University of Rochester

Laboratory XI
The Speed of Waves

Department of Physics & Astronomy

PHYSICS 123 - 183

Waves and Modern Physics

Name: ______________________ **Date:** ______________________

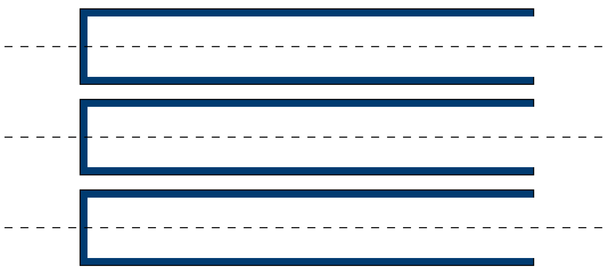
Collaborators: ______________________ **Lab Section:** ______________

PRELAB EXERCISES (2 points)

This prelab must be completed and handed in to the lab TA at the start of the lab.

Question 1 **1 point**

Sound within a hollow tube can form a standing wave. If the end of the tube is open, the standing sound wave must exhibit an antinode there, and if it is closed, the wave must exhibit a node there. Consider a tube of length L with one open and one closed end. Draw the first three harmonics of either the standing *pressure wave* or *displacement wave* of the air in the tube (indicate which one you have chosen to represent). Express each harmonic's wavelength in terms of L.

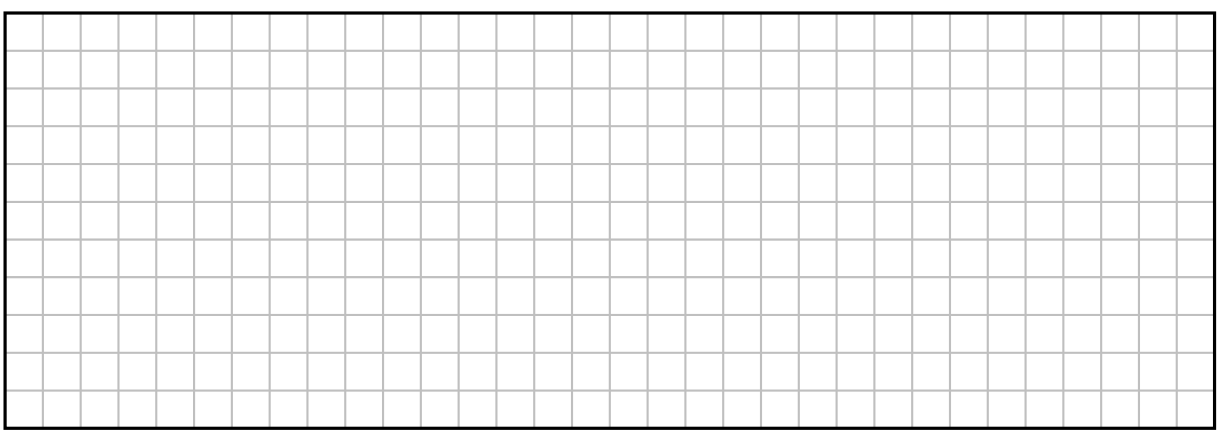


Question 2 **1 point**

Consider a traveling wave with wavelength $\lambda = 3\,\text{m}$ and frequency $f = 4\,\text{Hz}$, described by the function

$$f(x,t) = 2\sin\frac{2\pi}{\lambda}(x + vt).$$

Plot $f(x,0)$ on the interval $3\,\text{m} \leq x \leq 6\,\text{m}$ below. Then plot the wave at $t = 250\,\text{ms}$ and $t = 500\,\text{ms}$. Be sure to add labeled axes, and pay attention to units.



What is the period of this wave? Which direction is it moving? How would you change $f(x,t)$ to make the wave move in the opposite direction?

Objective

Measure the speed of sound in air by finding the resonant lengths of a closed tube at a known frequency. In the second part, drive standing waves on strings of different linear densities under varying tension, and verify the dependence of wave speed on these parameters.

