theoretical work on superconductivity in 1972.

Experiment

Ohm's Law

> Note: students at **EVEN**-numbered tables will do Ohm's law first, and superconductivity second. Students at **ODD**-numbered tables should skip to superconductivity and do this section second.

A Note about Resistor Color Codes

Carbon composition resistors (CCRs) of the type you will use are made with hundreds of different resistance values, tolerances, and power ratings. They are very inexpensive and bought in bulk, and in an electronics stockroom, it is common to find drawers filled with hundreds to thousands of different resistors. To keep things straight, the value of a resistor may be read from a code of color bands printed on its exterior, as shown in Figure 5. In the lab, you will typically see resistors with four bands, although 3-band, 5-band, and 6-band codes are also used.

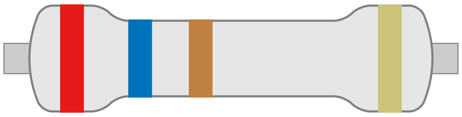


Figure 5: *Color codes for a* 260 Ω *resistor with* 5% *tolerance.*

Figure 5 is a 4-band resistor. To read the code, start from the first color band where the bands are bunched at one end (in this case, the red band). The first two bands represent the first two digits in the value of *R*. The third color band represents a multiplier to yield the final resistance value in ohms. If there is a fourth band present, it is a measure of the tolerance of the resistor, giving the uncertainty in the resistor's printed resistance value.

Table 1: *Resistor color codes (4-band resistors)*

Color	Significant Figures		Multiplier	Tolerance
⬛ black	0	0	×1	
🟫 brown	1	1	×10	±1% (F)
🟥 red	2	2	×100	±2% (G)
🟧 orange	3	3	×1 k	±0.05% (G)
🟨 yellow	4	4	×10 k	±0.02% (P)
🟩 green	5	5	×100 k	±0.5% (D)
🟦 blue	6	6	×1 M	±0.25% (C)
🟪 violet	7	7	×10 M	±0.1% (B)
▨ gray	8	8	×100 M	±0.01% (L)
⬜ white	9	9	×1 G	
🟨 gold			0.1	±5% (J)
▨ silver			0.01	±10% (K)

To read the resistance shown in Fig. 5, refer to Table 1. The first two bands (red and blue) represent the numbers 2 and 6. The third band (brown) represents the multiplier 10. The fourth band (gold) represents a tolerance of 5%. Thus, the resistance is

$$R = 26 \times 10 = 260\,\Omega \pm 5\% = 260\,\Omega \pm 13\,\Omega.$$

Thus, if you connect a digital multimeter to the resistor and measure its resistance, you should expect to read a value in the range of about 247 Ω to 273 Ω.

A Single Resistor: Procedure

Connect your electronic components into a single-resistor circuit as shown in Figure 6. The voltmeter (circled "V") measures the potential difference across the resistor R without interrupting the flow of current. The ammeter (circled "A") measures current and must be placed in series so that all current flows through it.

To wire the circuit: run one wire from the power supply output to one end of the resistor, then from the other end of the resistor into the milliamp (mA) port of the ammeter, and finally from the Common/Ground port of the ammeter back to the power supply ground. Connect the voltmeter across the resistor by tapping one lead into each end. Use a resistor of at least 1000 Ω, selected using the color code in Table 1.

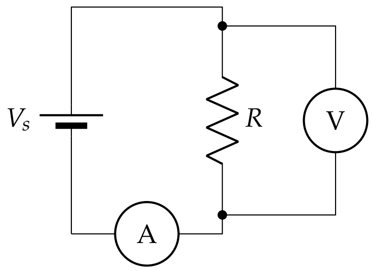


Figure 6: *A single-resistor circuit.*

1. Raise the supply voltage V_s from 0 V to 10 V in steps of 1 V, taking readings of the voltage and current at each step and recording them in Data Table 1.

   Note: if you get negative values for voltage or current, disregard the minus sign and just record the number.

2. Continue measuring until you have 10 data points ranging from 1 V to 10 V.

3. Measure the resistance of R using the ohmmeter capability of the digital multimeter. Record your data in Data Table 1.

Does a Light Bulb Obey Ohm's Law? Procedure

In this section, you will determine whether a common incandescent light bulb obeys Ohm's law. You will set up the light bulb in place of the resistor in the circuit shown in Fig. 6.

CAUTION: after you substitute the light bulb for the resistor, you **MUST** change the ammeter wires to the highest current setting allowed. A failure to do so could blow the fuse in your multimeter, wasting your time while we find a replacement fuse. Ask your TA or TI if you are unsure of what to do.

