University of Rochester

Laboratory XII

Geometrical Optics

DEPARTMENT OF PHYSICS & ASTRONOMY

PHYSICS 123 - 183
WAVES AND MODERN PHYSICS

Name: ______________________ **Date:** ______________________

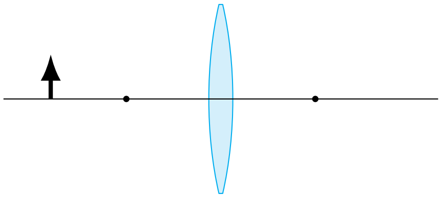
Collaborators: ______________________ **Lab Section:** ____________

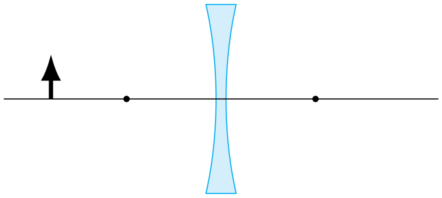
PRELAB EXERCISES (2 points)

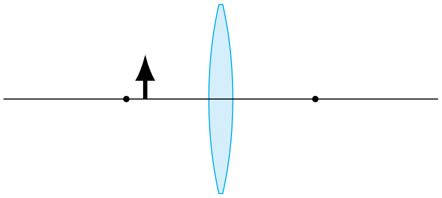
This prelab must be completed and handed in to the lab TA at the start of the lab.

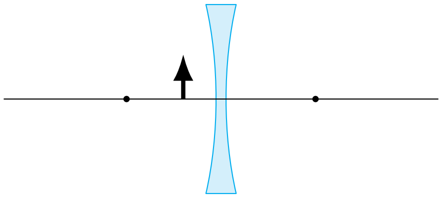
Question 1 **1 point**

Draw the location of the image of an object placed near converging and diverging lenses, pictured below. Foci are indicated as black dots. Make sure you include at least two rays in each diagram, and answer whether the images are *real* or *virtual*, *erect* or *inverted*, and *reduced* or *magnified*.









Question 2 **1 point**

You will be making many measurements of magnification in this laboratory. Explain how you will make this measurement to come up with a value for magnification.

Objective

> *Verify the thin lens equation by measuring image distances for converging and diverging lenses at various object positions, and characterize the difference between real and virtual images. Use combinations of lenses to construct simple optical systems such as a telescope and a compound microscope, measuring their magnifications.*

Theory

Lenses produce different types of images, depending on the position of the object relative to the lens and its focal point. In the following discussion, we use the descriptions of *real* or *virtual*, *upright* or *inverted*, and *reduced* or *magnified* to describe these images. The fundamental distinction between real and virtual images is that light rays converge to form a real image, while light rays diverge from a virtual image. Furthermore, real images may be projected onto a screen, while virtual images must be viewed through a lens.

The Thin Lens Equation

Consider a thin *converging* lens with focal length f, similar to that shown in Figure 1. An object of height h is placed at a distance p to the left of the lens. It produces an image of height h' at a distance i from the lens.

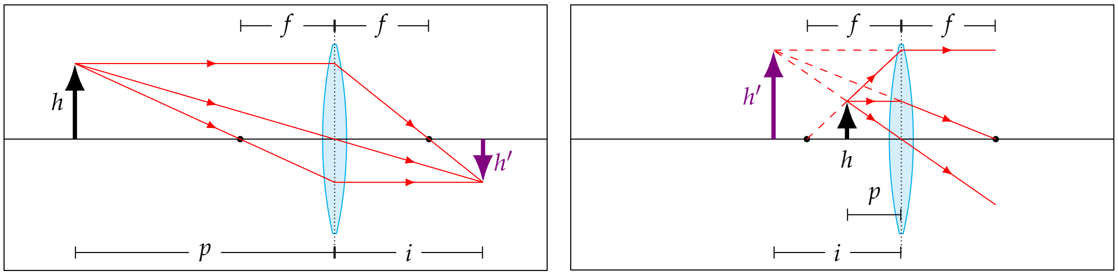


Figure 1: *Left: a thin converging lens of focal length f, with an object at a distance $p > f$, produces a real inverted image at position i behind the lens. Right: if the object is placed at $p < f$, a virtual upright image is produced at position i in front of the lens.*