Theory

Waves are ubiquitous in physics. They can be subdivided into two classes, *longitudinal* and *transverse* (see Fig. 1). Acoustic waves (sound) are longitudinal waves, propagating via the collective motion of atoms within some *medium*, such as air. In such a longitudinal wave, the atoms oscillate parallel to the direction of motion of the wave. In contrast, in transverse waves, the medium oscillates perpendicular to the wave's direction of motion. Examples of transverse waves include a vibrating string that is fixed at both ends, surface waves on a body of water, and electromagnetic waves.

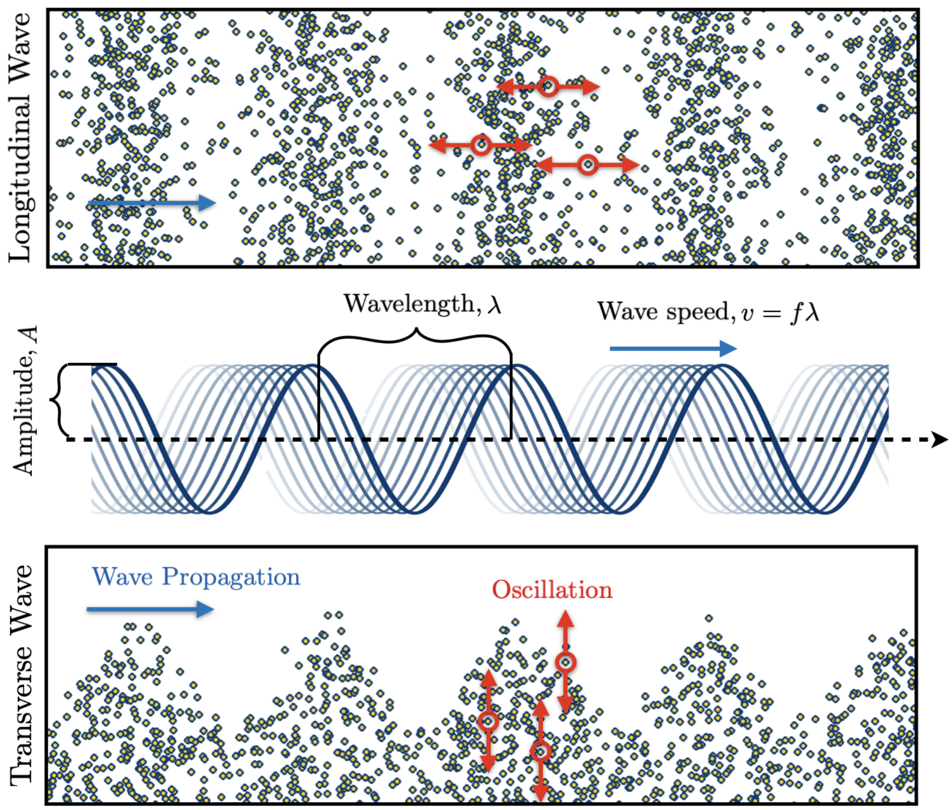


Figure 1: *Propagation of longitudinal waves (top) and transverse waves (bottom), showing the definition of wavelength λ (center).*

Waves oscillate both in space and in time. Spatially, a wave is described by a wavelength λ, given in units of length, and an amplitude A, given in units that depend on the nature of the wave phenomenon. Temporally, the wave is described by its period τ, which gives the time it takes for a wave to complete a single oscillation. For a traveling wave, this is also the time it takes the wave to advance by one wavelength. The inverse of the period is the wave's frequency f, given in units of Hertz (Hz) or seconds^{-1}, describing the number of cycles per second in the oscillation.

The speed of a traveling wave v satisfies

$$v = f\lambda. \tag{1}$$

While eq. (1) tells you how the frequency, wavelength, and wave speed are related, it does not explain why a given material supports a given wave speed.

For example, longitudinal acoustic waves traveling through gases, fluids, and solids are affected by the compressibility of the medium, as defined by the bulk modulus[1] K_s, and the density of the medium, ρ. The speed of an acoustic wave will be

$$v = \sqrt{\frac{K_s}{\rho}}. \tag{2}$$

A uniform string of mass M and length L, when pulled taut under tension T, will have a wave speed of

$$v = \sqrt{\frac{T}{M/L}} = \sqrt{\frac{T}{\mu}}, \tag{3}$$

where $\mu = M/L$ is the linear mass density. Stringed musical instruments support standing waves of fixed wavelength λ. Given the relationship between f, λ, and v in eq. (1) and between v and T in eq. (3), you can see why musicians adjust the tension in their strings ("tuning") until the desired frequencies emerge.

