# Information-thermodynamics in the quantum regime

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Quantum Thermodynamics
$QT^2$
Quantum Transport

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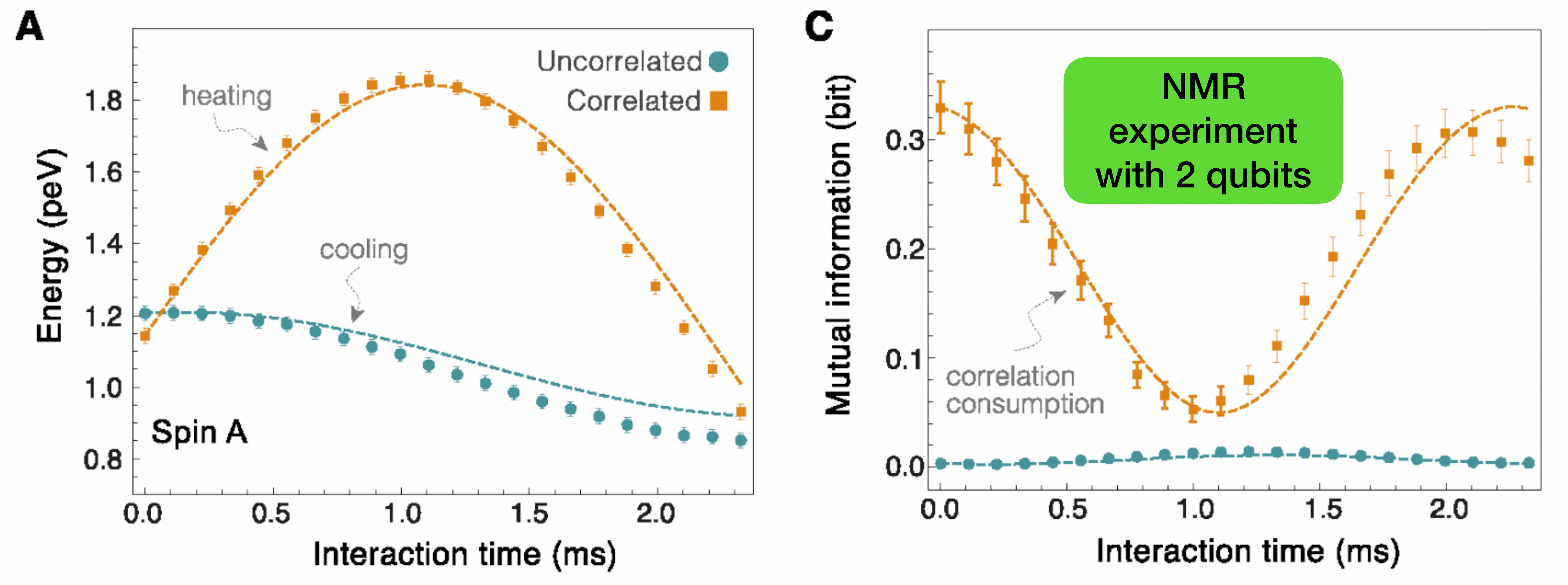
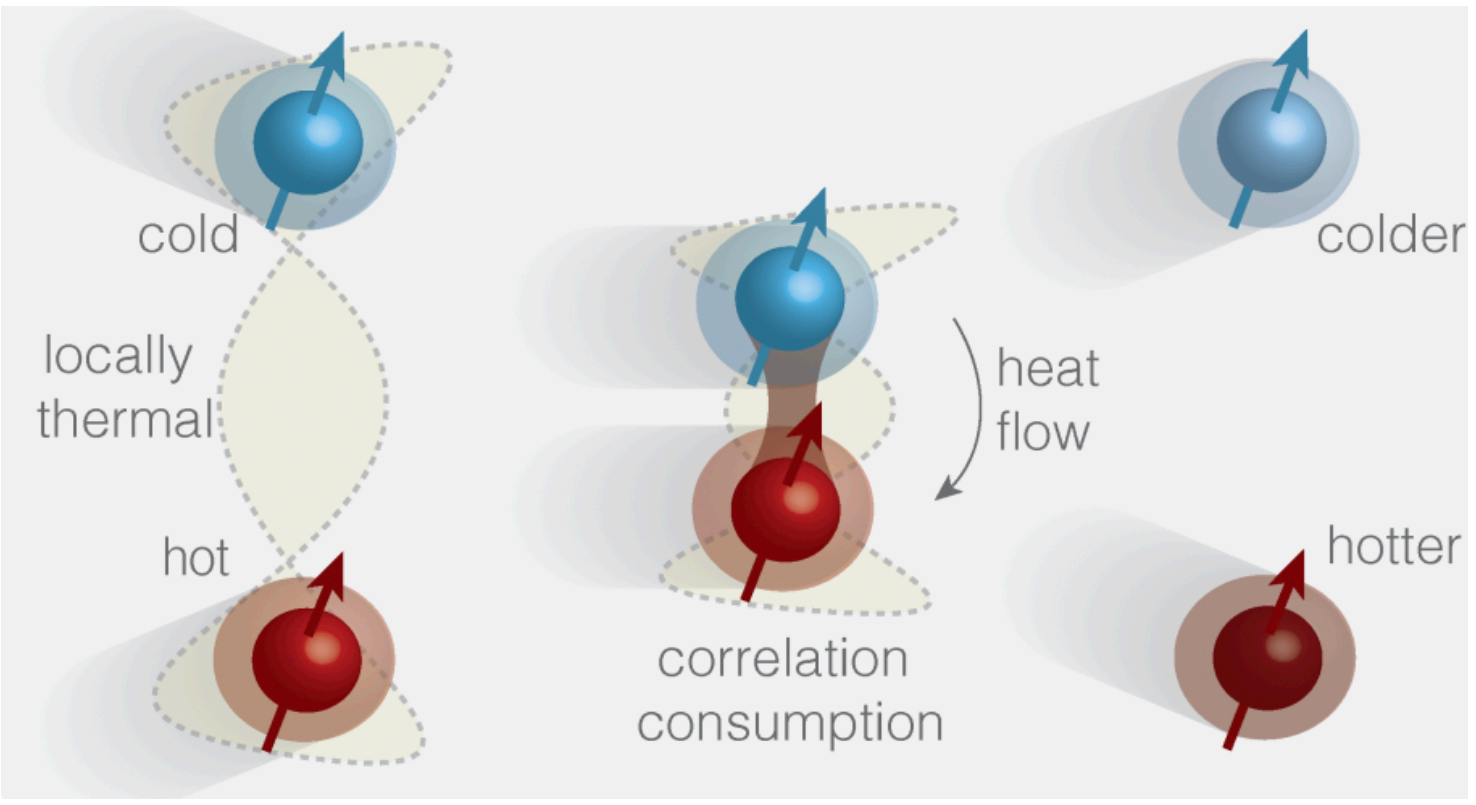
# Heat flows from hot to cold

- To break that, we must pay a **price**: fridges consumes electricity.
  - *“Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.”* (Clausius’ statement of the 2nd law)
- In the quantum domain, **information** is also a resource.




Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, "**Reversing the direction of heat flow using quantum correlations**”, *Nature Communications*, **10**, 2456 (2019)

# Thermodynamics is a pragmatic theory

- It deals with palpable quantities (heat, work, etc.)
  - Describing how they can be converted, stored, transported and manipulated.
- And it offers *simple* guidelines on which processes can or cannot happen.
- 1st law: $\Delta E_A + \Delta E_B = W_{\text{ext}}$

  2nd law: $\Sigma = \dfrac{\Delta E_A}{T_A} + \dfrac{\Delta E_B}{T_B} \geqslant 0$

