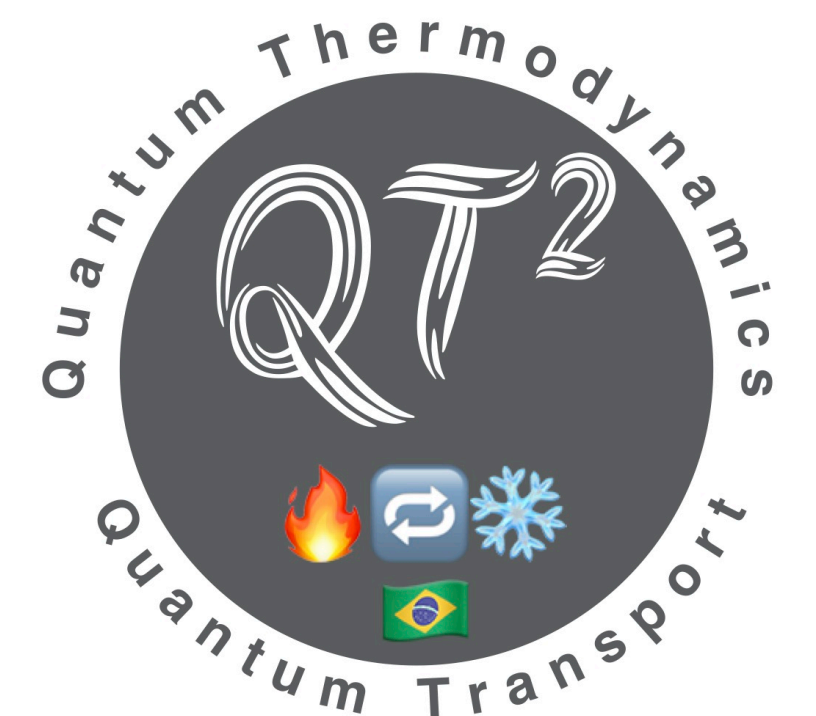


Information-thermodynamics in the quantum regime

Gabriel T. Landi - Universidade de São Paulo

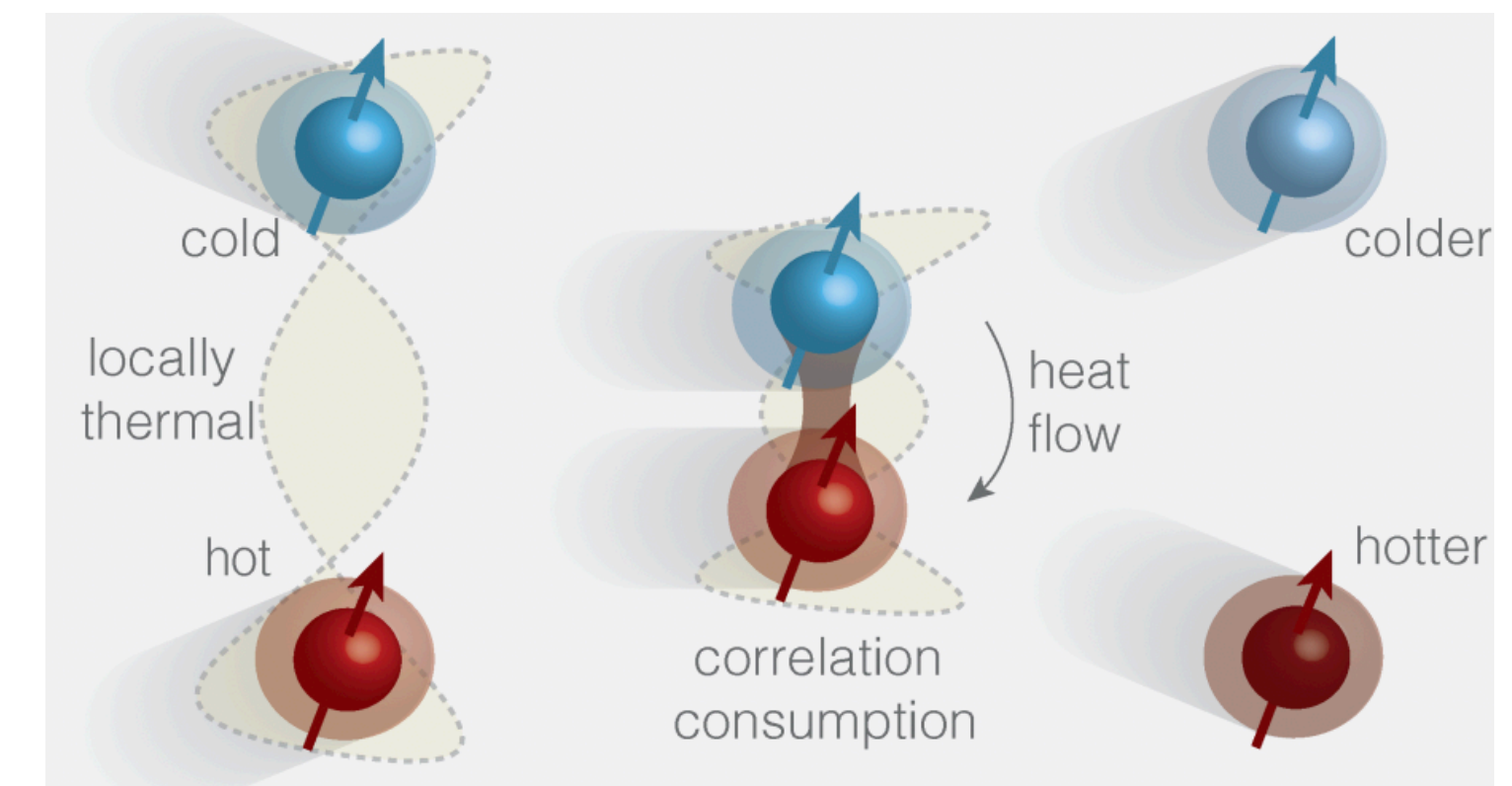
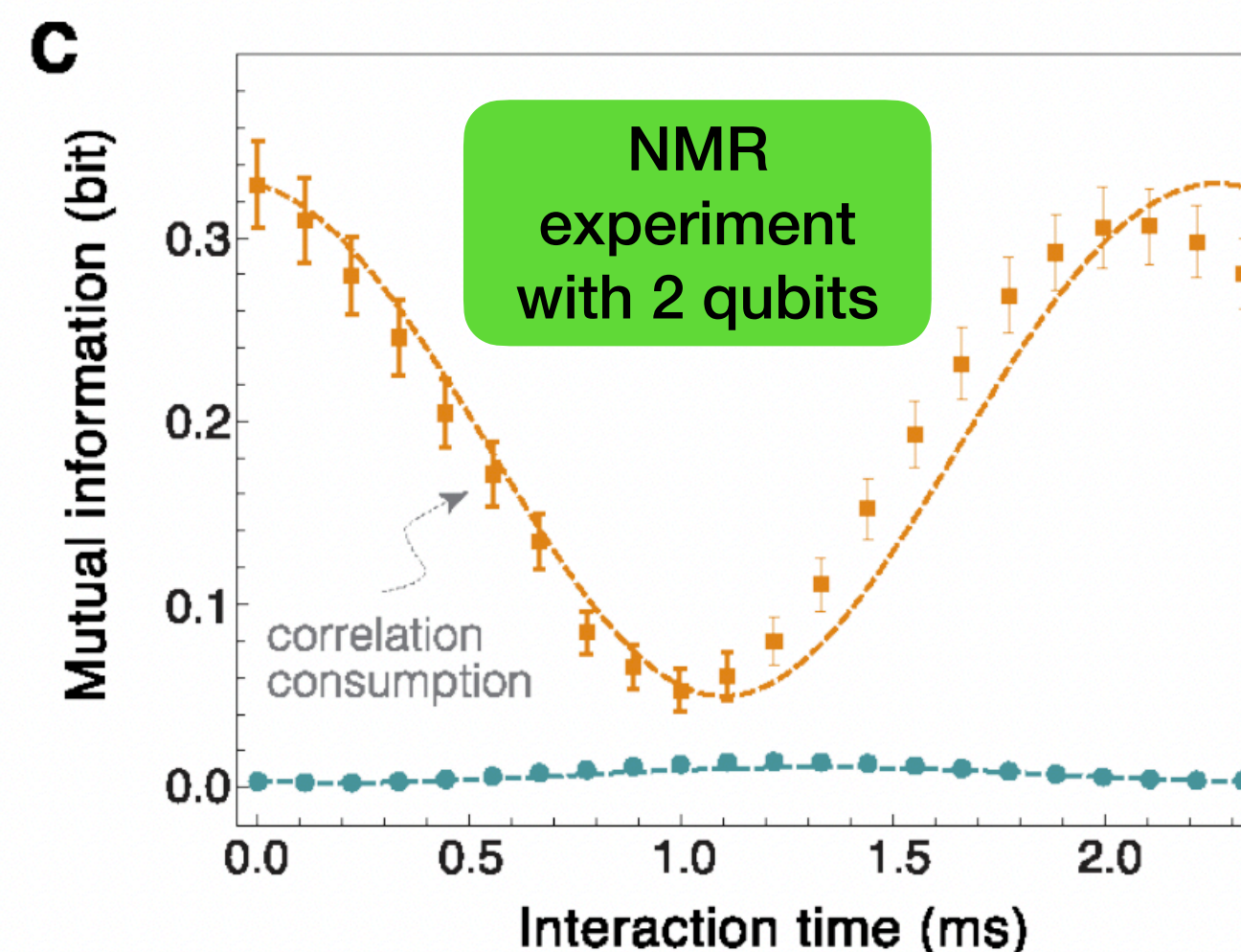
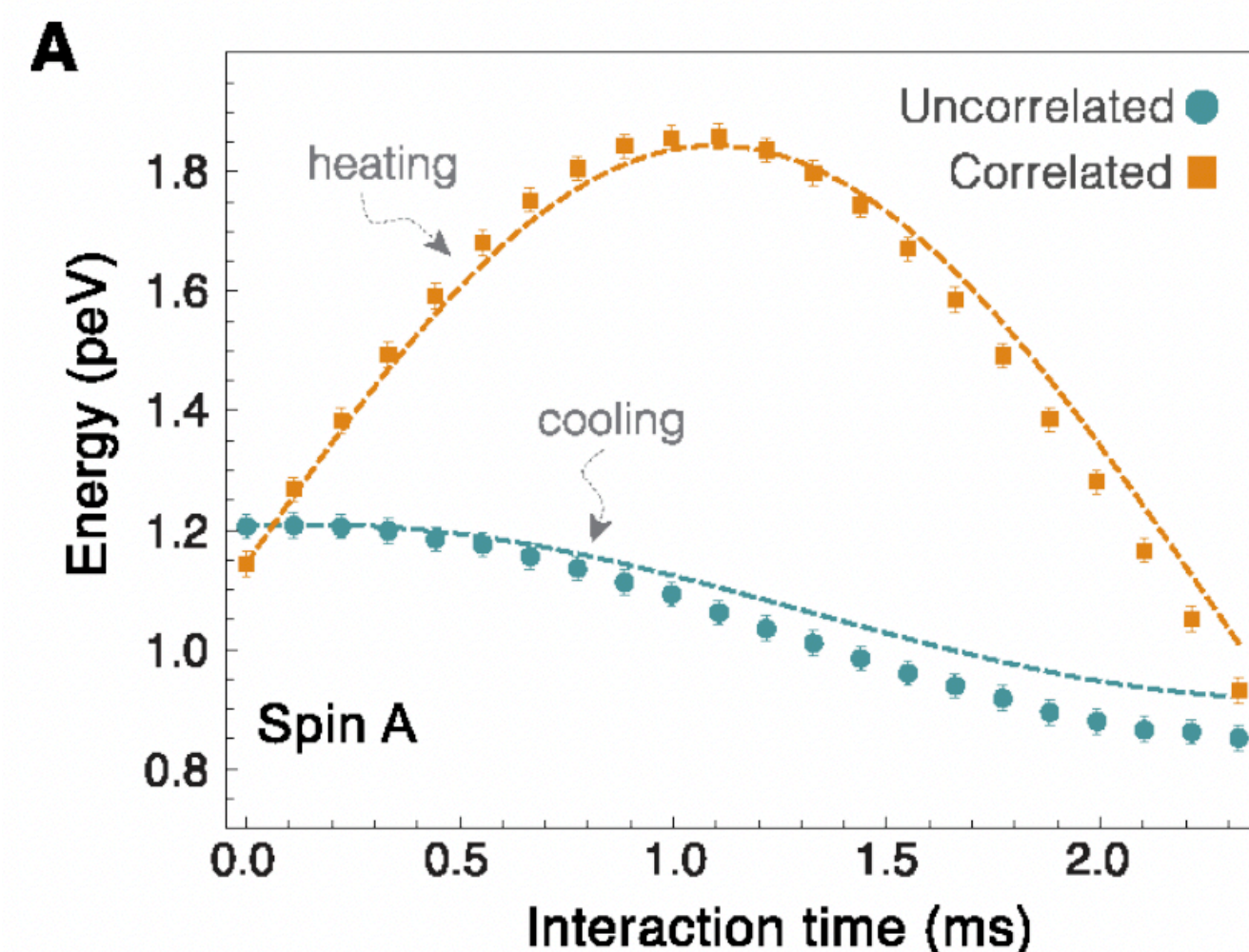
March 21st, 2022 - University of Rochester



www.fmt.if.usp.br/~gtlandi

Heat flows from hot to cold

- To break that, we must pay a **price**: fridges consumes electricity.
 - “Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.” (Clausius’ statement of the 2nd law)
- In the quantum domain, **information is also a resource.**



Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, “Reversing the direction of heat flow using quantum correlations”, *Nature Communications*, **10**, 2456 (2019)

Partovi, M. H., *Phys. Rev. E*, **77**, 021110 (2008)

Jennings, D. & Rudolph, T., *Phys. Rev. E*, **81**, 061130 (2010)

 **How to incorporate information in thermodynamics ?**

2nd law of thermodynamics

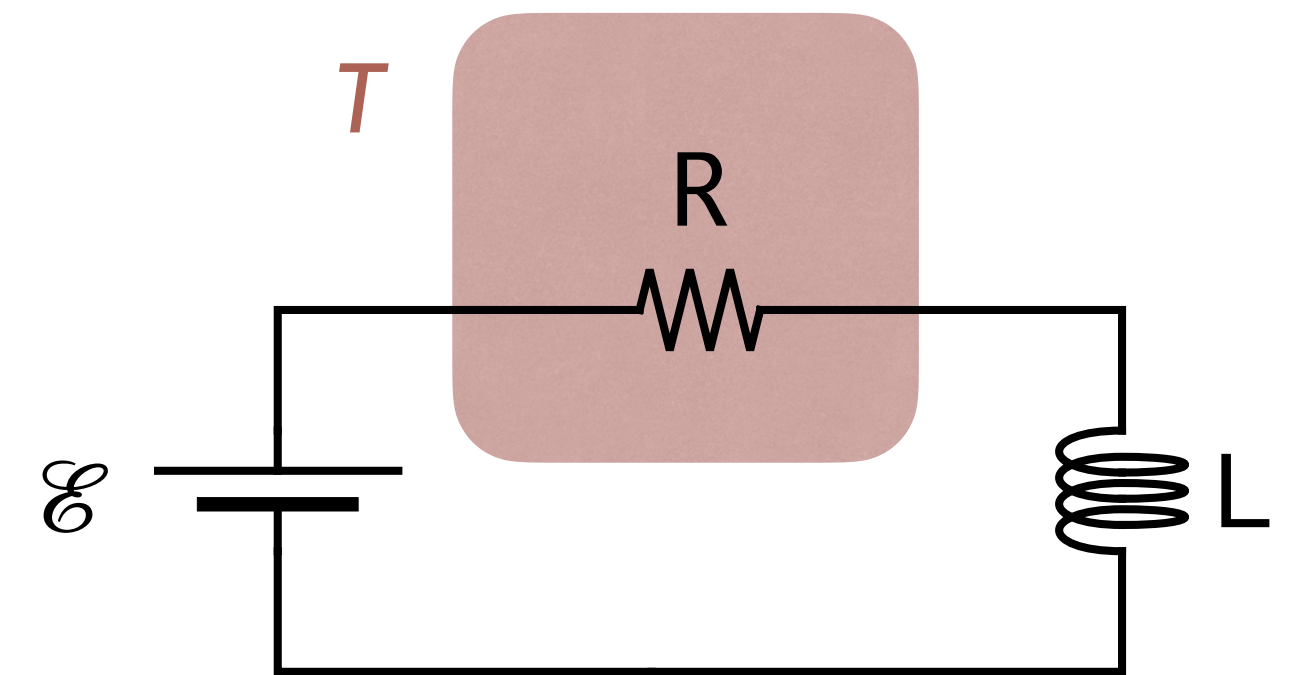
- *Every physical process is accompanied by an irreversible production of entropy.*
 - Entropy does not satisfy a conservation law.
- But the form of Σ depends on the problem in question.
- Ex: heat exchange ($\beta = 1/k_B T$)

$$\Sigma = Q (\beta_A - \beta_B) \geq 0$$

➔ $T_A > T_B \rightarrow Q < 0$
(hot system loses energy; heat flows from hot to cold).

Gabriel T. Landi and Mauro Paternostro, “**Irreversible entropy production, from quantum to classical**”, *Review of Modern Physics*, **93**, 035008 (2021)

ENTROPY PRODUCTION $\Sigma \geq 0$



$$\dot{\Sigma} = \frac{\mathcal{E}^2}{RT}$$

As long as there is juice in the battery, there is entropy being produced.

Why entropy production matters

- **Cyclic heat engine:**

1st law: $Q_h + Q_c = W_{\text{ext}}$

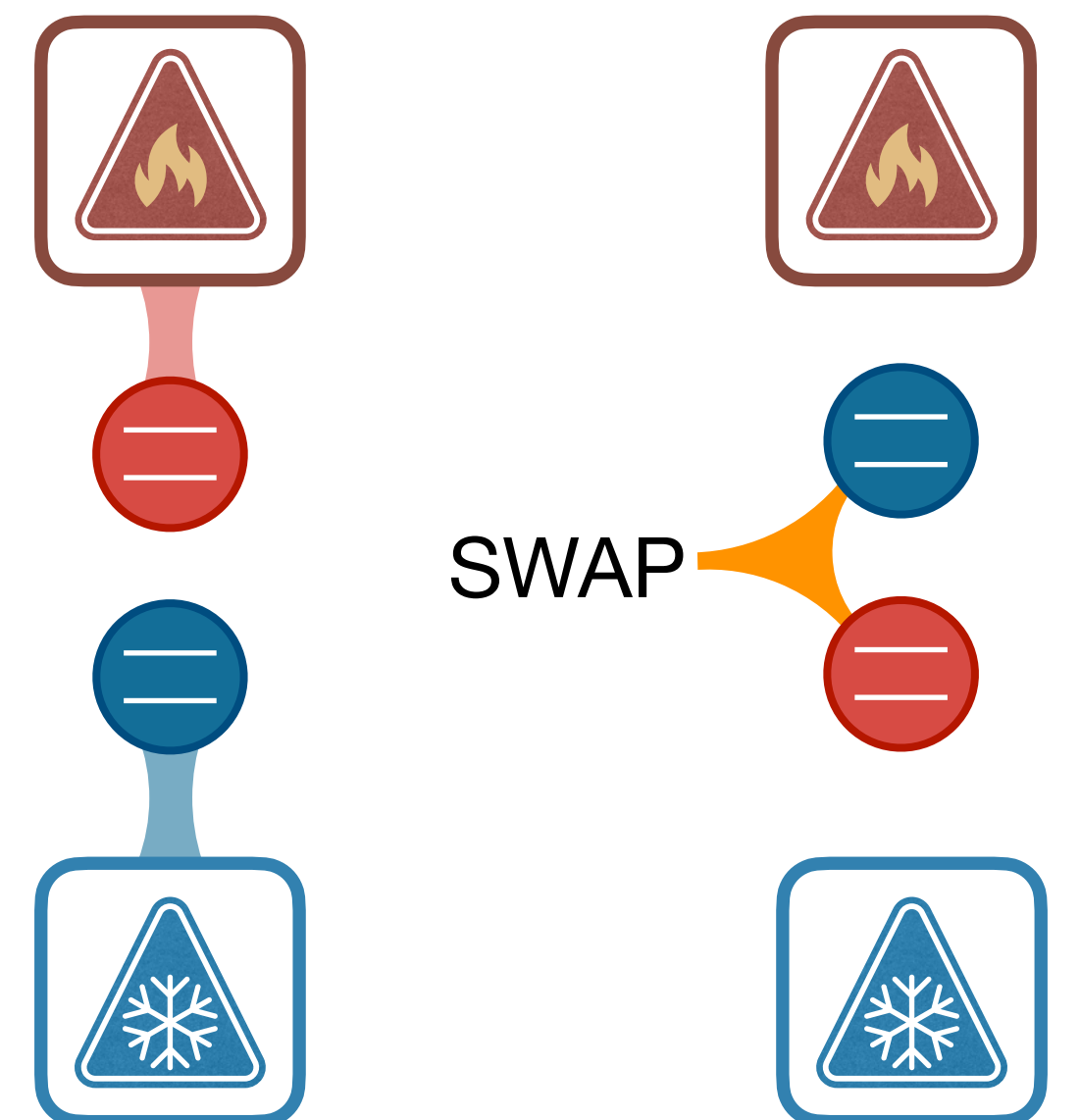
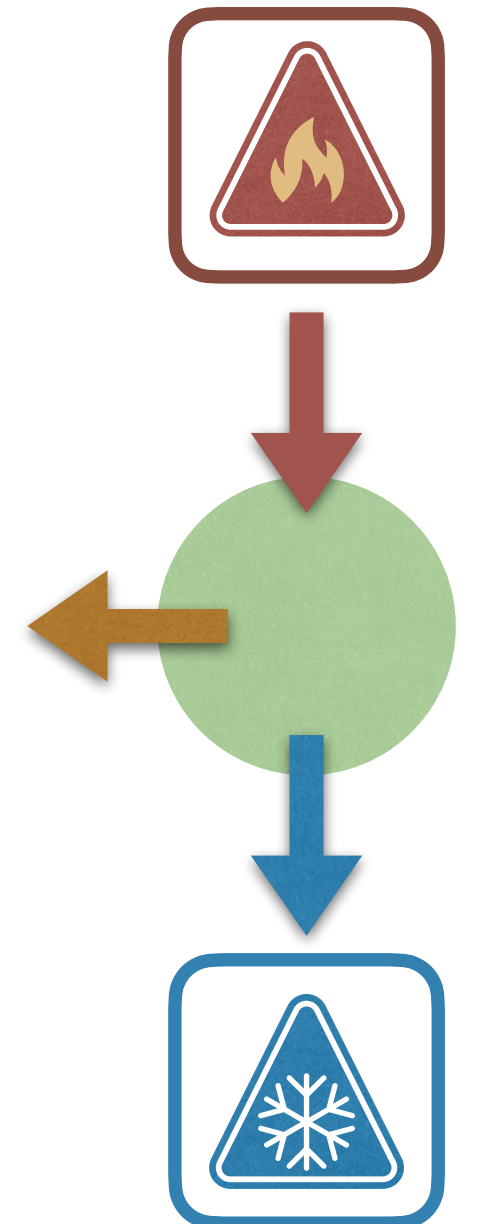
2nd law: $\Sigma = \beta_h Q_h + \beta_c Q_c \implies \eta = \frac{W}{Q_h} = \eta_c - \frac{T_c}{Q_h} \Sigma$

Efficiency is reduced from $\eta_c = 1 - T_c/T_h$ by a factor proportional to the entropy production.

- **2-stroke quantum engines:**

$$\Sigma = \frac{1}{\epsilon_c} (\beta_h \epsilon_h - \beta_c \epsilon_c) Q_h \geq 0$$

SWAP unitaries do work.



- **Landauer's erasure:**

$$\Sigma = \Delta S_{\text{bit}} + \beta \Delta Q_E \geq 0$$

Minimum cost to erase information of a bit: $\Delta Q_E \geq k_B T \ln 2$.

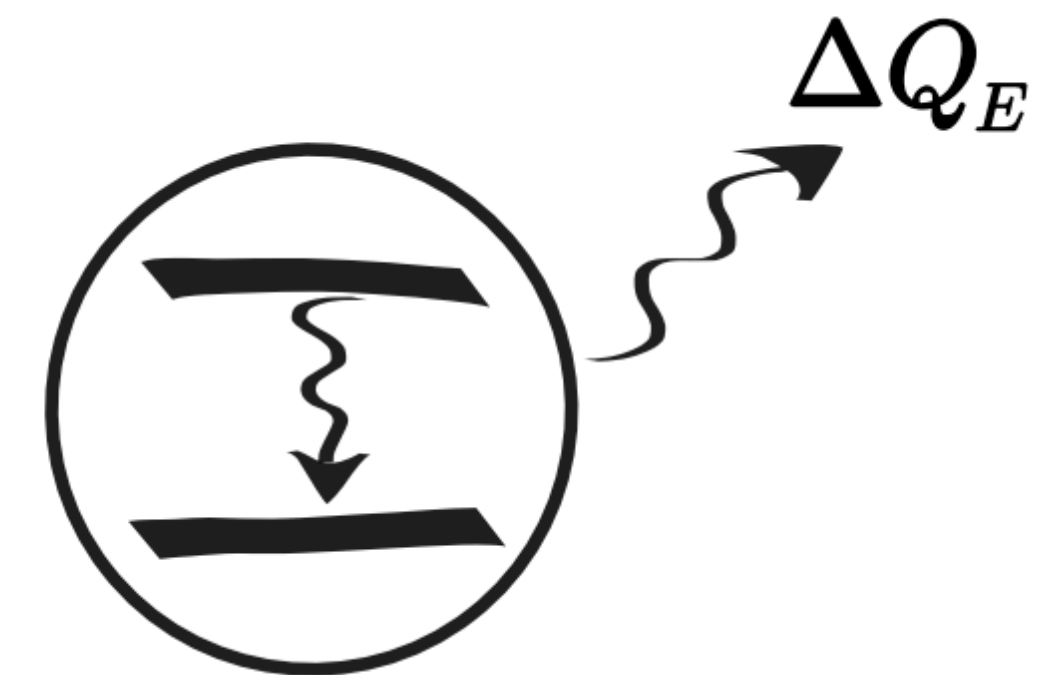
- What about $T \simeq 0$? (very relevant for quantum computation).

- Spontaneous emission is an erasure task.

- We proved that even at $T = 0$ there is still a finite cost.

- Ex: when eraser = waveguide

$$\Delta Q_E \geq k_B T \ln 2 + \frac{3\hbar c}{\pi L} \ln^2(2)$$

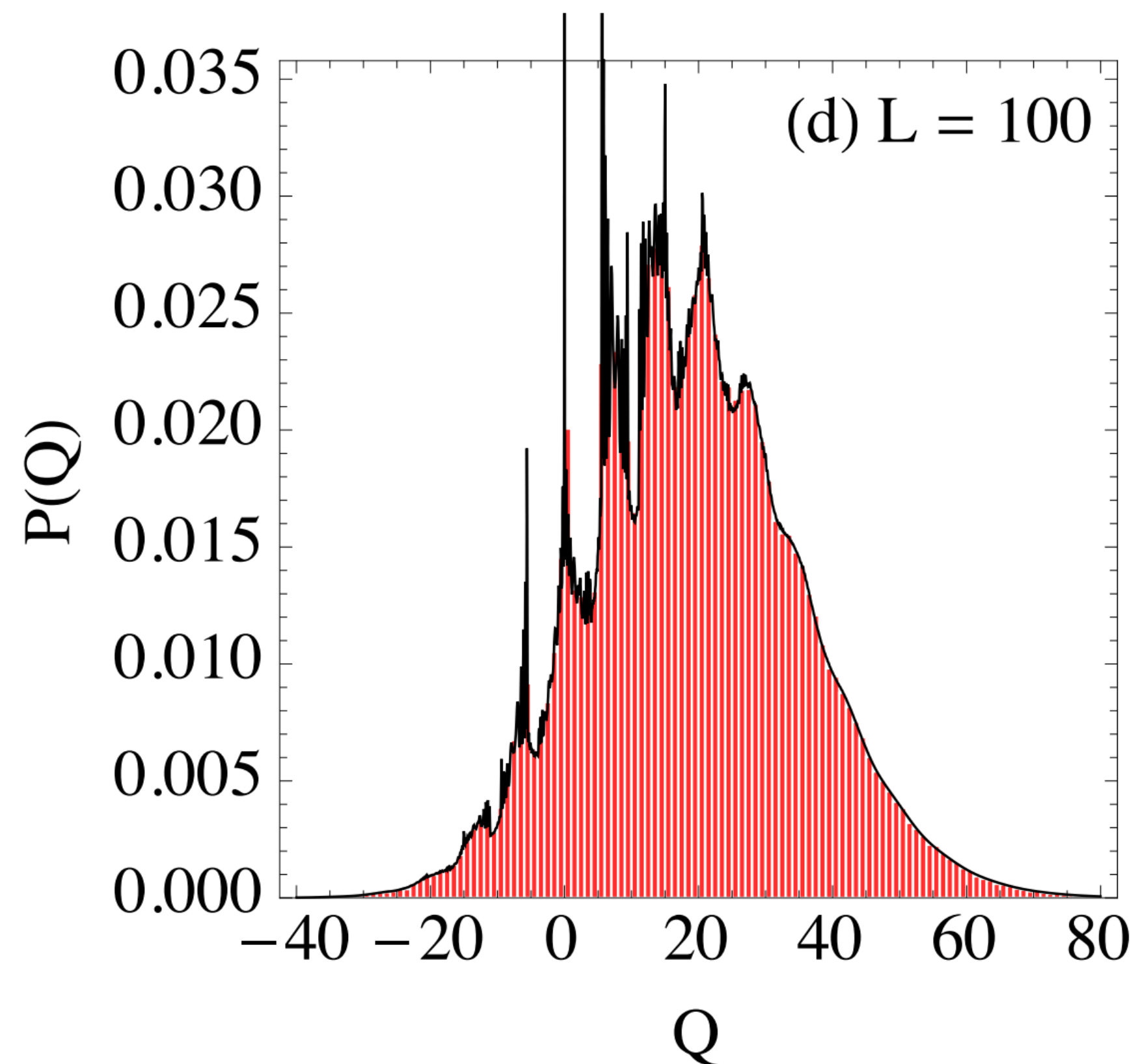
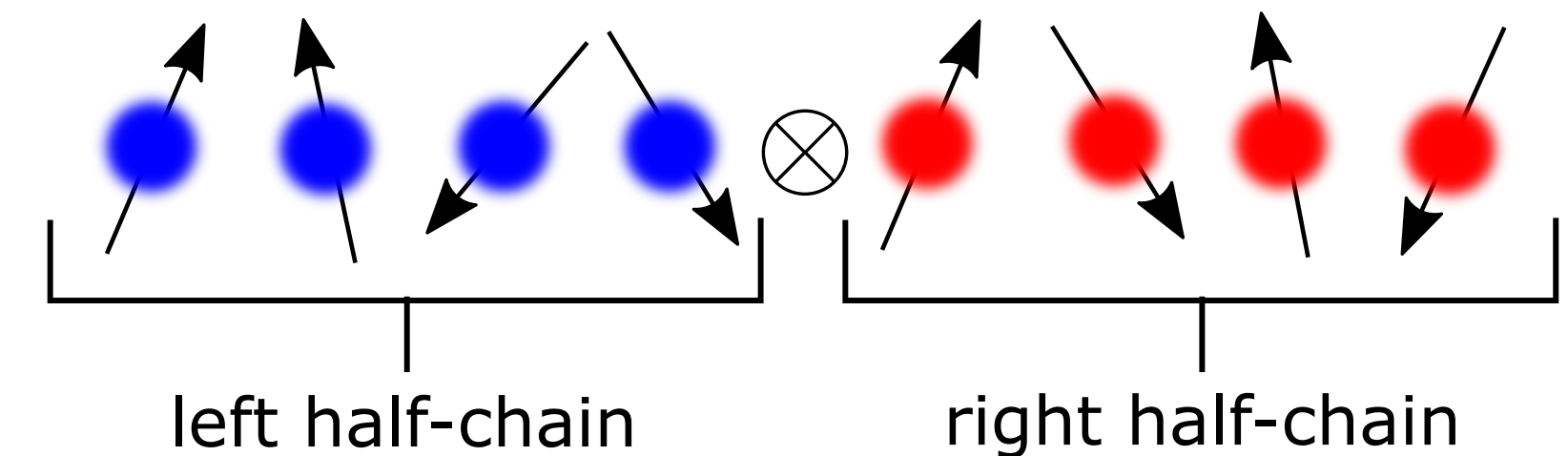


R. Landauer, IBM Journal of Research and Development **5**, 183 (1961).

André M. Timpanaro, Jader P. Santos, and Gabriel T. Landi, Phys. Rev. Lett. **124**, 240601 (2020)

Fluctuations are significant in the micro-world

- Macro-world: heat flows from hot \rightarrow cold.
- Micro-world: heat *usually* flows from hot \rightarrow cold.



Heat Exchange Fluctuation Theorem

$$P(-Q) = e^{-(\beta_A - \beta_B)Q} P(Q)$$

Implies 2nd law:

$$\langle Q \rangle (\beta_A - \beta_B) \geq 0$$

Jarzynski and Wójcik, Phys. Rev. Lett. 92, 230602 (2004)

G. E. Crooks, Journal of Statistical Physics, 90, 1481–1487 (1998)

- **Thermodynamic uncertainty relations:**

Fluctuation-dissipation trade-off (valid arbitrary)

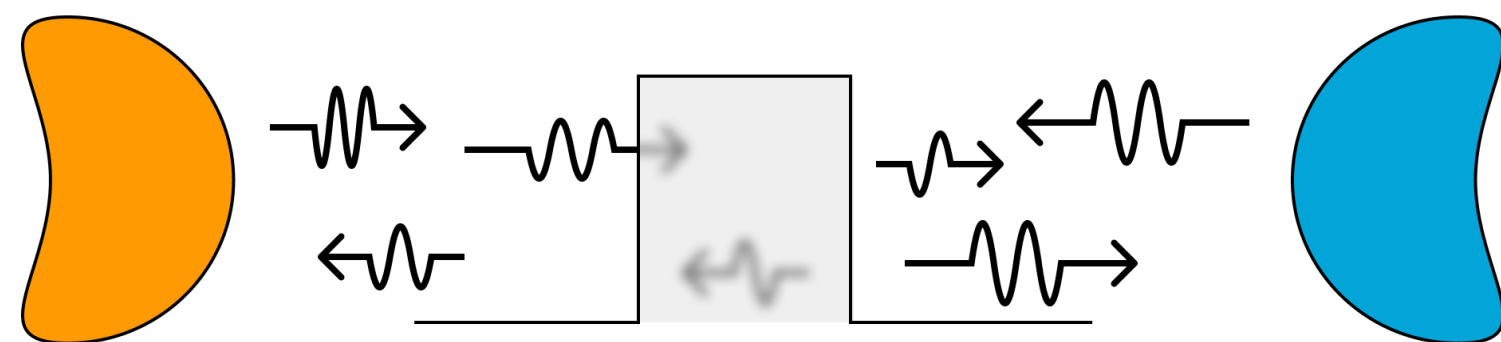
- Counter-intuitive: to reduce dissipation, one has to increase fluctuations

- Irreversible is good (when fluctuations are big)

- TURs only hold for classical Markov processes.

- Can be violated in quantum coherent transport.

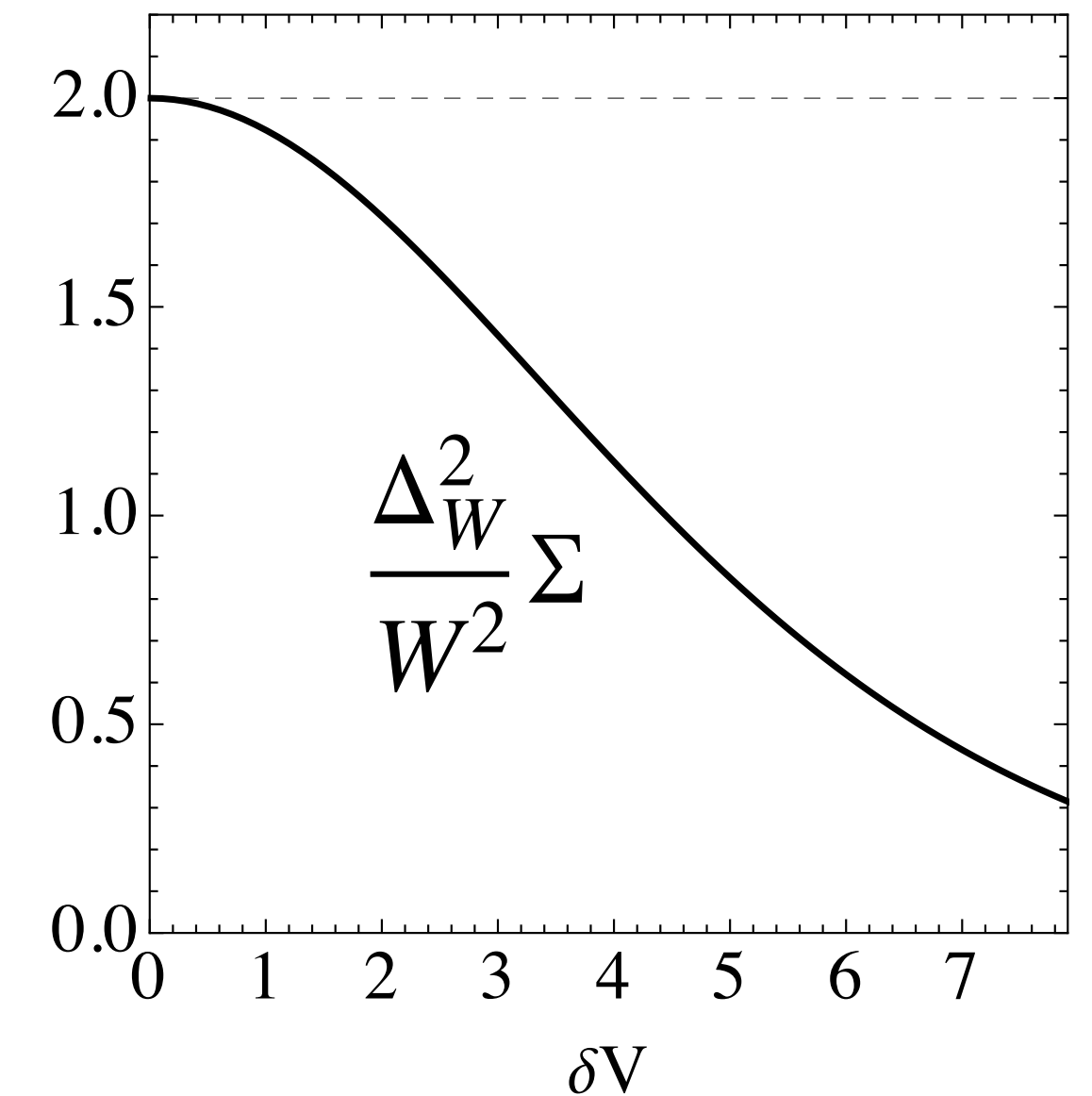
- e.g. scattering through quantum dots
(quantum thermoelectricity)



A. C. Barato, U. Seifert, *Physical Review Letters*, **114**, 158101 (2015)
P. Pietzonka and U. Seifert, *Phys. Rev. Lett.*, **120**, 190602 (2017)

 **Major open questions in the field:**

- Why quantum coherence?
- What limits the precision in the quantum world?
- Is there a trade-off?



Andre M. Timpanaro, Giacomo Guarnieri, and Gabriel T. Landi, arXiv 2106.10205

Information theoretic formulation of entropy production

Gabriel T. Landi and Mauro Paternostro, “**Irreversible entropy production, from quantum to classical**”, *Review of Modern Physics*, **93**, 035008 (2021)

Information theoretic formulation of entropy production

- **Operational notion of irreversibility:** *a process is irreversible because some information becomes inaccessible in practice.*
- Entropy production for a system in contact with a bath:

$$\Sigma = I'(S : E) + D(\rho'_E || \rho_E)$$

Mutual Information:

$$I'(S : E) = S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE})$$

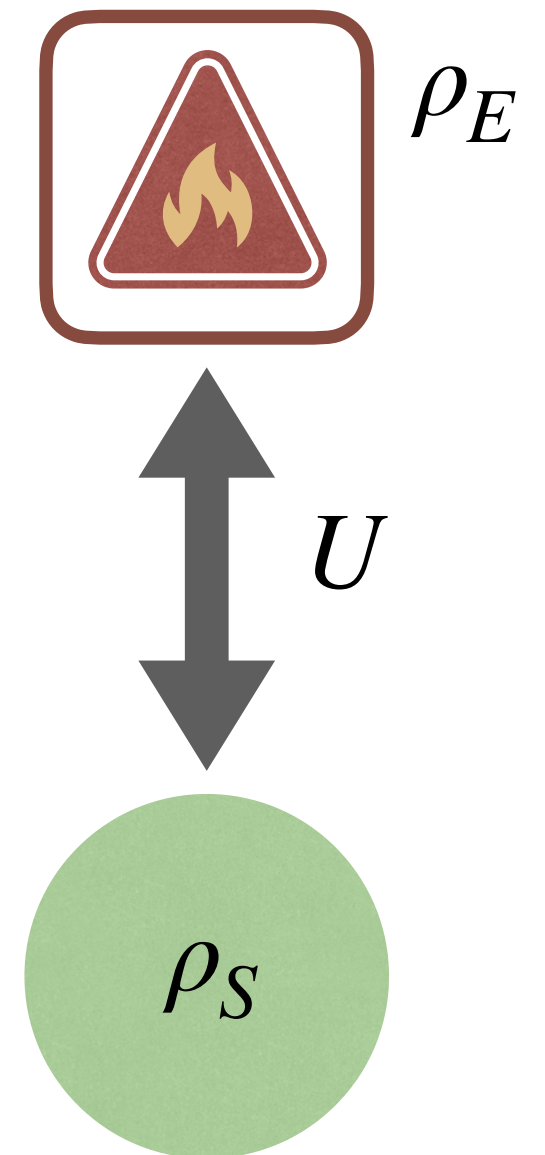
Quantifies all correlations
(classical + quantum)

Relative entropy

$$D(\rho'_E || \rho_E) = \text{tr}(\rho'_E \ln \rho'_E - \rho'_E \ln \rho_E)$$

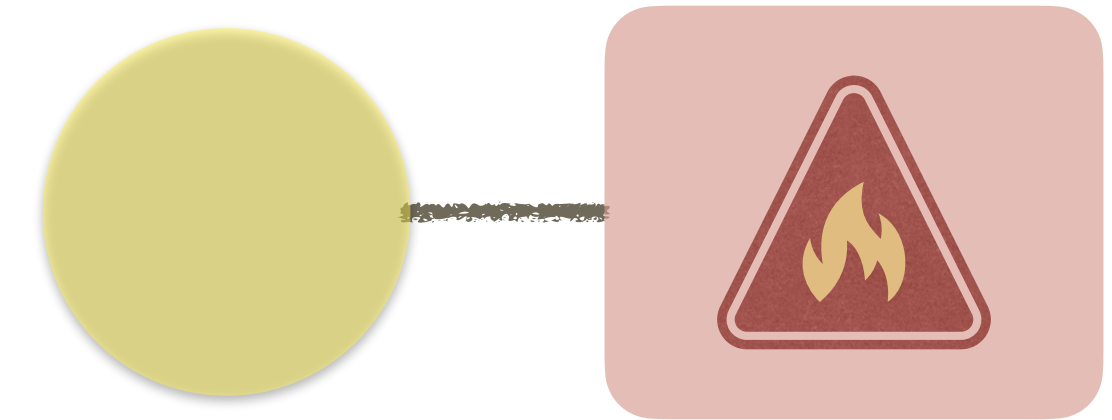
“Distance” between density matrices

$$\rho'_{SE} = U(\rho_S \otimes \rho_E)U^\dagger$$



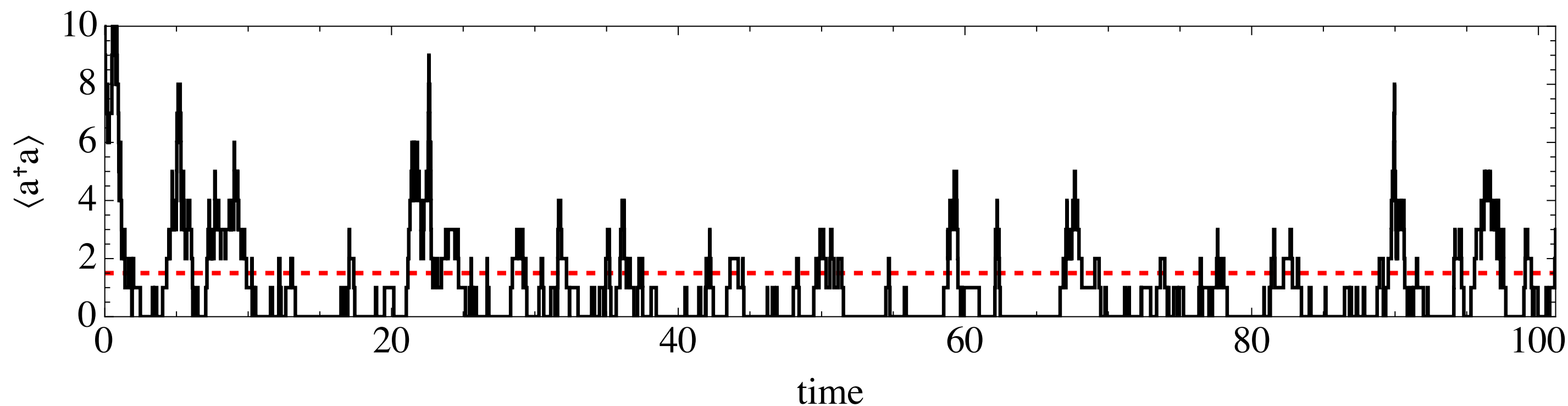
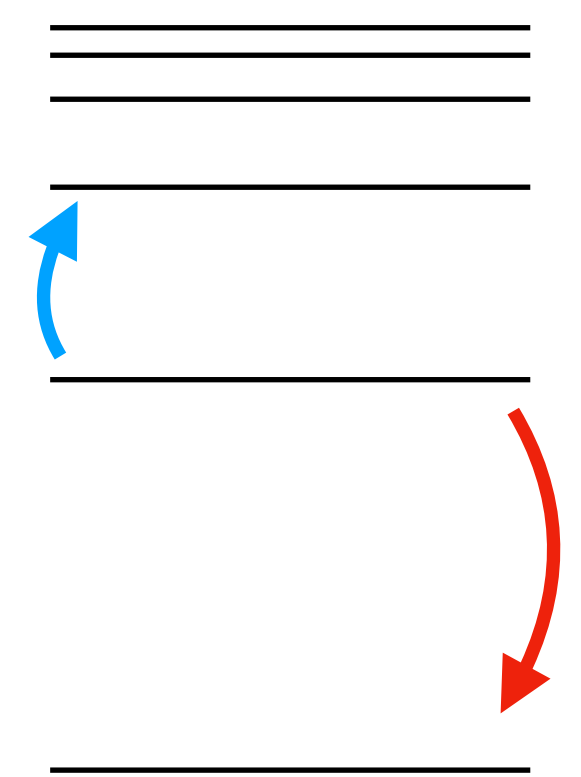
Describes an enormous
variety of processes!
(maybe a complicated U)

Relaxation towards equilibrium



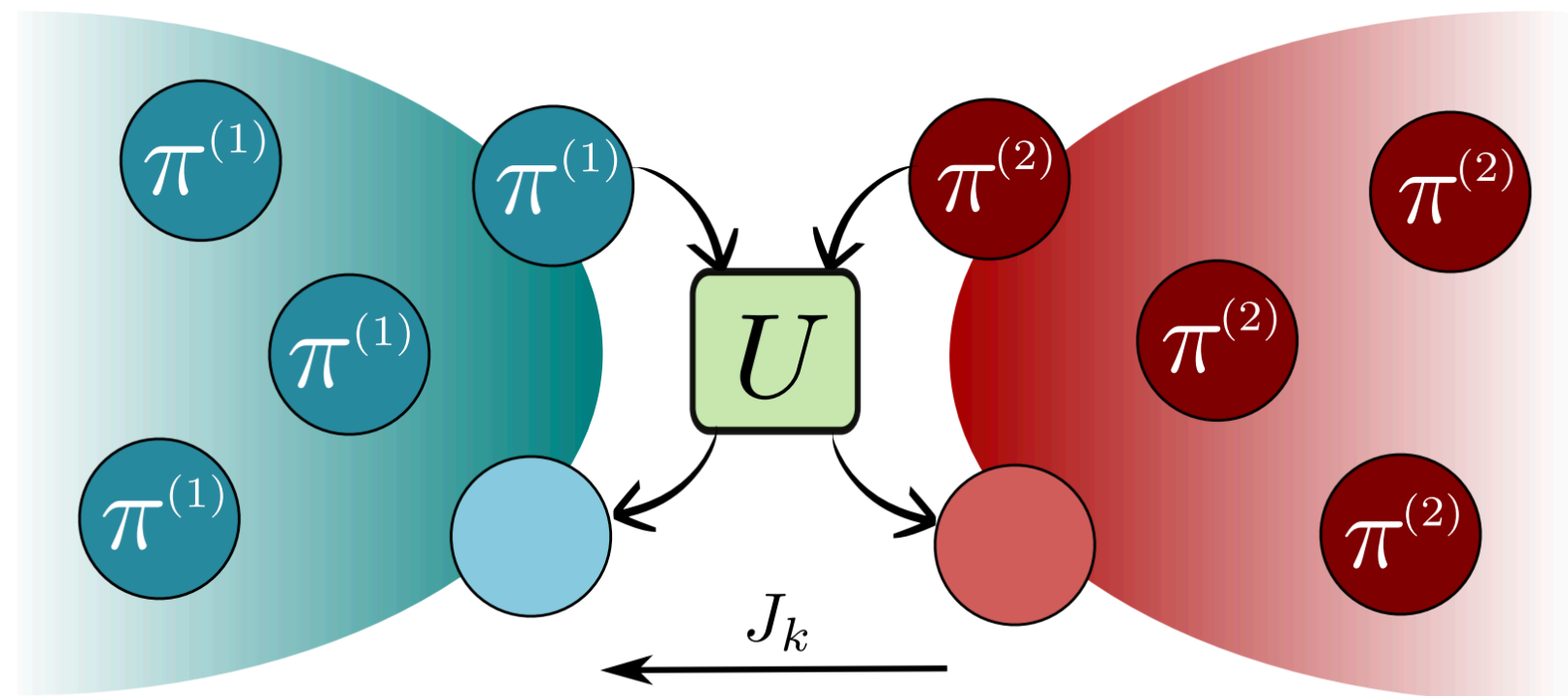
- Imagine an atomic system relaxing towards equilibrium.
 - Population in energy eigenstates fluctuate until they reach thermal equilibrium.
- In addition: destroy any superpositions (**decoherence**).
- Entropy production rate can be split as

$$\Sigma = \Sigma_{\text{pop}} + \Sigma_{\text{coh}}$$



Additional entropy production due to coherence:
Dissipation of information, without dissipation of energy.

