



University of Rochester

Laboratory X **Electronic Circuits**

DEPARTMENT OF PHYSICS & ASTRONOMY

PHYSICS 122 - 142 - 182
ELECTRICITY AND MAGNETISM

Name: _____ Date: _____

Collaborators: _____ Lab Section: _____

PRELAB EXERCISES (2 points)

This prelab must be completed and handed in to the lab TA at the start of the lab.

Question 1

1 point

How will you determine the capacitance of the unknown capacitor using the oscilloscope in the first part of the experiment? Explain.

Question 2

1 point

In theory, what should be the slope of the graph you will make of your data when you plot I/Q versus resistance in the second part of the experiment? What value should the y -intercept have?

Objective

The purpose of this lab is to study the concepts of capacitance, resistance, and inductance in real circuits, to observe electrical resonance in an RLC circuit, and to develop proficiency with the oscilloscope.

Theory

You will study RC circuits and then resonant RLC circuits, using an oscilloscope — the most versatile electronic measuring instrument — to investigate the characteristics of capacitors and resonant circuits.

RC Circuits: Charging a Capacitor

A capacitor, a device typically made of two conducting plates, stores electric charge. Each plate will have a charge of the same magnitude, q , but of opposite sign, creating an electric field and a potential difference, or voltage V_C , between the plates. The charge stored on the capacitor is proportional to the voltage across it, and the constant of proportionality C is called the capacitance¹:

$$q = CV_C. \quad (1)$$

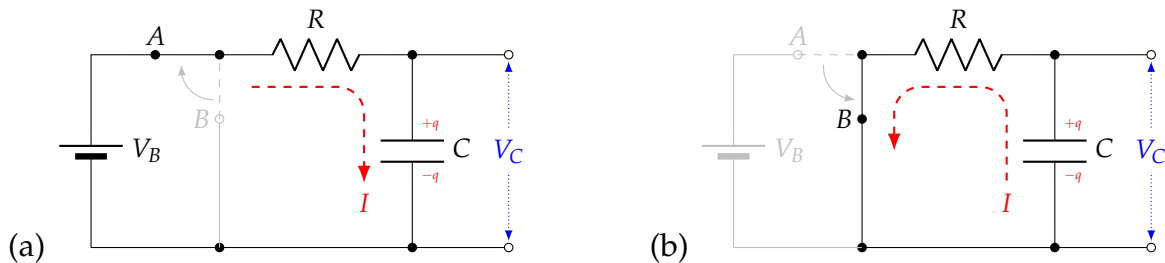


Figure 1: A series RC circuit with battery V_B , resistor R , capacitor C , and a switch. (a) When the switch is closed to point A, current I flows from the battery through R , **charging** the capacitor C . (b) When the switch is closed to point B, the battery is removed from the circuit and I is reversed, **discharging** the capacitor C through the resistor.

Figure 1a shows a battery with voltage V_B in series with a resistor and capacitor. When the switch to the battery is closed to point A, current flows through the resistor and capacitor, storing charge on the capacitor. By Kirchhoff's loop rule, the voltages across the circuit must sum to zero:

$$\begin{aligned} V_B - V_R - V_C &= 0 \\ V_B - IR - \frac{q}{C} &= 0, && \text{using Ohm's law and eq. (1),} \\ V_B - R \frac{dq}{dt} - \frac{q}{C} &= 0, && \text{using } I \equiv dq/dt. \end{aligned} \quad (2)$$

The variable quantities in eq. (2) are charge q and time t . The equation can be solved by "separation of variables," isolating q and t on opposite sides of the equation and integrating:

$$\begin{aligned} \frac{dq}{dt} &= \frac{CV_B - q}{RC} \\ \int_0^q \frac{dq'}{CV_B - q'} &= \int_0^t \frac{dt'}{RC}, && \text{separating variables,} \\ -\ln\left(\frac{CV_B - q}{V_B C}\right) &= \frac{t}{RC}, && \text{integrating,} \\ \frac{CV_B - q}{CV_B} &= e^{-t/RC}, && \text{exponentiating both sides.} \end{aligned} \quad (3)$$

¹The unit of capacitance is called the Farad (F). In consumer electronics, most capacitors have $C \leq 1$ nF (1 nanofarad).

Solving eq. (3) for q gives the charge as a function of time:

$$q(t) = Q \left(1 - e^{-t/\tau}\right), \quad (4)$$

where $Q = CV_B$ is the final amount of charge stored in the capacitor, and $\tau = RC$ is the time constant of the RC circuit, in dimensions of seconds. For example, if $R = 1 \text{ k}\Omega$ and $C = 1 \text{ nF}$, then

$$\tau = (1 \text{ k}\Omega)(1 \text{ nF}) = 1 \times 10^{-6} \text{ s} = 1 \mu\text{s}.$$

The current as a function of time is found by differentiating eq. (4):

$$I(t) = \frac{dq}{dt} = \frac{V_B}{R} e^{-t/RC} = I_0 e^{-t/\tau}. \quad (5)$$

The current starts large, with $I(0) = V_B/R$ by Ohm's law, and exponentially decays to zero as the capacitor reaches its final charge $Q = CV_B$. The behavior of $q(t)$ and $I(t)$ is plotted in Fig. 2.

