Planet-Civ Update

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Model 0: Constant Growth Rates

$$C_v \frac{dT}{dt} = \psi(1 - A) - I + \nabla \cdot (\kappa \nabla T) \tag{1}$$

$$\frac{dP}{dt} = \epsilon N \tag{2}$$

$$\frac{dN}{dt} = \alpha_{net} N \tag{3}$$

- $P = pCO2\ (ppm), \quad (P_0 = 284)$
  - $\epsilon = 4.5 * 10^{-5} = carbon\ footprint\ per\ capita =$ amount of pCO2 added to the atmosphere by a million people in one year
- $N = population\ (*10^6)$
  - $\alpha_{net} = \alpha_{birth} - \alpha_{death} = .02 - .01 = .01 = net/relative\ growth\ rate$
- $T = temperature\ (K)$
- $t = time\ (yr), \quad (t_0 = 1880)$

Model 1: Non-constant Growth Rates

Goal: To try to replicate our planets growth rates by adding a maximum value that cannot be exceeded. Additions in $pCO2$ would act as a proxy for technology, thus would increase the birth rates. Any deviations in global temperatures would make the planet less habitable, thus would increase the death rates.

- E_n = constant determining the relationship between changes in $pCO2$ and changes in birth rates ('technological abilities')
- F_r = constant determining the relationship between changes in temperature and changes in death rates ('fragility')
- T_{eq} = equilibrium temperature (ie: temperature at which the energy balance model finished balancing)
- $\alpha_{birth,0}/\alpha_{death,0}$ = initial birth/death rates

Equation 1 is called.

$$\alpha_{birth} = \alpha_{birth,0} + E_n(pco2 - pco2_0)$$

$$\alpha_{death} = \alpha_{death,0} + F_r(T - T_{eq})^2$$

if($\alpha_{birth} \leq \alpha_{birth,max}$) then

- $\alpha_{net} = \alpha_{birth} - \alpha_{death}$
- Call: $\frac{dN}{dt} = \alpha_{net} N$

if($\alpha_{birth} > \alpha_{birth,max}$) then

- $\alpha_{net} = \alpha_{birth,max} - \alpha_{death}$
- Call: $\frac{dN}{dt} = \alpha_{net}$

Equation 2 is called.

Issues

i) Although this helped to replicate our populations changes in growth rates, the sharp transition once the maximum growth rate is reached made for some very odd behavior.

ii) Could not use this model to replicate both the growth rates of our population as well as the amount of people.

iii) Makes ridiculously large numbers for the population.

Model 2: Non-Constant Growth Rates and Technological Abilities

Goal: Instead of having a maximum growth rate, this model has a maximum population, which acts as a carrying capacity for the growth rates. So that as the population approaches its maximum, the growth rate approaches zero. Also, multiplied by the net growth rate and the per-capita carbon footprint by E_n, so that increases in technology both increase the growth rate as well as the per-capita carbon footprint. Finally, made E_n obey a differential equation which is proportional to the fractional increase in the population, with a proportionality factor representing the rate of the civilizations technological advancements.

- N_{max} = maximum population (carrying capacity)
- ΔN = annual change in population
- α_{tech} = rate of the civilizations technological advancements, has effect of increasing the amplitude of the model (higher population/growth rates).

Set the birth and death rates to their initial values, then start the main loop, where each loop represents one year.