Transport of non-Abelian charges



- Classical transport: **energy and particles.**
- Quantum domain: excitations may not commute.
- Single-mode radiation: energy & squeezing

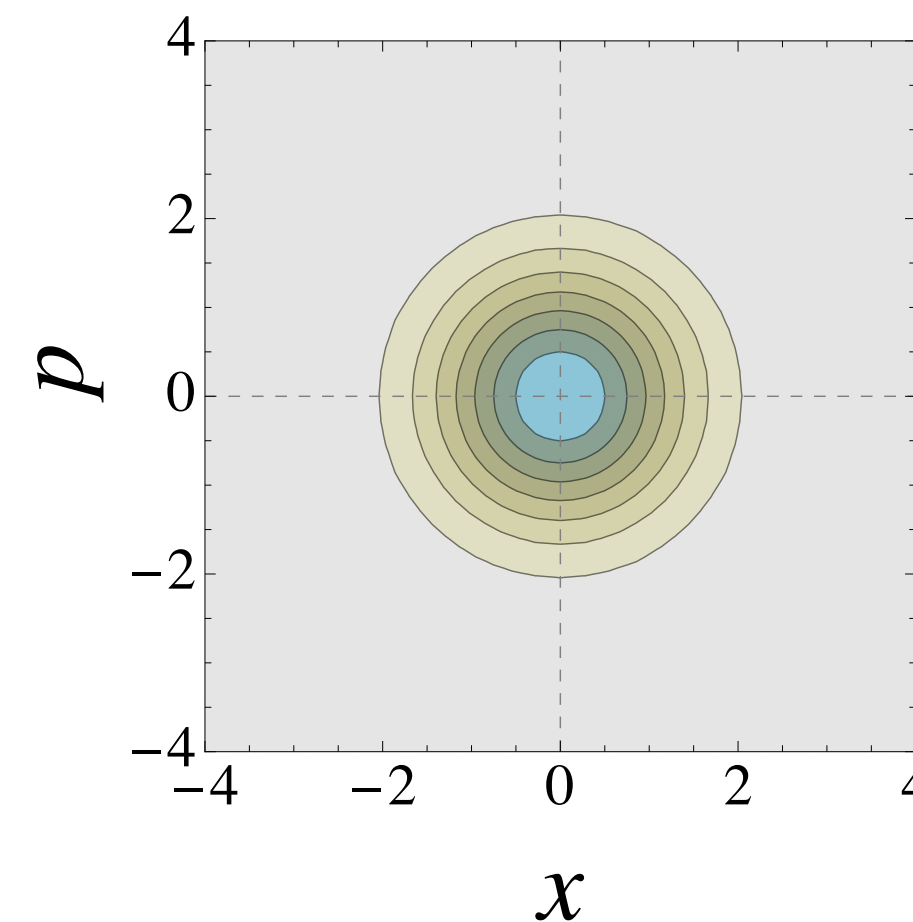
$$H = \frac{\omega}{2}(p^2 + x^2), \quad A = \frac{\omega}{2}(p^2 - x^2)$$

Entropy production is *reduced* due to non-commutativity

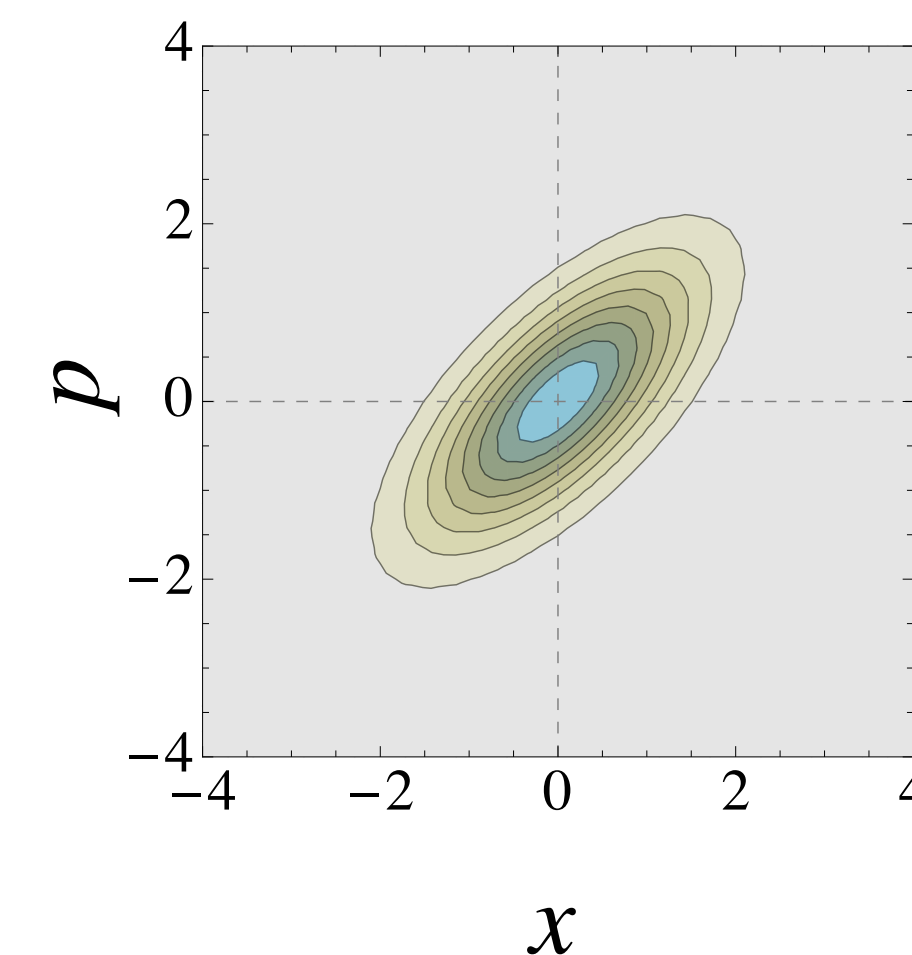
$$\Sigma = \Sigma_{\text{comm}} - I(\rho, D)$$

$$D = \sum_k \delta\lambda_k \Delta Q_k$$

Thermal state

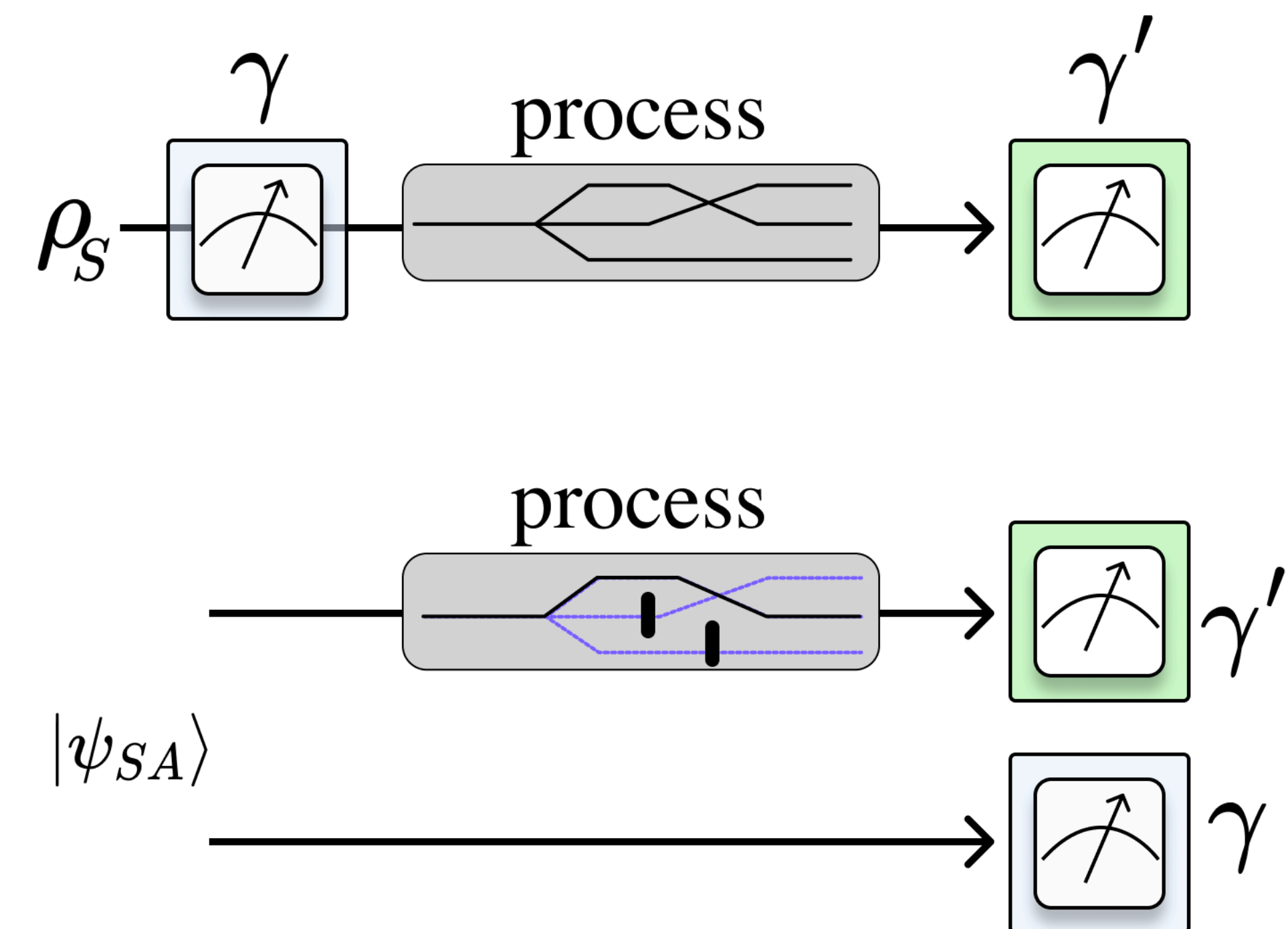
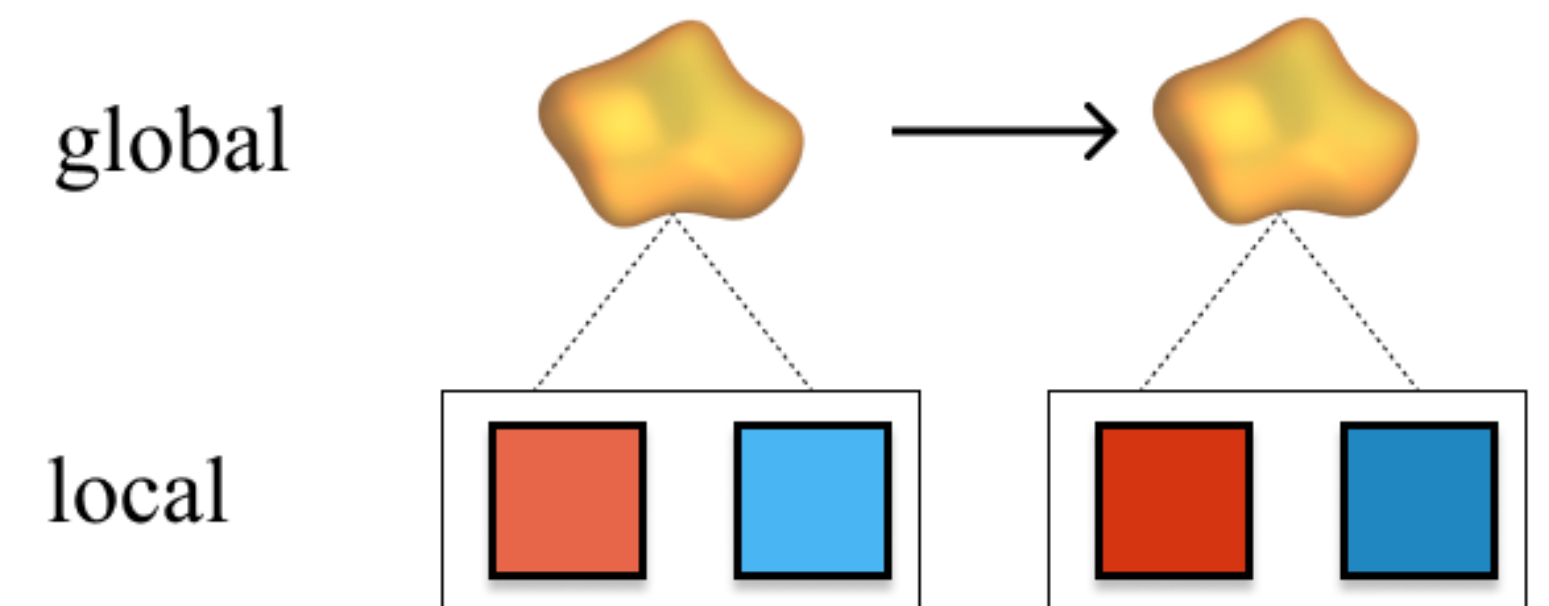


Squeezed state



Two-point measurements destroy quantum features

- Measurements in quantum mechanics are invasive.
 - Destroy initial quantum coherences.
- Fundamental limitation of Quantum + Thermodynamics.
- Can be overcome using an **entangled ancilla**, which is only measured at the end.
- Or if one has access to **identical copies** of a quantum system.