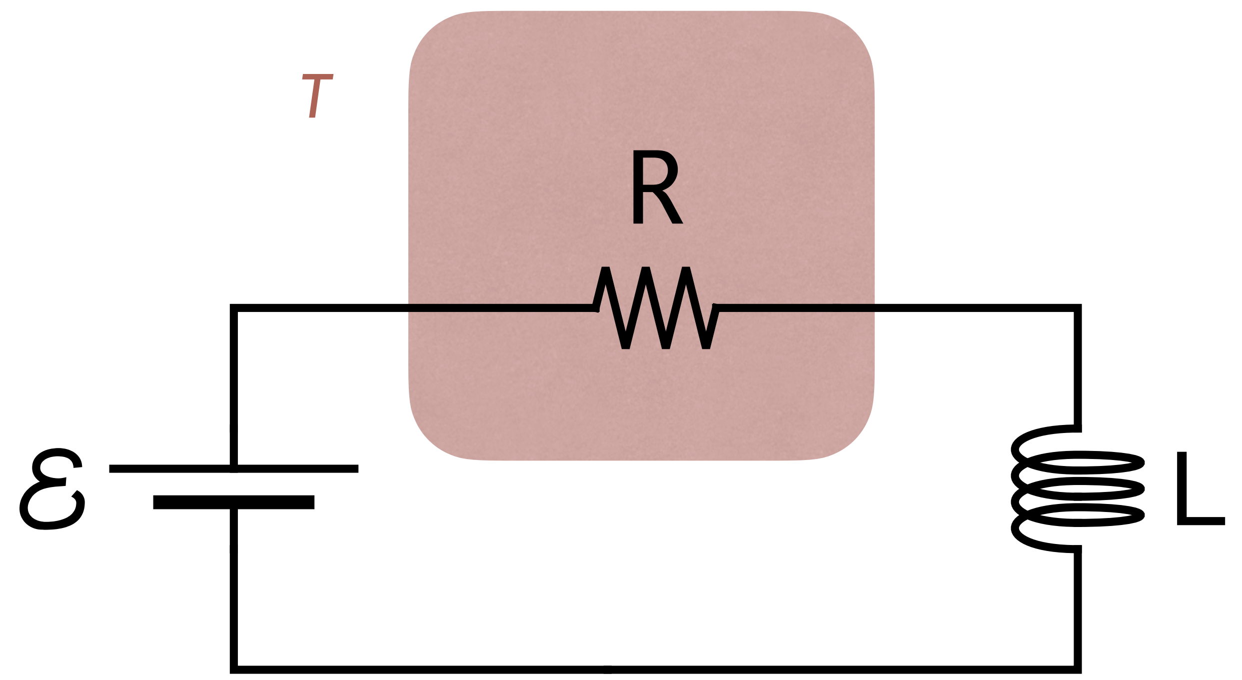
How to reformulate the laws of thermodynamics to include information?

- Ex: if $W_{\text{ext}} = 0$ then $\Delta E_A = -\Delta E_B := Q$ and

$$\Sigma = Q\left(\frac{1}{T_A} - \frac{1}{T_B}\right) \geqslant 0$$

  - $T_A > T_B \quad \rightarrow \quad Q < 0$ (hot system looses energy; heat flows form hot to cold).

# Example: electric circuit



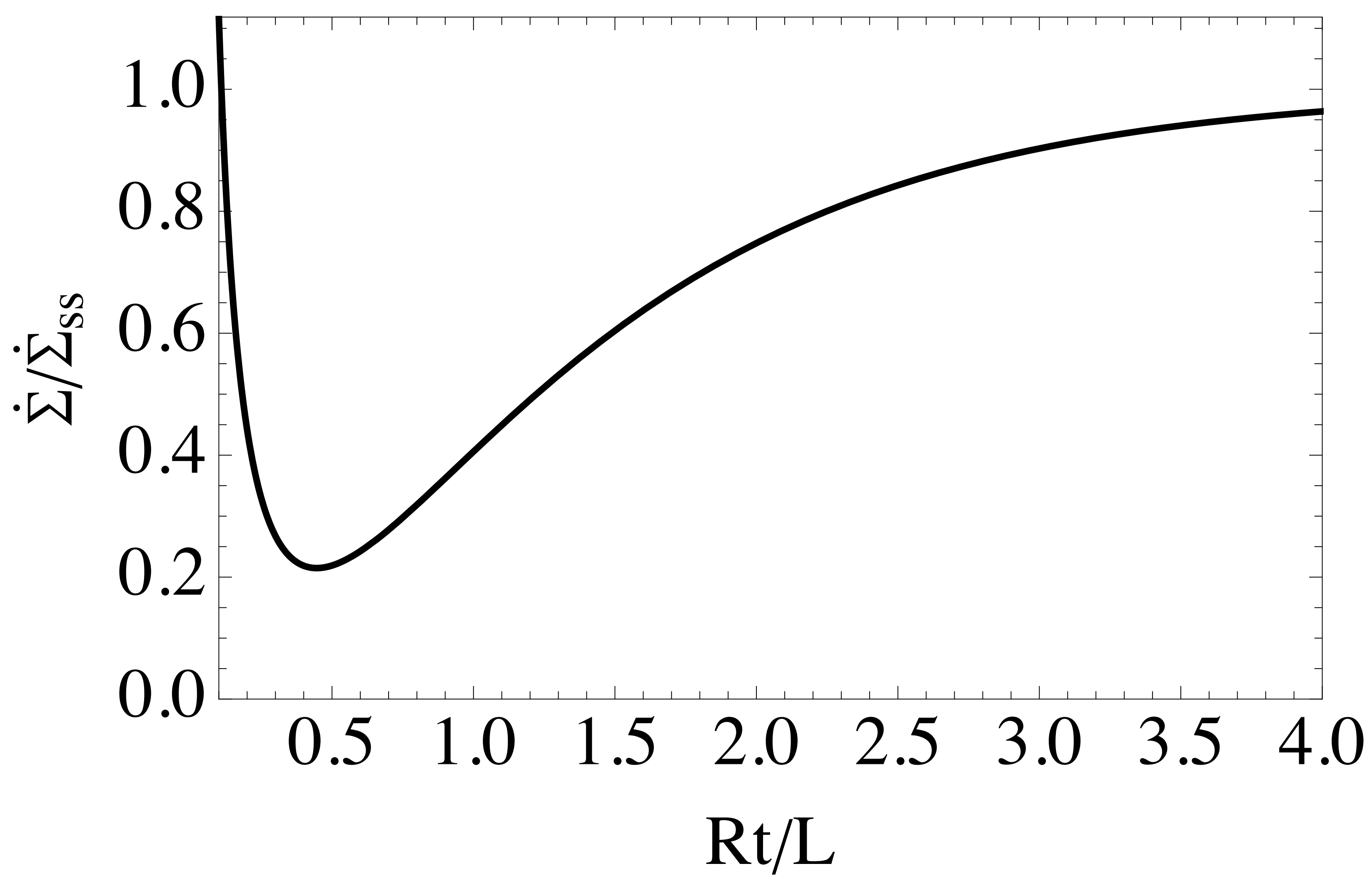

- Entropy production is applicable across physical systems.
- RL circuit dissipates energy in the resistor.
- Entropy production *rate* at the moment where the circuit is turned on:

$$\dot{\Sigma} = \frac{\mathcal{E}^2}{RT}(1 - e^{-Rt/L})^2 + \frac{R}{L}\frac{e^{-2Rt/L}}{e^{2Rt/L} - 1}$$

- In the long-time limit it tends to a non-equilibrium steady-state:

$$\dot{\Sigma}_{\text{ss}} = \frac{\mathcal{E}^2}{RT}$$

- As long as there is juice in the battery, there is entropy being produced.



# Entropy production is fully information theoretic

- Entropy production for a system in contact with a bath:

$$\Sigma = \Delta S_S + \beta Q$$

- Can also be written as $\Sigma = I'(S:E) + D(\rho'_E || \rho_E)$

**Mutual Information:**

$$I'(S:E) = S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE})$$

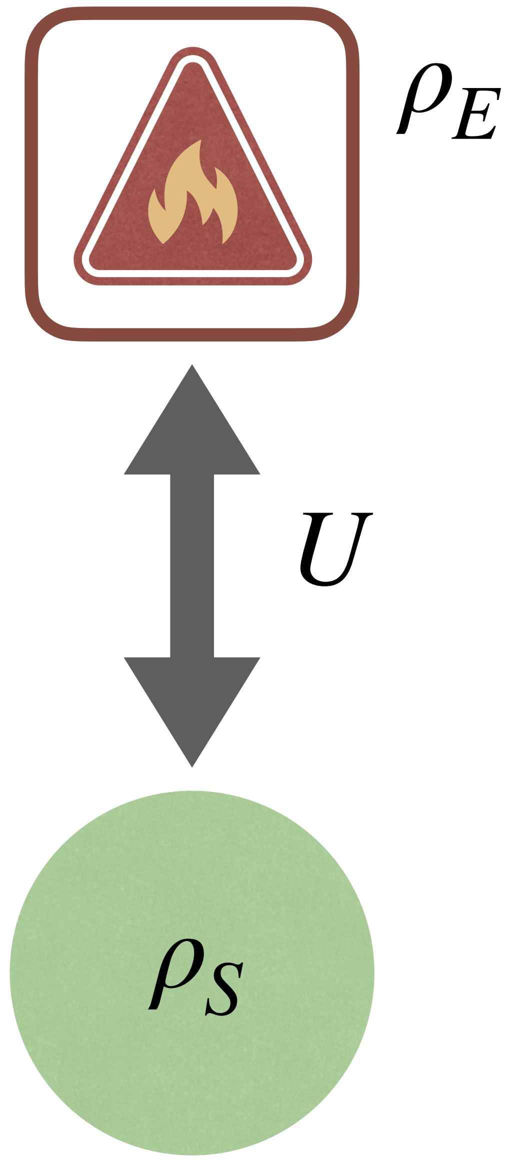
**Quantifies all correlations (classical + quantum)**

**Relative entropy**

$$D(\rho'_E || \rho_E) = \mathrm{tr}\left(\rho'_E \ln \rho'_E - \rho'_E \ln \rho_E\right)$$

**"Distance" between density matrices**

$$\rho'_{SE} = U(\rho_S \otimes \rho_E)U^\dagger$$



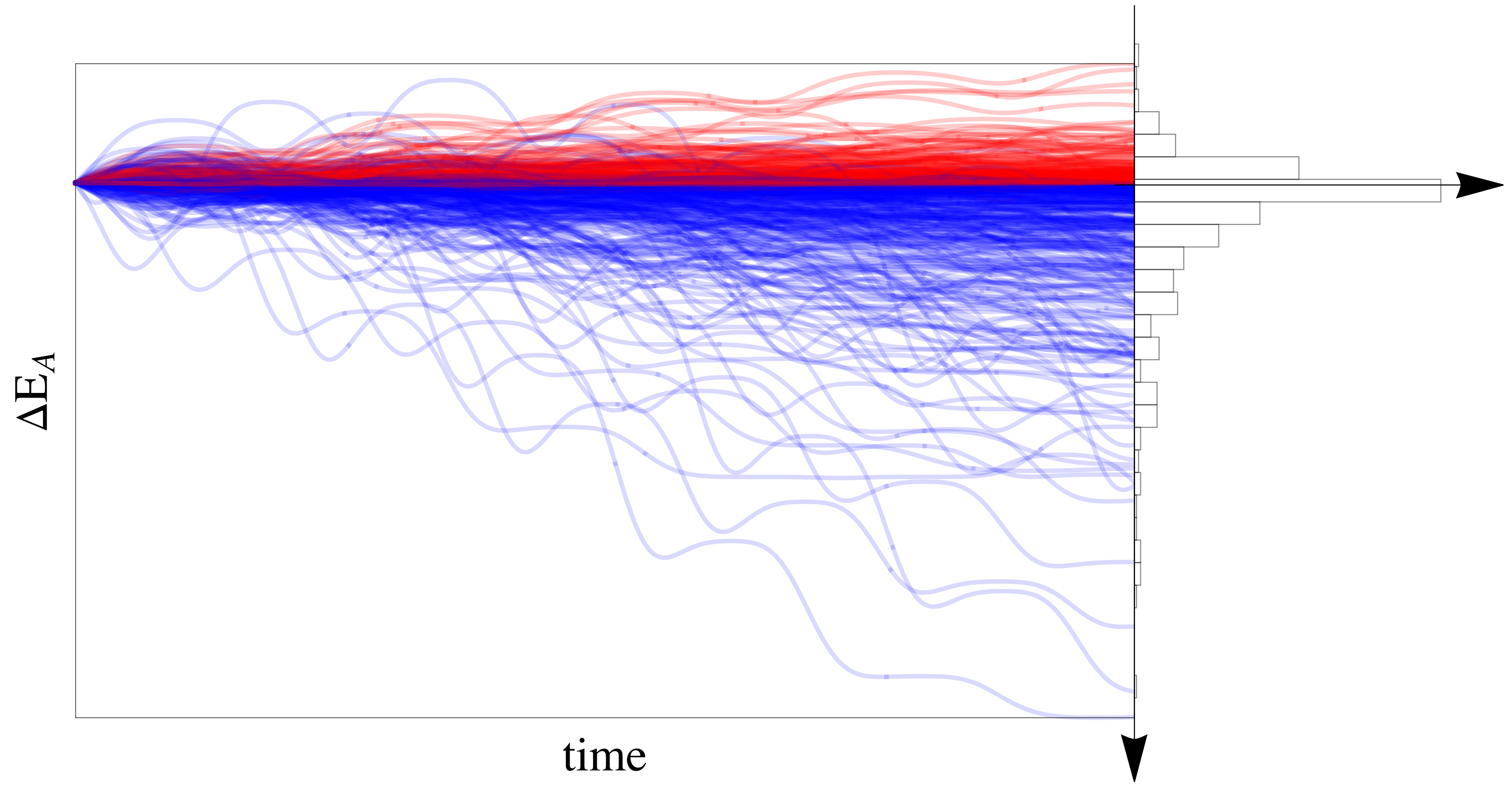

Describes an enormous variety of processes! (maybe a complicated $U$)


M. Esposito, K. Lindenberg, C. Van den Broeck, "**Entropy production as correlation between system and reservoir**". *New Journal of Physics*, **12**, 013013 (2010).

Gabriel T. Landi and Mauro Paternostro, "**Irreversible entropy production, from quantum to classical**", *Review of Modern Physics*, **93**, 035008 (2021)

# Fluctuations are significant in the micro-world

- Macro-world: heat flows from hot → cold: “*arrow of time*”.
- Micro-world: heat ***usually*** flows from hot → cold.




Lectures from Prof. George Porter

**Heat Exchange Fluctuation Theorem**

$$P(Q) = e^{(\beta_A - \beta_B)Q} P(-Q)$$

**Implies 2nd law:**

$$\langle Q \rangle \left( \frac{1}{T_A} - \frac{1}{T_B} \right) \geqslant 0$$

Jarzynski and Wójcik, Phys. Rev. Lett. 92, 230602 (2004)

# Fully quantum fluctuation theorems

- We put forth a theory encompassing Heat exchange in the presence of general quantum correlations.

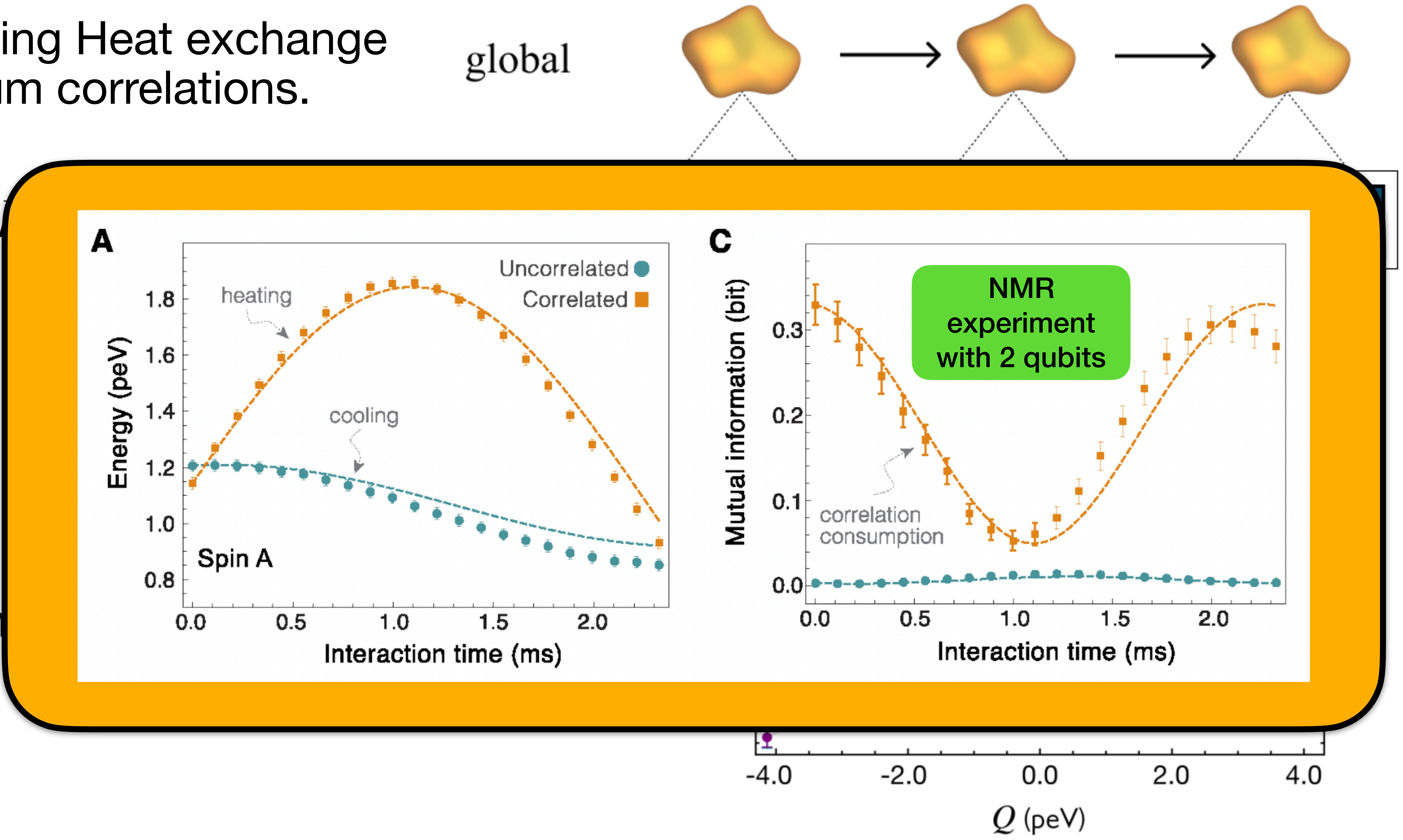
$$\frac{P(\Gamma)}{P(\Gamma^*)} = \exp\Big\{(\beta_A - \beta_B)Q - \Delta$$

- From this one can show that

$$(\beta_A - \beta_B)\langle Q\rangle \geqslant \langle \Delta I\rangle$$

- Correction to the heat distribution

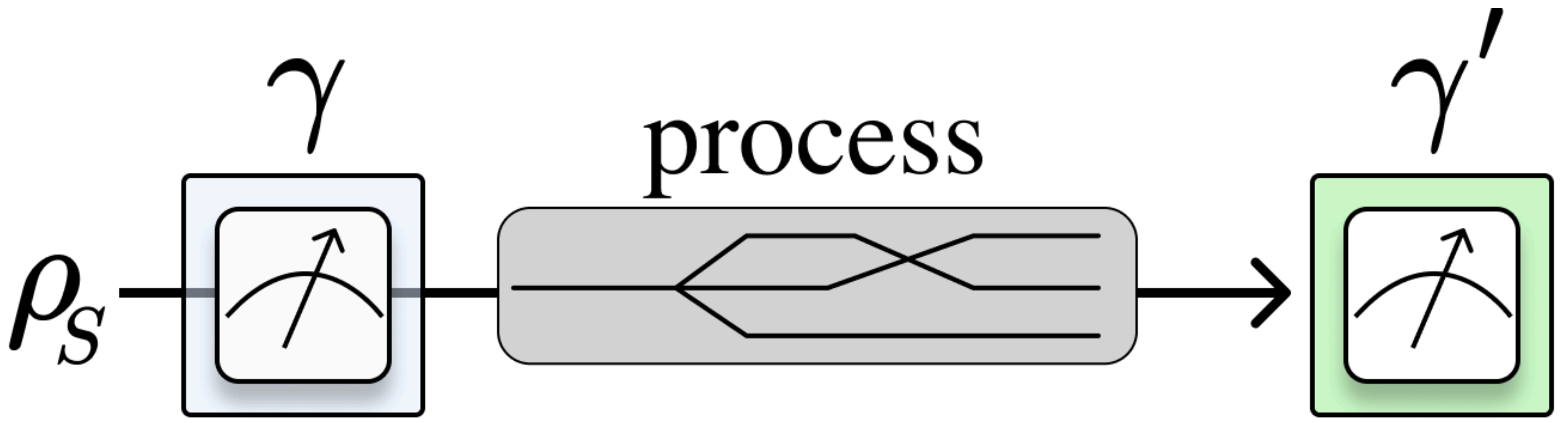
$$\frac{P(Q)}{P(-Q)} = \frac{e^{(\beta_A-\beta_B)Q}}{\Psi(Q)}$$

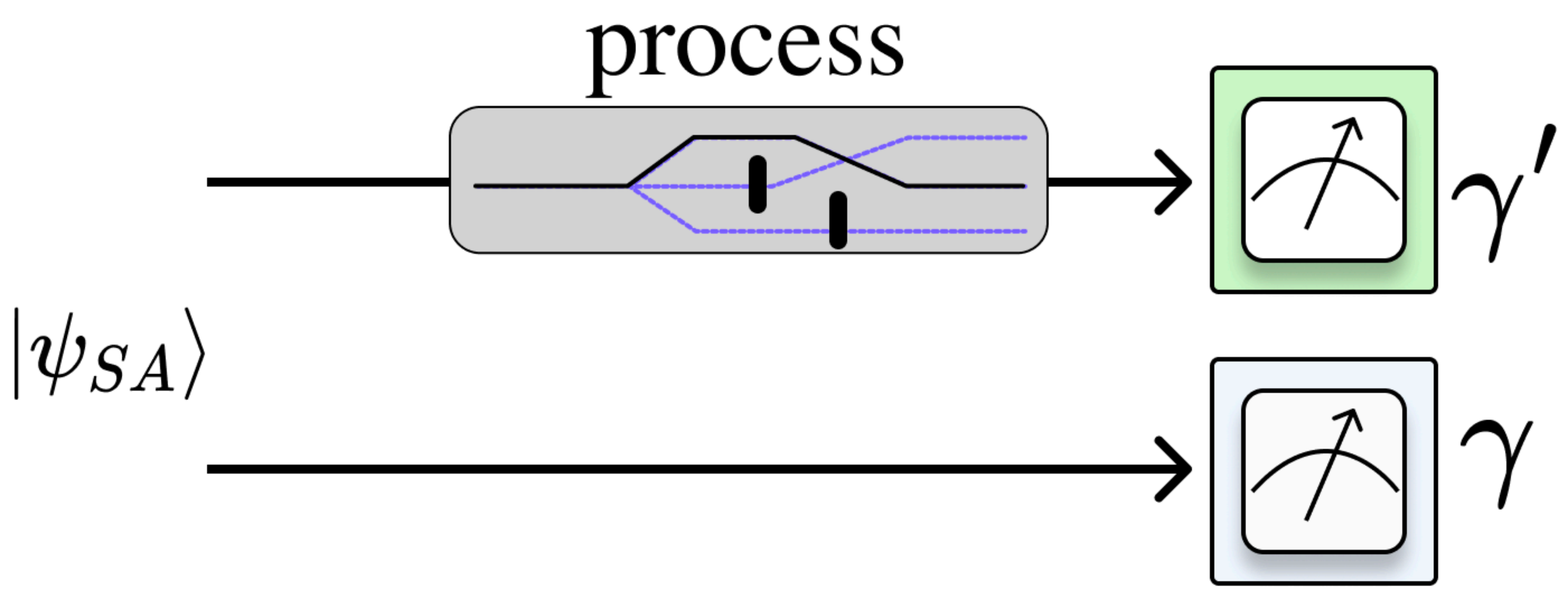


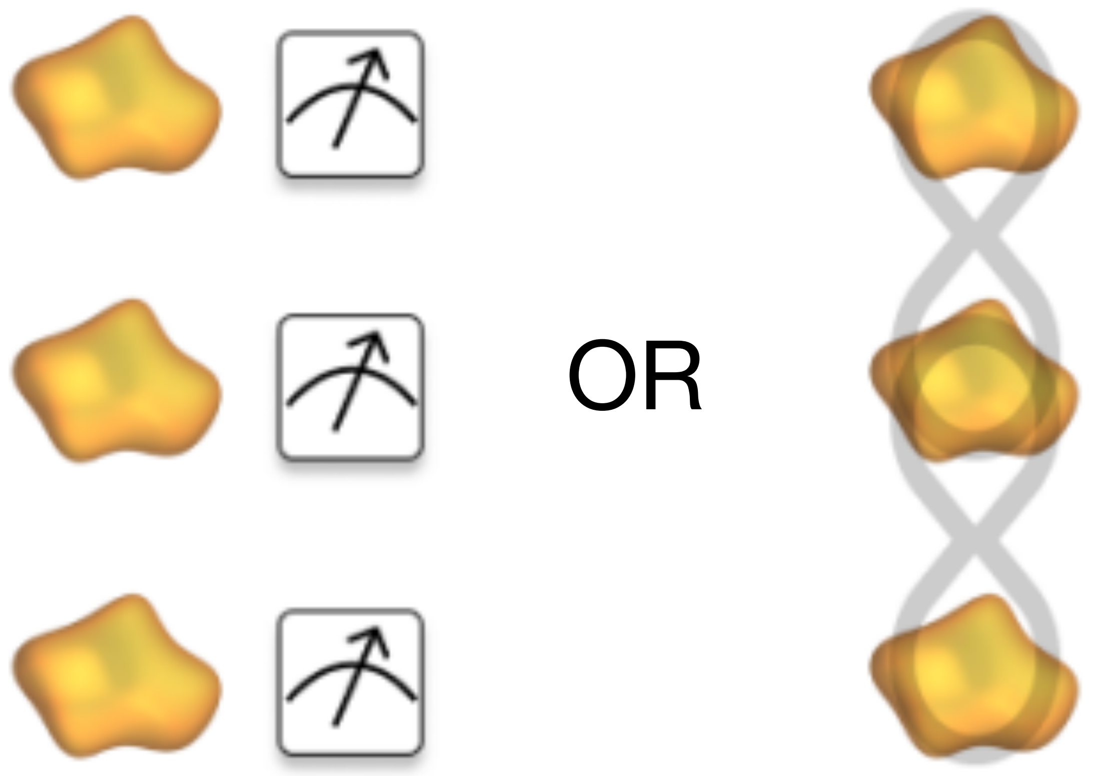

K. Micadei, G. T. Landi, E. Lutz, "**Quantum fluctuation theorems beyond two-point measurements**", *Phys. Rev. Lett*. **124**, 090602 (2020)
Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Roberto M. Serra, Eric Lutz, "**Experimental validation of fully quantum fluctuation theorems**", *Phys. Rev. Lett.*, **127**, 180603 (2021).

# Two-point measurements destroy quantum features

- Measurements in quantum mechanics are invasive.
  - Destroy initial quantum coherences.
- Fundamental limitation of Quantum + Thermodynamics.
- We recently showed that this can be overcome if one has access to ***identical copies*** of a quantum system.
- Or using an ***entangled ancilla***, which is only measured at the end.



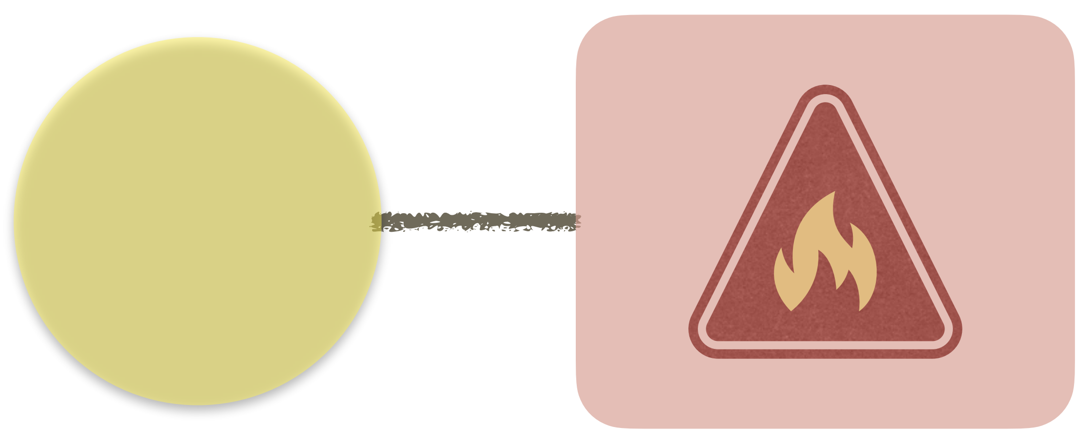


Kaonan Micadei, Gabriel T. Landi, Eric Lutz, "**Extracting Bayesian networks from multiple copies of a quantum system**," arXiv:2103.14570

Gabriel H. Aguilar, Thaís L. Silva, Thiago E. Guimarães, Rodrigo S. Piera, Lucas C. Céleri, Gabriel T. Landi, "**Two-point measurement of entropy production from the outcomes of a single experiment with correlated photon pairs**", arXiv:2108.03289 (2021)

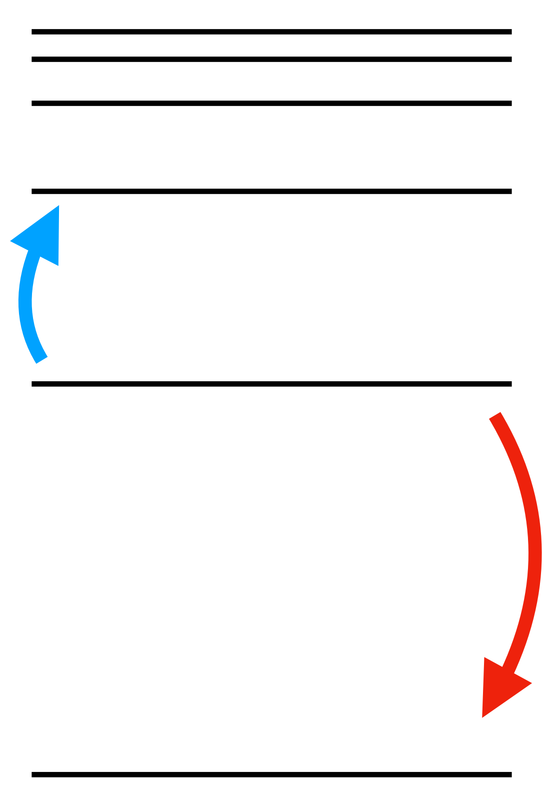
# Quantum features & entropy production

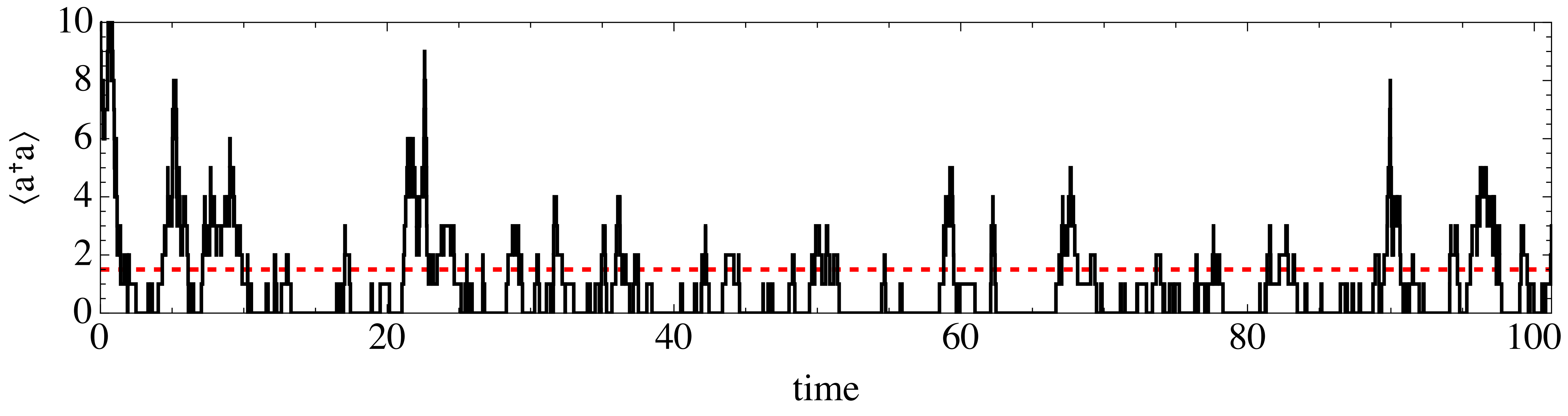
# Relaxation towards equilibrium



- Class of physical processes: **thermal operations.**
    - Population in energy eigenstates fluctuate until they reach thermal equilibrium.
- In addition: destroy any superpositions (**decoherence**).
- Entropy production rate can be split as