i) Call: $\dfrac{dT}{dt} = EBM(P)$

ii) $\alpha_{birth} = \alpha_{birth,0}$

iii) $\alpha_{net} = (\alpha_{birth} - \alpha_{death})(1 - \dfrac{N}{N_{max}})$

iv) Call: $\dfrac{dN}{dt} = E_n \alpha_{net} N$

v) Call: $\dfrac{dE_n}{dt} = \alpha_{tech}(\dfrac{\Delta N}{N})$

vi) Call: $\dfrac{d\alpha_{death}}{dt} = F_r(T - T_{eq})^2$

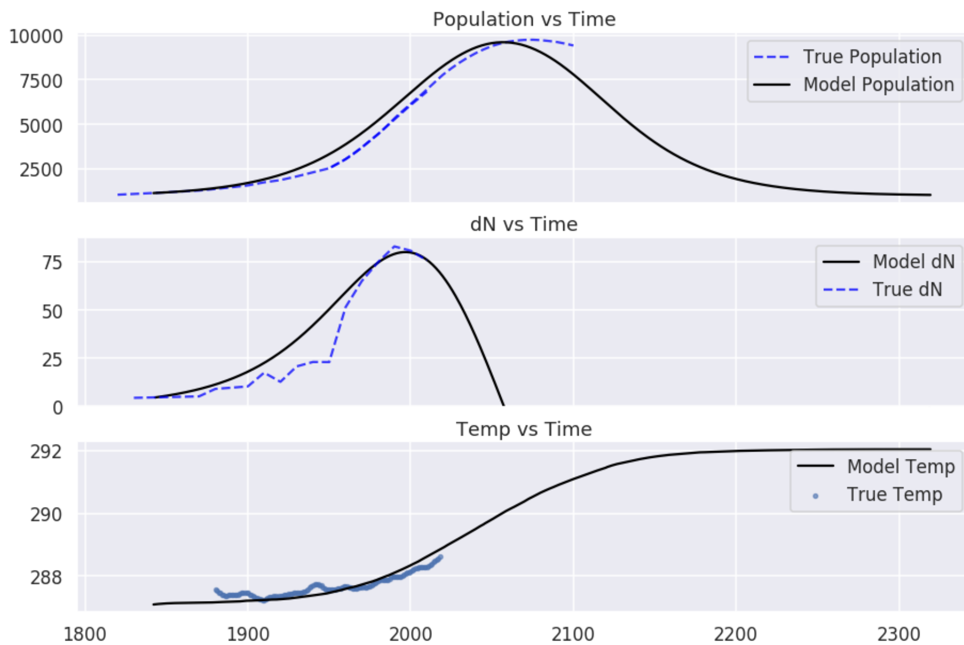
vii) Call: $\frac{dP}{dt} = E_n \epsilon N$

viii) If time has reached then end program, if time hasn't reached the end then go back to the the first step.

Conclusion: This model did the best job of replicating our civilizations changing population and growth rates, as well as the temperature of our planet. Once downside is that population falls off faster in our model than is projected to in real life. Also, their are too many constants. The values for the relevant inputs are:

- $N_{max} = 7800 * 5$, $\alpha_{birth,0} = 0.019$, $\alpha_{death,0} = 0.015$, $E_{n,0} = 1$, $\alpha_{tech} = 7$, $F_r = 2.25 * 10^{-10}$, $\epsilon = 4.5 * 10^{-5}$

  And the resulting output looks like:



Model 3: Variable Death Rate and Maximum Population

Set the birth and death rates to their initial values, then start the main loop, where each loop represents one year.

i) Call: $\frac{dT}{dt} = EBM(P)$

ii) $\alpha_{birth} = \alpha_{birth,0}$

iii) $\alpha_{death} = \alpha_{death,0} + F_r(T - T_{eq})^2$

iv) Call: $\frac{dN}{dt} = min(\alpha_{birth}N,\ \alpha_{death,0}N_{max}) - \alpha_{death}N$

v) Call: $\frac{dP}{dt} = \epsilon N$

vi) If time has reached the end, then end program, if time hasn't reached the end, go back to the the first step.

(Note: made population have a minimum of 1 million people, to avoid values of 10^{-100})

- Increases in N_{max} result in increases in the peak population and the time it takes to reach this peak.
- Increases in F_r result in dramatic increases in the death rates

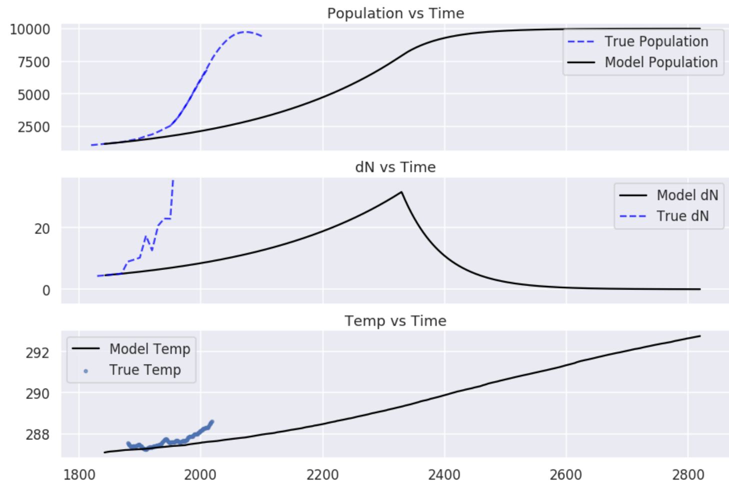
Conclusion: This model did a good job recreating the overall structure of the desired data, yet in both cases, the population and the growth rate don't increase nearly as fast as the data suggests they should. This could be resolved by adding a term onto the birth rate equation, as to model our exponentially increasing technological abilities.

Example Inputs/Output

If we use the inputs:

- $\alpha_{birth,0} = 0.019$
- $\alpha_{death,0} = 0.015$
- $F_r = 5 * 10^{-50}$
- $N_{max} = 10$ billion ppl

The resulting output looks like:



If we use the inputs:

- $\alpha_{birth,0} = 0.019$
- $\alpha_{death,0} = 0.015$
- $F_r = 5 * 10^{-3}$
- $N_{max} = 10$ billion ppl

The resulting output looks like: