Planet-Civ Update

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# 1 Variables, Constants and Units

- $T$ = Average Global Temperature (Kelvin)
  - $T_{eq}$ = Equilibrium (initial) Temperature, calculated with the energy balance model ($Kelvin$)
  - $\Delta T$ = Temperature Range in which humans can survive (higher values correspond to lower fragility)[1]
  - $D$ = Orbital Distance (AU)
- $P$ = Global Carbon Dioxide Partial Pressures (ppm)
  - $P_0$ = Initial Carbon Dioxide Partial Pressures (ppm)[2]
  - $\epsilon$ = Annual Per-Capita[3] Carbon Footprint ($\frac{ppm}{10^6 ppl*yr}$)
  - $\Delta P$ = A proportionality factor between the birth rate and changes in $pCO_2$. (higher values correspond to less technologically efficient civilizations (ie: must burn more fossil fuels in order to increase the birth rate))[4]
- $N$ = Global Population ($x10^6$ ppl)
  - $N_0$ = Initial Global Population ($x10^6$ ppl)
  - $N_{max}$ = Maximum Allowed Global Population ($x10^6$ ppl)
  - $\alpha_{birth,0}/\alpha_{death,0}$ = Initial Per-Capita[3] Birth/Death Rates ($1/yr$)
  - $\alpha_{birth}/\alpha_{death}$ = Current Per-Capita[3] Birth/Death Rates ($1/yr$)

[1] $\Delta T = \sqrt{\frac{\alpha_{death,0}}{F_r}}$

[2] $x\ ppm * \left(\frac{1Bar}{10^6 ppm}\right) = y\ bar$

[3] Per-Capita Meaning Per-Million People

[4] $\Delta P = \frac{\alpha_{birth,0}}{E_n}$

# 2 Model 1

## 2.1 Dimensionless Parameter ($\Lambda$)

$$\Lambda = \frac{\alpha_{birth,0}\Delta T}{\epsilon N_{max}\frac{dT}{dP}} = \frac{\textit{Rate of Temperature Change}}{\textit{Maximum Climate Forcing}} \tag{1}$$

- $\Lambda >> 1 \implies$ Corresponds to a civilization having a low risk of an Anthropocene
- $\Lambda << 1 \implies$ Corresponds to a civilization having a high risk of an Anthropocene
- $\Lambda = 1 \implies N_{avg,max}$

## 2.2 Model 1 Outline

First, let the energy balance model reach an equilibrium between incoming and outgoing radiation, this gives us the equilibrium temperature. The model continues by setting the initial temperature to this equilibrium value, as well as setting the birth and death rates to their initial values. The main loop now begins, where each loop represents one year.[5]

i) Call[6]: $\frac{dT}{dt} = EBM(P)$

ii) $\alpha_{birth} = \alpha_{birth,0}\left[1 + \frac{P - P_0}{\Delta P}\right]$

iii) $\alpha_{death} = \alpha_{death,0}\left[1 + \left(\frac{T - T_{eq}}{\Delta T}\right)^2\right]$

iv) Call: $\frac{dN}{dt} = min(\alpha_{birth}N,\ \alpha_{death,0}N_{max}) - \alpha_{death}N$

v) Call: $\frac{dP}{dt} = \epsilon N$

   a) If time has reached the end, program is finished

   b) If time hasn't reached the end, go back to the first step.

## 2.3 Linear Regressions of Temp vs pCO2

For 7 different distances, I first found the value of pCO2 which would make that planet's temp be around room temperature, fixing this value as the initial temp for this distance. Then made a loop in which initially the EBM is run with this value of pCO2, but after every loop I incremented this value slightly, then exited the loop once the equilibrium temperature became greater than 10 degrees higher than the initial temperature. After all distances have been looped, I ran a linear regression (using scipy.stats.linregress) for the data from each distance to find the relationship between changes in pCO2 and changes in global temperature.

$$T = 5.820 * 10^{-2}\left(\frac{P}{ppm}\right) + 323 \qquad \textbf{\textit{(0.94 AU)}}$$

$$T = 1.283 * 10^{-2}\left(\frac{P}{ppm}\right) + 310 \qquad \textbf{\textit{(0.96 AU)}}$$

$$T = 5.892 * 10^{-3}\left(\frac{P}{ppm}\right) + 297 \qquad \textbf{\textit{(0.98 AU)}}$$

$$T = 1.097 * 10^{-3}\left(\frac{P}{ppm}\right) + 294 \qquad \textbf{\textit{(1.00 AU)}}$$

