Editors' Suggestion

Experimental Assessment of Entropy Production in a Continuously Measured Mechanical Resonator

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The information on a quantum process acquired through measurements plays a crucial role in the determination of its nonequilibrium thermodynamic properties. We report on the experimental inference of the stochastic entropy production rate for a continuously monitored mesoscopic quantum system. We consider an optomechanical system subjected to continuous displacement Gaussian measurements and characterize the entropy production rate of the individual trajectories followed by the system in its stochastic dynamics, employing a phase-space description in terms of the Wigner entropy. Owing to the specific regime of our experiment, we are able to single out the informational contribution to the entropy production arising from conditioning the state on the measurement outcomes. Our experiment embodies a significant step towards the demonstration of full-scale control of fundamental thermodynamic processes at the mesoscopic quantum scale.

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The fundamental connections between information and thermodynamics dates back to the seminal contributions by Maxwell, Szilard, and Landauer [1]. The process of acquiring information can impact the entropic balance of a given physical process. Such information must thus be accounted for when formulating the second law, and considered on equal footing to other thermodynamic quantities, such as heat and work. This is particularly relevant for processes involving microscopic systems, which are fundamentally dominated by fluctuations: the acquisition of information through measurements introduces additional stochasticity and makes the overall process strongly dependent on the monitoring methodology.

Let us briefly illustrate the building blocks of the formulation of thermodynamics at the stochastic level. The changes dS in the entropy of a system subjected to a process can be attributed both to a flow of entropy ϕ between the system and its surroundings and to a contribution π associated to the irreversible production of entropy [2]. We can thus write

$$dS = \phi + \pi, \tag{1}$$

where both ϕ and π are stochastic quantities that fluctuate with each repetition of the experiment. Averaging over many realizations yields the entropy flux and production rates, Φ and Π, respectively. The second law enforces $\Pi \geq 0$ with $\Pi = 0$ when the system is in equilibrium. The introduction of measurement and feedback processes profoundly affects the above statements, as it has been studied in both classical [3–7] and quantum contexts [6,8–10]. Experimental assessments of such modifications have been reported recently in a variety of systems, including classical Brownian particles [11], superconducting qubits [12,13], trapped ions [14,15], and nuclear magnetic resonance [16,17]. Frameworks for describing the dynamics of *continuously monitored* quantum systems have been developed [18–22] and fundamental fluctuation theorems involving heat, work, and entropy for continuously monitored quantum two-level systems have been studied [23–26], including assessments at the single-trajectory level [27,28].

When assessing a monitored system, one must distinguish between the unconditional evolution and the dynamics conditioned on the measurement records [6,9,29]. It can be shown that the average entropy flux rate of the conditional dynamics Φ_c equals the unconditional one Φ_{uc} [30]. However, the same is not true for the entropy production rate that governs the irreversibility of the process. Acquiring information can only make the process more reversible, so that the average entropy production $\int^t \Pi_c d\tau$ of the *conditional trajectories* will be smaller than that of the unconditional one $\int^t \Pi_{\mathrm{uc}} d\tau$. Their difference is precisely associated with an information-theoretic term and can be written as [7,29,30]

$$\Pi_c = \Pi_{\mathrm{uc}} + \dot{\mathcal{I}}, \tag{2}$$

where $\dot{\mathcal{I}}$ is the net rate at which information is acquired through measurement. This term thus captures the fundamental effect of the measurement, providing a valuable link between information and thermodynamics.

Despite its importance, thus far there has been no experimental assessment of the influence of such information theoretic contribution, arising from quantum measurements, to the entropy production. In this Letter, we fill such a gap by reporting the experimental observations of the impact of weak continuous measurements on the nonequilibrium thermodynamics of a mesoscopic mechanical resonator [31]. Our system consists of a nanomechanical resonator coupled to an optical cavity and exposed to the effects of both electromagnetic and phononic environments. We continuously monitor its position by means of homodyne measurements on the output optical field [31]. Combining the phase-space formalism laid down in Ref. [30] with state retrodiction methods [31], we are able to characterize the entropy production at the level of individual quantum trajectories. It should be noted that, in the phase-space formalism, we chose the Wigner entropy as the entropic measure which, despite its limitations to Gaussian dynamics, presents several advantages when considering nonstandard thermodynamics setups [32] (see the discussion in the following). Remarkably, we are able to single out precisely the contribution of the measurement influence to the entropy production.