1. Raise the supply voltage V_s from 0 V to 6 V in steps of 0.6 V, taking readings of the voltage and current at each step and recording them in Data Table 2.

2. You should have 10 pairs of data points ranging from 0 V to 6 V. In the Postlab exercises, explore whether or not there is a linear relationship between V and I for the bulb.

A Voltage Divider with Series Resistors: Procedure

Set up the circuit shown in Fig. 7, using a 220 Ω in series with a 1 kΩ resistor. You will also need to connect the ammeter in series between the resistors and the power supply ground. If you are still using the ammeter's high-current port from your measurements of the I-V curve of the light bulb, switch your leads to use the low-current port. You will measure voltages across each of the two resistors using a voltmeter.

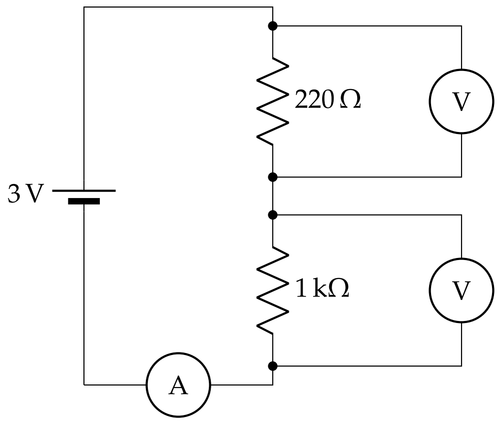


Figure 7: *The circuit used to study a voltage divider.*

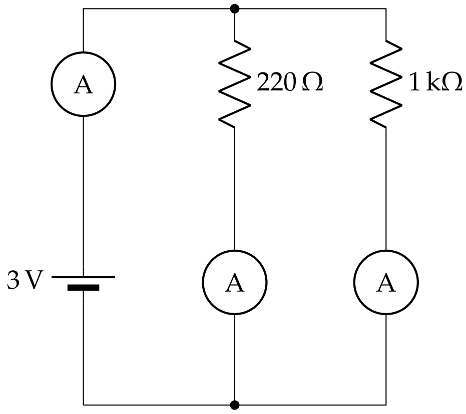


Figure 8: *The circuit used to study a current divider.*

1. Set up your circuit as shown in Fig. 7. Use the resistor color codes in Table 1 to choose a 220 Ω resistor and a 1 kΩ resistor.
2. Set the DC supply voltage to 3 V.
3. Measure the current I_s from the supply and record the information in Data Table 3.
4. Measure the voltage drop across each resistor using the voltmeter and record the result in Data Table 3.
5. Record the actual values of each resistor in Data Table 3.

A Current Divider with Parallel Resistors: Procedure

1. Assemble the circuit shown in Fig. 8.

   Note that if you don't have two ammeters to simultaneously measure the current in each branch, you will need to swap an ammeter between the branches. (**Don't forget to turn off the supply voltage before disconnecting and reconnecting elements in the circuit**.)
2. Set the DC supply voltage to 3 V.
3. Using your current meter(s), measure the current from the supply voltage I_s, the current through the first resistor I_1, and the current through the second resistor I_2. Record your results in Data Table 4.

Superconductivity

Note: students at **ODD**-numbered tables will do superconductivity first and Ohm's law second. Students at **EVEN**-numbered tables should do Ohm's law first, and this section second.

Safety Precautions

In this measurement, you will be handling liquid nitrogen, which boils at 77 K (−196.15 °C). The liquid nitrogen can cause severe "burns" due to its low temperature, so precautions must be taken. Read the following instructions carefully and follow all safety procedures. If you are unsure how to follow the safety procedures while doing this lab, please ask the lab TAs and TIs for help. **Those who do not follow the precautions will be removed from the lab and will not receive credit for the experiment.**

1. Wear safety glasses at all times in the presence of liquid nitrogen.
2. Do not allow any liquid nitrogen to touch your body.
3. Do not touch any items that have been immersed in liquid nitrogen until they warm to room temperature.

4. Keep liquid nitrogen away from water – it will flash boil and splash onto you.
5. Be very careful to avoid overfilling or spilling liquid nitrogen from its container.
6. Never store liquid nitrogen in a container with a tight-fitting lid. Pressure build-up from evaporating nitrogen can cause the container to explode.
7. The superconductors are made of metal oxides that may be toxic, so only handle them with the tweezers provided. Never touch the superconductors with your bare hands, and wash your hands with soap and water after completing the experiment.