Standing Waves

Several waves can occupy the same space at the same time and add together, leading to the phenomenon of **interference**. When the crests of two waves align, they add together, making a larger wave by *constructively* interfering with each other. When the peaks align with troughs, the waves cancel one another, *destructively* interfering.

When two waves of the same amplitude and wavelength, but traveling in opposite directions, meet, they form a **standing wave**. This is what occurs when you pluck a string fixed at both ends. The plucking generates waves that move in opposite directions, hit the ends of the string, reflect, and interfere with each other to produce a wave that looks like it isn't moving either to the left or to the right: a standing wave.

When observing a standing wave on a string of length L, such as those shown in Figure 2, one notices stationary points that do not move, such as where the ends are clamped. These stationary points are called **nodes**. Between every two nodes, there is a piece of string that moves with maximum amplitude, called an **antinode**.

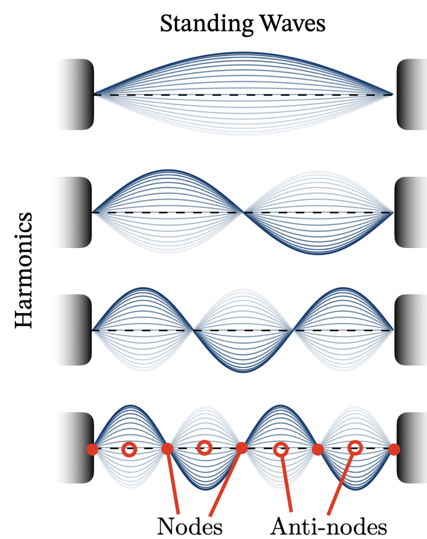


Figure 2: *The first four harmonics of a standing wave.*

Because of the fixed *boundary conditions* on the string, only certain standing waves can be supported. The lowest-frequency standing wave, called the **fundamental frequency** or the first harmonic, corresponds to one half-wavelength ($L = \lambda/2$), with two nodes at the clamped ends and one antinode in the center. The next-highest frequency, called the second harmonic, corresponds to a full wavelength ($L = \lambda$) fitting on the string, with a node in the center. The third harmonic corresponds to $L = 3\lambda/2$, with two nodes in the middle of the string between the fixed endpoints. The fourth harmonic corresponds to two full wavelengths ($L = 2\lambda$), with three nodes between the endpoints. And so on.

It is also possible to set up standing longitudinal waves, for example, in air-filled tubes. Musical instruments such as organs, clarinets, flutes, and other tubes can be driven to resonance and support standing acoustic waves. At resonance, the tube will create a tone (the fundamental frequency) and overtones (the higher harmonics). If the tube is closed at one end, you will find an air displacement node (the air cannot oscillate against the boundary) and an air pressure antinode. At the open end of the tube is an air displacement antinode, since the air molecules at the opening oscillate longitudinally at maximum amplitude, and an air pressure node, since atmospheric pressure acts as a boundary condition at the opening.

[1]In solids, K_s is called the coefficient of stiffness; in gases, it is called the modulus of bulk elasticity.

Experiment

Measuring Sound Speed in Air

In this experiment, you will measure the speed of sound indirectly by measuring the wavelength of a specific frequency of sound produced by a tuning fork. The resulting sound waves will enter a cylindrical cavity whose length you control, as shown in Figure 3. You will find the harmonics by changing the length of this cavity, which then allows you to infer the wavelength of the sound waves. With the frequency and wavelength measured, you can infer the sound speed using eq. (1).