The object and image distances are related to the focal length by the **thin lens equation**

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i}. \tag{1}$$

If we draw three rays from the object through the nearest focus, through the center of the lens, and parallel to the lens axis and trace the refracted rays through the lens, the point where the refracted rays converge gives the image position i and the image height h'. The magnification of the lens is then defined by

$$m = \frac{h'}{h} = -\frac{i}{p}. \tag{2}$$

If $m < 0$, then the resulting image is inverted, as in the left panel of Fig. 1. If $m > 0$, the image is upright, as shown in the right panel.

A Simple Projector

If $p > f$, as in the left panel of Fig. 1, the image is real but inverted. It can be projected onto a screen placed at a distance i behind the lens. Using eqs. (1) and (2), it is straightforward to show the conditions under which the lens will magnify or reduce the image size on the screen:

- Magnification: when $p > f$ and $p \leq 2f$, the image is enlarged; i.e., $h' \geq h$ and $|m| \geq 1$.
- Demagnification: when $p > 2f$, the image is reduced; i.e., $h' < h$ and $|m| < 1$.

A Simple Magnifier

The right panel of Fig. 1 shows how placing an object at location $p < f$ can produce a virtual upright image with $h' > h$. By placing the object at $p \approx f$ and holding the lens up to your eye, you will see a magnified image of the object in focus. To understand how this works, we need to explain some limitations of the eye, which is itself an optical system that uses a lens.

The human eye can focus on objects located anywhere from infinity down to a special point called the **near point** p_n. While the location of the near point varies from person to person, the average in the adult population is

$$\overline{p}_n \approx 25\,\text{cm}$$

from the front of the eye. If an object is placed closer to your eye than p_n, the rays from the object are too divergent for the eye to focus. However, placing a converging lens between the object and the eye makes the rays entering the eye less divergent. In fact, if the object is placed at the focal point of the lens, the rays entering the eye appear to "come from infinity," and the eye has no problem focusing such rays on the retina. Furthermore, if $f < p_n$ and the object is placed at f, the image of the object appears magnified.

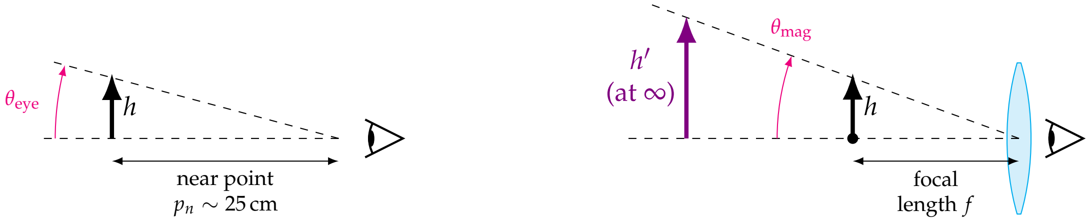


Figure 2: *Definition of the angles used in magnification power. Left: The naked eye observes an object at its near point subtending an angle* θ_{eye}. *Right: when a convex lens is placed in front of the eye, and the object is located at the focal point of the lens such that* f *is smaller than the near point* p_n*, the image subtends a larger angle* θ_{mag}*, leading to a magnified virtual image.*

Figure 2 shows the geometry of an object of height h placed at the near point of the eye. Suppose that $h \ll p_n$; for example, you are looking at very small text on a page at a distance $p = p_n \approx 25\,\text{cm}$. The angle subtended by the height of the text h at your eye is given by

$$\theta_{\text{eye}} \approx \tan\left(\theta_{\text{eye}}\right) = \frac{h}{p_n} \approx \frac{h}{25\,\text{cm}} \tag{3}$$

Now, if you put a converging lens in front of your eye and move the object to the point $p = f$, where $f < p_n$, the lens will produce a virtual upright image that appears to be at $i = \infty$ (you can verify this with eq. (1)), and will be in focus at your eye. We still have $h \ll f$, so the angle subtended by the object at distance $p = f$ is

$$\theta_{\text{mag}} \approx \tan\left(\theta_{\text{mag}}\right) = \frac{h}{f}. \tag{4}$$