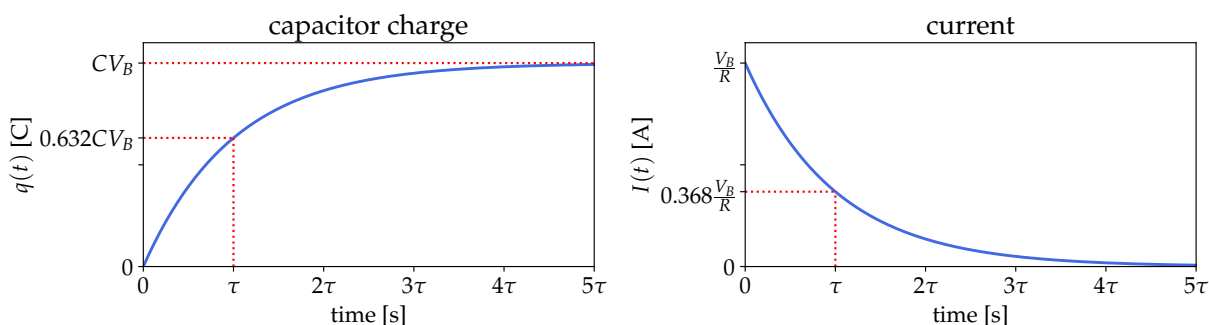


Figure 2: Charge on the capacitor (left) and current in the circuit (right) vs. time for the charging capacitor (Fig. 1a).

Combining eqs. (1) and (4) gives the voltage on the charging capacitor as a function of time:

$$V_C(t) = V_B \left(1 - e^{-t/\tau}\right). \quad (6)$$

RC Circuits: Discharging a Capacitor

Once we have charged the capacitor up to Q , suppose we disconnect the battery by closing the switch to point B , as shown in Figure 1b. In this case, the current will run in the opposite direction as the capacitor discharges across the resistor. By Kirchhoff's loop rule,

$$\begin{aligned} -V_R - V_C &= 0 \\ -IR - \frac{q}{C} &= 0, && \text{using Ohm's law and eq. (1),} \\ -R \frac{dq}{dt} - \frac{q}{C} &= 0, && \text{using } I \equiv dq/dt. \end{aligned} \quad (7)$$

Separating variables and solving for q gives

$$q(t) = Q e^{-t/\tau}, \quad (8)$$

$$I(t) = \frac{dq}{dt} = -\frac{Q}{RC} e^{-t/\tau}. \quad (9)$$

Both the charge on C and the current in the circuit exponentially decay to zero. The negative sign in the current indicates that it reverses direction through the resistor when the capacitor discharges. The behavior of $q(t)$ and $I(t)$ is plotted in Fig. 3.

Combining eqs. (1) and (8) gives the voltage on the discharging capacitor as a function of time:

$$V_C(t) = V_B e^{-t/\tau}. \quad (10)$$

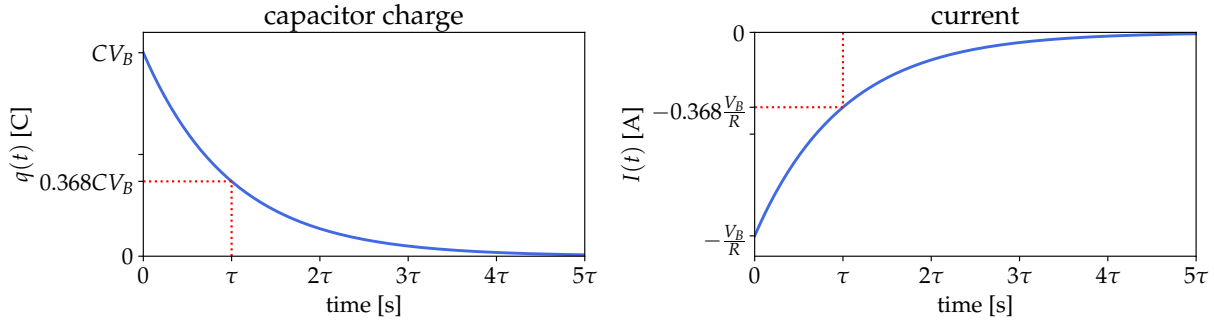


Figure 3: Charge on the capacitor (left) and current in the circuit (right) vs. time for the discharging capacitor (Fig. 1b). Note that the current still exponentially decays to zero, but with the opposite sign.

Continuous Charging and Discharging with a Square Wave

Suppose we replace the battery in the circuit with a function generator, as shown in Figure 4.

We can set the generator to drive the RC circuit with a square wave of period T , which has the functional form

$$V_s(t) = \begin{cases} V_s, & 0 < t \leq \frac{T}{2}, \\ 0, & \frac{T}{2} < t \leq T. \end{cases} \quad (11)$$

The square wave provides a DC voltage V_s for the first half of the period, and 0 V for the second half of the period. The capacitor charges up when the square wave input is nonzero, and then discharges through the resistor when the input is 0 V.