Gabriel H. Aguilar, Thaís L. Silva, Thiago E. Guimarães, Rodrigo S. Piera, Lucas C. Céleri, Gabriel T. Landi, arXiv:2108.03289 (2021)

Kaonan Micadei, Gabriel T. Landi, Eric Lutz, arXiv:2103.14570

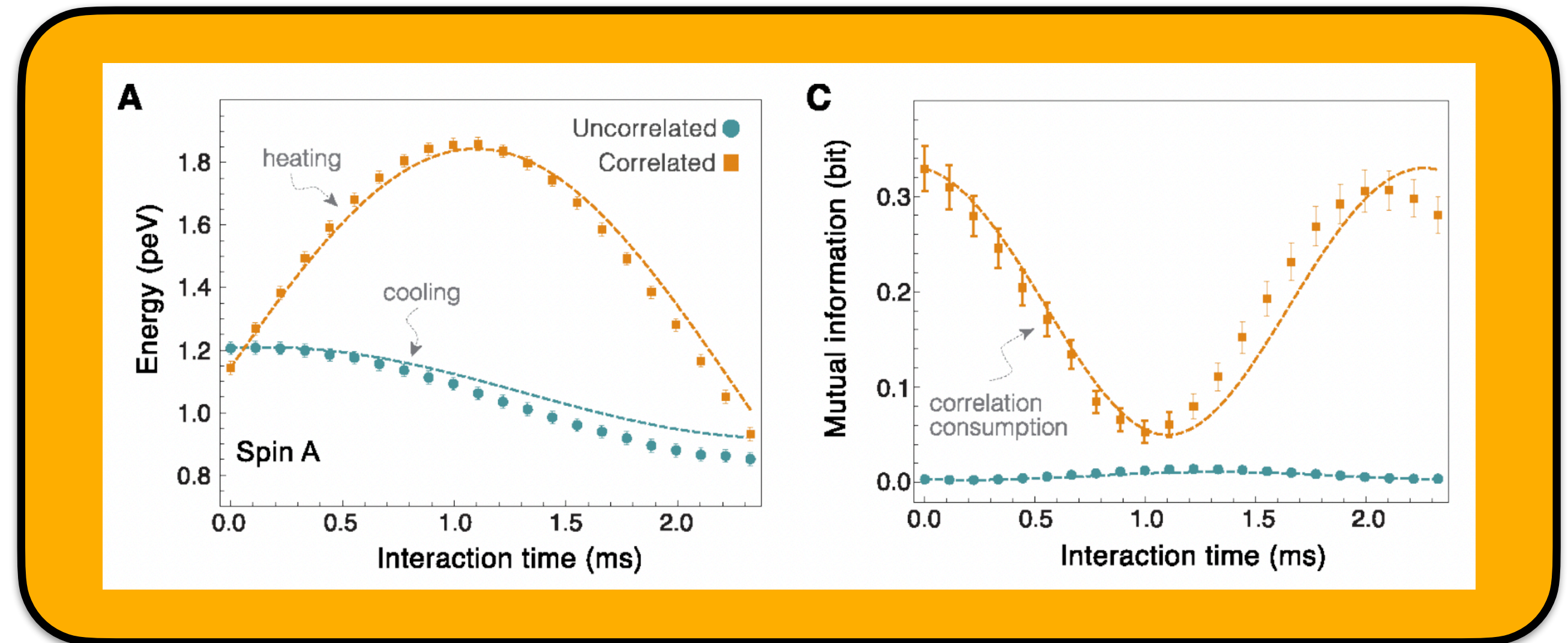
Fully quantum fluctuation theorems

- We put forth a theory encompassing Heat exchange in the presence of general quantum correlations.

$$\frac{P(\Gamma)}{P(\Gamma^*)} = \exp \left\{ (\beta_A - \beta_B)Q - \Delta I - S_A - S_B - \gamma \right\}$$

Consequence:

$$(\beta_A - \beta_B)\langle Q \rangle \geq \langle \Delta I \rangle$$

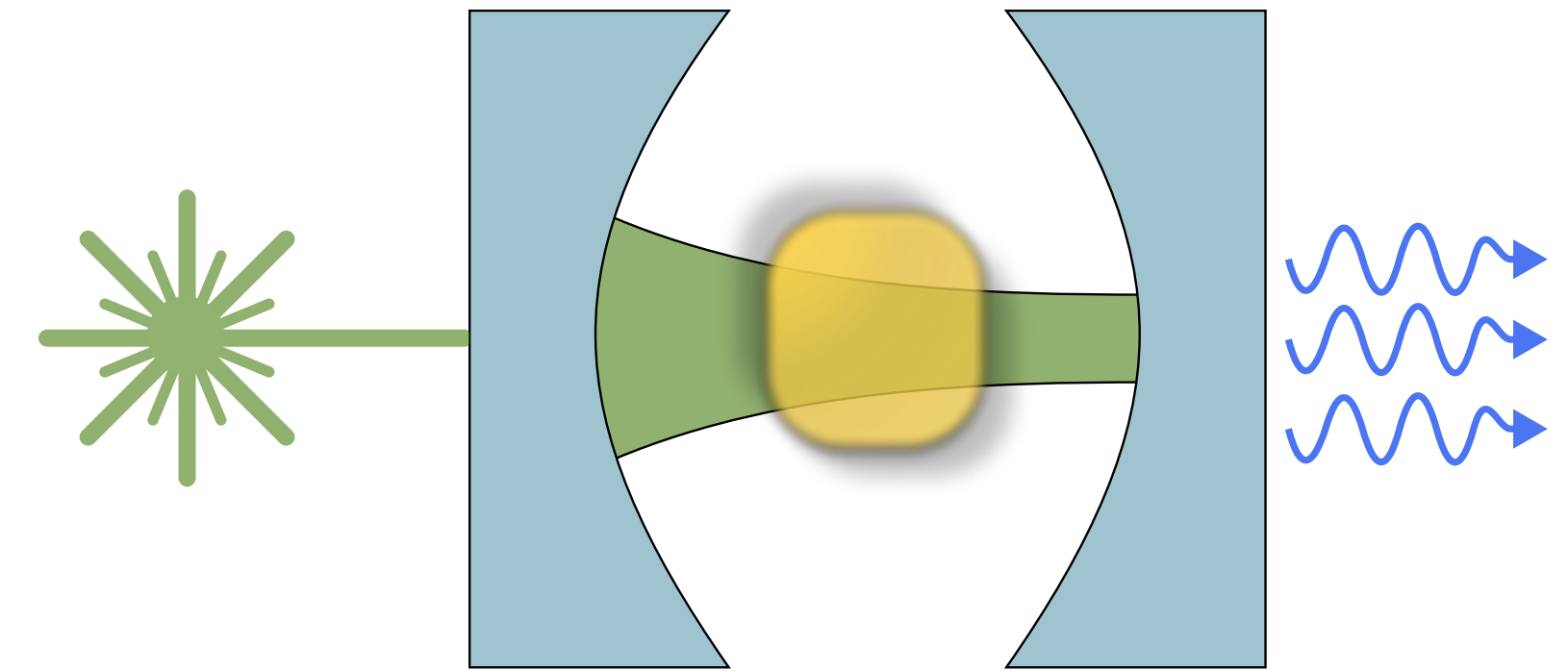


K. Micadei, G. T. Landi, E. Lutz, “Quantum fluctuation theorems beyond two-point measurements”, *Phys. Rev. Lett.* **124**, 090602 (2020)

Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Roberto M. Serra, Eric Lutz, “Experimental validation of fully quantum fluctuation theorems”, *Phys. Rev. Lett.*, **127**, 180603 (2021).

Quantum phase space

- Many quantum experiments are done using optical cavities with semi-transparent mirrors.
- Photons leaking out \simeq zero temperature bath.
 - Spontaneous emission: excitations can leave, but not return.
- 2nd law is buggy @ $T = 0$: $\Sigma = Q \left(\frac{1}{T_A} - \frac{1}{T_B} \right)$.
 - Does not include vacuum fluctuations (*present in every measurement*).
- We reformulated the entropy production problem in terms of quantum phase space & the *Wigner function*.



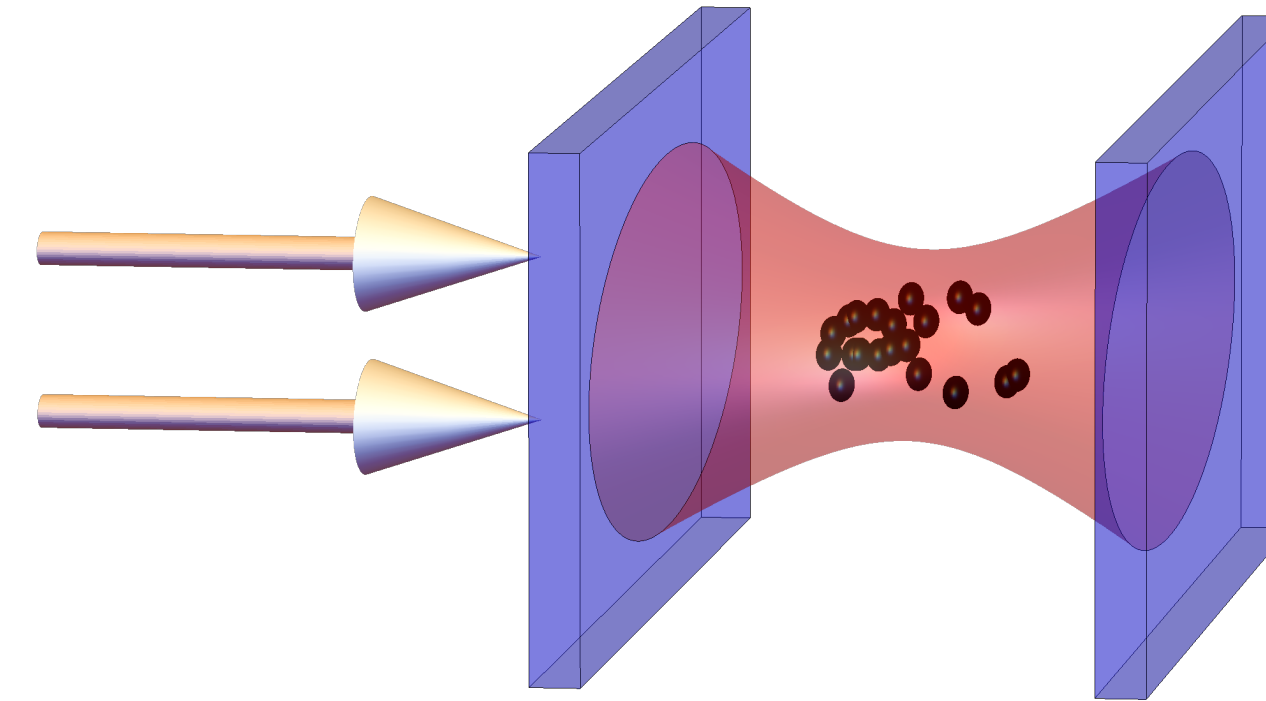
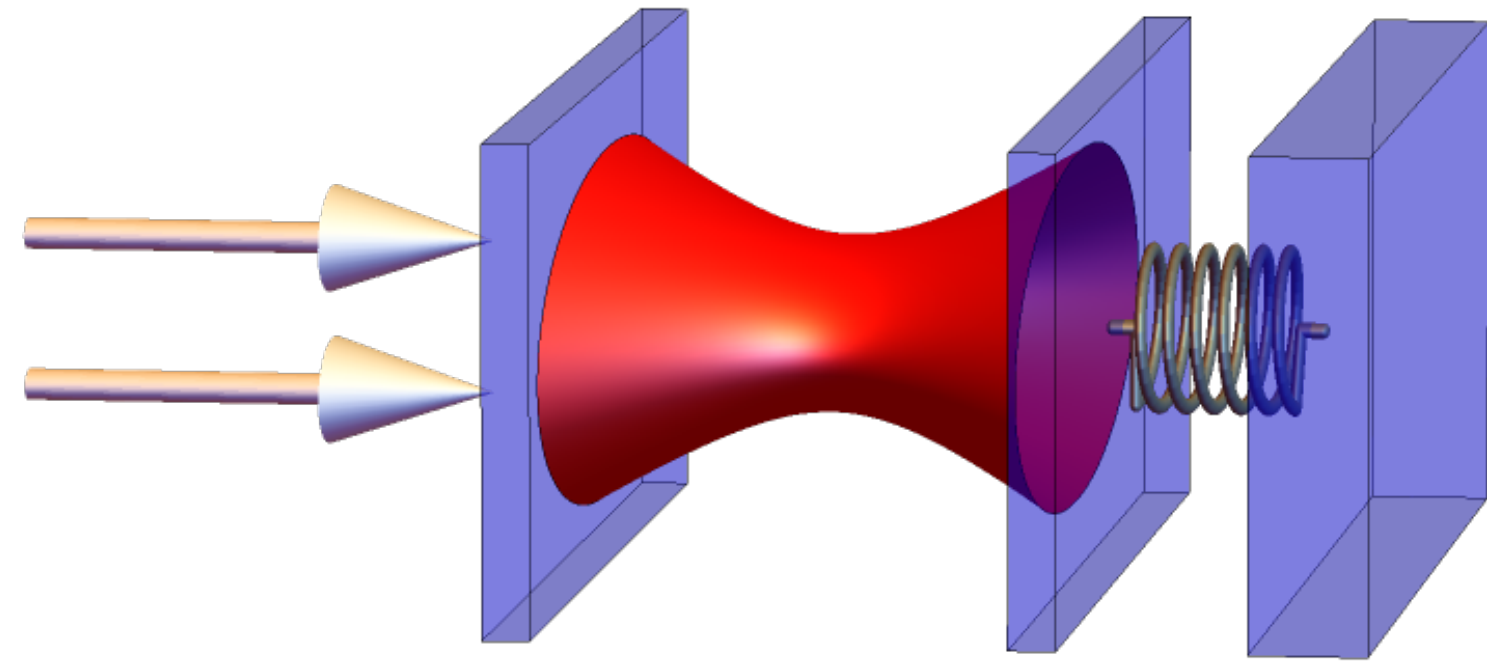
$$\Sigma = Q \left(\frac{1}{T_A^{\text{eff}}} - \frac{1}{T_B^{\text{eff}}} \right)$$

$$T^{\text{eff}} = \omega(\bar{n} + 1/2), \quad \bar{n} = \frac{1}{e^{\beta\omega} - 1}$$

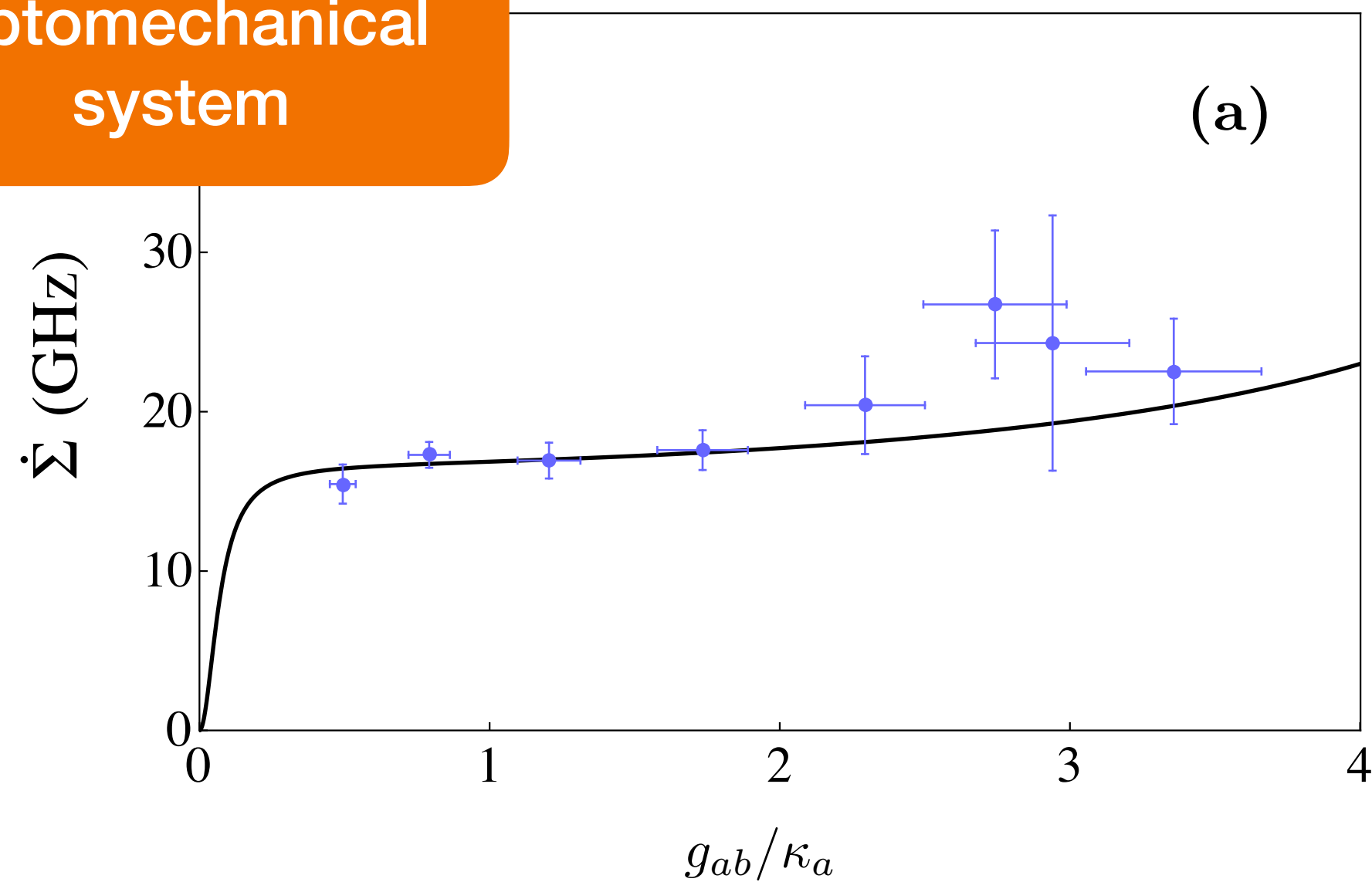
High temperatures: $\omega(\bar{n} + 1/2) \simeq T$.