The magnifying power of a simple lens, or its **angular magnification**, is defined by the ratio of θ_{mag} to θ_{eye}, where θ_{mag} is the apparent angular size of the object when placed at distance $p = f$ from the lens, and θ_{eye} is the apparent angular size of the object at your naked eye when the object is at your near point:

$$m_\theta = \frac{\theta_{\text{mag}}}{\theta_{\text{eye}}} = \frac{h/f}{h/p_n} = \frac{p_n}{f} \approx \frac{25\,\text{cm}}{f}. \tag{5}$$

A Compound Microscope

To view very small objects, you can build a compound microscope using nothing more than two coaxial lenses separated by some distance. The lens closest to the object is called the **objective lens**, while the lens closest to your eye is called the **eyepiece**[1].

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[1] Hopefully, the reasons for these names are obvious.

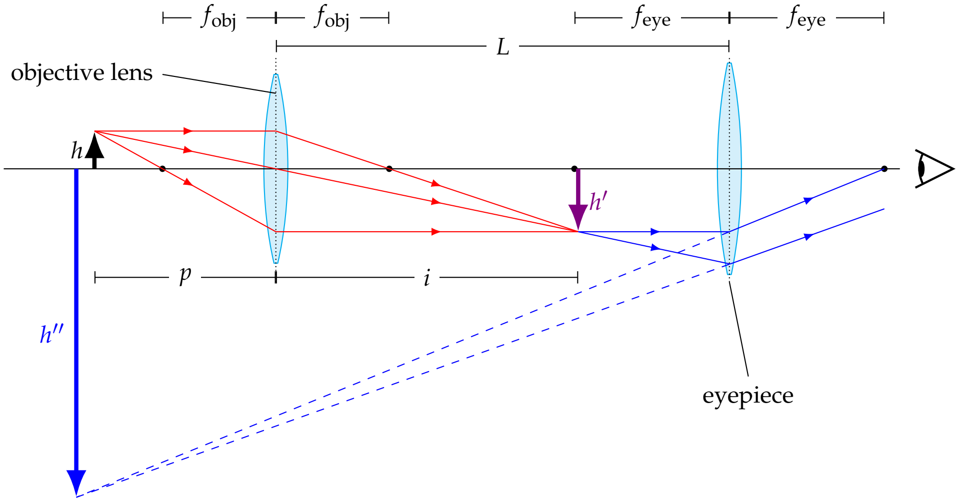

Figure 3: *A basic compound microscope with an objective lens of focal length f_{obj}, an eyepiece of focal length f_{eye}, and a barrel length L between the objective and eyepiece. Rays traced through the objective lens are drawn in red; rays traced through the eyepiece are drawn in blue.*

The workings of the compound microscope are shown in Fig. 3. An object of height h (black arrow) is positioned in front of an objective lens of focal length f_{obj} such that $f_{\text{obj}} < p < 2f_{\text{obj}}$. The **objective acts like a projector** and produces a magnified, inverted real image of size h' near the focal length of the eyepiece (violet arrow). Following eq. (2), the objective provides a linear magnification

$$m = -\frac{i}{p} = -\left(\frac{L - f_{\text{eye}}}{p}\right).$$

The real image from the objective acts as the object for the eyepiece. With h' located at f_{eye}, **the eyepiece acts like a simple magnifier** and produces a virtual image of height h'' (blue arrow) with rays that appear nearly parallel to the user. The angular magnification of the eyepiece, following eq. (5), is

$$m_\theta = \frac{p_n}{f_{\text{eye}}} \approx \frac{25\,\text{cm}}{f_{\text{eye}}}.$$

The total magnification of the compound microscope is the product of the linear magnification of the objective and the angular magnification of the eyepiece:

$$M = m m_\theta = -\left(\frac{L - f_{\text{eye}}}{p}\right)\left(\frac{25\,\text{cm}}{f_{\text{eye}}}\right). \tag{6}$$

An Astronomical Telescope

An astronomical telescope, shown in Figure 4, has a similar layout to the compound microscope, but instead of magnifying very nearby small features, it is designed to magnify very distant objects. It is designed such that $f_{\text{obj}} \gg f_{\text{eye}}$. For very distant objects ($p \to \infty$), we can treat rays from the object as if they are parallel. In this case, the objective focuses the rays at $i = f_{\text{obj}}$, as you would predict from the thin lens equation, producing a real inverted image of height h'. As in the compound microscope, the eyepiece acts like a simple magnifier, producing a virtual image of size h'' with rays nearly parallel at your eye. That is, the virtual image is "at infinity."

To estimate the magnification power of the telescope, we consider the angular magnification of the objective and the eyepiece. This differs from the compound microscope, where we considered the linear magnification of the objective lens. For an astronomical telescope, where the object of interest is effectively at $p = \infty$, and the object size could be gigantic[2], it is much more sensible to use the apparent angular size.

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[2] If we are looking at a planet or a star through the telescope, the object's size is much greater than the image size.

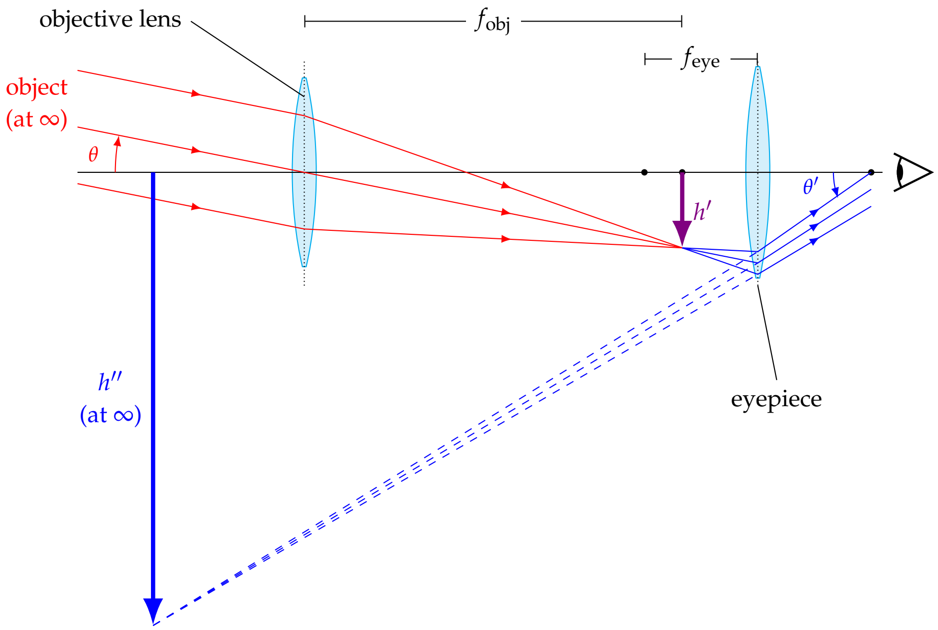

Figure 4: *A basic astronomical telescope with an objective lens of focal length f_{obj} and an eyepiece of focal length f_{eye}. An object at $p = \infty$ sends parallel rays through the objective, which are focused at $i = f_{\text{obj}}$ (red arrows). The eyepiece produces a virtual image at $i' = \infty$ (blue arrows).*

The apparent angular size of the real image from the objective is

$$\theta \approx \tan\theta = \frac{h'}{f_{\text{obj}}},$$

where we use a small-angle approximation. The apparent angular size of the virtual image from the eyepiece is

$$\theta' \approx \tan\theta' = \frac{h'}{f_{\text{eye}}}.$$

The total magnification of the telescope is thus

$$M = \frac{\theta'}{\theta} \approx \frac{\tan\theta'}{\tan\theta} = \frac{h'/f_{\text{eye}}}{h'/f_{\text{obj}}} = -\frac{f_{\text{obj}}}{f_{\text{eye}}}, \tag{7}$$

where the minus sign indicates that the image is inverted.