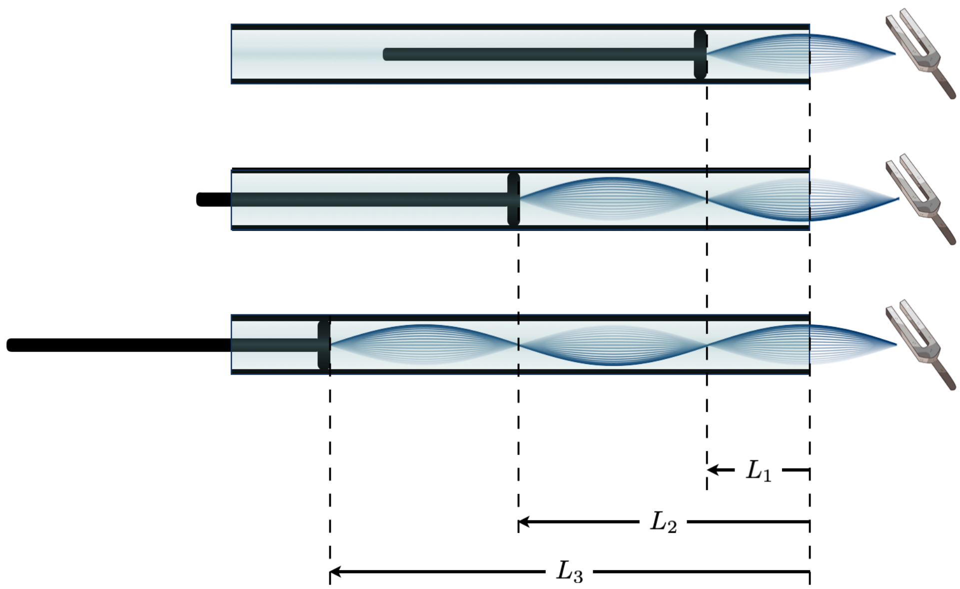


Figure 3: *Tuning fork and open tube with an adjustable length L. The diagram shows the displacement waves of air.*

Procedure

1. Set up the apparatus as shown in Fig. 3. Position the tuning fork at the end of the plastic tube with its edge about 1 cm from the open end of the tube.
2. Set up the piston close to the opening of the plastic tube to start the cavity length as small as possible.
3. Firmly strike the tuning fork with the rubber mallet. **Do not strike the fork on the table, or you could damage it.**
4. Move the piston away from the opening to increase the effective length of the resonant cavity. When you hear the tone suddenly get louder, you have reached a resonance and found an air displacement node.
5. Record the distance of each node from the open end of the tube in Data Table 1 in the Postlab exercises. Include an estimate of the uncertainty of your measurements.
6. Record the frequency of the tuning fork in Data Table 1.
7. Repeat the previous steps with at least two other tuning forks with different frequencies.

The Sonometer: Measuring Waves on a Stretched String

The sonometer is an apparatus used for exploring the wave properties of vibrating strings. It includes mounts to install strings of different linear densities μ, a bridge that allows us to vary string length L, and weights that allow control of the tension T in the strings. A schematic view of the sonometer is shown in Figure 4.