Our experiment probes both the relaxation dynamics and the steady state. The latter, in particular, configures an informational steady state, where information acquired from the measurement is constantly counterbalancing noise introduced by the environment. In addition to the net rate of information gain $\dot{\mathcal{I}}$, we are also able to single out the *differential* information gain $\mathcal{G}(t)$, which represents the rate at which information must be acquired in each small time step in order to maintain this steady state. Our work thus embodies a step forward towards the full characterization of quantum mesoscopic irreversibility and its control via suitably arranged measurements.

Experimental setup.—The experimental system is provided by an ultracoherent soft-clamped membrane resonator [cf. inset of Fig. 1]. The central defect, embedded in a phononic crystal, supports a localized, "soft-clamped" mechanical mode [33] at the resonance frequency $\Omega_m/(2\pi) = 1.14$ MHz. Once cooled to a temperature of $T = 11$ K, we find for this mode a quality factor $Q = \Omega_m/\Gamma_m = 1.03 \times 10^9$, where Γ_m is the energy dissipation rate. The mechanical system is dispersively coupled to the frequency of a Fabry-Perot cavity mode (linewidth $\kappa/(2\pi) = 18.5$ MHz), with vacuum optomechanical coupling rate $g_0/(2\pi) = 129$ Hz. The cavity mode is pumped by an external probe laser to an averaged photon occupancy $\bar{n}_{\text{cav}}$. We assume that the semiclassical steady state of the nonlinear dynamics has been reached and, when speaking of different modes, we refer to the fluctuations

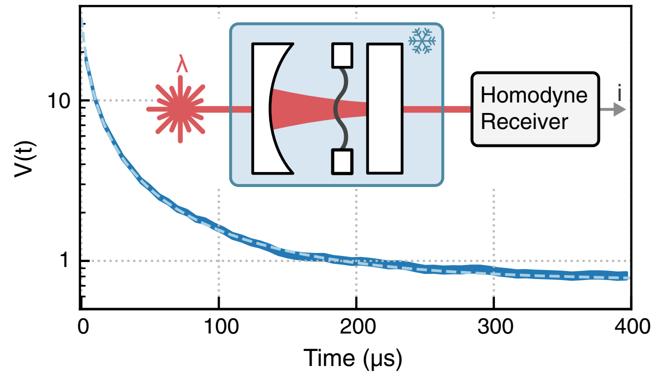

FIG. 1. Conditional mechanical evolution. Measured conditional variance $V(t)$ (blue line), from the initial unconditional value $V_{\text{uc}} \approx 34$ to the steady state $V_{\text{ss}} \approx 0.8$. The dashed line is a theoretical prediction. The inset shows a sketch of the experimental system, which comprises a cryogenic optomechanical cavity resonantly driven by a coherent probe laser. The mechanical resonator is in thermal contact with two baths: a thermal, cryogenic bath and the optical bath. The output field is continuously monitored by means of a homodyne receiver. The photocurrent i is used to estimate the conditional mechanical state.

around such a mean steady state, as it is common practice [34]. In this linearized interaction regime, the effective, multiphoton optomechanical coupling, enhanced by the average cavity photon occupancy, is $g = g_0\sqrt{\bar{n}_{\text{cav}}}$ and the fluctuations of the system evolve according to a Hamiltonian that is quadratic in the system's fluctuations. This ensures that all the states remain Gaussian.

We use an auxiliary light field to stabilize the system and provide precooling of other mechanical modes. Such beam also introduces additional damping and cooling on the mode of interest, effectively changing its thermal environment. In addition, any small detuning of the probe beam from the cavity resonance causes additional damping. We account for these effects by introducing the effective energy damping rate and bath occupancy $\Gamma_m/(2\pi) = 19$ Hz and $\bar{n}_{\text{th}} = 14$, respectively. The total thermal decoherence rate is thus $\gamma = \Gamma_m(\bar{n}_{\text{th}} + 1/2) = 2\pi \times 265$ Hz.

The quantum measurement is performed by imprinting, through the optomechanical interaction, the mechanical displacement in the phase quadrature of the probe laser. Such quadrature is measured by a phase-sensitive measurement of the output field, implemented using a balanced homodyne receiver with detection efficiency $\eta_{\text{det}} = 74\%$ [cf. inset of Fig. 1].