A Terrestrial Telescope (or Spyglass)

A terrestrial telescope, also known as a *spyglass* or an *opera glass*, is a two-lens system used for imaging distant objects. Like the astronomical telescope, it uses a long-focus objective lens and a short-focus eyepiece, but in the terrestrial telescope, a concave (diverging) lens replaces the convex eyepiece. The magnification of a terrestrial telescope is given by eq. (7).

The first known terrestrial telescope, with 3× magnification, was built by the Dutch lensmaker Hans Lippershey in 1608. Due to the concave lens at the eyepiece, the telescope produces an upright virtual image, making it useful for surveying and military applications. In 1609, Galileo Galilei refined Lippershey's design by replacing the concave eyepiece with a convex lens, allowing him to build astronomical telescopes with 30× magnification. Using his device, Galileo rapidly published several revolutionary discoveries, including proof of the heliocentric model using the observed phases of Venus, mapping out the surface of the Moon, discovering the moons of Jupiter, observing Neptune, discovering Saturn's rings, and discovering that the Milky Way (which appears nebulous to the naked eye) is full of stars.

Experiment

Equipment

In this experiment, we use an optical bench: a magnetized track upon which various combinations of objects and lenses can be mounted. A picture of the track is shown in Figure 5. The equipment needed for this lab is listed below:

- ☐ Box: Multiple lenses, holders, screen, object
- ☐ Light source
- ☐ 2 lengths of track

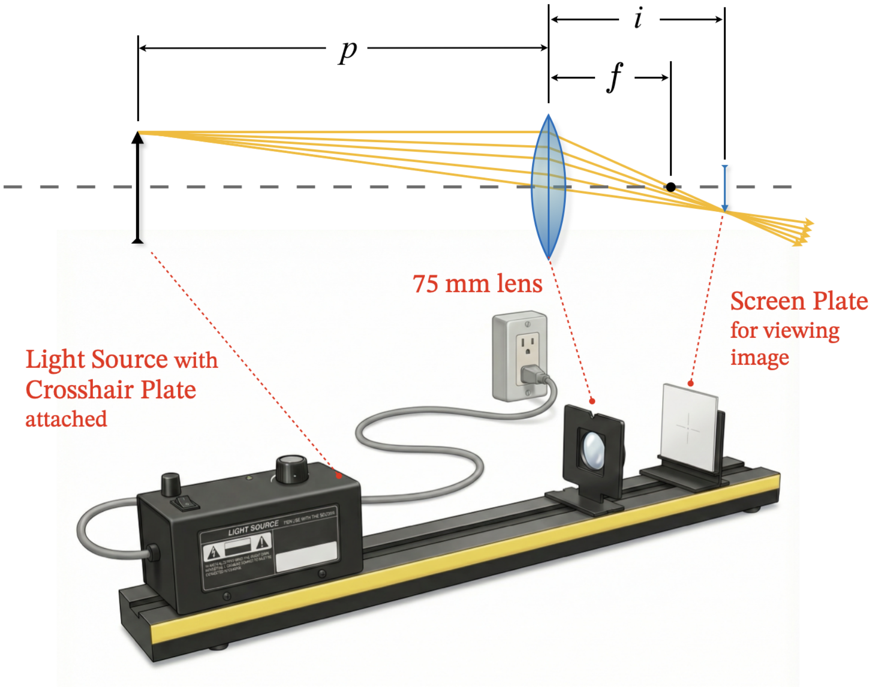

Figure 5: *Setup to test a projector lens and study the thin lens equation in eq. (1).*

The Projector

In the first part of the experiment, you will build a **projector** using a single convex lens, an object to project, and a screen on which to view the projected image. A projector is a simple lens system that focuses an image onto a distant screen. By measuring pairs of object and image distances, you will be able to experimentally determine the focal length of the convex lens using the thin lens equation in eq. (1). Note that the thin-lens equation is an approximation, so your results may not agree exactly with those predicted by eq. (1).