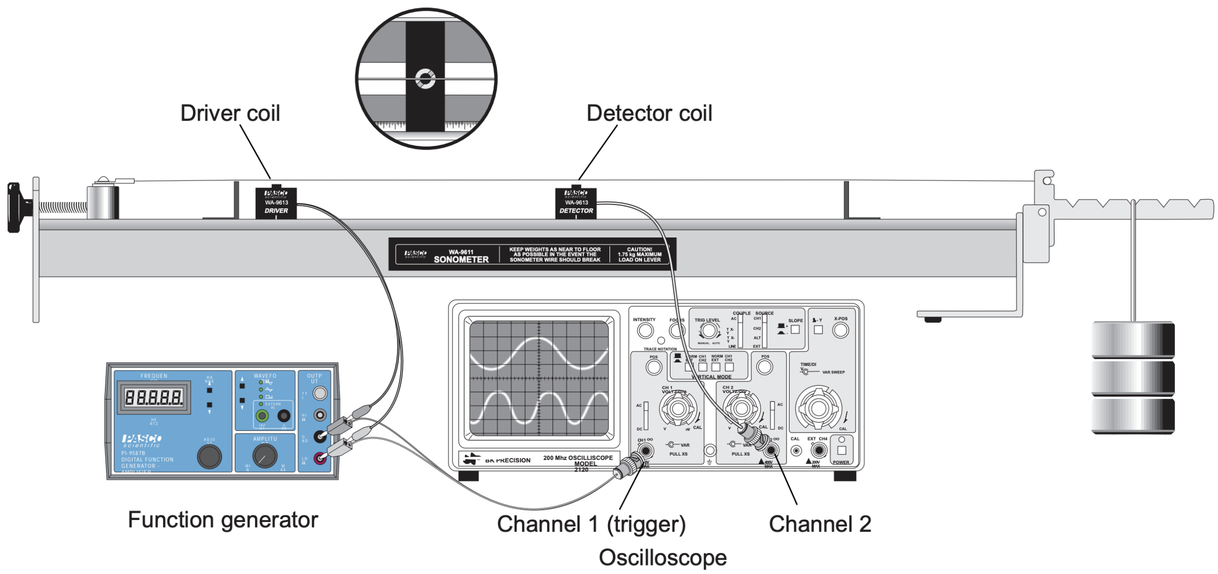


Figure 4: *The sonometer used to measure wave velocity on strings under tension. Image credit: PASCO Scientific.*

A **driver coil** connected to a function generator is used to generate standing waves on the string. The driver coil drives the string vibrations at any frequency the function generator will produce, but note that the frequency observed on the wire is usually **twice the driver frequency** because the driver electromagnet exerts a force on the wire twice per cycle. A **detector coil** allows you to view the vibration of the string on an oscilloscope.

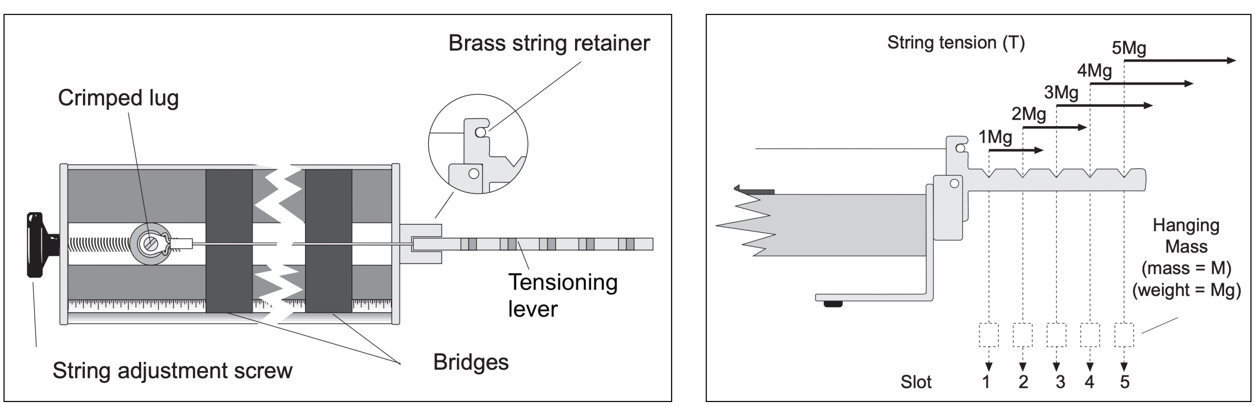


Figure 5: *The sonometer adjustment screw (left) and tensioning lever (right). Credit: PASCO Scientific.*

A set of color-coded strings with known and unknown linear mass densities is available in the lab. The strings and their properties are summarized in Table 1.

Procedure: Measuring Wave Speed in a Wire of Known Density

1. Set the amplitude of the function generator to its maximum value to see the largest string vibrations.

Color Code	**Diameter** (inches)	**Linear Density** μ (g/cm)
White	0.010	0.39
Blue	0.014	unknown
Yellow	0.017	1.12
Red	0.020	unknown
Black	0.022	1.84

Table 1: *Properties of the strings used with the sonometer.*

2. Start with the bridges 60 cm apart. Use any of the included strings with a known linear density (see Table 1) and hang a mass of approximately 1 kg, being sure to include the mass of the hanger itself, from the first slot of the tensioning lever (see Figure 5).

3. Adjust the string adjustment knob so that the tensioning lever is horizontal. Position the driver coil 5 cm from one of the bridges and position the detector coil near the center of the wire. Look up the properties of your chosen string in Table 1 and record the length, tension, and linear density in Data Table 2.

4. Calculate the resonant frequency of the fundamental harmonic, f_0, using

$$v = \lambda_0 f_0 = \sqrt{\frac{T}{\mu}}$$

and record the result in Data Table 2.