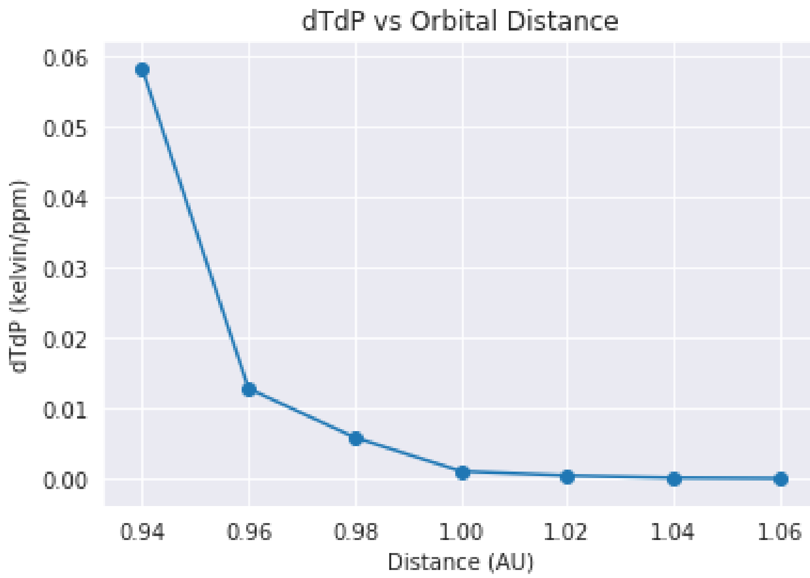
$$T = 4.829 * 10^{-4}\left(\frac{P}{ppm}\right) + 291 \qquad \textbf{\textit{(1.02 AU)}}$$

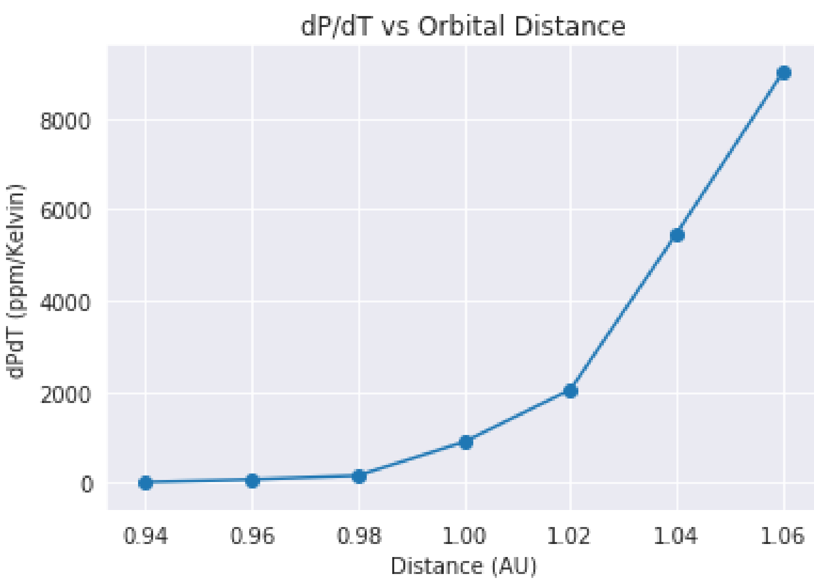
[5] Note: made population have a minimum of 1 million people, to avoid values of $10^{-100}$

[6] $EBM(P) = \frac{\psi(1 - A) - I + \nabla \cdot (\kappa \nabla T)}{C_v}$

$$T = 1.814 * 10^{-4}\left(\frac{P}{ppm}\right) + 294 \qquad \textbf{\textit{(1.04 AU)}}$$

$$T = 1.103 * 10^{-4}\left(\frac{P}{ppm}\right) + 294 \qquad \textbf{\textit{(1.06 AU)}}$$




Above plot (dPdT) is interesting because the y-axis is essentially showing on average how much added $pCO2$ it would take at this orbital distance to raise the climate by one degree Kelvin/Celsius.

## 2.4 Example: Modeling Earth ($t_0 = 1820,\ P_0 = 284,\ N_0 = 1,129$)

### 2.4.1 Input Values

- $N_{max} = 13$ billion people
- $\alpha_{birth,0} = 0.019\ yr^{-1}$
- $\alpha_{death,0} = 0.015\ yr^{-1}$
- $\Delta T = 1.73K$
- $\Delta P = 6.3 * 10^{-5}\ Bar$
- $\epsilon = 0.00019\ \frac{ppm}{10^6 ppl*yr}$
- $\Lambda = 3.94$

### 2.4.2 Output Plots

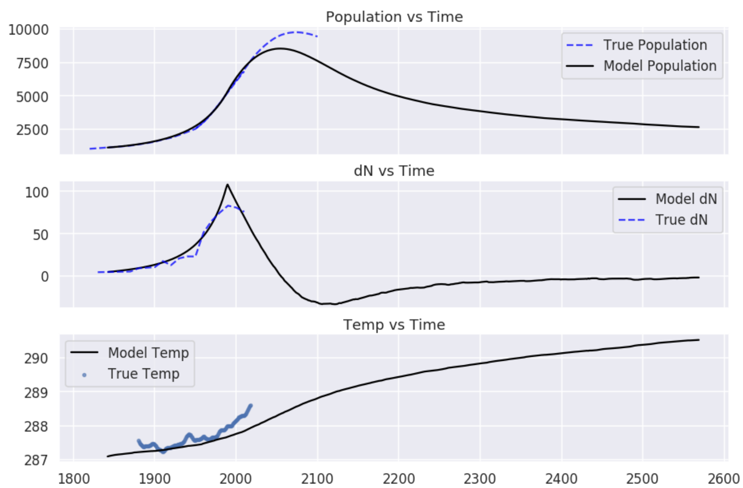



Figure 1: Model Output (solid black line) vs Real Global Data (dotted blue line) for 750 Years