The behavior is identical to the RC circuit shown in Fig. 1, with the capacitor charging and discharging as if we were flipping the mechanical switch back and forth between points A and B. However, in this case, rather than manually connecting and disconnecting a DC source to the RC circuit, an oscillator in the function generator flips the voltage on and off automatically.

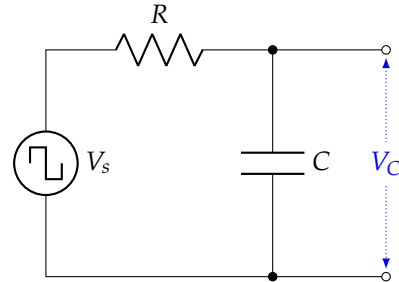


Figure 4: RC circuit driven by a square wave from a function generator.

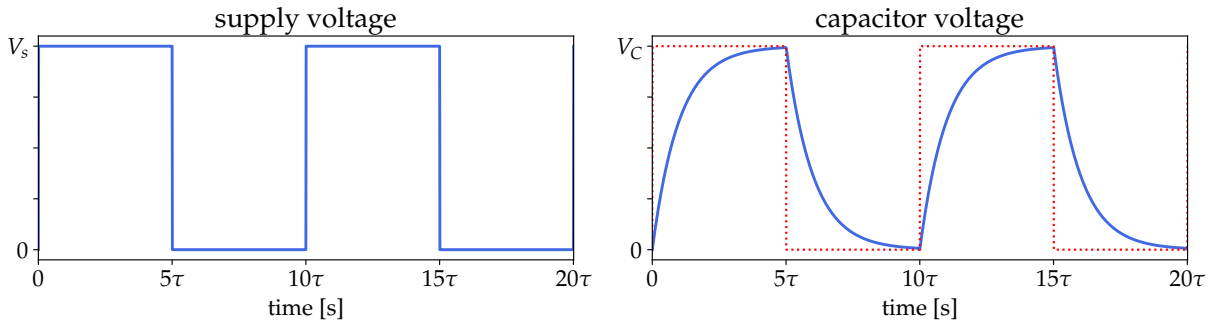


Figure 5: Left: the input voltage V_s of the square wave given in eq. (11). Right: the capacitor voltage V_C over time as it is charged and discharged by the square wave input.

Figure 5 shows a square wave with period $T = 10\tau = 10RC$ and the corresponding voltage across the capacitor as it repeatedly charges and discharges.

The RLC Circuit: A Damped Resonant Oscillator

Suppose that we connect a resistor R , an inductor L , and a capacitor C in series and drive them with a sinusoidal voltage from a function generator, as in Figure 6a. How does the circuit behave?

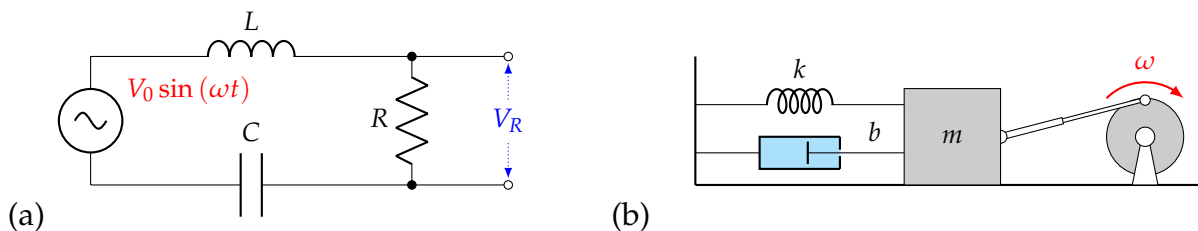


Figure 6: (a) A series RLC circuit driven by a sinusoidal voltage $V(t) = V_0 \sin(\omega t)$ from a function generator. (b) A mechanical analog of the driven RLC circuit, where a mass and spring with a damper are driven on a frictionless surface by a sinusoidal force $F(t) = F_0 \sin(\omega t)$ from a motor turning at speed ω .

Start with Kirchhoff's loop rule and sum the voltages around the loop to zero:

$$L \frac{dI}{dt} + IR + \frac{q}{C} = V_0 \sin(\omega t). \quad (12)$$

Since $I = dq/dt$, we can rewrite this as a second-order differential equation in q :

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = V_0 \sin(\omega t). \quad (13)$$

Since the behavior of the system may not be obvious at first glance, consider an analogous mechanical system shown in Fig. 6b. A mass m is attached to a spring with constant k and a damper² with damping coefficient b . A rod attached to a rotating crankshaft applies a sinusoidal force to the mass, causing it to oscillate. Summing the forces on the mass gives

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = F_0 \sin(\omega t). \quad (14)$$

This is an oscillator with a **mechanical resonance**: when the frequency of the driving motor ω matches the natural frequency of the mass-spring system $\omega_0 = \sqrt{k/m}$, the amplitude of the oscillations of the mass m reaches a maximum. When the motor is turned off, the damping cylinder quickly damps out the oscillations. Verify that with a suitable change of variables, eqs. (13) and (14) are identical. Thus, the RLC circuit must also be a damped oscillator with a resonance.