5. Slowly increase the frequency of the signal to the driver coil, starting at 25 Hz. Listen for an increase in the volume of the sound from the sonometer and/or an increase in the amplitude of the detector signal on the oscilloscope; these indicate a resonance. Find the lowest frequency at which resonance occurs — this is the fundamental harmonic. Measure the period τ of the standing wave using the detector coil and oscilloscope by counting the number of divisions between successive peaks. There will be some uncertainty $\Delta\tau$ in this measurement based on how precisely you can read the oscilloscope. The resonant frequency is then $f = 1/\tau$. Record all results in Data Table 2.

6. Continue increasing the frequency of the driver coil to find successive resonant frequencies (at least 2 or 3). Think about how the frequencies and wavelengths for harmonics are related to the fundamental frequency and wavelength.

7. Record the number of antinodes and the measured and calculated resonance frequency for each harmonic in Data Table 2. Think about how the number of antinodes is related to the order of the harmonic.

8. When done with the wire, put it back in the appropriate location.

Procedure: Finding the Linear Density of an Unknown Wire

In this section of the lab, you will determine the linear density of an unknown wire using the sonometer.

1. Replace the wire on the sonometer with a color-coded unknown wire. With the setup otherwise the same as in the previous section, measure the fundamental harmonic f_0 at 5 different tensions. Vary the tension by keeping the mass the same but changing its location on the tensioning lever (refer to Figure 5). Each time you move the mass, readjust the string adjustment knob so the tensioning lever remains horizontal.

2. Record the color code of the unknown wire, as well as the node spacing and the period of the wave as picked up by the detector coil using the oscilloscope, for each of the five tensions in Data Table 3.

3. When done with the wire, put it back in the appropriate location.
4. Calculate the wave speed in Data Table 4 and use it to compute the unknown linear mass density μ of the wire in the Postlab exercises.

Name: ______________________ **Date:** ______________________

Collaborators: ______________________ **Lab Section:** ______________

Postlab Exercises (20 points)

Submit the postlab to the TA at the end of the lab.

Measuring Sound Speed in Air (8 points)

Question 3 **1 point**

Record the frequency of the tuning forks and the distances to the first, second, and third nodes in Data Table 1.

Data table 1: *Node lengths in the resonant air column, driven by a tuning fork.*

Frequency f (Hz)	**Node Distance** L_1 (m)	**Node Distance** L_2 (m)	**Node Distance** L_3 (m)

Question 4 **2 points**

Calculate the average distance between consecutive nodes ($L_{i+1} - L_i$) for each frequency. Recall that in a standing wave, adjacent nodes are separated by exactly half a wavelength, so this distance equals $\lambda/2$. Show a sample calculation below.

Question 5 **2 points**

Calculate the speed of sound in air for each of the three tuning forks. Show your work.

Question 6 **2 points**

From your three measurements of the sound speed, compute the average velocity of sound in air and its uncertainty using the standard deviation of your values. Show your work.

Question 7 **1 point**

Using your average sound speed and its uncertainty from the previous question, compare your result with the value predicted by the following formula for the speed of sound in dry air as a function of temperature:

$$v(T) \approx v_0\,(1 + 0.6T)\,, \qquad v_0 = 331.5\,\text{m/s at } 0\,^\circ\text{C},$$

where T is the air temperature in degrees Celsius. (Note: room temperature is about 20 °C).

Sonometer: Standing Waves in a Wire of Known Density (2 points)

Question 8 **1 point**

Using eq. (1) and eq. (3), compute the resonant frequency of the fundamental harmonic. Show your work.

Question 9 **1 point**

Record the number of antinodes and the resonant frequency (calculated and measured) for a given length of string L, a given tension T, and a given linear density μ in Data Table 2.

$L =$ ____________ m $T =$ ____________ N $\mu =$ ____________ kg/m

Data table 2: *Measurements of the wire of known density.*

Number of antinodes	Calculated resonant frequency (Hz)	Measured period τ (s)	Uncertainty in measured period $\Delta\tau$ (s)	Measured resonant frequency (Hz)

Sonometer: Standing Waves in a Wire of Unknown Density (10 points)

Question 10 **1 point**

Record the color code of the unknown wire below. In Data Table 3, record the tension, the node spacing, and the period of the wave.