Experimental Setup and Measurement Procedure

The apparatus for exploring the Meissner effect using a YBCO superconductor is shown in Figure 9. It consists of a foam bath with 9 terminals for connecting power supplies and measurement devices. Refer to the figure when making all connections.

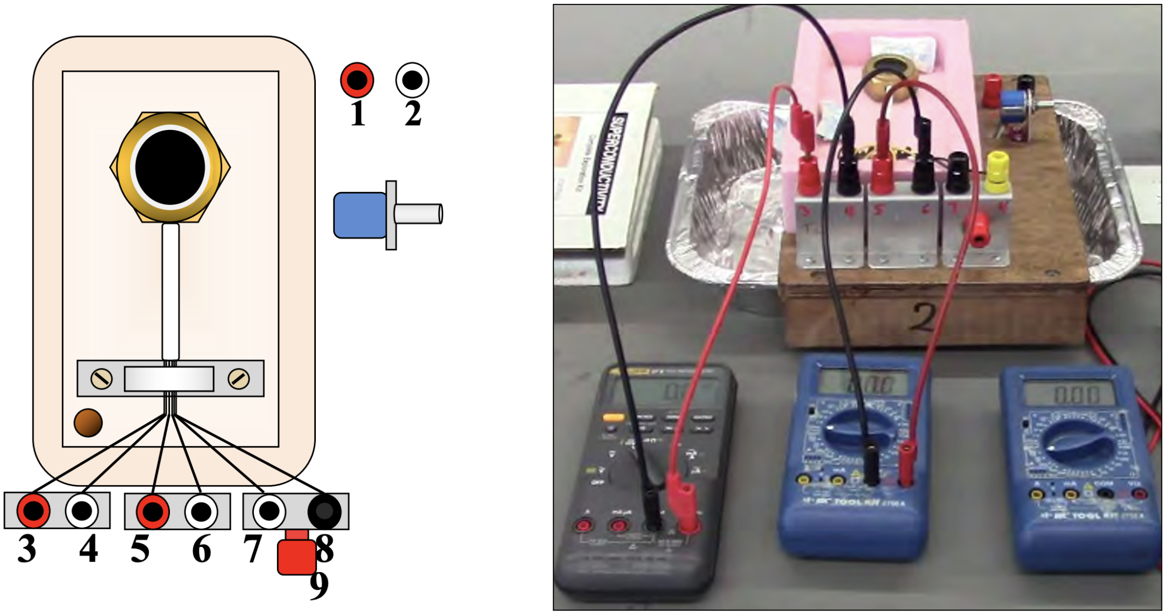


Figure 9: *Top-down view of the apparatus for studying the Meissner effect, with connection terminals labeled (left); photograph of the wired apparatus (right).*

1. Set up a multimeter to measure DC voltage in the range 20 mV and attach leads to terminals 3 and 4.
2. Set up a second multimeter to measure DC voltage in the 2 mV range and attach the leads to terminals 5 and 6.
3. Set up third multimeter to measure current in the 10 A range. Do not use the 200 mA setting, or you could blow the fuse of the multimeter.
4. Wire the terminals of a 6 V battery to leads 1 and 2. Alternatively, set the DC power supply to 6V and wire the power supply's terminals to leads 1 and 2.
5. Use the potentiometer (the blue box with a metal twist throttle located below leads 1 and 2) to set the current to a maximum of 0.3 A. Adjust the variable resistance as needed and make note of your final value in Data Table 5 in the Postlab exercises.
6. **Cooling**: Make sure the cork is secured in the foam and the aluminum foil tray is under the drainage pipe. Pour liquid nitrogen over the superconductor to bring the temperature below the critical temperature.

7. You will measure its temperature with the voltmeter connected to leads 3 and 4, which will measure the superconductor's thermal conductivity. As the superconductor cools, the conductivity should go from about 0 mV at room temperature to about 6 mV when it is in equilibrium with the liquid nitrogen. When the multimeter connected to leads 5 and 6 reads 0.0 mV, the superconductor is sufficiently cooled.
8. Now, using a pair of tweezers, pick up one of the cubic magnets provided and place it on top of the superconductor disk. Try gently spinning the magnet with the tweezers. Observe whether the magnet is touching the surface of the disk, remembering to keep your fingers and your face from getting too close to the liquid nitrogen bath. Leave the cubic magnet there and continue to the next step.
9. When you are ready to start taking data, use a cell phone camera to start a VIDEO RECORDING containing the values of all 3 meters. Then **USE THE TWEEZERS** to remove the cork so the liquid nitrogen drains from the foam bath into the aluminum tray. Be mindful not to accidentally touch the tray, which will get cold.
10. Watch the Fluke meter's voltage, which is the thermal conductivity. If the thermal conductivity does not decrease after about a minute, aim an incandescent light bulb at the apparatus to speed up the warming process. You may also start using the hair dryer on a low setting to warm up the system.
11. As the superconductor warms, keep recording the 3 meters' values. Also, watch for the levitating magnet to fall and make a record (for instance by waving your hand in front of the camera) so you can record the temperature where this occurs.
12. Once the thermal conductivity drops below 4 mV, stop the video recording. Disconnect the battery and various meters from the circuit. Use a hairdryer on the "low" setting to evaporate any remaining liquid nitrogen in the foil tray, maintaining a safe distance to avoid splashing any liquid nitrogen.
13. **Recording Measurements** Make recordings of the thermal conductivity (leads 3 and 4) and potential (leads 5 and 6) in Data Table 5. Your first row of measurements should be the highest thermal conductivity measured, around 6mV. After the thermal conductivity drops by 0.2 mV, record all three meters' data in Data Table 5. Keep recording rows of data until the thermal conductivity falls below 4 mV. Make a mark next to the row where the magnet fell.

Analysis of Critical Temperature

1. Fill in the second column of Data Table 5 by converting the thermal coupling values to temperature using Table 2. Interpolate temperatures if your measurements are between the values in the table.
2. Use Ohm's law to fill in column 5 of Data Table 5 and plot the two shaded columns as instructed in the Postlab exercises.

Name: ______________________________ **Date:** ____________________

Collaborators: ______________________________ **Lab Section:** ____________

POSTLAB EXERCISES (20 points)

Submit the postlab to the TA at the end of the lab.

Ohm's Law (15 points)

Question 3 **½ point**

Enter the single-resistor data into Data Table 1.

Data table 1: *Single resistor data.*

$R =$ ____________________

Power Supply Setting	**Measured Voltage** (V)	**Measured Current** (A)

Question 4 **½ point**

Enter the light bulb data into Data Table 2.

Data table 2: *Bulb data.*

Power Supply Setting	**Measured Voltage** (V)	**Measured Current** (A)

Question 5 **½ point**

Enter the series resistor data into Data Table 3.

Data table 3: *Series resistor data.*

R_1	
R_2	
V_s	
I_s	
V_1	
V_2	

Question 6 **½ point**

Enter the parallel resistor data into Data Table 4.

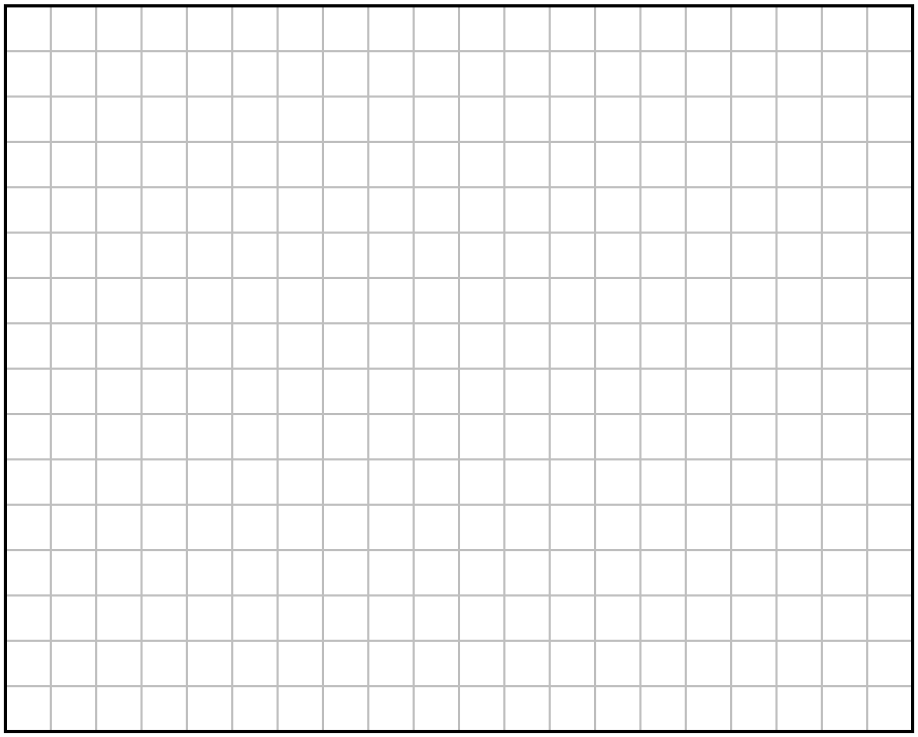
Data table 4: *Parallel resistor data.*

R_1	
R_2	
V_s	
I_s	
I_1	
I_2	

Question 7 **2 points**

Single resistor analysis: take the single resistor data from Data Table 1 and plot voltage versus current in Graph 1, including labels and units. Is the relationship linear? If so, draw the best-fit line.