The steady-state solution to eq. (12) is

$$I(t) = \frac{V_0}{Z} \cos(\omega t + \varphi), \quad (15)$$

where

$$Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2} \quad \text{is the circuit's } \mathbf{impedance}, \text{ and} \quad (16)$$

$$\tan \varphi = \frac{\omega L - 1/(\omega C)}{R} \quad \text{is the } \mathbf{phase\ angle} \text{ of the current.} \quad (17)$$

You will be measuring the voltage across R , given by Ohm's law:

$$V_R(t) = I(t)R = \frac{V_0 R}{\sqrt{R^2 + (\omega L - 1/(\omega C))^2}} \cos(\omega t + \varphi). \quad (18)$$

²The damper is just a fluid-filled cylinder with a piston that resists the motion of the mass with a force proportional to its velocity.

Notice that the amplitude of $V_R(t)$ is a function of the driving frequency of the function generator ω . Moreover, the amplitude reaches a maximum at the frequency ω_0 where

$$\omega_0 L - 1/(\omega_0 C) = 0,$$

since the denominator is minimized when this term vanishes. Solving for ω_0 gives

$$\omega_0 = 2\pi f_0 = \sqrt{\frac{1}{LC}}. \quad (19)$$

This is the resonant frequency of the RLC circuit.

Q Factor of the Resonant Circuit

In the RLC circuit, L and C produce oscillations while the resistance dissipates energy and damps the oscillations, much like the damper in the mechanical system of Fig. 6b. If we turn off the driving voltage, the time it takes for the oscillations to die down depends on the size of R relative to L and C . If R is small, relatively speaking, the oscillations could take quite some time to stop after the driving voltage is removed.

Figure 7 shows the **response curve** of the RLC circuit as a function of driving frequency, as measured by the amplitude of the voltage across the resistor, V_R . As you can see, the amplitude peaks at $\omega_0 = \sqrt{1/LC}$. The width of the response curve in the vicinity of ω_0 , denoted $\Delta\omega$, is measured where the amplitude reaches half of the peak power at the resistor. The peak power in the resistor is

$$P_{\max} = \frac{V_{\max}^2}{R},$$

so half the peak power corresponds to a voltage amplitude of

$$V_{\max}/\sqrt{2} \approx 0.708V_{\max}.$$

Thus, $\Delta\omega$ is sometimes called the full width at half-maximum (FWHM) of the response curve. The ratio of ω_0 to $\Delta\omega$ is called the **Q-factor**³:

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f} = \frac{1}{R} \sqrt{\frac{L}{C}}. \quad (20)$$

You can express Q in terms of frequency f or angular frequency ω . (In the experiment, you will measure frequencies f .)

Physically, Q represents the number of oscillations it takes for the oscillator to die down once the driving voltage is switched off. For example, the response curve in Fig. 7 corresponds to an RLC circuit with $Q = 3$, meaning that if we switch off the function generator, the oscillations in V_R will die out after about 3 oscillations.

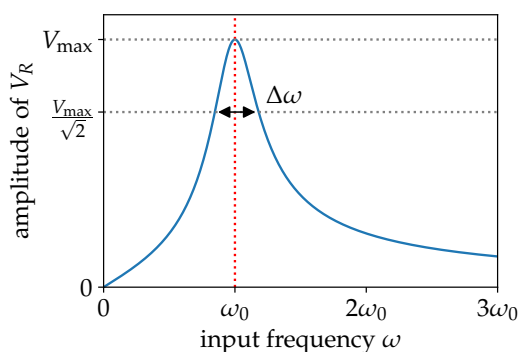


Figure 7: The response curve of an RLC circuit driven at frequencies ω around the resonance frequency ω_0 . The circuit has $Q = 3$; see eq. (20).

³The metric Q can be defined for any oscillating system driven to resonance, including buildings and bridges under wind loads, acoustic instruments, laser cavities, etc.

Experiment

Observe RC Charging on the Oscilloscope

In the first part of the experiment, you will connect the function generator, the resistor decade box, and the capacitor decade box in series to make a closed circuit like the one shown in Figure 8.

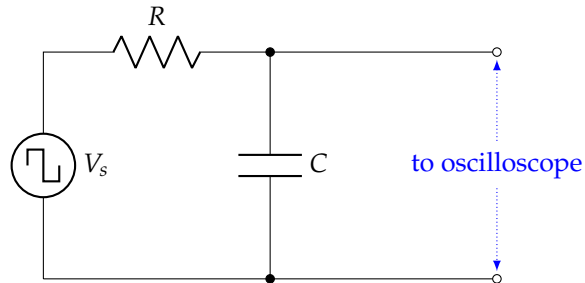


Figure 8: RC circuit driven by a square wave from a function generator.

Procedure

1. Set the decade boxes to the values $R = 3000 \Omega$, $C = 0.01 \mu\text{F}$ and the signal generator to output a square wave with frequency $f = 3000 \text{ Hz}$ and amplitude 3 V .
2. Adjust the oscilloscope so the whole display is used and draw the trace you observe on Graph 1 in the Postlab exercises. Determine both the charging and discharging time constant by measuring the time taken for the voltage to change from its peak value to $1/e = 0.368$ of its final value.
3. Average the charging time constant and the discharging time constant, and compare with the theoretical value of $\tau = RC$.
4. Change the value of either R or C by 30% and repeat the previous steps, plotting the scope trace in Graph 2 in the Postlab exercises.
5. Get an unknown capacitor from the TA and substitute it for the capacitance decade box. Adjust the oscilloscope for a convenient display, draw the trace you observe in Graph 3 in the Postlab exercises, and measure the time constant τ , leaving R fixed. Use the result to obtain an experimental value for the capacitor, C .
6. Get an unknown resistor from the TA and substitute it for the resistance decade box. Plot the scope trace in Graph 4 in the Postlab exercises, determine τ , and using τ and the known value of C , determine the unknown value of R .
7. Turn off the oscilloscope and disconnect all cables.

Resonant Frequency and Response Curve of an RLC Circuit

In the second part of the experiment, you will set up the series RLC circuit with the function generator and oscilloscope as shown in Figure 9.

Procedure

1. Assemble the circuit shown in Figure 9 using the values indicated for inductance, resistance, and capacitance.

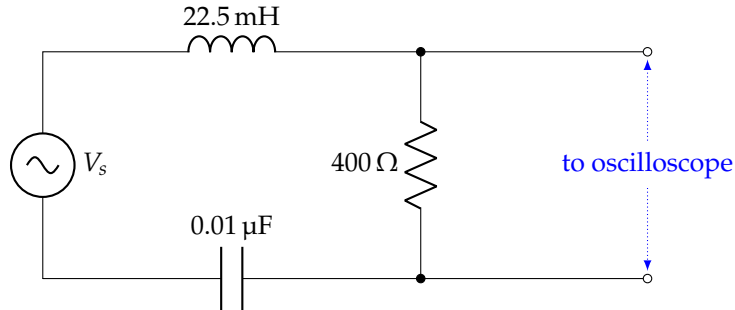


Figure 9: RLC circuit driven by a sinusoidal wave from a function generator.

2. Calculate the resonant frequency f_0 (not the angular frequency ω_0) in Hz. Set the signal generator to a frequency near f_0 with a voltage amplitude of about $V_s = 3\text{ V}$.
 Note that you can set the signal generator output precisely by unhooking the RLC circuit from the generator and directing the output only to the oscilloscope. Set the oscilloscope to the **AC Volts** setting, with the sensitivity set to the 20 V setting. The AC Volts setting is the one with a symbol that looks like $V \sim$.
3. With the RLC circuit set up, measure the voltage across the resistor V_R . Now vary the frequency f from the function generator to find the maximum voltage (it will likely not match the full 3 V you are inputting from the signal generator), and record the frequency of the maximum $f_{0,\text{meas}}$. This frequency will probably be a little different from your calculated f_0 .
4. Take many measurements – at least four on each side of $f_{0,\text{meas}}$ – of V_R in the vicinity of the resonant frequency $f_{0,\text{meas}}$, ensuring that you tune the signal generator through a broad enough range of frequencies that you observe V_R drop off by at least one-third on each side from the peak voltage. Record your data in Data Table 1 in the Postlab exercises.
5. Set $R = 200\ \Omega$ and repeat all your measurements, recording your data in Data Table 1.
6. Set $R = 100\ \Omega$ and repeat all your measurements, recording your data in Data Table 1.

Name: _____ Date: _____

Collaborators: _____ Lab Section: _____

POSTLAB EXERCISES (20 points)

Submit the postlab to the TA at the end of the lab.

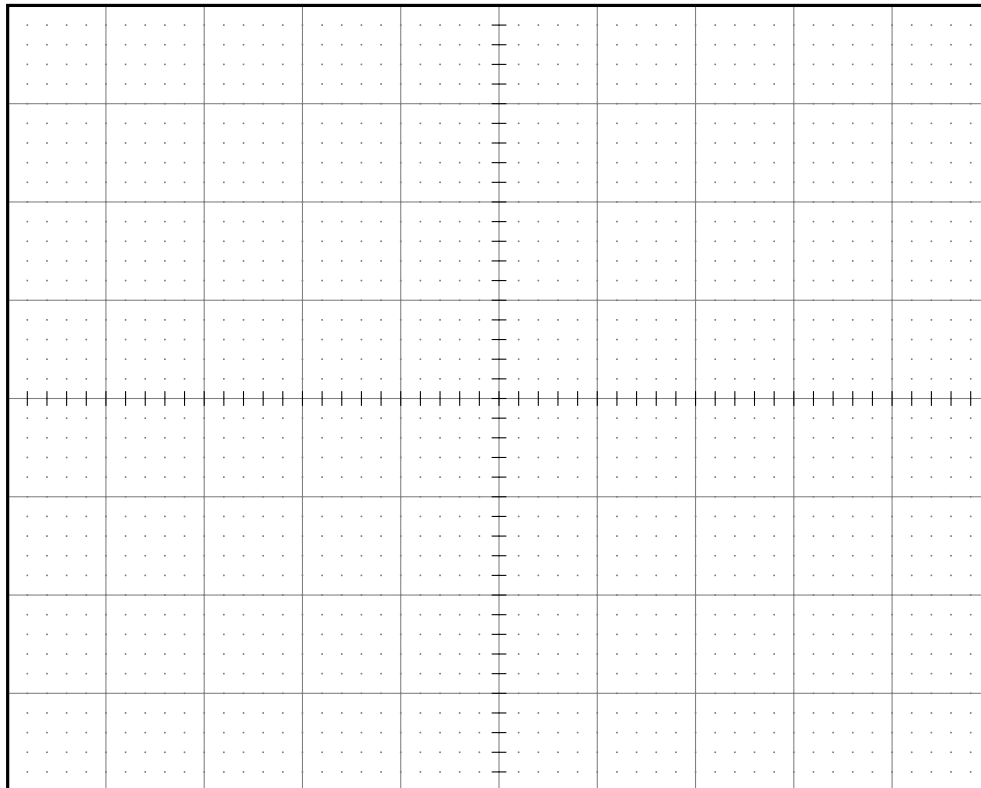
RC Circuit (9 points)