Our experiment operates in the nonresolved-sideband regime $\Omega_m \ll \kappa$, which enforces a separation of time scales and allows the cavity mode to be adiabatically eliminated [31]. In a frame rotating at frequency Ω_m and within the rotating wave approximation, the conditional dynamics of the mechanical mode alone is well described by the stochastic master equation (SME) [35,36]

$$d\rho_c = (\mathcal{L}_{\mathrm{th}} + \mathcal{L}_{\mathrm{qba}} + \mathcal{L}_{\mathrm{stoc}})\rho_c dt. \quad (3)$$

The term $\mathcal{L}_{\mathrm{th}}\rho = \Gamma_m(\bar{n}_{\mathrm{th}} + 1)\mathcal{D}[\hat{c}]\rho + \Gamma_m\bar{n}_{\mathrm{th}}\mathcal{D}[\hat{c}^\dagger]\rho$ in Eq. (3) describes the contact with the effective mechanical phonon bath. Here $\hat{c} = (\hat{X} + i\hat{Y})/\sqrt{2}$ is the annihilation operator of the mechanical mode (written in terms of quadratures $\hat{X}, \hat{Y}$) and $\mathcal{D}[\hat{A}]\rho = \hat{A}\rho\hat{A}^\dagger - \{\hat{A}^\dagger\hat{A}, \rho\}/2$. The quantum measurement backaction is described by $\mathcal{L}_{\mathrm{qba}}\rho_c = \Gamma_{\mathrm{qba}}[\mathcal{D}(\hat{c}) + \mathcal{D}(\hat{c}^\dagger)]\rho_c$, which results in radiation pressure force fluctuations with a decoherence rate $\Gamma_{\mathrm{qba}} = 4g^2/\kappa \approx 2\pi \times 0.36$ kHz. Finally, the term

$$\mathcal{L}_{\mathrm{stoc}}\rho_c = \sqrt{\eta_{\mathrm{det}}\Gamma_{\mathrm{qba}}}(\boldsymbol{\mathcal{H}}\rho_c) \cdot d\mathbf{W} \quad (4)$$

with the vectors $\boldsymbol{\mathcal{H}}\rho_c = (\mathcal{H}(\hat{X})\rho_c, \mathcal{H}(\hat{Y})\rho_c)^T$ and $d\mathbf{W} = (dW_X, dW_Y)^T$, describes the stochastic contribution to the dynamics stemming from conditioning upon the measurement outcomes, with $\mathcal{H}(\hat{A})\rho = \hat{A}\rho + \rho\hat{A}^\dagger - \langle\hat{A} + \hat{A}^\dagger\rangle_\rho\rho$, and $dW_{X,Y}$ independent real Wiener increments [19,36].

The unconditional mechanical dynamics is retrieved by neglecting $\mathcal{L}_{\mathrm{stoc}}\rho_c$. The corresponding Gaussian steady-state density matrix is characterized by vanishing first cumulants, $\mathbf{r}_{\mathrm{uc}} = (\langle\hat{X}\rangle_{\mathrm{uc}}, \langle\hat{Y}\rangle_{\mathrm{uc}})^T = \mathbf{0}$, and a diagonal covariance matrix with elements $V_{\mathrm{uc}} = \langle\hat{X}^2\rangle_{\mathrm{uc}} = \langle\hat{Y}^2\rangle_{\mathrm{uc}} = \bar{n}_{\mathrm{th}} + 1/2 + \Gamma_{\mathrm{qba}}/\Gamma_m$. We use such state density matrix as the initial preparation in all experiments reported below. This is the natural steady state of the optomechanical system, thus its preparation requires only to wait for the initial brief transient to decay, before conditioning upon the measurement outcomes. The conditional dynamics described by Eq. (3), on the other hand, has both first and second cumulants evolving nontrivially according to

$$d\mathbf{r}(t) = -\frac{\Gamma_m}{2}\mathbf{r}dt + \sqrt{4\eta_{\mathrm{det}}\Gamma_{\mathrm{qba}}}V(t)d\mathbf{W}, \quad (5)$$

$$\dot{V}(t) = \Gamma_m[V_{\mathrm{uc}} - V(t)] - 4\eta_{\mathrm{det}}\Gamma_{\mathrm{qba}}V(t)^2. \quad (6)$$

The ensuing dynamics gives the covariance $\boldsymbol{V}(t) = V(t)\mathbb{1}$ where we have introduced the identity matrix $\mathbb{1}$ and the c-number variance $V(t) = \langle\hat{X}^2\rangle - \langle\hat{X}\rangle^2 = \langle\hat{Y}^2\rangle - \langle\hat{Y}\rangle^2$ [37]. The first cumulants thus evolve stochastically, while the second ones obey a deterministic nonlinear evolution. It should be noted that, the process entailed by this model is dynamically stable, as it can be easily verified following the criteria discussed in Ref. [44]. This ensures the convergence of any quantity integrated over long-time windows.

The last term in Eq. (6) is associated with the information acquired by the measurement, and we dub it *innovation*. It is nonpositive as acquired information can never increase the uncertainty about the mechanical motion. According to Eq. (6), the initial unconditional variance V_{uc} evolves into the conditional steady-state value $V_{\mathrm{ss}} = -\mu + \sqrt{\mu(\mu + 2V_{\mathrm{uc}})}$ with $\mu = \Gamma_m/(8\eta_{\mathrm{det}}\Gamma_{\mathrm{qba}})$. Owing to the innovation term, $V_{\mathrm{ss}} \leq V_{\mathrm{uc}}$ given that $\mu > 0$. The continuous weak measurements thus lead to a conditional steady-state density matrix with a higher purity than the unconditional one [45,46]. This is an instance of measurement-based cooling and was experimentally demonstrated in Ref. [31]. The conditional first cumulants $\mathbf{r}(t)$ in Eq. (5) are related to the experimental homodyning measurement outcomes $\mathbf{i}(t)$ through $\mathbf{i}(t)dt = \sqrt{4\eta_{\mathrm{det}}\Gamma_{\mathrm{qba}}}\mathbf{r}(t)dt + d\mathbf{W}$. In practice, the latter relation is used to express $d\mathbf{W}$ in terms of the outcomes, $\mathbf{i}(t)$, and substituted in Eq. (5). This yields a recursive relation used to experimentally filter the data $\mathbf{i}(t)$ to obtain the first cumulant $\mathbf{r}(t)$. The conditional variance $V(t)$, however, evolves independently of the specific measurement outcomes. To assesses it experimentally we thus employ a prediction-retrodiction method [31], which reconstructs $V(t)$ by combining data on $\mathbf{r}(t')$ acquired at earlier ($t' < t$) *and* later times ($t' > t$). Such future outcomes can be used to obtain a retrodicted trajectory, $\mathbf{r}_b(t)$, [47] using an experimental filter similar to what has been derived from $\mathbf{r}(t)$ [37]. The fluctuations of the difference $\mathbf{d}(t) = \mathbf{r}(t) - \mathbf{r}_b(t)$ over an ensemble of independent realizations can be shown to be directly connected to $V(t)$ according to the relation [37]

$$V_d(t) = V(t) + V_{\mathrm{ss}} + \Gamma_m/(4\eta_{\mathrm{det}}\Gamma_{\mathrm{qba}}). \quad (7)$$

In the limit of high cooperativity ($\Gamma_{\mathrm{qba}} \gg \Gamma_m$) and large detection efficiency ($\eta_{\mathrm{det}} \approx 1$) the last term can be neglected. The values of V_{ss} and $V(t)$ are then readily obtained as $V_{\mathrm{ss}} = V_d(\infty)/2$ [31] and $V(t) = V_d(t) - V_{\mathrm{ss}}$, respectively.

Figure 1 shows the evolution of $V(t)$ from the initial unconditional value V_{uc}, all the way to the steady-state value V_{ss}. The experimental data compare very well to the theoretical prediction provided by Eq. (6), thus strongly corroborating the suitability of our model.

Entropy production along individual trajectories.—We are now in a position to assess the thermodynamics of the system at the level of individual quantum trajectories. Our setup is not a standard thermodynamic system due to the presence of the optical cavity, which acts as a nonthermal bath. The usual formulation of entropy production thus does not apply. Despite this, it is possible to employ an alternative put forth in Ref. [32], which makes use of quantum phase-space methods and is adequate for the description of Gaussian dynamics. This approach has already been successfully applied to the experimental characterization of the mean entropy production in the dynamics of open mesoscopic systems [48]. In Ref. [30], the method was extended to account for the presence of quantum-limited detectors continuously monitoring the system.

When applied to our experimental endeavors [37], such theoretical framework shows that the conditional entropy flux and production rates, defined in Eq. (1), can be written in terms of the first and second cumulants as

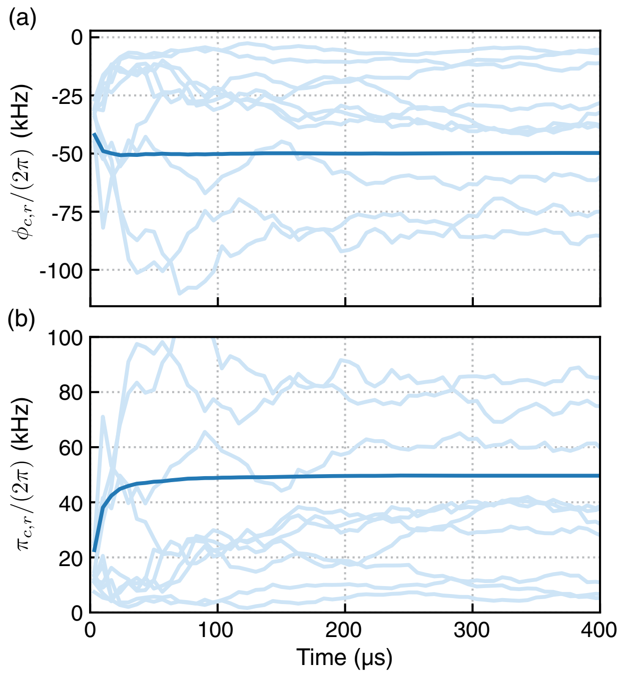

FIG. 2. Stochastic entropy flux and production rates. (a) The stochastic entropy flux rates (light blue) for a sample of ten trajectories. The dark blue line is the ensemble average over all the trajectories. (b) The stochastic entropy production rates (light blue) and the ensemble average (dark blue), for the same sample of trajectories.

$$\phi_{c,\mathbf{r}} = \frac{\Gamma_m}{n_{\text{th}} + 1/2}[(\bar{n}_{\text{th}} + 1/2) - \theta(t)] - 4\Gamma_{\text{qba}}\theta(t),$$
$$\pi_{c,\mathbf{r}} = \Gamma_m\left[\frac{\theta(t)}{\bar{n}_{\text{th}} + 1/2} + \frac{V_{\text{uc}}}{V(t)} - 2\right] + 4\Gamma_{\text{qba}}[\theta(t) - \eta_{\text{det}}V(t)], \tag{8}$$

where $\theta(t) = V(t) + \boldsymbol{r}(t)^{\mathrm{T}}\boldsymbol{r}(t)/2$ encompasses all the stochastic contributions [cf. Eq. (5)]. We can experimentally reconstruct such quantities by means of the measured stochastic trajectories $\mathbf{r}(t)$ and the inferred conditional variance $V(t)$. We show in Fig. 2 some realizations of the stochastic entropy flux and production rates. Despite the low thermal occupancy of $\bar{n}_{\text{th}} \approx 14$ phonons, these quantities fluctuate substantially, highlighting the essential role of fluctuations in the thermodynamics of the system.

We also average them over 3600 trajectories, yielding the conditional flux and production rates $\Phi_c = \mathbb{E}(\phi_{c,\boldsymbol{r}})$ and $\Pi_c = \mathbb{E}(\pi_{c,\boldsymbol{r}})$, which are shown in Fig. 2, dark blue. These quantities can be readily computed from our model by noting that, owing to Eqs. (5) and (6) and our choice of initial conditions, we have $\mathbb{E}[\theta(t)] = V_{\text{uc}}$. From Fig. 2 we gather that both Φ_c and Π_c relax monotonically towards the new steady-state values. However, even at the steady state, the entropy production rate Π_c does not vanish due to the nonequilibrium nature of the stationary state, where the effects of the thermal bath, measurement backaction, and information gain compete with each other.

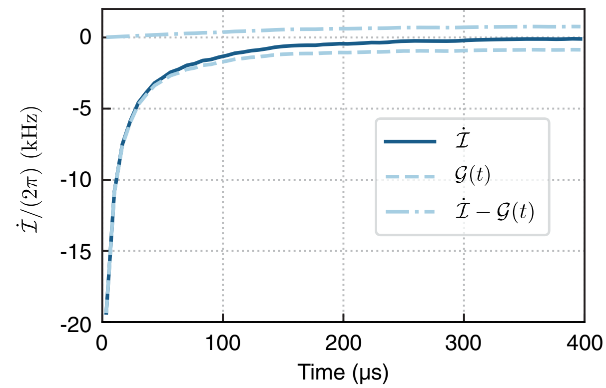

FIG. 3. Informational contribution to the entropy production rate. We obtain the informational contribution (dark blue) from the entropy production. The dashed (dot-dashed) line is the differential gain of information due to the measurement (loss of information due to noise input by the phonon bath).

Information gain.—The influence that monitoring the system has on the irreversibility of the dynamics is encoded in the mismatch between the conditional entropy production rate Π_c and the unconditional one Π_{uc} [cf. Eq. (2)]. Such mismatch is quantified by the net rate of information gain achieved through measuring

$$\dot{\mathcal{I}} = \Gamma_m(V_{\text{uc}}/V(t) - 1) - 4\eta_{\text{det}}\Gamma_{\text{qba}}V(t). \tag{9}$$

The temporal behavior of $\dot{\mathcal{I}}$ reconstructed from the experimental data is shown in Fig. 3. As in our case the system is prepared in the steady state of the unconditional dynamics, the first and second cumulants in the absence of monitoring remain constant in time, and the unconditional rate of entropy production keeps the value $\Pi_{\text{uc}} = \Gamma_m[V_{\text{uc}}/(\bar{n}_{\text{th}} + 1/2) - 1] + 4\Gamma_{\text{qba}}V_{\text{uc}}$ (cf. Ref. [37] for further details). We can thus subtract such value from Π_c in Fig. 2 to obtain the net rate of acquired information due to the continuous monitoring.

As the quantity $-\int_0^\infty \dot{\mathcal{I}}dt$ quantifies the mutual information between system and detector [30], and given that $\dot{\mathcal{I}}$ vanishes in the (conditional) steady state [cf. Fig. 3], such quantity tends to a constant in the long-time limit. This is intuitively understood from the fact that, in the steady state, monitoring the system does not add any additional information. If, however, the monitoring process suddenly stops, the conditional steady state will not be sustained and the system will heat-up back towards V_{uc}. Constant monitoring is thus necessary to maintain the conditional steady state. In other words, even at the steady state, information is constantly being acquired, but noise is constantly being introduced by the phonon bath. It is thus interesting to identify which of the terms in $\dot{\mathcal{I}}$ is responsible for the incremental gains of information required to maintain the conditional steady state.

This concept can be readily understood from inspecting Eq. (9), which consists of the competition between the noise introduced by the phonon bath (at rate Γ_m) and the gain of information (proportional to the detection efficiency $\eta_{\rm det}$). We can thus quite naturally introduce the *differential gain* $\mathcal{G}(t) \coloneqq -4\eta_{\rm det}\Gamma_{\rm qba}V(t)$ and notice that, in light of the interpretation of the last term in Eq. (6) as an innovation rate, $\mathcal{G}(t)$ is the contribution of this innovation to $\dot{\mathcal{I}}$. The behaviors of $\dot{\mathcal{I}}$ and $\mathcal{G}(t)$ inferred from the experimental data are shown in Fig. 3: the initial closeness of $\mathcal{G}(t)$ to $\dot{\mathcal{I}}$ suggests that the early stages of the dynamics are strongly affected by the differential information gain. As the dynamics approaches the steady state, however, the contribution from $\mathcal{G}(t)$ become less significant. However, while $\dot{\mathcal{I}} \to 0$, $\mathcal{G}(t)$ tends to the (in general small) non-null value $\mathcal{G}(\infty) = -4\eta_{\rm det}\Gamma_{\rm qba}V_{\rm ss}$, which thus represents the gain of information per unit time that the detector must acquire in order to maintain the steady state.

Conclusions.—We have investigated the effects of weak continuous measurements on the thermodynamics of a mesoscopic mechanical system. By employing a phase-space formalism [30] and the retrodictive techniques used in Ref. [31], we have connected pivotal thermodynamic quantities, such as entropy production and flux rates along individual dynamical trajectories, to accessible experimental data. The working point of our experiment has enabled us to single out the contributions to the entropy production of the system due solely to the information acquired by monitoring the system. Such contribution decreases in time as the system reaches a nonequilibrium steady state.

Our endeavors demonstrate the key role played by measurements in influencing the energetics of a quantum system. Remarkably, they showcase the intricate interplay between fundamental energy-exchange processes and information in setting up (and sustaining) the dynamical and steady-state features of a process. Such influences can be further explored by assessing whether the control of informational terms to entropy production stemming from suitable measurement strategy could be used as an effective tool for quantum state engineering [49]. Another interesting direction would address composite systems endowed with initial quantum correlations and the experimental study of their effects, in conjunction with continuous monitoring, on the thermodynamics of the systems.