Data table 3: *Measurements of the wire of unknown density.*

Tension T (N)	Node spacing (cm)	Period τ (s)	Uncertainty in period $\Delta\tau$ (s)

Question 11 **3 points**

Calculate the velocity and the uncertainty in the velocity of the wave for each of the five tensions using the measured wavelength and the period of the fundamental harmonic. (Hint: how do you relate the period of the fundamental harmonic as measured on the oscilloscope to its frequency?) Propagate the uncertainty in the period $\Delta\tau$ to the velocity using

$$\Delta v = \frac{\lambda\, \Delta\tau}{\tau^2},$$

which assumes the wavelength has no uncertainty. Compute the uncertainty in the square of the velocity as

$$\Delta(v^2) = 2v \cdot \Delta v.$$

Add your results to Data Table 4, showing work for one calculation out of the five.

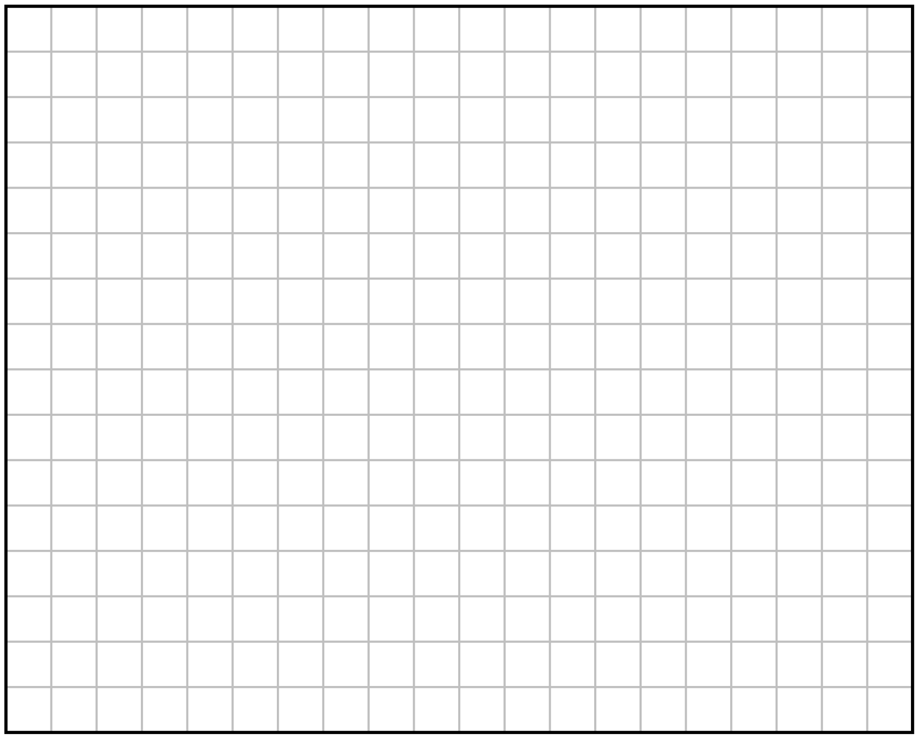
Data table 4: *Calculations of the wire of unknown density.*

Tension T (N)	λ (m)	f (Hz)	v (m/s)	Δv (m/s)	$\Delta(v^2)$ (m^2/s^2)

Question 12 **3 points**

Plot v^2 versus T in Graph 1. Include labels, units, and the uncertainty on v^2 for each point.

Graph 1: *Plot of v^2 vs. T.*



Is the relationship linear? Find the slope of the line. The slope is $1/\mu$, where μ is the linear density of the unknown wire. Show your work.

Question 13 **2 points**

Calculate the uncertainty in $1/\mu$ as follows. In Graph 1, draw the minimum and maximum slopes m_{min} and m_{max} that are consistent with the upper and lower error bars of the data. Use these slopes to compute an average slope and an uncertainty on the average:

$$\overline{m} = \frac{|m_{\text{max}} + m_{\text{min}}|}{2},$$
$$\Delta\overline{m} = \frac{|m_{\text{max}} - m_{\text{min}}|}{2}.$$

Question 14 **1 point**

How does the value of μ from the slope compare to the accepted value of the color-coded wire? (Ask your TA for the accepted value of μ for your unknown wire.) Do the values agree within experimental uncertainties?