Procedure

1. Use the 75 mm focal length convex lens (#14), a crossed arrow target as the object (#8), a viewing screen (#11), and 2 element holders (#22) to hold the lens and screen. Set up the optical bench as in Figure 5. Place the lamp at one end of the bench and direct its illumination parallel to the bench axis using the knob on the top. You may attach the crossed arrow target directly to the lamp housing. Place the lens 10 cm away from the crossed arrow target (i.e., set $p = 10\,\text{cm}$). Place the screen on the opposite side of the lens from the target. Adjust the screen to sharply focus the projected image.

2. Measure the distance from the center of the lens to the plane of the screen, i, and the distance from the center of the lens to the object, p, and record these values in Data Table 1 in the Postlab exercises. The image may seem to be in sharp focus over some measurable distance. Estimate this uncertainty in the image distance, Δi, and record it in Data Table 1.

3. Describe the image using the terms real or virtual, erect or inverted, magnified or reduced. It may be difficult to identify each pair definitively, but you should see a trend emerge.

4. Repeat steps 2 and 3 for many different object distances, exploring the range from 10 cm to 20 cm in increments of 2 cm. Then complete the calculations in the Postlab exercises.

The Magnifier

Figure 6: *The setup for testing the magnification of a converging lens.*

Procedure

1. Estimate the location of your eye's near point by holding the crossed arrow target at arm's length and slowly bringing it towards your eye, with one eye closed. Keep your eye relaxed and do not try too hard to keep the target in focus. As the target moves closer to your eye, there will be a point at which you can no longer focus easily on it. This is your eye's near point. Measure this distance from eye to target, taking care not to poke your eye out!

2. Place a convex lens between your eye and the target (still at the near point). Verify that you can now focus on the target. The convex lens has allowed you to bring the object closer to your eye, thus making it appear larger. This is how a simple magnifier works. For the rest of this experiment, whenever an equation includes the population-average near point of $\overline{p}_n = 25\,\text{cm}$, substitute the measured value for your eye instead.

3. Set up the magnifier as shown in Fig. 6. Use the crossed arrow target as your object. Place the 75 mm convex lens about 2 cm away from the object and view it through the lens. Increase the object distance (the distance between the lens and the object) until the size of the image is maximized, yet still in focus. This should occur when the object distance is approximately equal to the focal length f. Note that beyond this point, the image becomes blurry.

4. Measure the magnification of the lens using the following procedure. While looking through the lens at the object, hold a ruler between your eye and the lens and measure the apparent height of the object (the height of an arrow, for instance). Repeat the measurement again, holding your eye, the ruler, and the object in the same place, but with the lens removed from its holder. It may be helpful to have one person make the visual measurement while another person removes (and replaces) the lens. You may need to repeat this a few times to get the hang of it. Record your data, focal length, and image observations in Data Table 2 in the Postlab exercises.

5. Repeat steps 2 to 4 with a 150 mm focal length convex lens.

The Compound Microscope

The compound microscope (Figures 3 and 7) uses two lenses to create a magnified image of a very small, very close object.

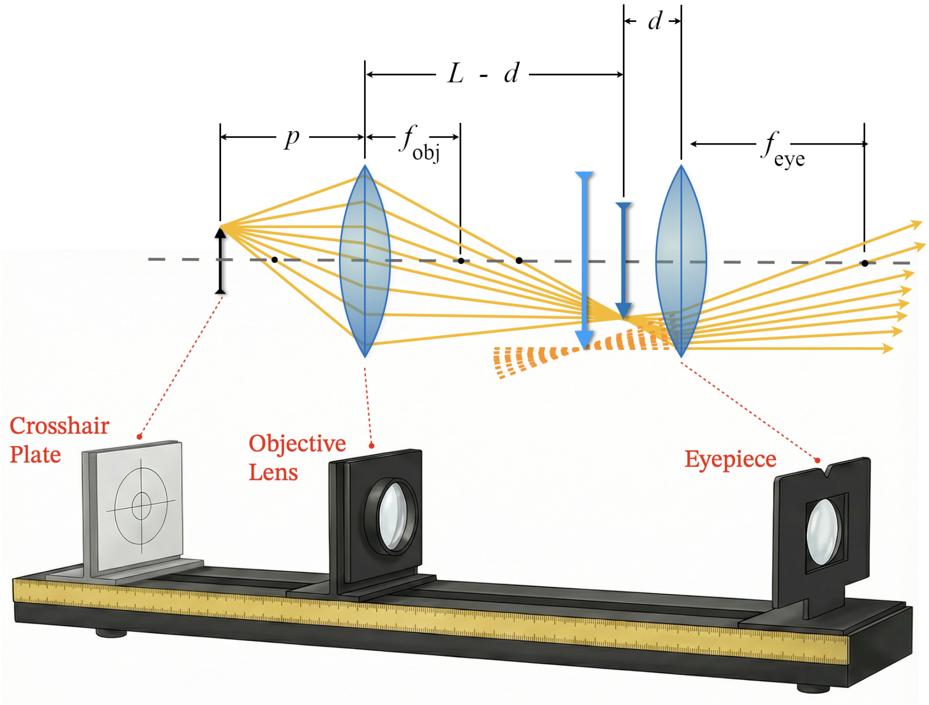

Figure 7: *Setup for the compound microscope. Refer to Fig. 3 and eq. (6).*

Procedure

1. Construct a compound microscope on the optical bench as in Figures 3 and 7. You may need to use 2 tracks to form one long bench. Use the 75 mm convex lens as the objective and the 150 mm convex lens as the eyepiece. Place the crossed arrow target 12.5 cm in front of the objective lens. Start with the 2 lenses about 50 cm apart. As you look through the microscope, move the eyepiece closer to the objective until you see a clearly focused image of the arrow target (perhaps only the center portion of the crossed arrow target).

2. Measure the magnification of the system using the same method as described in step 4 of the magnifier procedure. This time, measure the apparent size of the arrows with both lenses in place, then with both lenses removed. Record your measured apparent heights in Data Table 3 in the Postlab exercises.

3. While looking through the microscope, move the objective lens closer to the target and adjust the eyepiece as needed to keep the object in focus as well as you can.

4. Record your data, the focal lengths, and your observations of the image in Data Table 3.

The Astronomical Telescope

The setup for an astronomical telescope is very similar to the compound microscope, except that you will remove the crosshair plate from the optical bench and look at objects at much greater distances.

Procedure

1. Construct an astronomical telescope on the optical bench as in Figure 4. Use the 75 mm convex lens as your eyepiece and the 150 mm lens as the objective lens. The distance between the two lenses will be approximately 225 mm, the sum of the two focal lengths. Look at some reasonably distant object, perhaps a poster on the farthest wall in the lab room. Adjust the distance between the lenses to bring the object into sharp focus.
2. Measure the magnification of the telescope using the same procedure as in the previous two sections. Measure the apparent height of the distant object with both lenses in place, then with both lenses removed.
3. Record your measurements, the focal lengths of both lenses, and observations of the image (inverted or upright, real or virtual, etc.) in Data Table 4 in the Postlab exercises.

The Terrestrial Telescope

Procedure

1. Construct a terrestrial telescope on the optical bench using the same objective as in the astronomical telescope (150 mm convex lens), but replace the convex eyepiece with a 49 mm concave eyepiece (#19). The focal length of the concave lens is $f_{\text{eye}} = -49\,\text{mm}$.
2. Put the eyepiece at one end of the optical bench. Place the objective about 20 cm from the concave eyepiece and aim the telescope at a distant object, perhaps the same one used for the astronomical telescope in the previous section. Slowly move the objective toward the eyepiece until the distant object comes into focus.
3. Measure the apparent magnification of the telescope using the same procedure as before. Measure the apparent height of the distant object with and without both lenses.
4. Record your measurements, the focal lengths of both lenses, and observations of the image (inverted or upright, real or virtual, etc.) in Data Table 5 in the Postlab exercises.

Name: ______________________ **Date:** ______________________

Collaborators: ______________________ **Lab Section:** ______________

POSTLAB EXERCISES (20 points)

Submit the postlab to the TA at the end of the lab.