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[1] J. M. R. Parrondo, J. M. Horowitz, and T. Sagawa, Thermodynamics of information, Nat. Phys. **11**, 131 (2015).
[2] R. C. Tolman and P. C. Fine, On the irreversible production of entropy, Rev. Mod. Phys. **20**, 51 (1948).
[3] T. Sagawa and M. Ueda, Generalized Jarzynski Equality Under Nonequilibrium Feedback Control, Phys. Rev. Lett. **104**, 090602 (2010).
[4] T. Sagawa and M. Ueda, Fluctuation Theorem with Information Exchange: Role of Correlations in Stochastic Thermodynamics, Phys. Rev. Lett. **109**, 180602 (2012).
[5] T. Sagawa and M. Ueda, Role of mutual information in entropy production under information exchanges, New J. Phys. **15**, 125012 (2013).
[6] S. Ito and T. Sagawa, Information Thermodynamics on Causal Networks, Phys. Rev. Lett. **111**, 180603 (2013).
[7] J. M. Horowitz and H. Sandberg, Second-law-like inequalities with information and their interpretations, New J. Phys. **16**, 125007 (2014).
[8] T. Sagawa and M. Ueda, Second Law of Thermodynamics with Discrete Quantum Feedback Control, Phys. Rev. Lett. **100**, 080403 (2008); Minimal Energy Cost for Thermodynamic Information Processing: Measurement and Information Erasure, Phys. Rev. Lett. **102**, 250602 (2009).
[9] K. Funo, Y. Watanabe, and M. Ueda, Integral quantum fluctuation theorems under measurement and feedback control, Phys. Rev. E **88**, 052121 (2013).
[10] T. Benoist, V. Jakšić, Y. Pautrat, and C.-A. Pillet, On entropy production of repeated quantum measurements. I. General theory, Commun. Math. Phys. **357**, 77 (2018).
[11] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki, and M. Sano, Experimental demonstration of information-to-energy conversion and validation of the generalized jarzynski equality, Nat. Phys. **6**, 988 (2010).
[12] Y. Masuyama, K. Funo, Y. Murashita, A. Noguchi, S. Kono, Y. Tabuchi, R. Yamazaki, M. Ueda, and Y. Nakamura, Information-to-work conversion by Maxwell's demon in a superconducting circuit-QED system, Nat. Commun. **9**, 1 (2018).

[13] N. Cottet, S. Jezouin, L. Bretheau, P. Campagne-Ibarcq, Q. Ficheux, J. Anders, A. Auffèves, R. Azouit, P. Rouchon, and B. Huard, Observing a quantum Maxwell demon at work, Proc. Natl. Acad. Sci. U.S.A. **114**, 7561 (2017).

[14] T. P. Xiong, L. L. Yan, F. Zhou, K. Rehan, D. F. Liang, L. Chen, W. L. Yang, Z. H. Ma, M. Feng, and V. Vedral, Experimental Verification of a Jarzynski-Related Information-Theoretic Equality Using a Single Trapped Ion, Phys. Rev. Lett. **120**, 010601 (2018).

[15] L. L. Yan, T. P. Xiong, K. Rehan, F. Zhou, D. F. Liang, L. Chen, J. Q. Zhang, W. L. Yang, Z. H. Ma, and M. Feng, Single-Atom Demonstration of Quantum Landauer Principle, Phys. Rev. Lett. **120**, 210601 (2018).

[16] P. A. Camati, J. P. S. Peterson, T. B. Batalhão, K. Micadei, A. M. Souza, R. S. Sarthour, I. S. Oliveira, and R. M. Serra, Experimental Rectification of Entropy Production by Maxwell's Demon in a Quantum System, Phys. Rev. Lett. **117**, 240502 (2016).

[17] J. P. P. Silva, R. S. Sarthour, A. M. Souza, I. S. Oliveira, J. Goold, K. Modi, D. O. Soares-Pinto, and L. C. Céleri, Experimental demonstration of information to energy conversion in a quantum system at the Landauer Limit, Proc. R. Soc. A **472**, 1 (2014).

[18] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, New York, 2002).

[19] H. M. Wiseman and G. J. Milburn, *Quantum Measurement and Control* (Cambridge University Press, Cambridge, England, 2010).

[20] J. J. Alonso, E. Lutz, and A. Romito, Thermodynamics of Weakly Measured Quantum Systems, Phys. Rev. Lett. **116**, 080403 (2016).

[21] J. Dressel, A. Chantasri, A. N. Jordan, and A. N. Korotkov, Arrow of Time for Continuous Quantum Measurement, Phys. Rev. Lett. **119**, 220507 (2017).

[22] C. Elouard, D. A. Herrera-Martí, M. Clusel, and A. Auffèves, The role of quantum measurement in stochastic thermodynamics, npj Quantum Inf. **3**, 9 (2017).

[23] P. G. Di Stefano, J. J. Alonso, E. Lutz, G. Falci, and M. Paternostro, Non-equilibrium thermodynamics of continuously measured quantum systems a circuit-qed implementation, Phys. Rev. B **98**, 144514 (2018).

[24] M. Naghiloo, J. J. Alonso, A. Romito, E. Lutz, and K. W. Murch, Information Gain and Loss for a Quantum Maxwell's Demon, Phys. Rev. Lett. **121**, 030604 (2018).

[25] P. M. Harrington, D. Tan, M. Naghiloo, and K. W. Murch, Characterizing a Statistical Arrow of Time in Quantum Measurement Dynamics, Phys. Rev. Lett. **123**, 020502 (2019).