$$\dot{\Sigma} = \dot{\Sigma}_{\mathrm{pop}} + \dot{\Sigma}_{\mathrm{coh}}$$

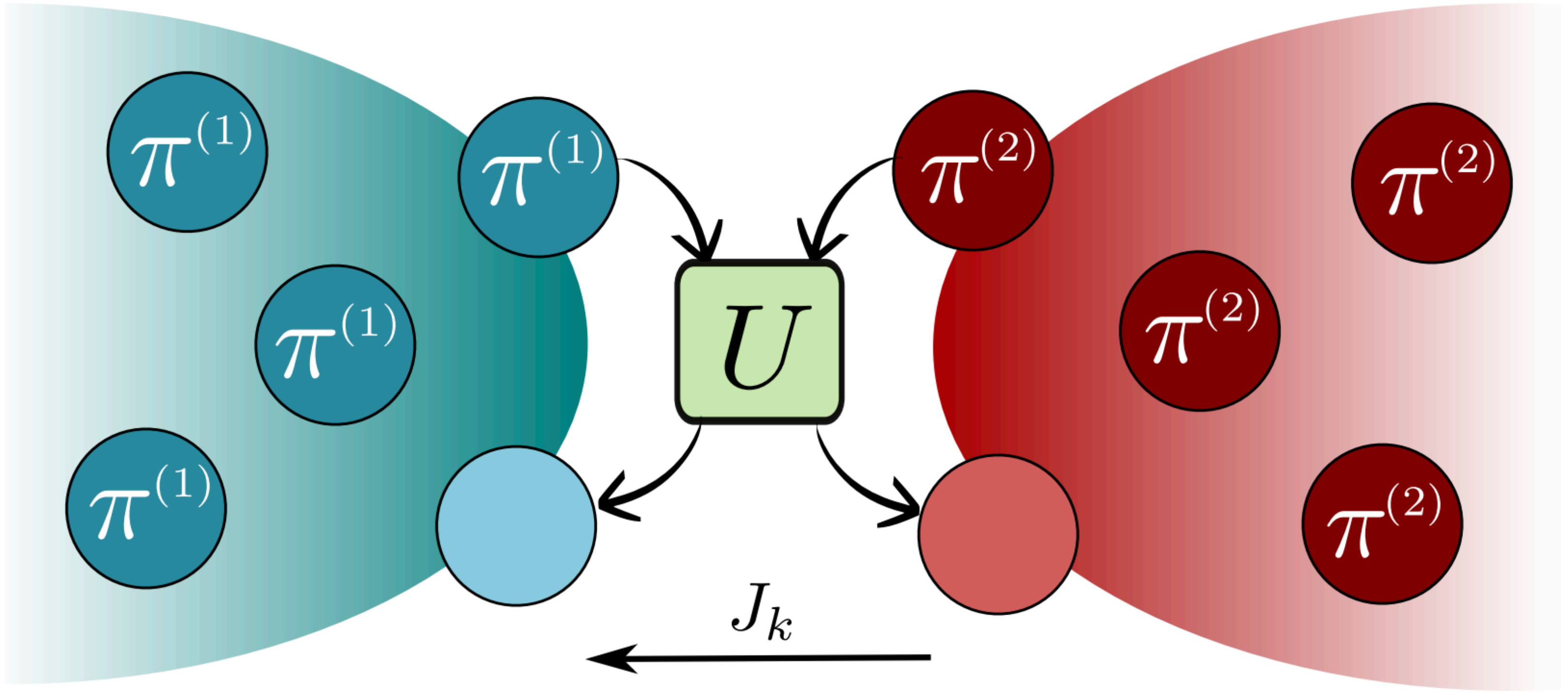





Additional entropy production due to coherence:
Dissipation of information, without dissipation of energy.

J. P. Santos, L. Céleri, GTL, M. Paternostro, npj Quantum Information **5**, 23 (2019)

# Transport of non-Abelian charges




- Classical transport: **energy and particles.**
- Quantum domain: excitations may not commute.
- Generalised Gibbs Ensemble:

$$\rho = \frac{1}{Z} \exp\Big\{ -\sum_k \lambda_k Q_k \Big\}$$

$Q_k$ = charges, $\lambda_k$ = affinities.

- Charge conservation condition:

$$[U, Q_k^{(A)} + Q_k^{(B)}] = 0, \qquad \forall k$$

**Entropy production is *reduced* due to non-commutativity**

$$\Sigma = \Sigma_{\text{comm}} - I(\rho, D)$$

$$D = \sum_k \delta\lambda_k \Delta Q_k$$

Gonzalo Manzano, Juan M. R. Parrondo, Gabriel T. Landi, "**Non-Abelian Quantum Transport and Thermosqueezing Effects**", *PRX Quantum* **3**, 010304 (2020)

# Thermosqueezing operations

- Single QHO:

$$\rho = \frac{1}{Z}\exp\Big\{-\beta H - \beta\mu A\Big\}, \qquad H = \frac{\omega}{2}(p^2 + x^2), \qquad A = \frac{\omega}{2}(p^2 - x^2)$$

- Two charges, $H$ (energy) and $A$ (asymmetry). Satisfy SU(1,1) algebra.

- Onsager coefficients:

$$J_Q = L_{QQ}\delta_\beta - L_{QA}\beta\delta_\mu \qquad J_A = L_{AQ}\delta_\beta - L_{AA}\beta$$

  - **Peltier:** gradient of squeezing generates a flow of
  - **Seebeck** ("squeebeck"?)**:** gradient of temperature

**Transport coefficients**

**Thermal conductance:** $\kappa = -\beta^2 L_{QQ}$

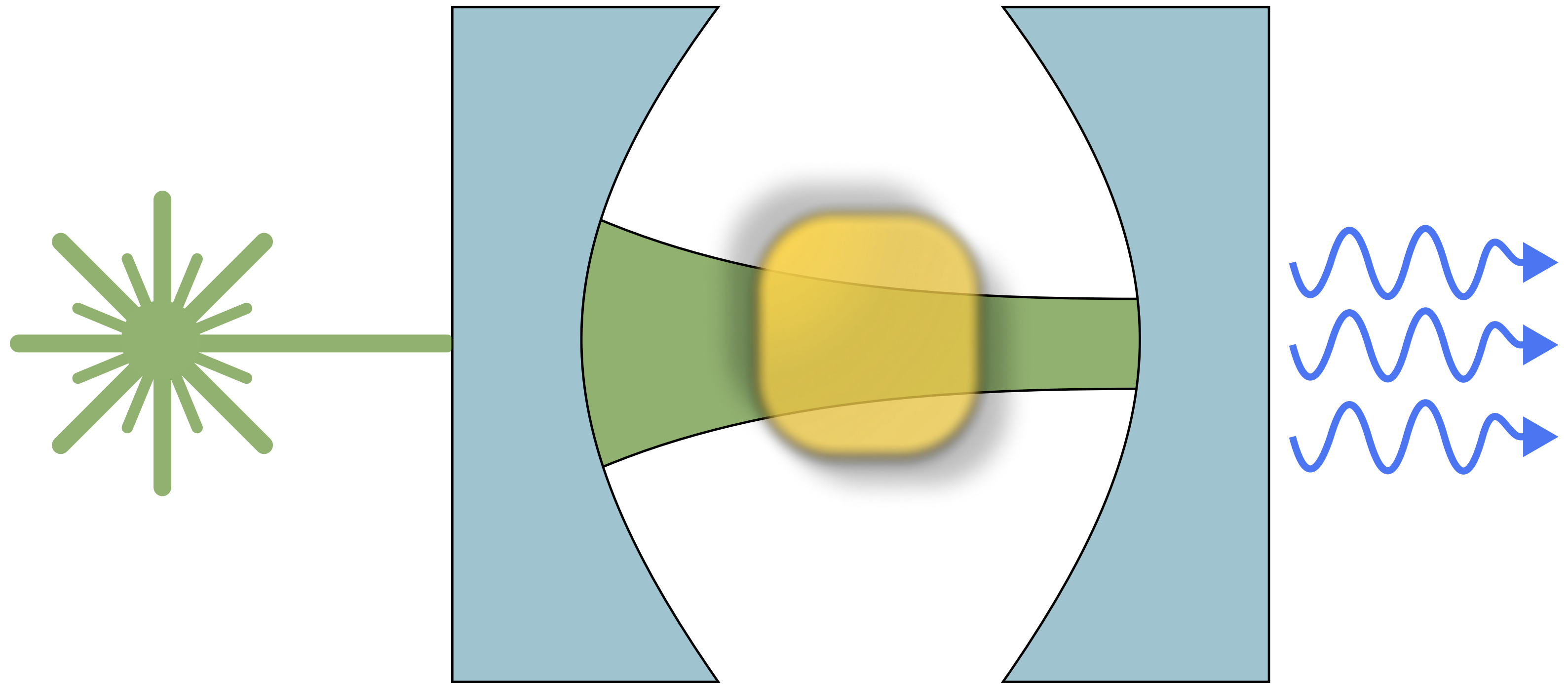
**Squeezing conductance:** $G = -\beta L_{AA}$

**Entropy production/dissipated heat reads**

$$\dot{Q}_{\text{diss}} = \Sigma/\beta = \kappa\delta T^2/T + J_A G$$

**New Joule-like heating term due to squeezing.**

# Quantum phase space



- Many quantum experiments are done using optical cavities with semi-transparent mirrors.