Zero temperature: $\omega(\bar{n} + 1/2) = \omega/2$.

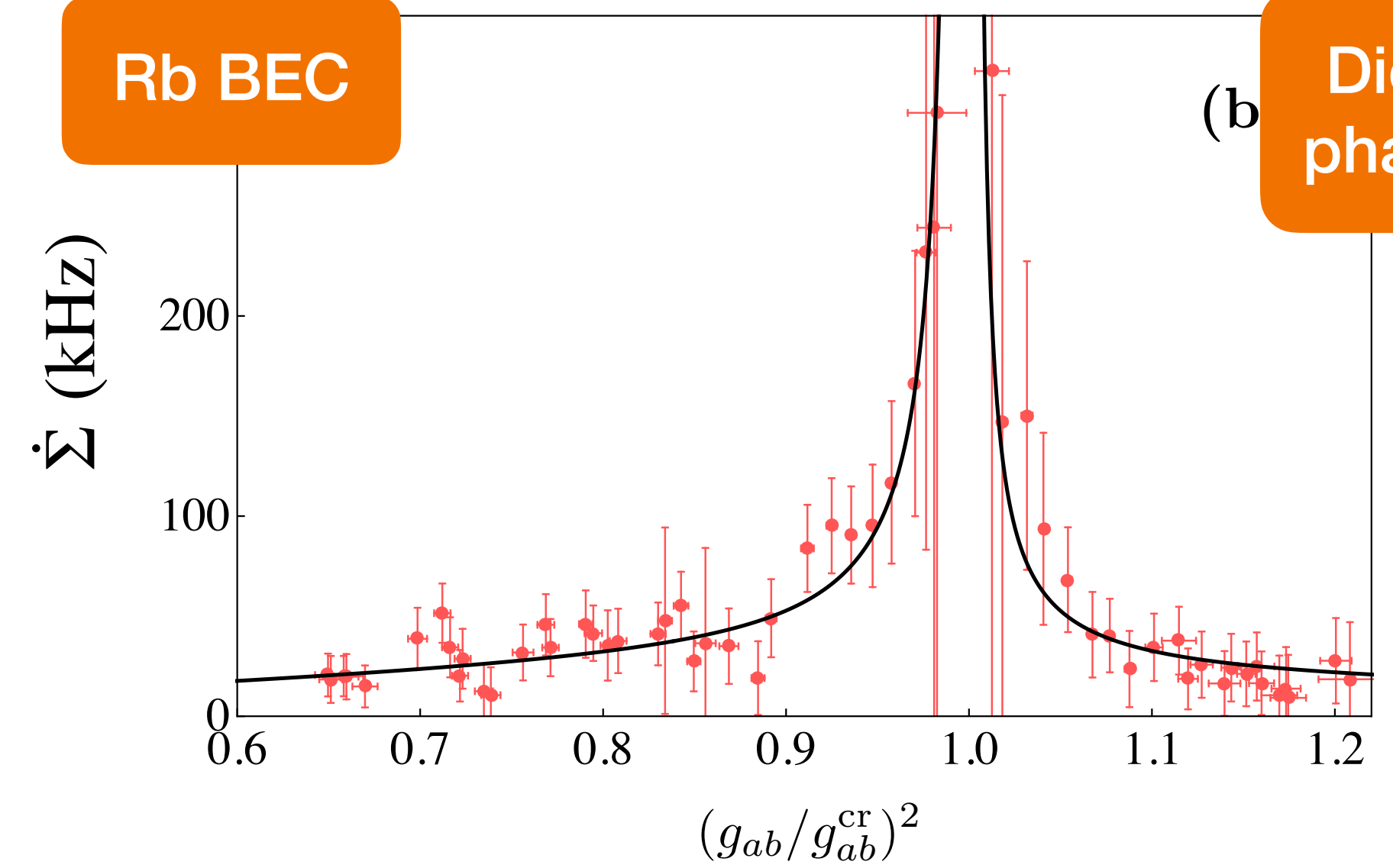
Experiments



Optomechanical system



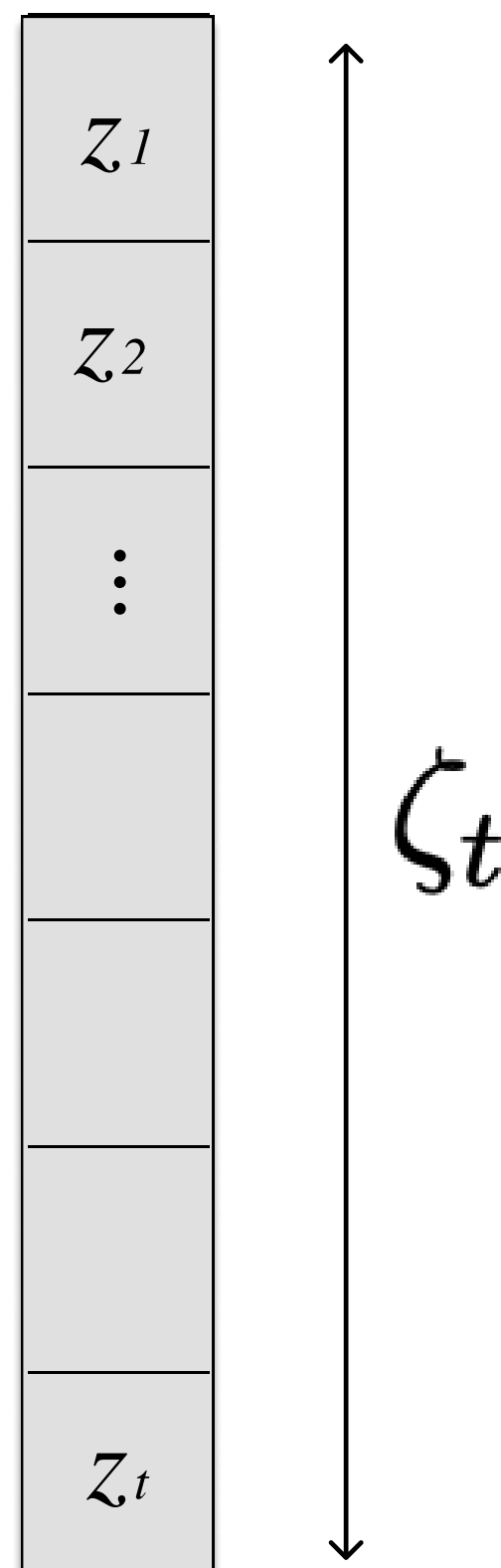
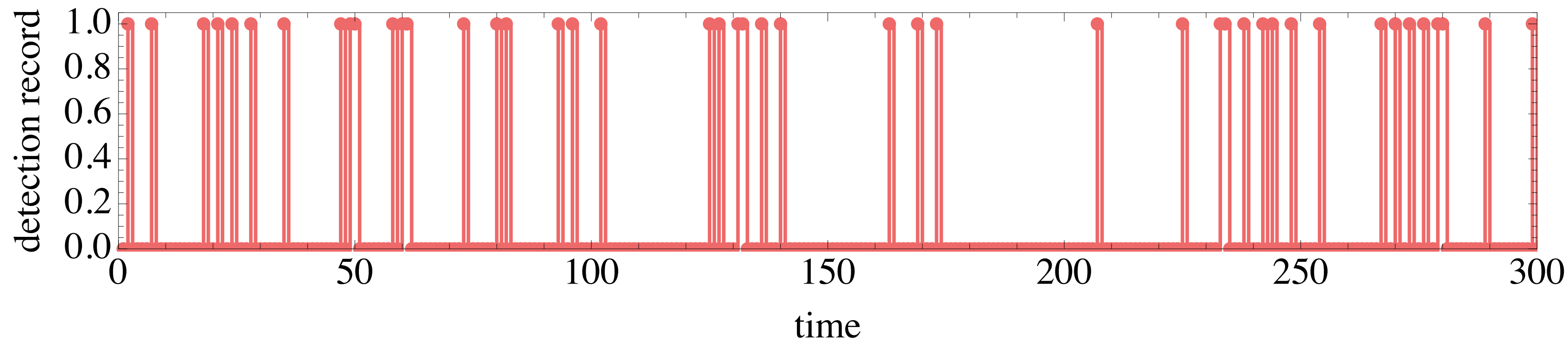
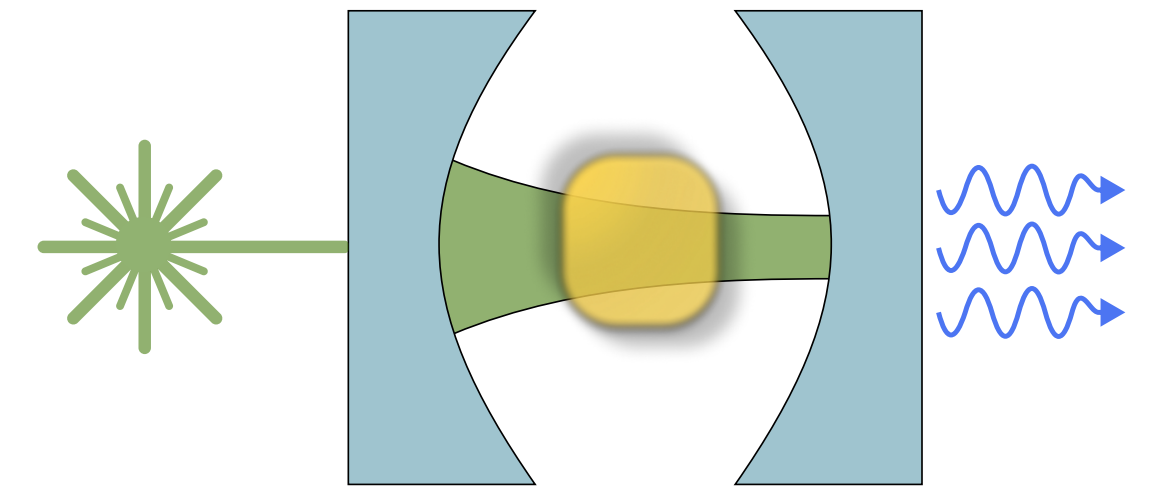
Rb BEC



Dicke quantum phase transition.

Continuously monitored quantum systems

- Continuous monitoring of photons that leak out of the cavity.
 - Individual clicks in the detector.
- Fundamental questions: what is entropy production *given* a detection record.
 - Operation: define thermodynamics in terms of what we can actually measure.
 - Includes *information* directly in the formulation.



Holevo information

- **Unconditional:** If we do not know the individual clicks: ρ_t
- **Conditional on the detection record:** $\rho_{t|\zeta_t}$
- **Holevo information:** accumulated information we learned from the detection.