Question 3

2 points

Known RC values: After setting up the RC circuit with $R = 3000 \Omega$ and $C = 0.01 \mu\text{F}$, sketch the pattern you see on the oscilloscope in Graph 1. Carefully note the value of 1 division of the time axis.

Graph 1: RC charge and discharge pattern for $R = 3000 \Omega$ and $C = 0.01 \mu\text{F}$.



Compute the expected value of τ below, and use the plot to estimate the time constant.

$R =$ _____ $C =$ _____ $RC =$ _____

Time of charging to within $1/e$ of max. value of V_C _____

Time of discharging to within $1/e$ of min. value of V_C _____

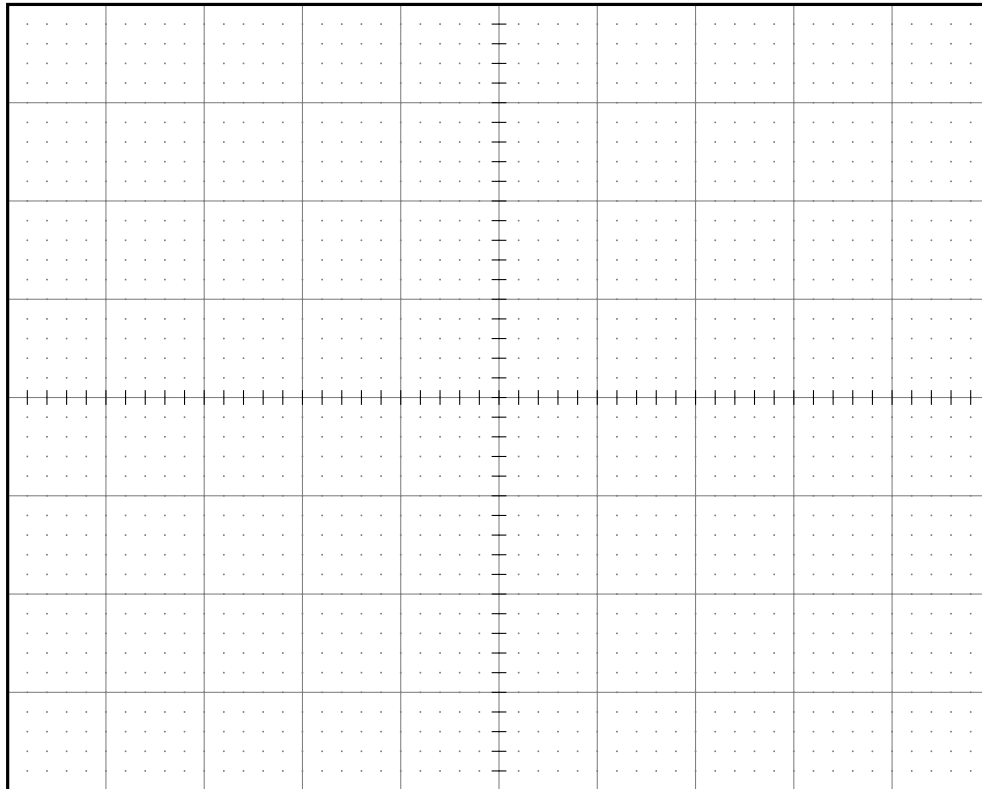
Average _____

Question 4

1 point

30% values data: change either R or C by 30% and sketch the resulting pattern in Graph 2.

Graph 2: RC charge and discharge pattern for R or C adjusted by 30%.



Compute the expected value of τ below, and use the plot to estimate the time constant.

$R =$ _____ $C =$ _____ $RC =$ _____

Time of charging to within $1/e$ of max. value of V_C _____

Time of discharging to within $1/e$ of min. value of V_C _____

Average _____

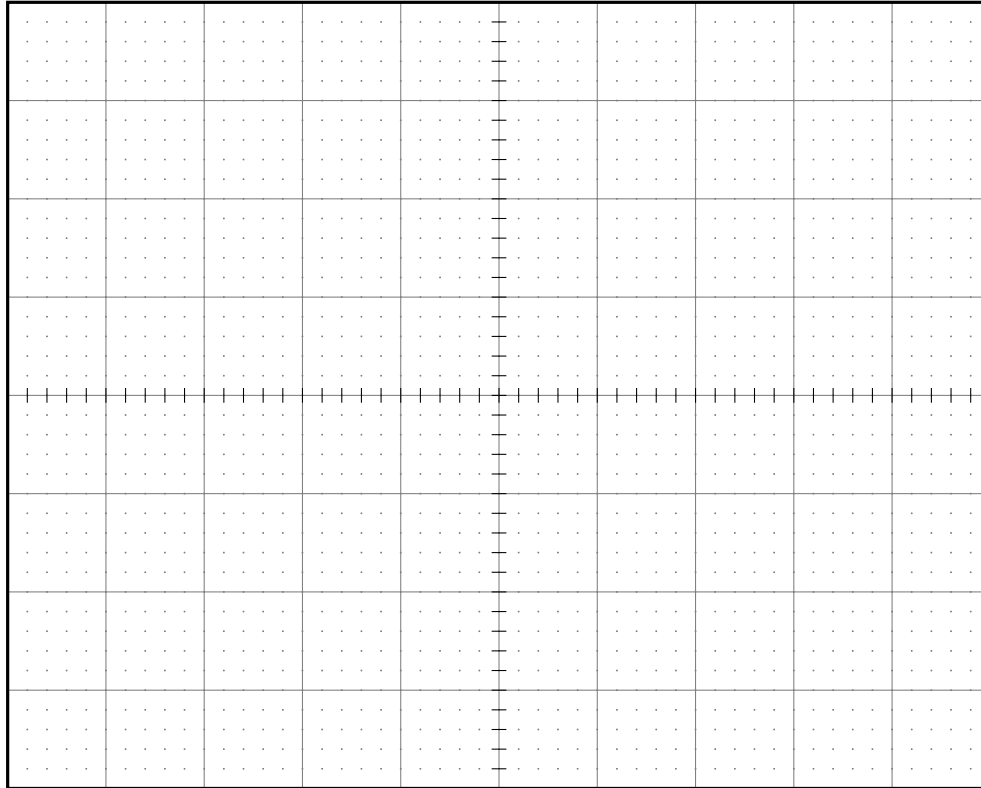
Question 5

1 point

Unknown capacitance data:

(a) (1/2 point) Swap in an unknown C and sketch the resulting pattern in Graph 3.

Graph 3: RC charge and discharge pattern for unknown capacitance C .



$R =$ _____ $C =$ **unknown** $RC =$ **unknown** _____
 Time of charging to within $1/e$ of max. value of V_C _____
 Time of discharging to within $1/e$ of min. value of V_C _____
Average _____

(b) (1/2 point) Calculate the unknown capacitance from your measured value of RC .

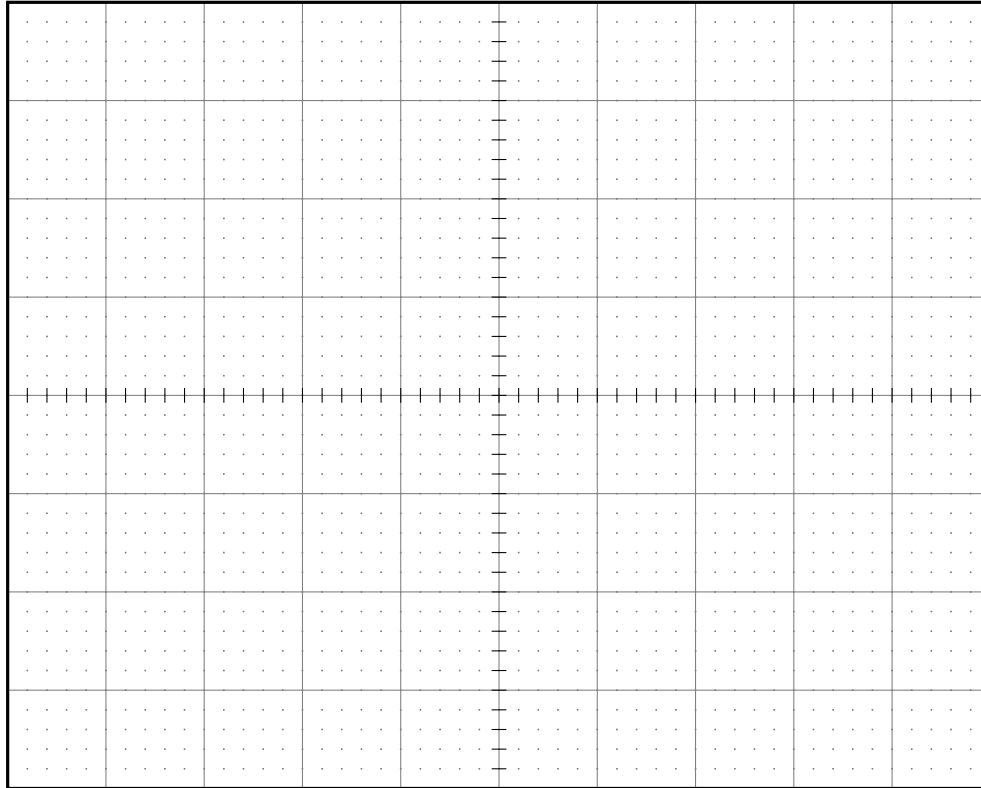
Question 6

1 point

Unknown resistance data:

(a) ($1/2$ point) Swap in an unknown R and sketch the resulting pattern in Graph 4.

Graph 4: *RC charge and discharge pattern for unknown resistor R.*



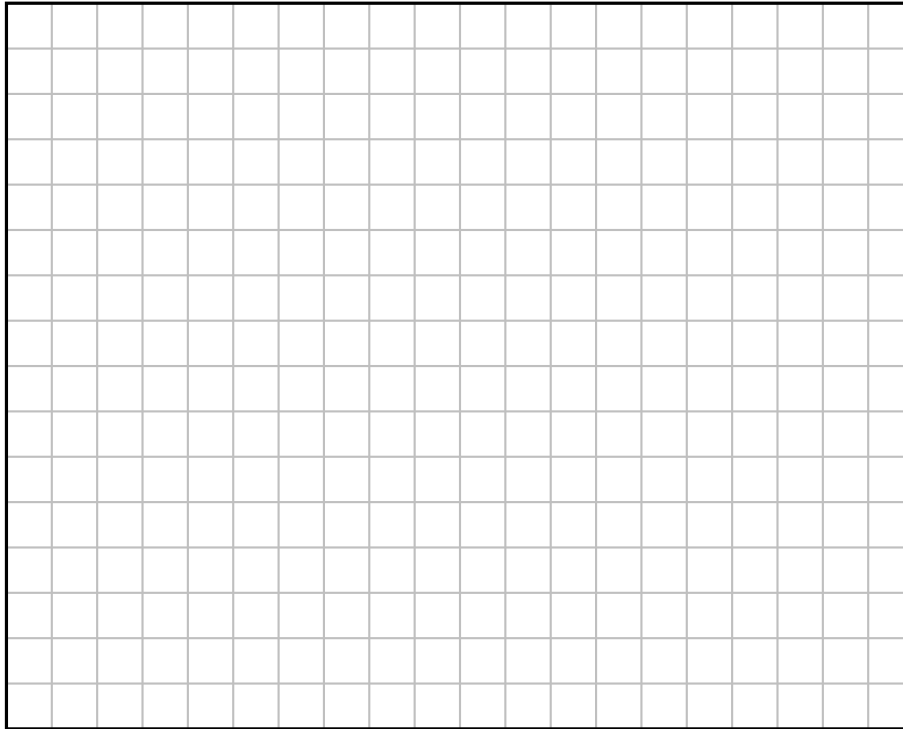
$R =$ unknown $C =$ _____ $RC =$ unknown
 Time of charging to within $1/e$ of max. value of V_C _____
 Time of discharging to within $1/e$ of min. value of V_C _____
 Average _____

(b) ($1/2$ point) Calculate the unknown resistance from your measured value of RC .

Question 10**3 points**

Plot resistor voltage V_R versus frequency f for all three values of R in Graph 5, using solid circles for $R = 400 \Omega$, hollow squares for $R = 200 \Omega$, and small x 's for $R = 100 \Omega$. Interpolate between the points in each separate set to produce smooth curves. You should find that $R = 400 \Omega$ produces the broadest response.

Graph 5: *Response function of the RLC circuit.*

**Question 11****1 point**

How do the resonant frequencies compare in the three response functions? Is this expected? Explain.

Question 12**1 point**

From your plots, measure the Q factor for each of the three curves, showing your work for at least one of the three values. Use $Q = f_0/\Delta f$ from eq. (20) to compute Q .

Question 13**1 point**

Calculate Q for each of the three values of R used ($100\ \Omega$, $200\ \Omega$, and $400\ \Omega$) using

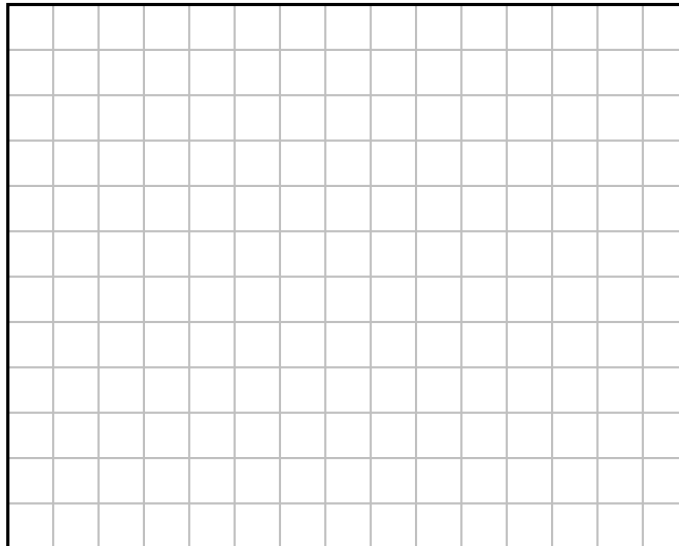
$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

from eq. (20). Show your work for at least one of the three calculations.

Question 14**2 points**

Using the values of Q that you measured from the plots of the response function, plot $1/Q$ versus R in Graph 6 below.

Graph 6: Plot of $1/Q$ versus R .



Does the plot of $1/Q$ versus R look like a straight line, as you would expect from eq. (20)? Draw a best-fit line and measure the slope. How close is your slope to its expected value?