- Photons leaking out $\simeq$ zero temperature bath.
  - Spontaneous emission: excitations can leave, but not return.

- 2nd law is buggy @ $T = 0$: $\dot{\Sigma} = \dfrac{\dot{Q}}{T}$.
  - Does not include vacuum fluctuations.

- We reformulated the entropy production problem in terms of quantum phase space: *Wigner function* and *Husimi Q function.*
  - Quantum Fokker-Planck equation.

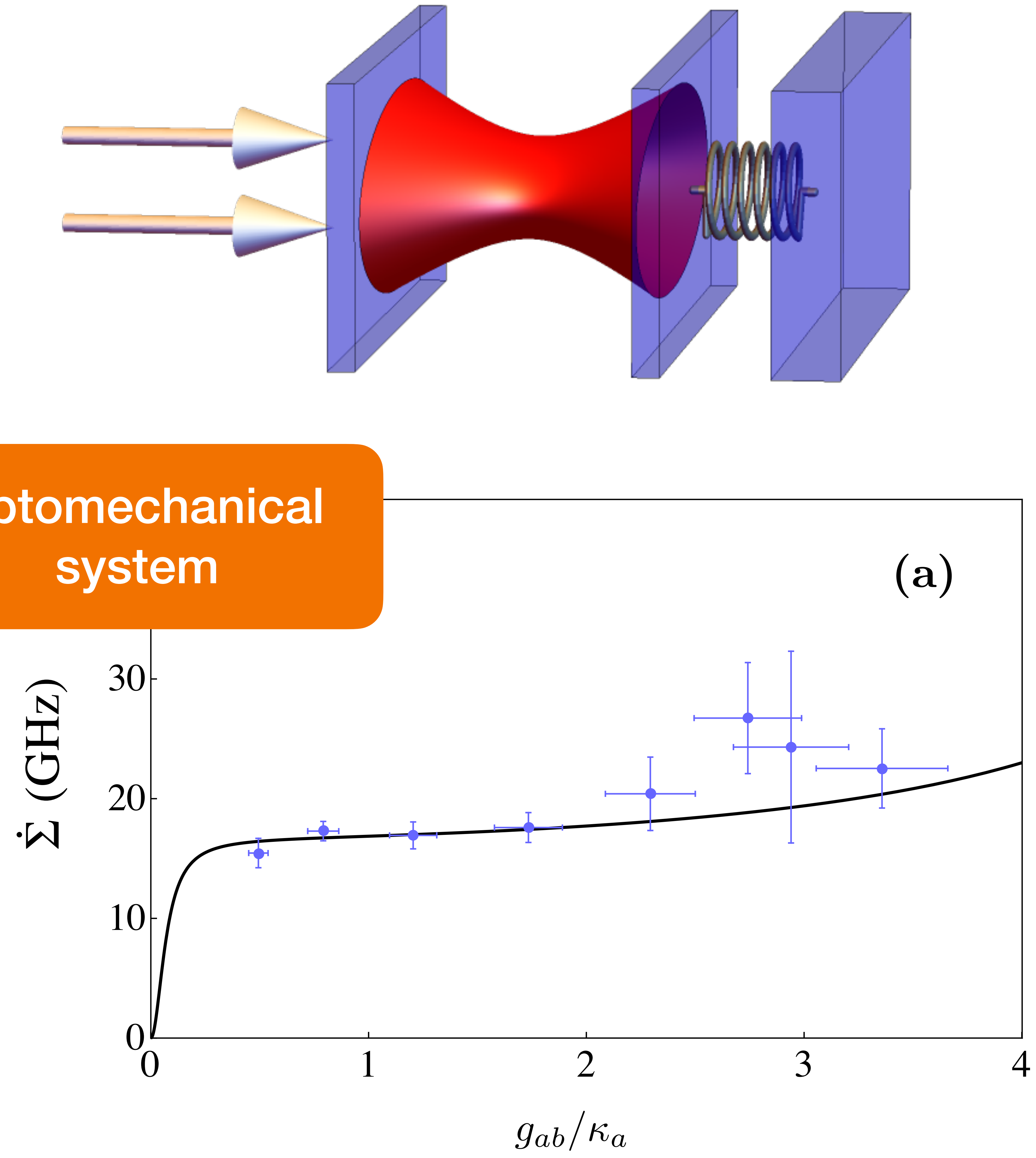
$$\dot{\Sigma} = \frac{\dot{Q}}{\omega(\bar{n} + 1/2)} \qquad \bar{n} = \frac{1}{e^{\beta\omega} - 1}$$

High temperatures: $\omega(\bar{n} + 1/2) \simeq T$.

Zero temperature: $\omega(\bar{n} + 1/2) = \omega/2$.

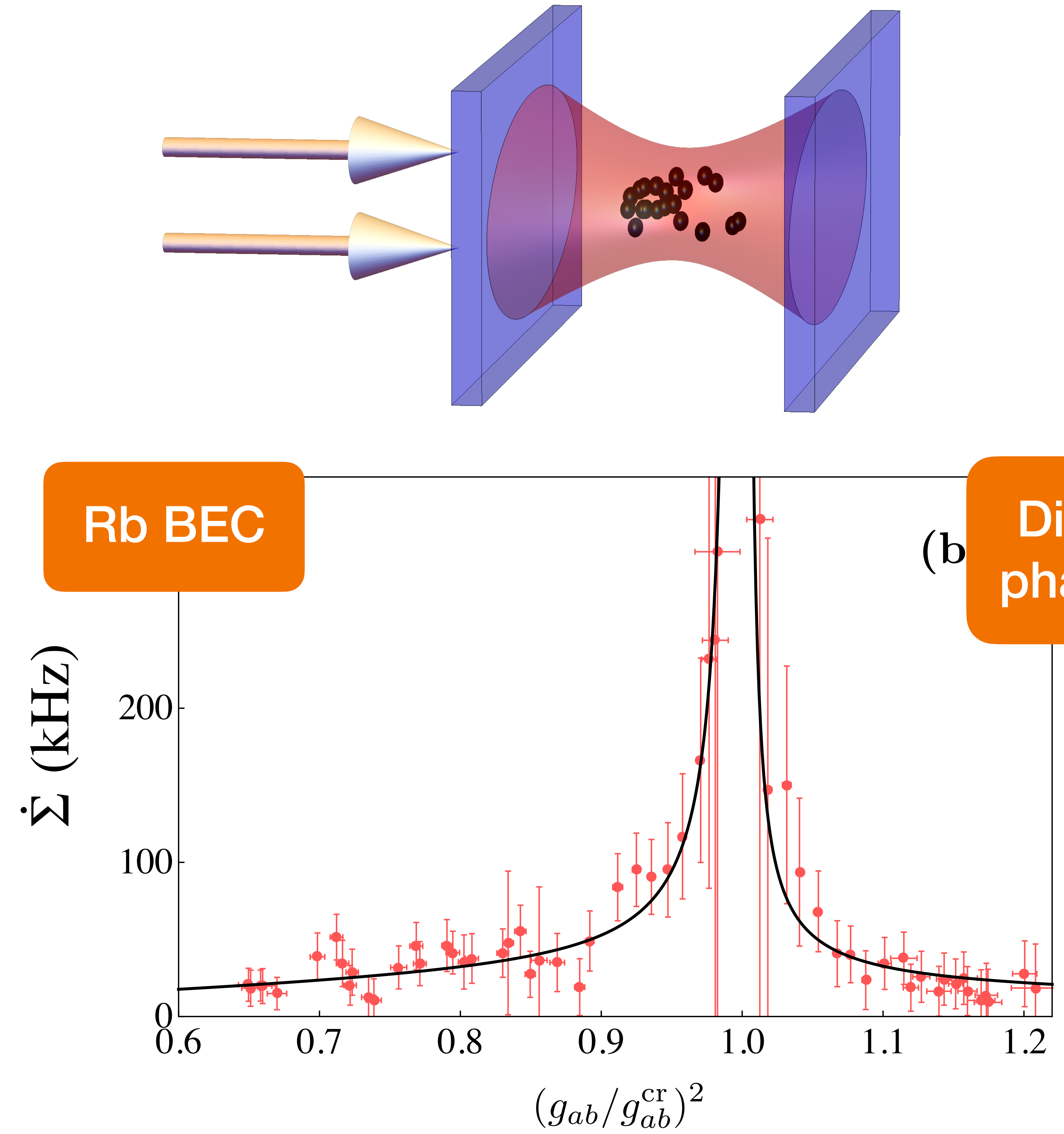
Jader P. Santos, G.T. Landi and Mauro Paternostro, Phys. Rev. Lett, **118**, 220601 (2017),