Graph 1: V *vs.* I *for the single resistor.*



Question 8 **1 point**

Calculate the slope of the best-fit line in Graph 1 and use it to estimate the value of R. Compare this to the value labeled on the actual resistor and the value you measured from the multimeter.

Question 9 **1 point**

Calculate the power dissipated in the resistor when $V = 6.0\,\mathrm{V}$.

Question 10 **2 points**

Single bulb analysis: take the single resistor data from Data Table 2 and plot voltage versus current in Graph 2, including labels and units.

Graph 2: V *vs.* I *for the light bulb.*

Question 11 **1 point**

Is there a linear relationship between voltage and current in Graph 2, compared to the linearity of the single resistor in Graph 1? (It should not be!)

Question 12 **1 point**

Calculate the resistance of the filament when $V = 6.0\,\mathrm{V}$ and when $V = 1.0\,\mathrm{V}$. Note that a non-linear function does not have a constant slope! To calculate the resistance at a certain voltage, draw a line tangent to the curve at that voltage. Then calculate the slope of the tangent line to find the instantaneous resistance.

Question 13 **1 point**

Intuitively, do you think the temperature of the filament is higher at $V = 6.0\,\mathrm{V}$ and $V = 1.0\,\mathrm{V}$? How does this information compare with what you learned about metal superconductors as the temperature decreases?

Question 14 **1 point**

Series resistance analysis: Use eq. (4) and verify if your measured V_1 is comparable to the expected value. Show all your work.

Question 15 **1 point**

From the value of I_s and V_s, calculate the equivalent or total resistance of the series resistor circuit. Compare this with the theoretical value using eq. (2).

Question 16 **1 point**

Parallel resistance analysis: use eq. (5) and verify if your measured I_1 is comparable to the expected value. Show all your work.

Question 17 **1 point**

From the value of I_s and V_s, calculate the equivalent or total resistance of the parallel resistor circuit. Compare this with the theoretical value using eq. (3).

Superconductivity (5 points)

Question 18 **2 points**

Fill in the measurements of the superconducting transition made as the superconductor is allowed to warm up. Use Table 2 to convert the measured thermal conductivity κ in mV to superconductor temperature T in K. Use Ohm's law to convert voltage and current measurements to resistance.

Data table 5: *Superconductor Temperature Data.*

Thermal Conductivity (mV)	**Temperature** (K)	**Voltage** (V)	**Current** (A)	**Resistance** (Ω)

Table 2: *Conversion of thermal conductivity κ to superconductor temperature T.*

κ (mV)	T (K)	κ (mV)	T (K)	κ (mV)	T (K)	κ (mV)	T (K)	κ (mV)	T (K)	κ (mV)	T (K)	κ (mV)	T (K)
6.92	70	6.29	80	5.9	90	5.52	100	5.16	110	4.81	120	4.46	130
6.85	71	6.25	81	5.86	91	5.48	101	5.13	111	4.77	121	4.42	131
6.78	72	6.21	82	5.83	92	5.44	102	5.09	112	4.74	122	4.39	132
6.71	73	6.17	83	5.79	93	5.41	103	5.06	113	4.70	123	4.35	133
6.64	74	6.13	84	5.75	94	5.37	104	5.02	114	4.67	124	4.32	134
6.56	75	6.09	85	5.72	95	5.34	105	4.99	115	4.63	125	4.28	135
6.49	76	6.05	86	5.68	96	5.30	106	4.95	116	4.60	126	4.25	136
6.42	77	6.01	87	5.64	97	5.27	107	4.91	117	4.56	127	4.21	137
6.37	78	5.97	88	5.6	98	5.23	108	4.88	118	4.53	128	4.18	138
6.33	79	5.93	89	5.56	99	5.20	109	4.84	119	4.49	129	4.14	139

Question 19 **3 points**

Take your measurements from Data Table 5 and plot of resistance versus temperature in Graph 3. Include labels and units. Do not draw a line of best fit. Mark on your plot the temperature where the levitating magnet fell. What is the critical temperature T_C of the superconductor?

Graph 3: *R vs. T for the Superconductor.*