The Projector (6 points)

Question 3 **1 point**

Add measurements from your study of the projector to Data Table 1 and compute all derived quantities. To estimate the uncertainty on $1/i$, use propagation of uncertainties:

$$\Delta(1/i) = \frac{\Delta i}{i^2}.$$

Data table 1: *Data for the projector measurements.*

Object distance p [cm]	Image distance i [cm]	Image distance uncertainty Δi [cm]	Magnified / Reduced Real / Virtual Inverted / Upright	$1/p$ [cm^{-1}]	$1/i$ [cm^{-1}]	$\Delta(1/i)$ [cm^{-1}]

Question 4 **1 point**

As the object distance p gets larger, the image distance i approaches some value. Estimate this value by increasing the object distance, and observe the image distance.

Question 5 **2 points**

In Graph 1, plot $1/i$ versus $1/p$. Draw the calculated uncertainties on $1/i$ as error bars. Include labels and units for the axes. Draw a best-fit straight line through your data points. Determine the slope and y-intercept of this line. Show your work below.

Graph 1: *Plot of* $1/i$ *vs.* $1/p$ *for the projector lens.*

Question 6 **1 point**

According to the plot in Graph 1, what is the focal length of the lens used in the projector? Use the thin lens formula in eq. (1) and the equation for a line ($y = mx + b$).

Question 7 **1 point**

Is the thin lens formula verified according to your data and Graph 1? Explain your answer.

The Magnifier (4 points)

Question 8 **1 point**

Add measurements from your study of the magnifier to Data Table 2.

Data table 2: *Data for the magnifier measurements.*

75 mm Lens		**150 mm Lens**	
height with lens:		height with lens:	
height without lens:		height without lens:	
$M_{75\text{ mm}}$ (measured):		$M_{150\text{ mm}}$ (measured):	

Question 9 **1 point**

Calculate the theoretical magnification of the 75 mm lens and the 150 mm lens using your near point. Show your work.

Question 10 **2 points**

How does your measured magnification compare to the theoretical magnification? Explain why it may or may not be so. Give specific reasons having to do with your measurement procedure.

The Compound Microscope (4 points)

Question 11 **1 point**

Add measurements from your study of the compound microscope to Data Table 3.

Data table 3: *Data for the compound microscope.*

f_{obj} [cm]:		height with lenses:	
f_{eye} [cm]:		height without lenses:	
Barrel length L [cm]:		M (measured):	

Question 12 **1 point**

Calculate the theoretical magnification of your microscope using eq. (6). Show all work.

Question 13 **1 point**

Is your calculated magnification consistent with the measured value? Why or why not?

Question 14 **1 point**

Why does the magnification increase as the objective lens is moved closer to the object? What focusing problems tend to develop as the magnification increases?

The Astronomical Telescope (3 points)

Question 15 **1 point**

Add measurements from your study of the astronomical telescope to Data Table 4.

Data table 4: *Data for the astronomical telescope.*

f_{obj} [cm]:		height with lenses:	
f_{eye} [cm]:		height without lenses:	
Upright or inverted?		M (measured):	

Question 16 **1 point**

Calculate the theoretical magnification of your astronomical telescope in eq. (7). Show your work.

Question 17 **1 point**

Compare your calculated magnification to the measured one. Compute the percentage difference using the expression

$$\left|\frac{M_{\mathrm{meas}} - M_{\mathrm{calc}}}{M_{\mathrm{calc}}}\right| \times 100\ [\%].$$

Is the percentage difference under 20%? If not, explain why not. Are the two values consistent with each other? Justify your answer.

The Terrestrial Telescope (3 points)

Question 18 **1 point**

Add measurements from your study of the terrestrial telescope to Data Table 5.

Data table 5: *Data for the terrestrial telescope.*

f_{obj} [cm]:		height with lenses:	
f_{eye} [cm]:		height without lenses:	
Upright or inverted?		M (measured):	

Question 19 **1 point**

Calculate the theoretical magnification of your terrestrial telescope in eq. (7). Show your work.

Question 20 **1 point**

What is the main difference between the two telescopes' performance (other than the eyepiece)? When would you use the astronomical telescope? When would you use the terrestrial telescope?