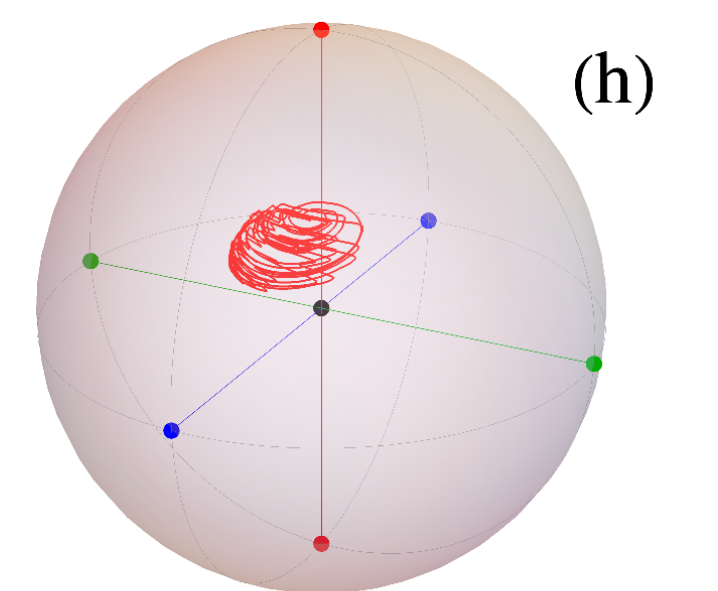
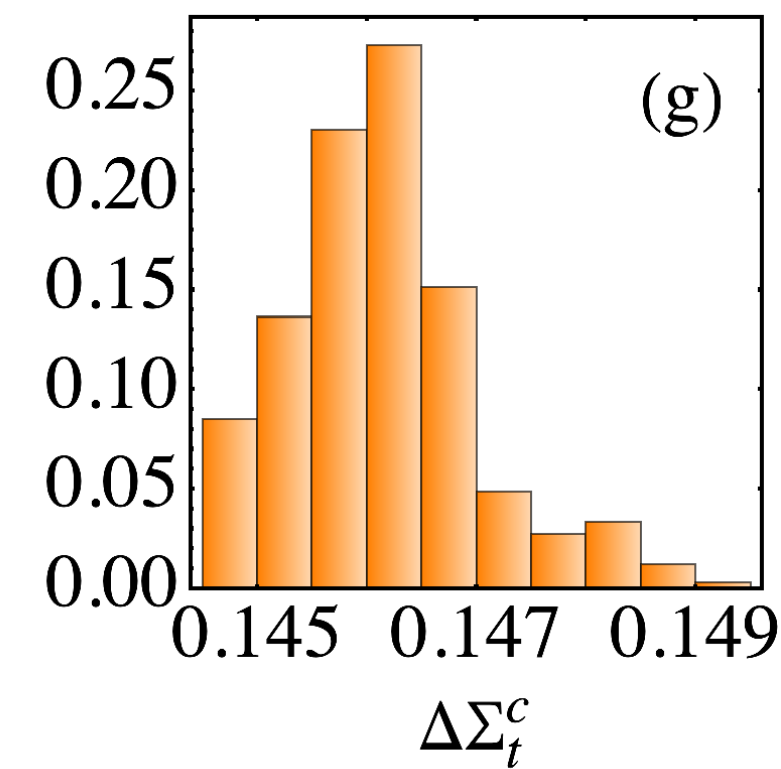
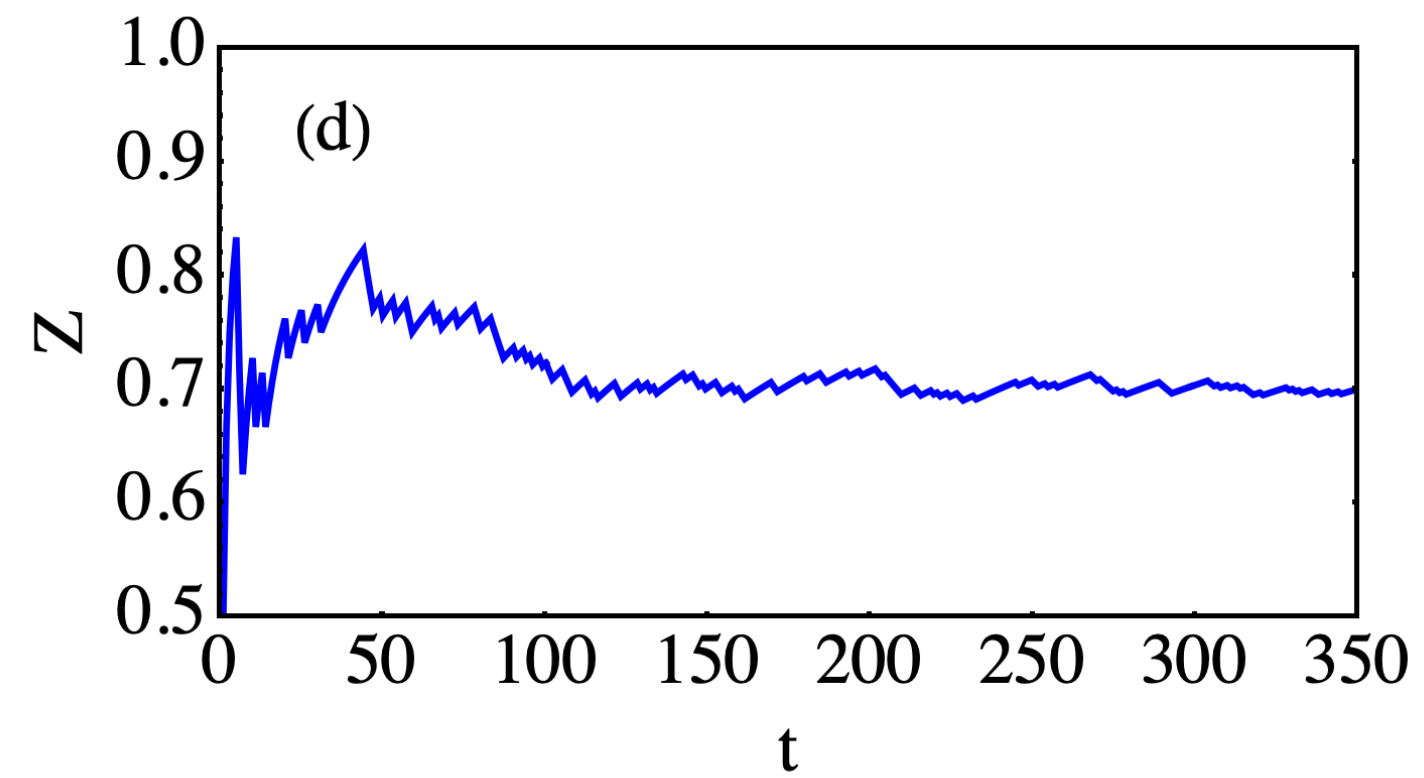
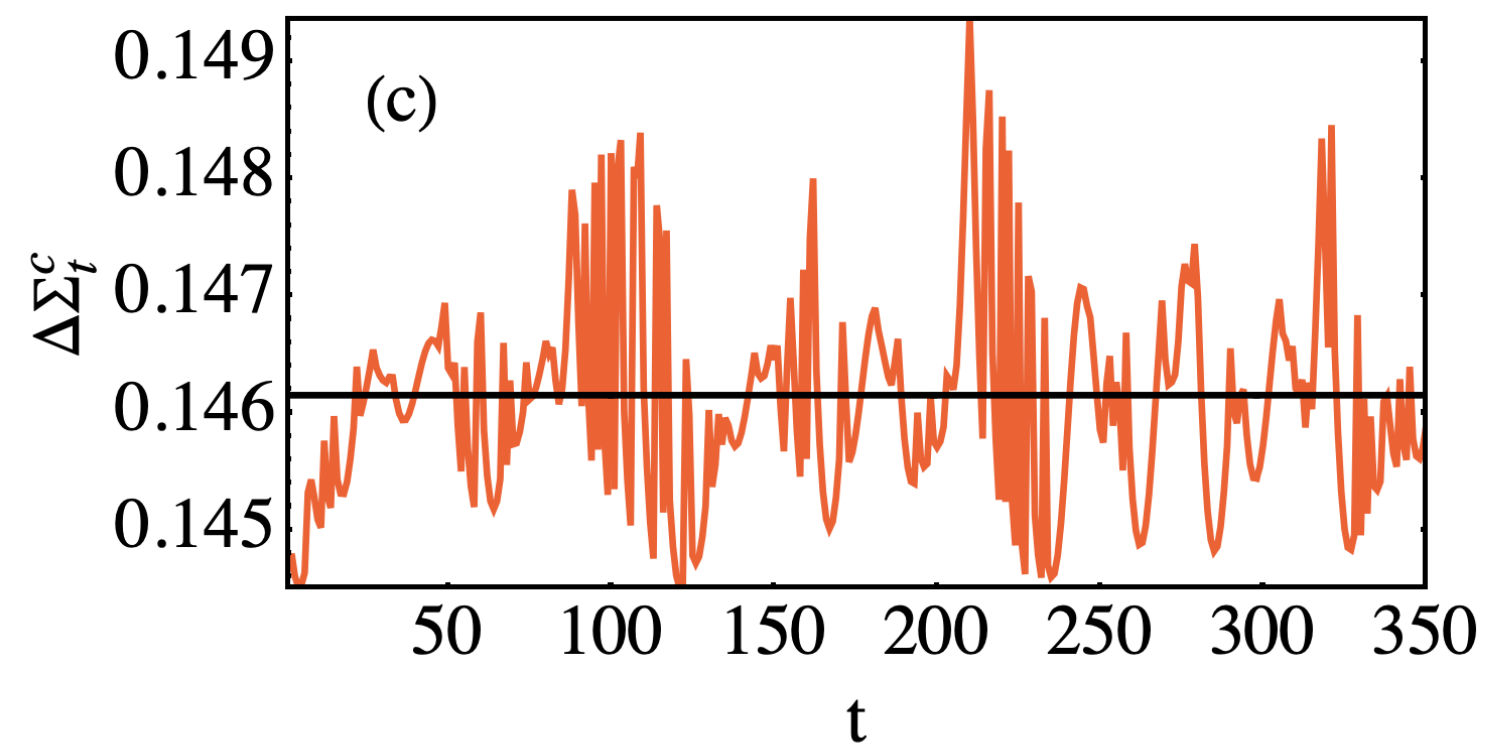
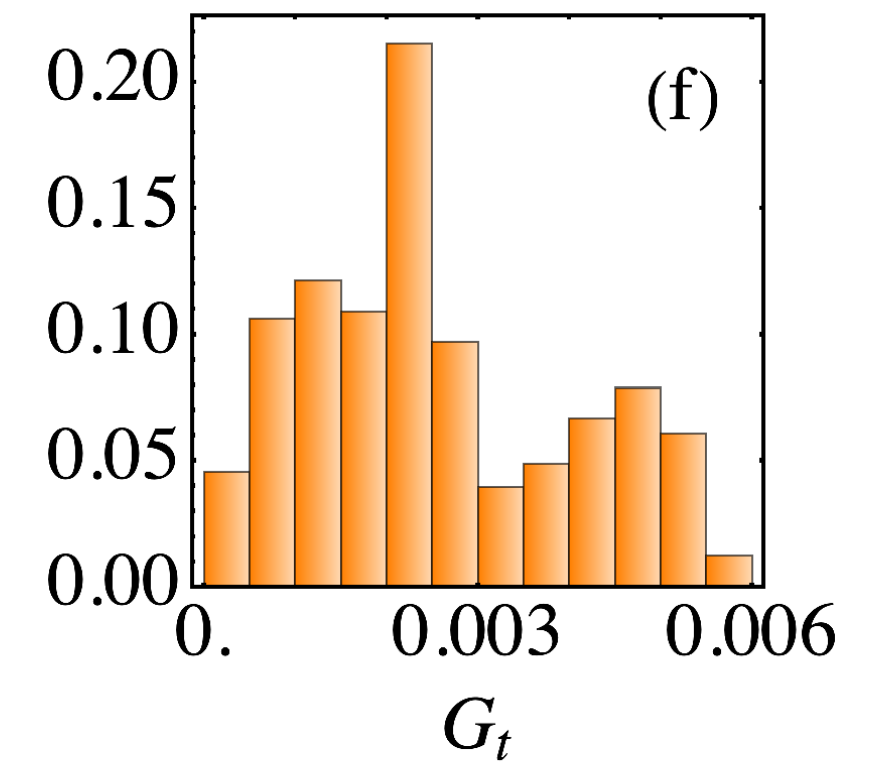
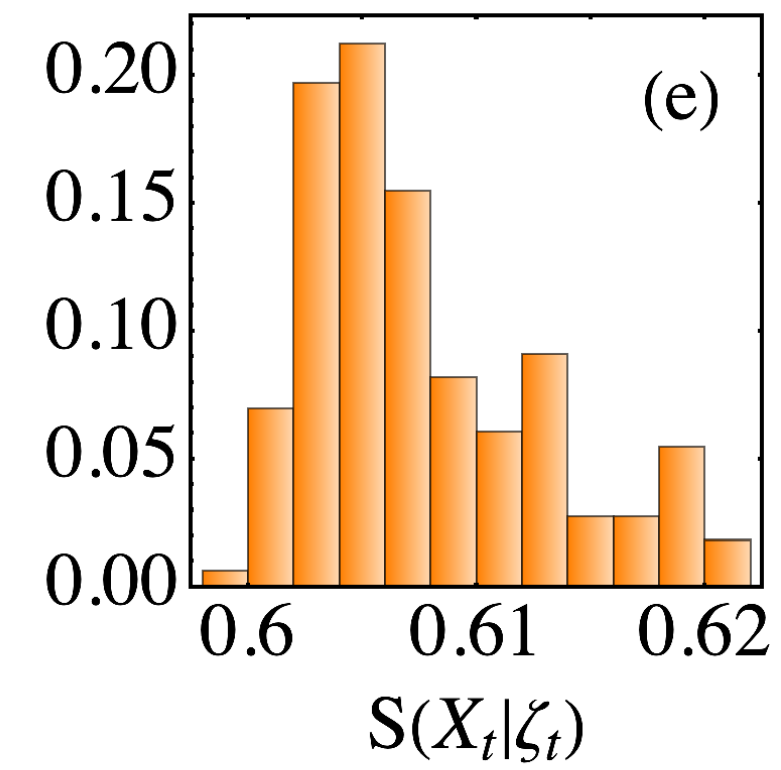
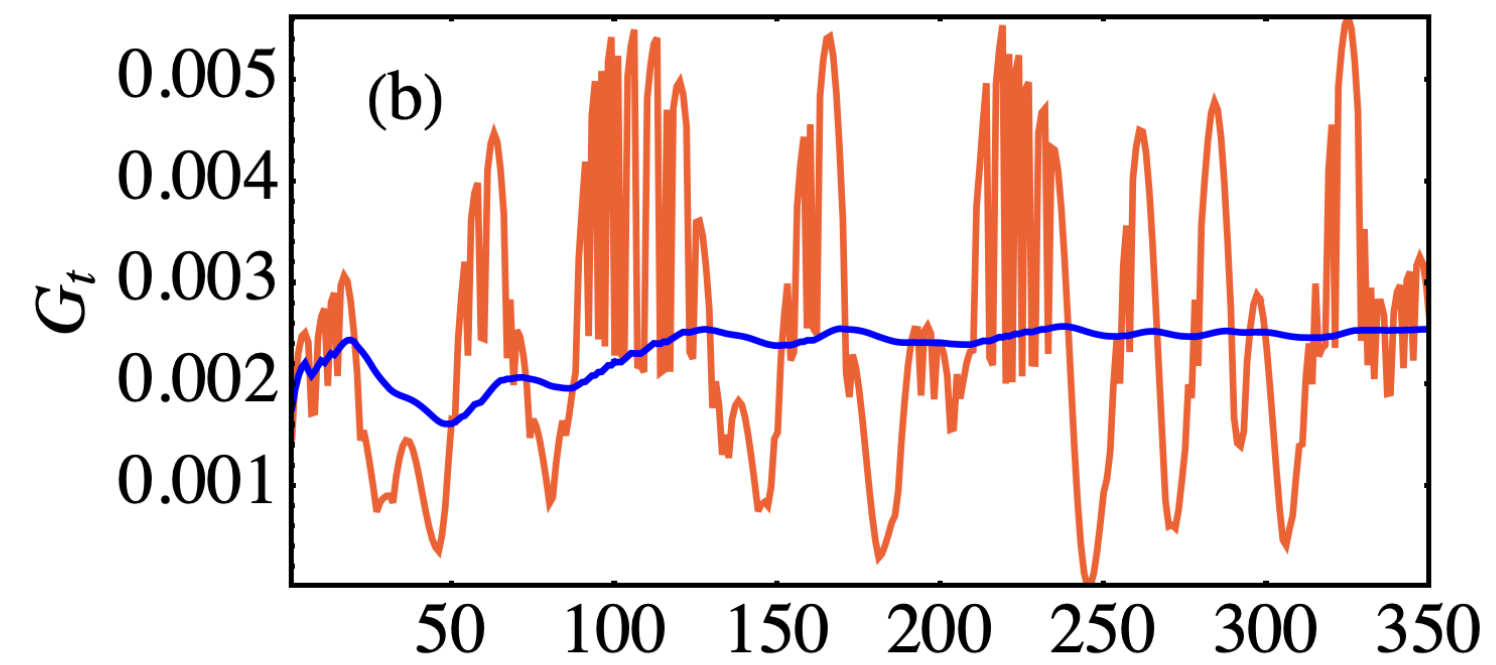
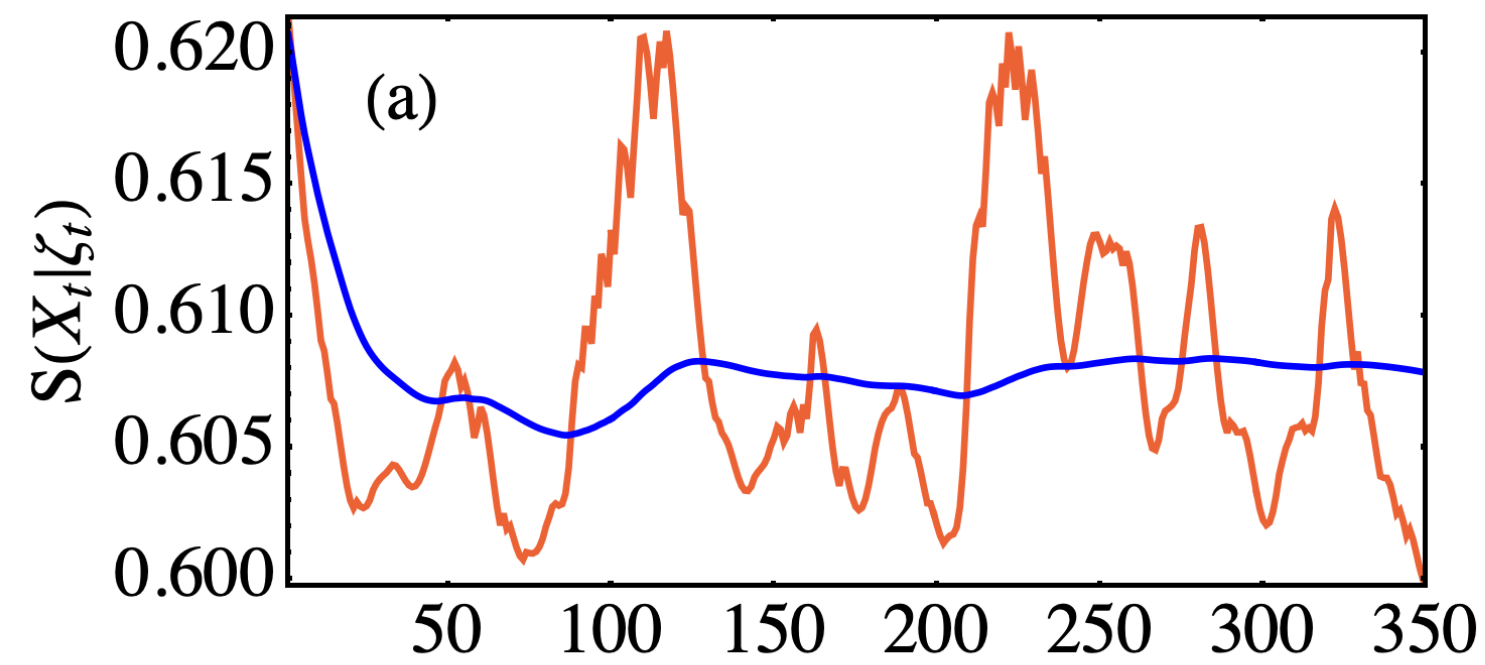
$$I(S_t : \zeta_t) = \sum_{\zeta_t} P(\zeta_t) D(\rho_{t|\zeta_t} || \rho_t)$$

- With each new detection

$$\Delta I_t = G_t - L_t = \text{gain} - \text{loss}$$

- Conditional entropy production

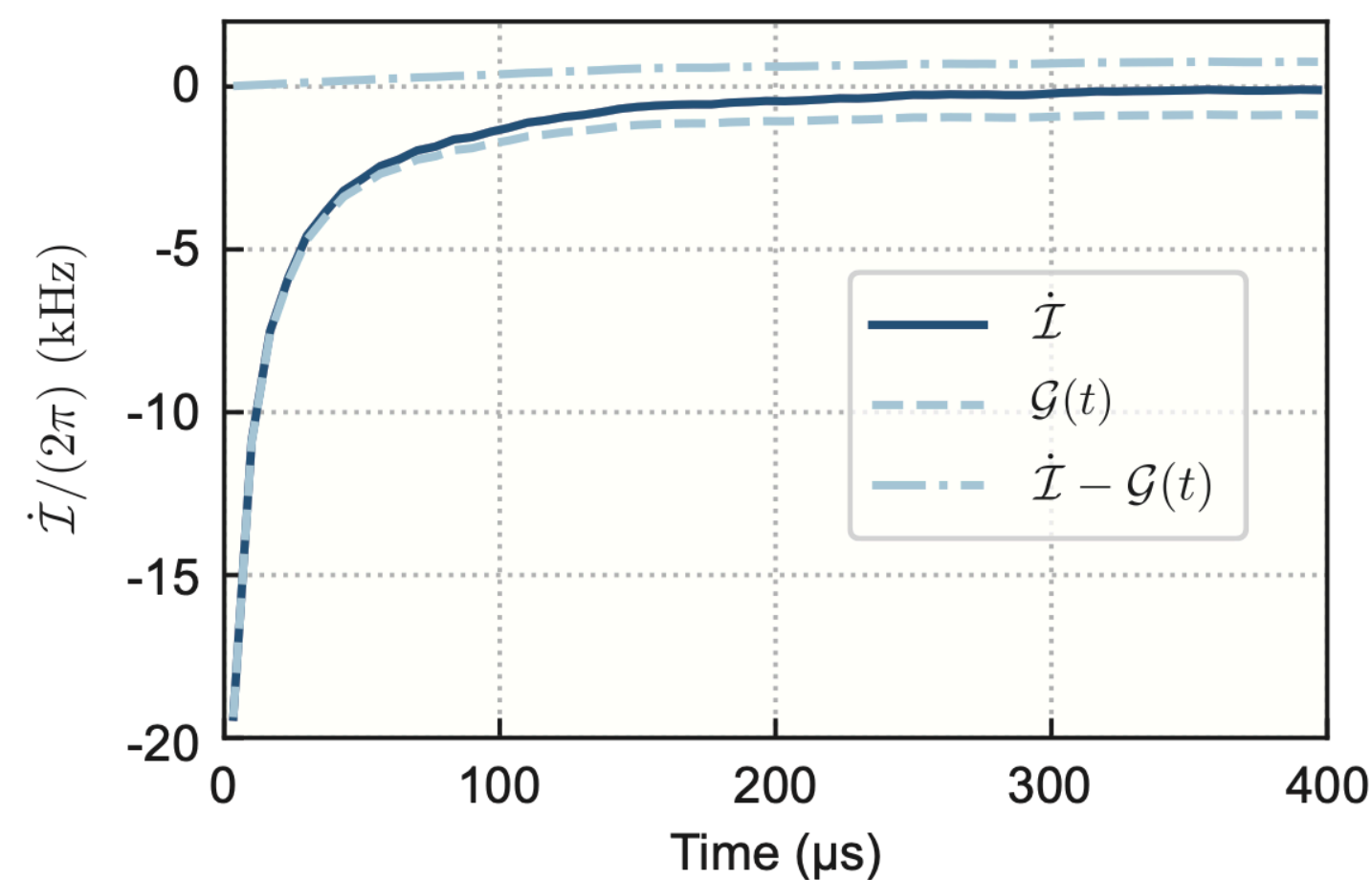
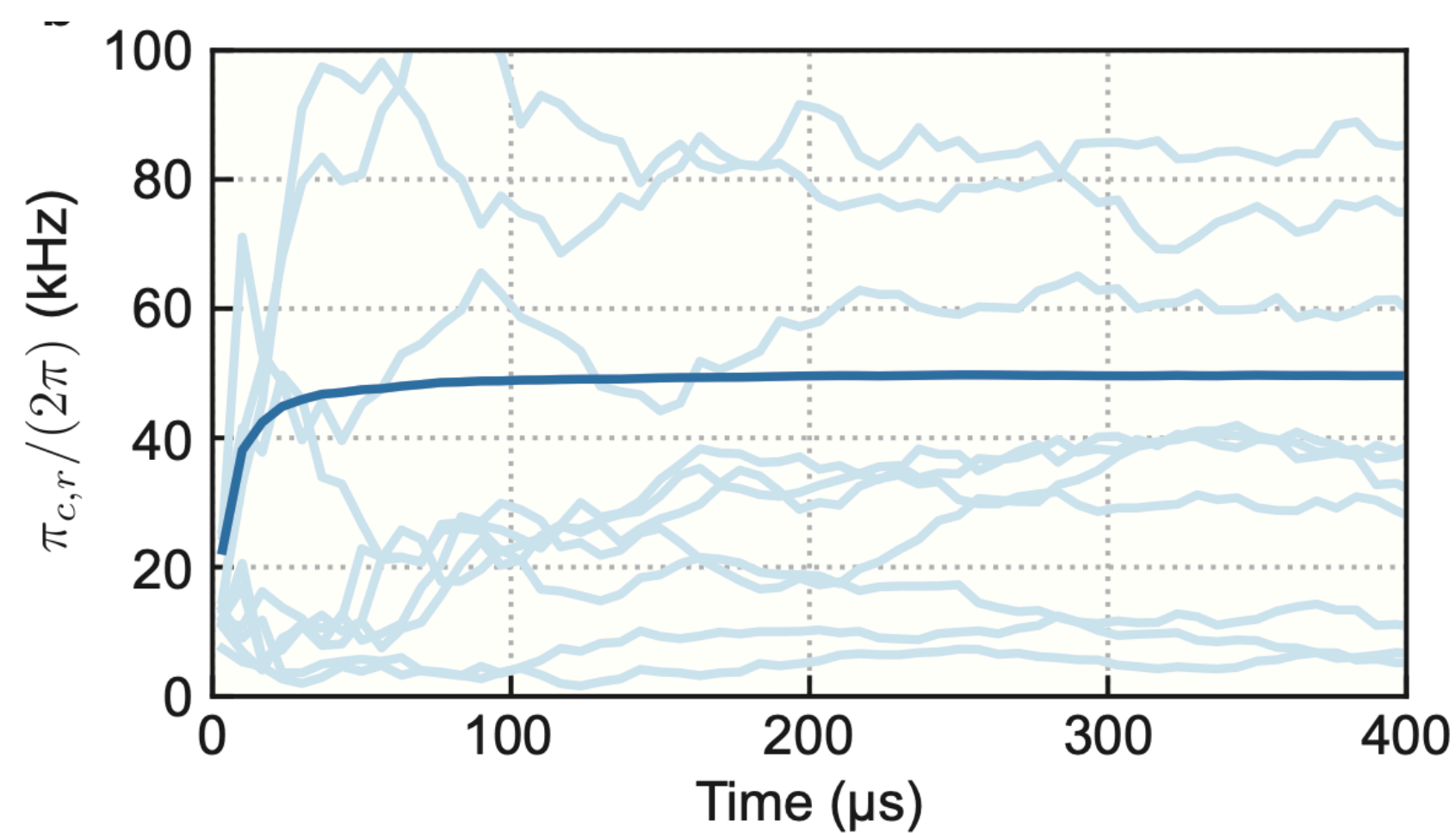
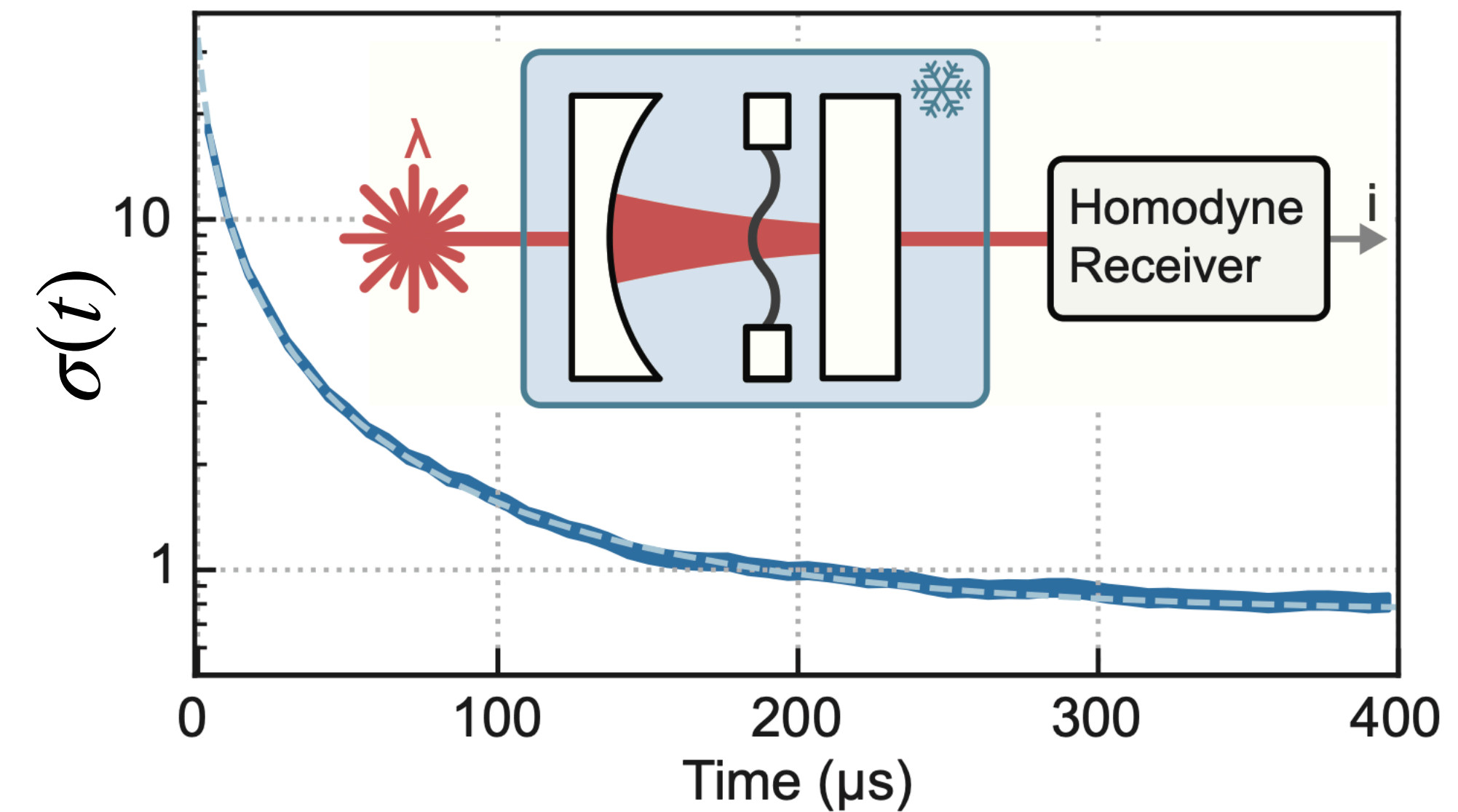
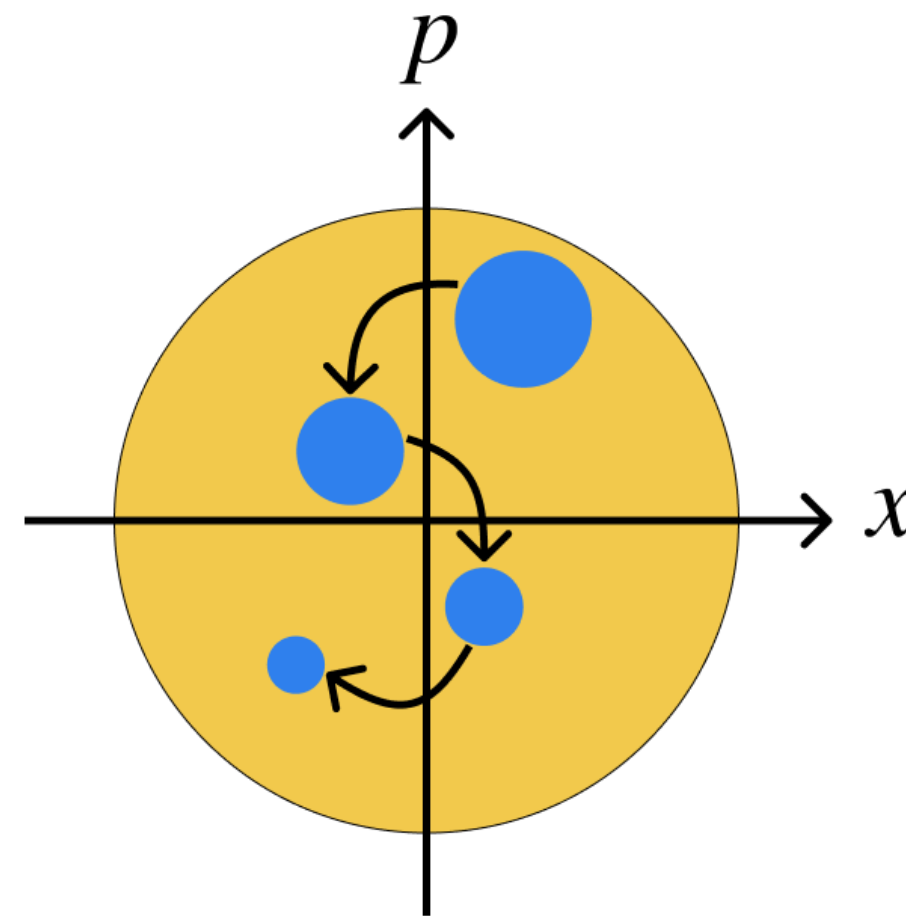
$$\Delta \Sigma^c = \Delta \Sigma^u - \Delta I$$



Alessio Belenchia, Luca Mancino, Gabriel T. Landi and Mauro Paternostro, “**Entropy production in continuously measured quantum systems**”, *npj Quantum Information*, **6**, 97 (2020).

Gabriel T. Landi, Mauro Paternostro and Alessio Belenchia, “**Informational steady-states and conditional entropy production in continuously monitored systems**”, *PRX Quantum* **3**, 010303, (2020).

Optomechanical setup



Informational steady-state:

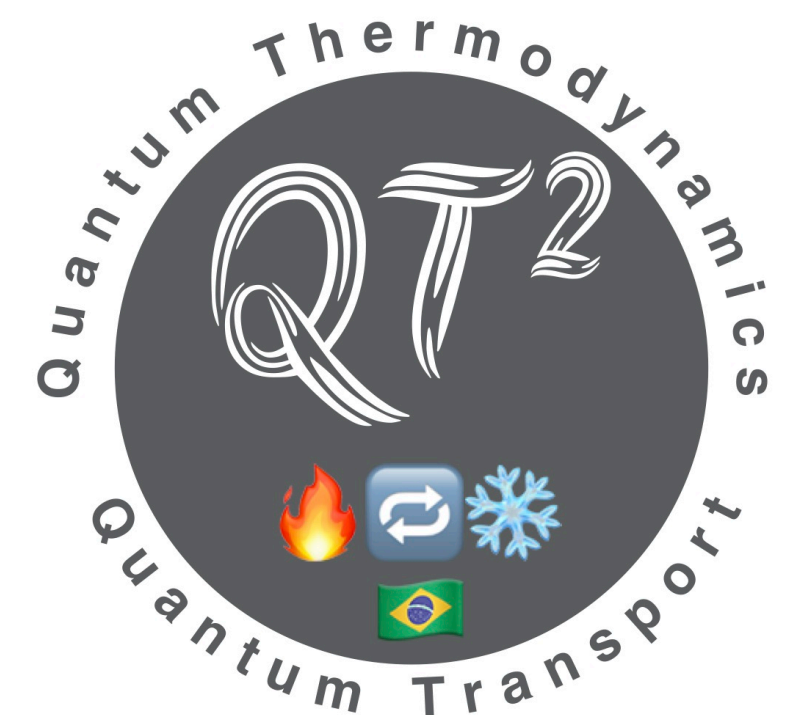
Conditional dynamics relaxes to a colder state, which can only be maintained by continuously monitoring S .

Massimiliano Rossi, Luca Mancino, Gabriel T. Landi, Mauro Paternostro, Albert Schliesser, Alessio Belenchia, **"Experimental assessment of entropy production in a continuously measured mechanical resonator"**, *Phys. Rev. Lett.* **125**, 080601 (2020)

Conclusions

- **Information is a thermodynamic resource.**
- How to incorporate information in the laws of thermodynamics is still an open question.
- I have focused on some recent developments, concerning the 2nd law (entropy production)
 - Quantum correlations.
 - Quantum coherence.
 - Non-Abelian charges.
 - Quantum phase space.
 - Continuous measurements.

Thank you!



Thermosqueezing operations

- Single-mode radiation:

$$\rho = \frac{1}{Z} \exp\{ -\beta H - \beta\mu A \}, \quad H = \frac{\omega}{2}(p^2 + x^2), \quad A = \frac{\omega}{2}(p^2 - x^2)$$

- Two charges, H (energy) and A (squeezing). Satisfy SU(1,1) algebra.