[26] M. Naghiloo, D. Tan, P. M. Harrington, J. J. Alonso, E. Lutz, A. Romito, and K. W. Murch, Heat and Work Along Individual Trajectories of a Quantum Bit, Phys. Rev. Lett. **124**, 110604 (2020).

[27] K. Murch, S. Weber, C. Macklin, and I. Siddiqi, Observing single quantum trajectories of a superconducting quantum bit, Nature (London) **502**, 211 (2013).

[28] S. Weber, A. Chantasri, J. Dressel, A. N. Jordan, K. Murch, and I. Siddiqi, Mapping the optimal route between two quantum states, Nature (London) **511**, 570 (2014).

[29] P. Strasberg and A. Winter, Stochastic thermodynamics with arbitrary interventions, Phys. Rev. E **100**, 022135 (2019).

[30] A. Belenchia, L. Mancino, G. T. Landi, and M. Paternostro, Entropy production in continuously measured quantum systems, arXiv:1908.09382.

[31] M. Rossi, D. Mason, J. Chen, and A. Schliesser, Observing and Verifying the Quantum Trajectory of a Mechanical Resonator, Phys. Rev. Lett. **123**, 163601 (2019).

[32] J. P. Santos, G. T. Landi, and M. Paternostro, Wigner Entropy Production Rate, Phys. Rev. Lett. **118**, 220601 (2017).

[33] M. Rossi, D. Mason, J. Chen, Y. Tsaturyan, and A. Schliesser, Measurement-based quantum control of mechanical motion, Nature (London) **563**, 53 (2018).

[34] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, Rev. Mod. Phys. **86**, 1391 (2014).

[35] A. Szorkovszky, A. C. Doherty, G. I. Harris, and W. P. Bowen, Mechanical Squeezing via Parametric Amplification and Weak Measurement, Phys. Rev. Lett. **107**, 213603 (2011).

[36] A. C. Doherty, A. Szorkovszky, G. Harris, and W. Bowen, The quantum trajectory approach to quantum feedback control of an oscillator revisited, Phil. Trans. R. Soc. A **370**, 5338 (2012).

[37] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.125.080601 for additional details on the theory and data analysis, which includes Refs. [38–43].

[38] M. L. Gorodetsky, A. Schliesser, G. Anetsberger, S. Deleglise, and T. J. Kippenberg, Determination of the vacuum optomechanical coupling rate using frequency noise calibration, Opt. Express **18**, 23236 (2010).

[39] K. Jacobs and D. A. Steck, A straightforward introduction to continuous quantum measurement, Contemp. Phys. **47**, 279 (2006).

[40] M. Esposito and C. Van den Broeck, Three faces of the second law. I. Master equation formulation, Phys. Rev. E **82**, 011143 (2010).

[41] S. G. Hofer and K. Hammerer, Quantum control of optomechanical systems, in *Advances in Atomic, Molecular, and Optical Physics* (Elsevier, New York, 2017), Vol. 66, pp. 263–374.

[42] G. D. Chiara, G. Landi, A. Hewgill, B. Reid, A. Ferraro, A. J. Roncaglia, and M. Antezza, Reconciliation of quantum local master equations with thermodynamics, New J. Phys. **20**, 113024 (2018).

[43] G. Landi *et al.* (to be published).

[44] H. M. Wiseman and A. C. Doherty, Optimal Unravellings for Feedback Control in Linear Quantum Systems, Phys. Rev. Lett. **94**, 070405 (2005).

[45] H. Maassen and B. Kümmerer, Purification of quantum trajectories, in *Institute of Mathematical Statistics Lecture Notes—Monograph Series* (Institute of Mathematical Statistics, Beachwood, OH, 2006), pp. 252–261.

[46] A. Barchielli and A. M. Paganoni, On the asymptotic behaviour of some stochastic differential equations for quantum states, Inf. Dim. Anal. Quant. Prob. Rel. Topics **06**, 223 (2003).
[47] J. Zhang and K. Mølmer, Prediction and retrodiction with continuously monitored gaussian states, Phys. Rev. A **96**, 062131 (2017).
[48] M. Brunelli, L. Fusco, R. Landig, W. Wieczorek, J. Hoelscher-Obermaier, G. Landi, F. L. Semião, A. Ferraro, N. Kiesel, T. Donner, G. De Chiara, and M. Paternostro, Experimental Determination of Irreversible Entropy Production in Out-of-Equilibrium Mesoscopic Quantum Systems, Phys. Rev. Lett. **121**, 160604 (2018).
[49] G. Cardona, A. Sarlette, and P. Rouchon, Exponential stabilization of quantum systems under continuous non-demolition measurements, Automatica **112**, 108719 (2020).