University of Rochester

Laboratory XIV
The Atomic Spectrum of Hydrogen

DEPARTMENT OF PHYSICS & ASTRONOMY

PHYSICS 123 - 183
WAVES AND MODERN PHYSICS

Name: ______________________________ **Date:** ______________________

Collaborators: ___________________________ **Lab Section:** ____________

PRELAB EXERCISES (2 points)

This prelab must be completed and handed in to the lab TA at the start of the lab.

Question 1 **1 point**

You will be using a diffraction grating in this lab exercise as a dispersive element in a spectrometer. When you begin to examine the Balmer series of atomic hydrogen, you will observe an indigo line, a red line, and a violet line as you move the spectrometer's telescope away from the zero angle (0th order) position.

What will be the sequence of the spectral lines, starting from the zero angle position? Explain why, showing some calculations or a diagram.

Question 2 **1 point**

Showing all work, what is the expected measured grating separation, d, if you use:

(a) ($1/2$ point) A 600 groove/mm grating?

(b) ($1/2$ point) A 300 groove/mm grating?

Objective

Use a diffraction grating spectrometer to resolve the visible emission lines of atomic hydrogen and measure their wavelengths. Calibrate the spectrometer with a sodium source and verify that the hydrogen wavelengths match the predictions of the Balmer formula, confirming the quantization of atomic energy levels.

Theory

One of the earliest successes of quantum mechanics was the explanation of the spectrum of atomic hydrogen. In 1853, physicist Anders Ångström observed that excited hydrogen gas emits light at regular discrete wavelengths. Mathematician Johann Balmer developed an empirical formula predicting the sequence of wavelengths in 1885:

$$\frac{1}{\lambda_{i,j}} = R\left(\frac{1}{n_i^2} - \frac{1}{n_j^2}\right) \tag{1}$$

where $n_i < n_j$ are both integers and R is a constant. The spectral lines are classified into series, each sharing a common value of the integer n_i. The left-hand side of eq. (1) is the wavenumber (inverse wavelength, usually quoted in units of cm^{-1}) of the jth member of series i. The named series of the spectrum of atomic hydrogen are listed in Table 1.

Series Name	Wavelength Range	Expression	Restriction
Lyman (*disc. 1906*)	Ultraviolet (UV): 91.2 - 121.6 nm	$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$	$n \geq 2$
Balmer (*disc. 1853*)	Near UV / Visible: 364.5 - 656.279 nm	$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$	$n \geq 3$
Paschen (*disc. 1908*)	Near IR - Infrared: 820.4 - 1875 nm	$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$	$n \geq 4$
Brackett (*disc. 1922*)	Infrared - Far IR: 1458 - 4051 nm	$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$	$n \geq 5$
Pfund (*disc. 1924*)	Very Far Infrared: 2279 - 7460 nm	$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$	$n \geq 6$
Humphreys (*disc. 1953*)	Very Far Infrared: 3282 - 12370 nm	$\frac{1}{\lambda} = R\left(\frac{1}{6^2} - \frac{1}{n^2}\right)$	$n \geq 7$

Table 1: *The first six series of observed spectral lines of hydrogen. Only the Balmer series is visible to the naked eye.*

The constant R, called the Rydberg constant after Swedish physicist Johannes Rydberg, is empirically measured to be

$$R = \frac{m_e e^4}{8\epsilon_0^2 h^3 c} = 1.097\,373 \times 10^{-2}\,\text{nm}^{-1}, \tag{2}$$

where m_e and e are the mass[1] and charge of the electron, ϵ_0 is the permittivity of free space, h is Planck's constant, and c is the speed of light.

In this lab, you will examine the spectrum of atomic hydrogen in the visible region. This restricts your test of the theory to three spectral lines of the Balmer series. By measuring the wavelengths of these lines and comparing them to the Balmer formula, you will confirm the quantum nature of atomic energy levels. You can also verify the value of the Rydberg constant by explicit calculation using eq. (2).

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[1] In some atoms, the nuclear mass must be included in the calculation of R, replacing m_e with the **reduced mass** $\mu^{-1} = m_e^{-1} + M_{\text{nucl}}^{-1}$.

The apparatus for this experiment consists of a telescope spectrometer with a diffraction grating as its dispersive element (see Fig. 1). A diffraction grating is a planar device with a certain number of parallel "grooves" per unit length, separated by a constant distance d. The diffraction grating acts like a prism: light passing through the grating is emitted into distinct angles θ depending on wavelength λ, according to the grating equation

$$d \sin\theta = m\lambda. \tag{3}$$

In eq. (3), $m = 1, 2, 3, \ldots$ is an integer called the diffraction order. If light incident on the grating contains more than one color, a spectrum is formed, since the various colors are diffracted to different angles. By carefully measuring the angle at which each color of light from excited hydrogen is diffracted, you will measure the hydrogen spectrum and thus infer its atomic energy levels.

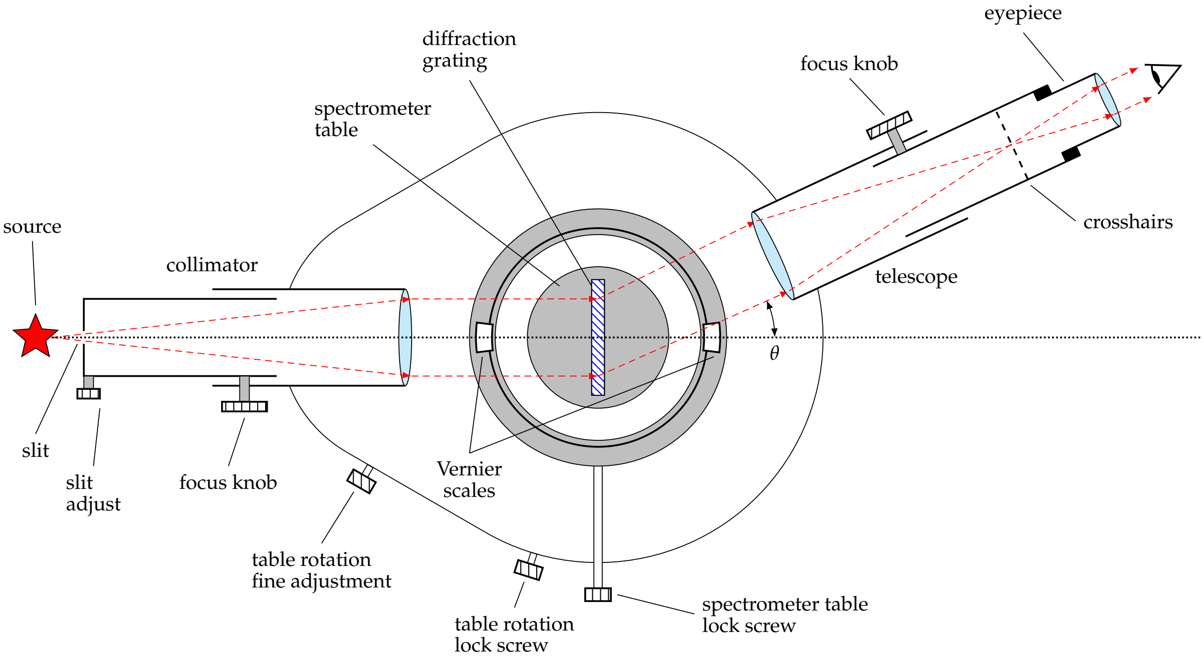

Figure 1: *Diagram of the spectrometer to be used in the lab.*

The physical grating is situated on one side or face of a slab of glass. The grating holder in your setup (see Fig. 1) should specify the face upon which the grating has been placed. The grating will also be labeled with the number of grooves per mm. This number is approximate only, and the true grating separation must be measured.

In the first part of the lab, you will align your spectrometer so that the telescope and collimator are focused properly on the slit image, and the grating is perfectly perpendicular to the optical path. Next, you will calibrate the groove separation of your particular grating by using a known wavelength of the sodium spectrum. Finally, you will measure the angles of some of the spectral lines of the Balmer series. These angles will be converted to wavelengths using the calibration value you obtain for the grating.

Equipment

- Spectrometer
- Diffraction Grating
- Sodium (Na) Lamp
- Hydrogen (H) Lamp
- Magnifying Glass

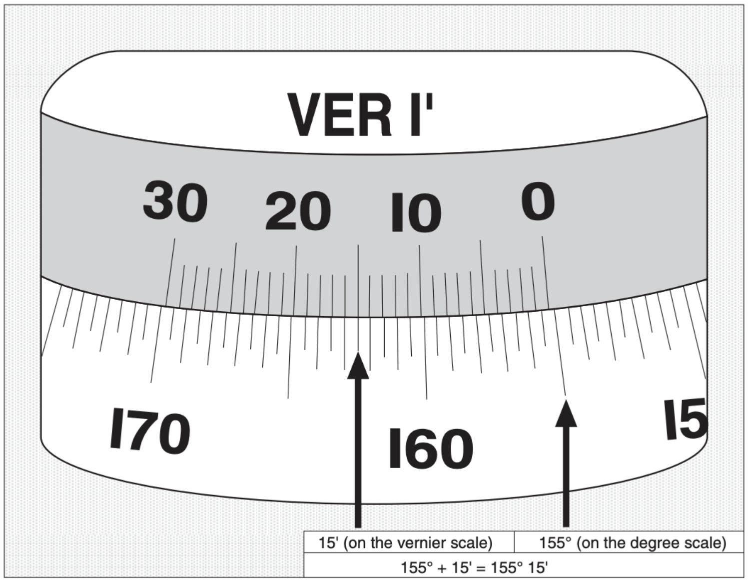

Figure 2: *The vernier scale and degree plate on the spectrometer. Credit: PASCO Scientific.*

Experiment

Measuring Angles Precisely with a Vernier Scale

To precisely measure the wavelengths of the hydrogen spectrum, we will need to record angles to the nearest arcminute ($1'$=$1/60°$). To do this, we use a vernier scale comprising a degree scale on the spectrometer table and a secondary scale accurate to $1'$. There are two windows on the table platform with degree and vernier scales, as shown in Fig. 1.

Figure 2 shows an example angle reading on the scale. To read the angle, first find where the zero point of the vernier scale aligns with the degree plate and record the value. If the zero point of the vernier scale is between two lines on the degree scale, use the smaller degree value. In Fig. 2, the zero point on the vernier scale is between the $155°$ and $155°30'$ marks on the degree plate, so the recorded value is $155°$.

Next, use the magnifying glass to find the line on the vernier scale that aligns most closely with any line on the degree scale. In Fig. 2, this is the line corresponding to a measurement of $15'$. Add this value to the coarse reading recorded above to get the correct measurement to within 1 minute of arc:

$$155° + 15' = 155°15' = 155.25°.$$

Alignment of the Spectrometer

The spectrometer needs to be aligned (see Fig. 3) to ensure accurate measurements of the sodium spectral calibration lines and the hydrogen spectral lines. Loosening the locking screw right below the telescope (not shown in Fig. 1) allows it to rotate. Loosening the lock screw on the long stalk below the spectrometer table (shown in Fig. 1) allows the grating to rotate. Loosening the table rotation lock screw allows the platform to rotate; when the platform rotates, the angles visible in the vernier windows change. Learn how to keep careful track of these changes.

Procedure

1. Turn on the Na lamp. It needs a few minutes to warm up. You will know it is ready when the emitted light has a warm yellow glow. Place the lamp in front of the slit, as shown in Fig. 3, and center it by eye as best you can.

2. Familiarize yourself with all of the controls on the spectrometer. Learn the effect of loosening and tightening each knob as described above. By mastering the operation of the spectrometer *now*, you will save yourself time and frustration if you loosen or tighten something incorrectly *later*.

3. Rotate the telescope so that it points at a far wall with no obstruction (i.e., it does not look at the slit). Focus the telescope at infinity by sighting a distant object on a far wall.