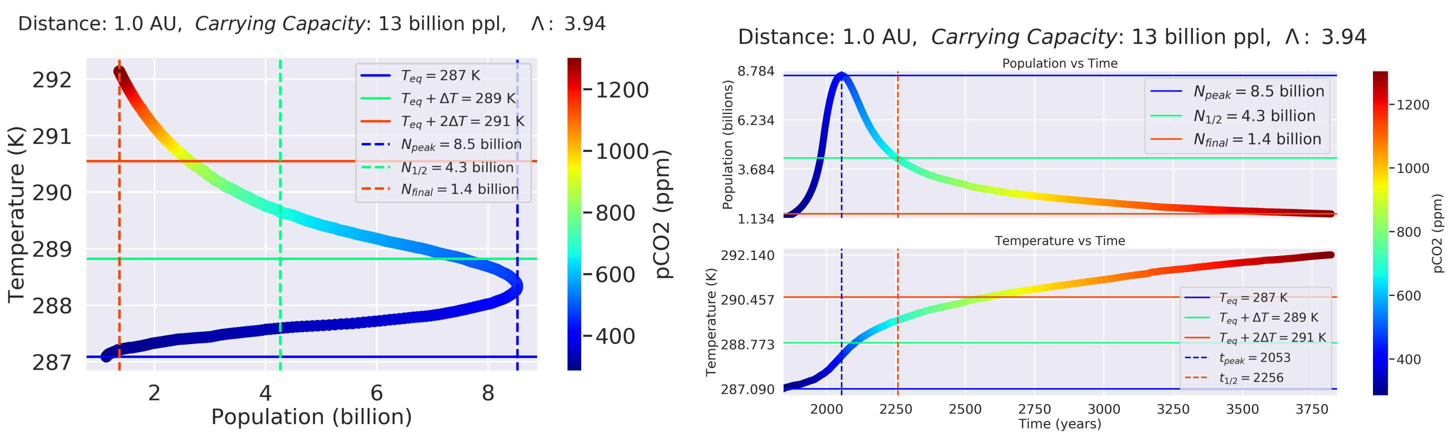



Figure 2: Model Output for 2000 Years

# 3 Analytic Model

## 3.1 Dimensionless Parameter ($\Gamma$)

First, we define the timescales:[7]

$$t_{growth} = \frac{1}{\alpha_{birth}} \tag{2}$$

$$t_{climate} = \frac{\Delta T}{\epsilon N_{max} \frac{dT}{dP}} \tag{3}$$

Then we can write our dimensionless parameter like:

$$\Gamma = \frac{t_{growth}}{t_{climate}} = \frac{\epsilon N_{max} \frac{dT}{dP}}{\alpha_{birth} \Delta T} = \frac{\textit{Timescale for Population Growth}}{\textit{Timescale for Climate to Change}} \tag{4}$$

- $\Gamma << 1 \implies$ Climate will change on timescales much longer than the average generation. Corresponds to a civilization having a low risk for an Anthropocene.
- $\Gamma = 1 \implies$ Climate will change within one generation.
- $\Gamma >> 1 \implies$ Climate will change on timescales much shorter than the average generation. Corresponds to a civilization having a high risk for an Anthropocene.

## 3.2 Analytic Model Outline

First, let the energy balance model reach an equilibrium between incoming and outgoing radiation, this gives us the equilibrium temperature. The model continues by setting the initial temperature to this equilibrium value, as well as setting the birth and death rates to their initial values. The main loop now begins, where each loop represents one year.[8]

i) Call[9]: $\frac{dT}{dt} = EBM(P)$

ii) Call: $\frac{dN}{dt} = min(\alpha_{birth} N,\ \alpha_{death} N_{max}) - \alpha_{death} N - \alpha_{birth} N \left( \frac{T - T_{eq}}{\Delta T} \right)^2$

iii) Call: $\frac{dP}{dt} = \epsilon N$

  a) If time has reached the end, program is finished

  b) If time hasn't reached the end, go back to the first step.

[7] Wouldn't we want this to be $t_{growth} = \frac{1}{\alpha_{growth}}$? (where $\alpha_{growth} = \alpha_{birth} - \alpha_{death}$)

[8] Note: made population have a minimum of 1 million people, to avoid values of $10^{-100}$

[9] $EBM(P) = \frac{\psi(1 - A) - I + \nabla \cdot (\kappa \nabla T)}{C_v}$