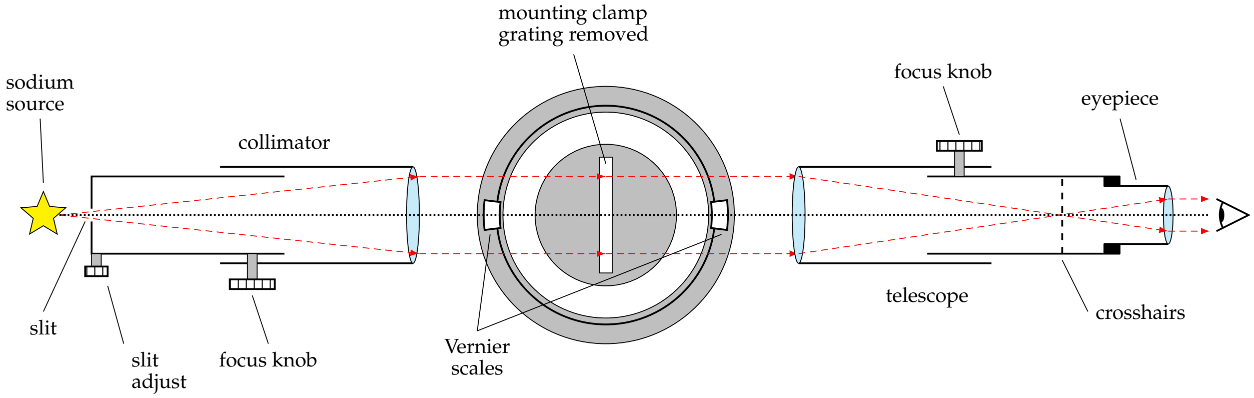

Figure 3: *Aligned orientation of the spectrometer to be used in alignment steps 4, 5, and 6.*

4. With the grating removed, align the telescope with the collimator/slit component as shown in Fig. 3. Adjust the slit and collimator to produce a sharp, narrow image of the source. Make sure you **do not** refocus the telescope.

5. Center the crosshairs on the image of the slit.

6. Record the angle displayed in the window in Data Table 1 in the Postlab exercises; this is your initial zero angle $\theta_{0,i}$. There are two vernier windows on the spectrometer platform; be sure to use the same window for the entire lab.

7. Replace the grating and ensure the grating lines or "rules" are perpendicular to the platform (i.e., they should point toward the ceiling). The grating face should face the slit side of the spectrometer. Record the number of lines/mm of your grating in Data Table 1 in the Postlab exercises. If you have the option, choose the 600 grooves/mm grating. With the grating fixed in place, rotate the telescope exactly 90° as shown in Fig. 4 and then fix the telescope in place.

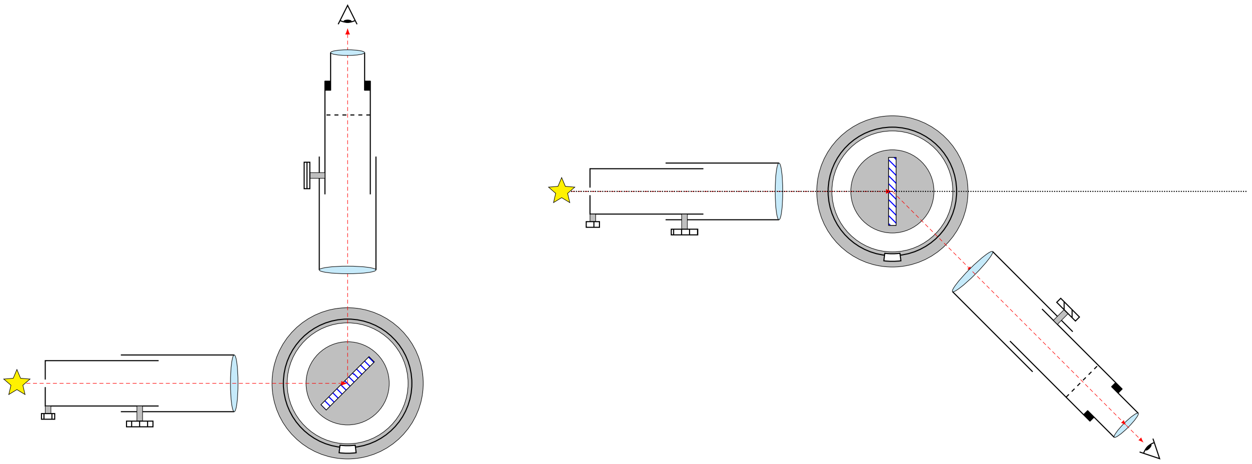

Figure 4: *Left:* 90° *orientation of the spectrometer for parts 7 and 8 of the alignment procedure. Note the* 45° *angle of the grating. Right:* 135° *orientation of the spectrometer for step 10 of the alignment procedure. Note the perpendicular alignment of the grating.*

8. Rotate the grating until the slit image of the Na light source is centered on the crosshairs. Use the zero-order spectrum (i.e., the reflection of the light bulb itself, not a spectral line). The incident angle of the light is now 45°.
9. Fix the grating in place, making sure that the crosshairs stay centered on the light.
10. Turn the telescope back 90° to its original position, as in Fig. 3. Then rotate **the platform** by 135° so that the light from the source is normal to the grating (see Fig. 4). The grating face is now facing the telescope side of the spectrometer, so that the light from the source passes through the grating last. The crosshairs should be centered on the slit. Record the final zero angle, $\theta_{0,f}$. You will subtract this angle from all subsequent angle measurements.

Calibration of the Diffraction Grating

In this part of the experiment, you will determine your grating separation by measuring the angle of the sodium D-lines. The sodium D-lines are a pair of yellow spectral lines, called a "doublet," with wavelengths of 589.0 nm and 589.6 nm. With care you should be able to resolve the doublet and measure a distinct angle for each line. It will help to use a 600 groove/mm grating and to minimize the slit width using the slit adjustment thumbscrew. If you cannot resolve the doublet, measure the average angle at which the yellow line appears and average the two wavelength values given above.

Procedure

1. Start with the telescope at the final zero angle position $\theta_{0,f}$. Move the telescope to the left until you find the first pair of yellow lines; this is the first-order spectrum ($m = 1$). Record this angle (or both angles if you can resolve the doublet) in Data Table 2. Now measure the first-order position(s) on the opposite (right) side and record them in Data Table 2. You can determine the grating spacing using eq. (3) and the average first-order angle(s). You do not need to measure the second-order lines ($m = 2$).
2. In filling out Data Table 2: the Measured Angle is the value read from the vernier scale; the Difference Angle is the positive difference between the Measured Angle and $\theta_{0,f}$; and the Average Angle is the mean of the left and right Difference Angles. Use the Average Angle to calculate the grating separation according to eq. (3).

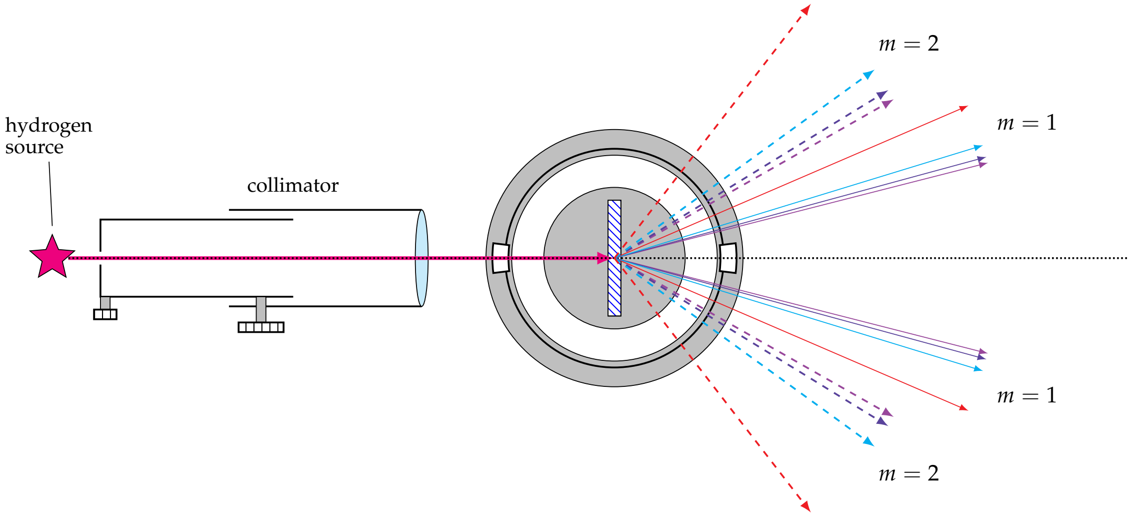

Figure 5: *Angles and diffraction orders of visible Balmer lines. Note that you may only be able to see three lines per order, not four.*

The Balmer Series of Hydrogen

With the spectrometer calibrated, you can now measure the Balmer series of atomic hydrogen. You should be able to observe the first (solid lines) and second (dashed lines) orders for three Balmer lines: a red line (Hα), a cyan line (Hβ), and a blue line (Hγ). You may also be able to resolve a violet line (Hδ). See Fig. 5 for the relationship between the first ($m = 1$) and second ($m = 2$) orders.

Procedure

1. Replace the sodium lamp with the hydrogen lamp. Center the hydrogen bulb on the spectrometer slit to maximize the intensity of the spectral lines. To clearly see the violet lines, you must block stray light; draping black cloth around the lamp or over the spectrometer is helpful.
2. Record the angles at which each spectral line appears. Measure both the first and second orders on both the left and right sides of the zero angle for the red, cyan, and blue lines, for a total of 12 measurements. Record your data in Data Table 3.

Name: ____________________ **Date:** ____________________

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POSTLAB EXERCISES (18 points)

Submit the postlab to the TA at the end of the lab.

Spectrometer Alignment and Calibration (5 points)

Question 3 **3 points**

Fill in your zero angles and the grooves/mm written on the grating.

Data table 1: *Zero angles and diffraction grating size used in the spectrometer.*

Quantity		**Value**	
Initial zero angle:	$\theta_{0,i} =$		
Final zero angle:	$\theta_{0,f} =$		
Grating grooves/mm:			grooves/mm

Next, fill in the calibration data from the sodium D-lines:

Data table 2: *Calibration data from sodium D-lines.*

Spectral Line	Measured Angle	Difference Angle	Wavelength (nm)	Order m	Grating sep. d (nm)	$\overline{d}$ (nm)
Left, Line 1						
Right, Line 1						
Left, Line 2						
Right, Line 2						

Question 4 **1 point**

Show an example calculation of how you determined the grating spacing, d, using data from Data Table 2. Calculate an average value for d.

Question 5 — 1 point

Calculate the uncertainty in grating separation, Δd, using the standard deviation of your two (or four) values for d.

The Balmer Series (13 points)

Question 6 — 2 points

Fill in the table with your measurements of the Balmer lines.

Data table 3: *Balmer series measurements.*

Spectral Line	Order	Measured Angle	Difference Angle	Average Angle	m	Wavelength (nm)	**Average Wavelength (nm)**
Red, Right	1st						
Red, Left	1st						
Red, Right	2nd						
Red, Left	2nd						
Cyan, Right	1st						
Cyan, Left	1st						
Cyan, Right	2nd						
Cyan, Left	2nd						
Blue, Right	1st						
Blue, Left	1st						
Blue, Right	2nd						
Blue, Left	2nd						

Question 7 **1 point**

Show the calculation for how you found the wavelength of the first-order blue line.

Question 8 **1 point**

Show the calculation for how you found the wavelength of the second-order red line.

Question 9 **2 points**

Calculate the uncertainty in wavelength $\Delta\lambda$ for the first-order cyan line using error propagation. Use the uncertainty in the grating separation that you determined earlier, and estimate a value for the uncertainty $\Delta\theta$, which you must convert to radians. Using the average angle θ when computing $\tan\theta$ in the expression below, compute $\Delta\lambda$:

$$\left(\frac{\Delta\lambda}{\lambda}\right)^2 = \left(\frac{\Delta d}{d}\right)^2 + \left(\frac{\Delta\theta}{\tan\theta}\right)^2. \tag{4}$$

Question 10 **1 point**

Show the calculation for the theoretical Balmer wavelength for $n = 3$ using Table 1 and eq. (2).

Question 11 **1 point**

Show the calculation for the theoretical Balmer wavelength for $n = 4$.

Question 12 **1 point**

Show the calculation for the theoretical Balmer wavelength for $n = 5$.

Question 13 **2 points**

Assume that the wavelength uncertainty $\Delta\lambda$ that you calculated for the cyan line also applies to the blue and red lines. Are the theoretical Balmer wavelengths consistent with your measured wavelengths within the uncertainties? Why or why not?

Question 14 **2 points**

Ordinary hydrogen has a nucleus consisting of one proton. However, naturally occurring hydrogen contains a small amount of deuterium, hydrogen having a nucleus consisting of one proton and one neutron. If deuterium existed in sufficient quantity in your source to produce observable spectral lines, would you expect to resolve its lines from those of ordinary hydrogen with our equipment? Let the neutron mass be equal to the proton mass, and let the proton mass be about 1840 times greater than the electron mass. You are expected to support your answer through a calculation.