# Experiments




Optomechanical system

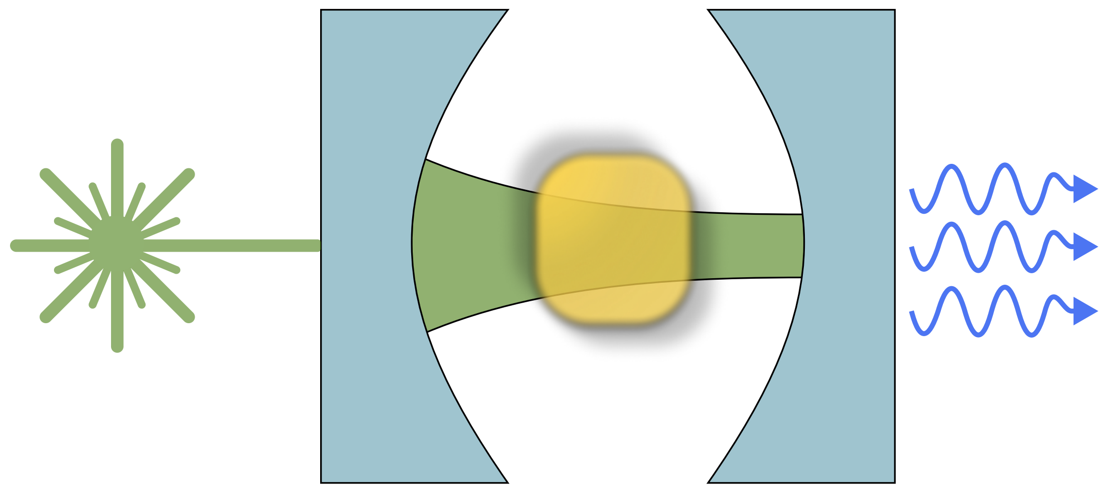
Rb BEC



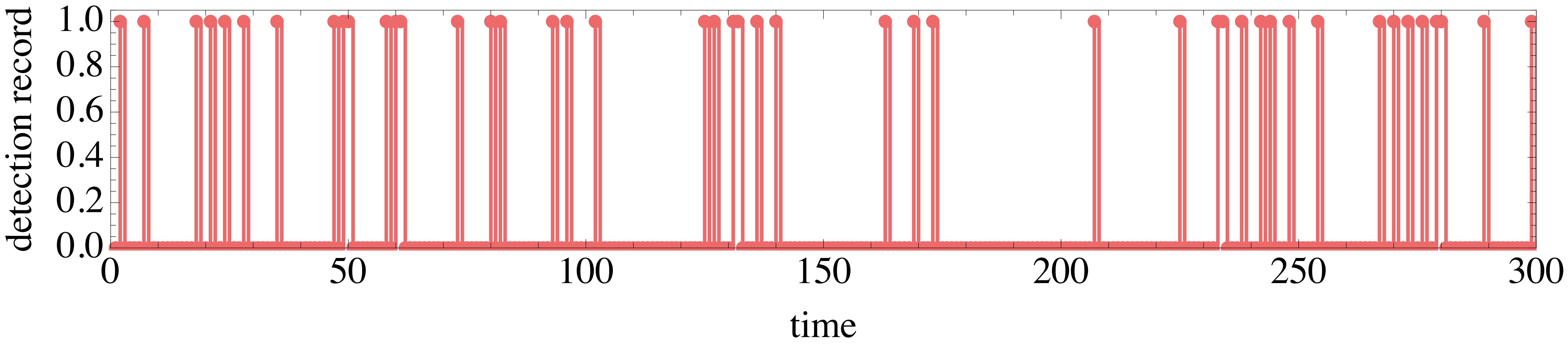

Dicke quantum phase transition.

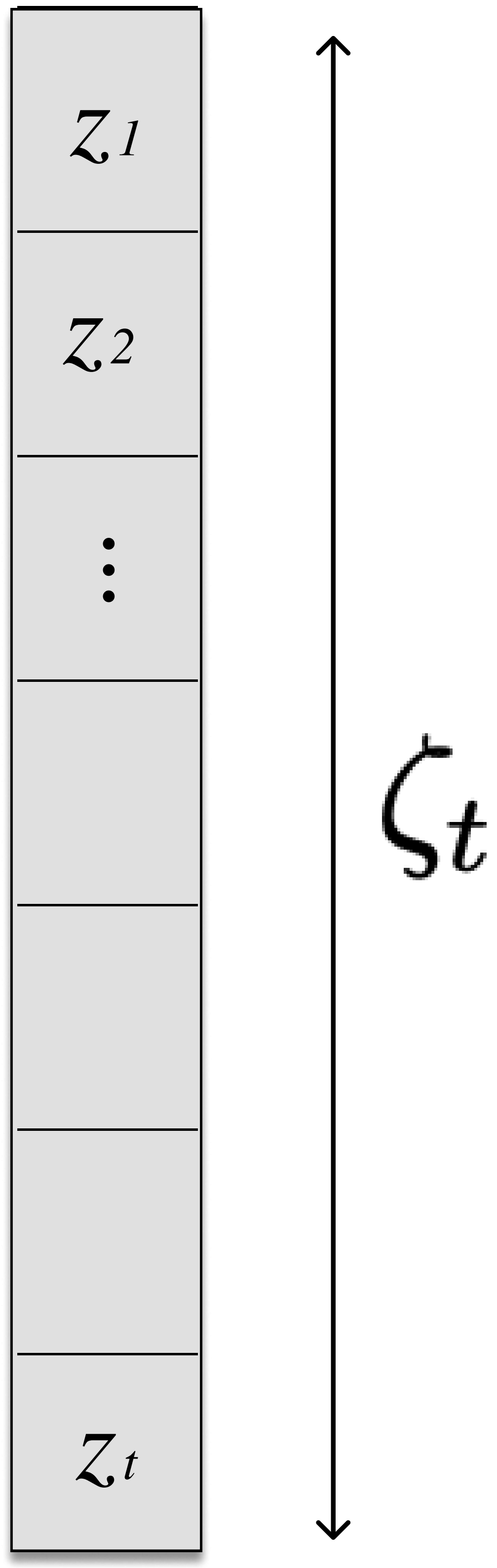
M. Brunelli, L. Fusco, R. Landig, W. Wieczorek, J. Hoelscher-Obermaier, GTL, F Semião, A. Ferraro, N. Kiesel, T. Donner, G. De Chiara, and M. Paternostro. *Phys. Rev. Lett.*, **121**, 160604 (2018)

# Continuously monitored quantum systems



- We can monitor the photons that leak out.
  - Individual clicks in the detector.
- Fundamental questions: what is entropy production *given* a detection record.
  - Operation: define thermodynamics in terms of what we can actually measure.
  - Includes *information* directly in the formulation.



# Holevo information

- **Unconditional:** If we do not know the individual clicks: $\rho_t$
- **Conditional on the detection record:** $\rho_{t|\zeta_t}$
- **Holevo information:** accumulated information we learned from the detection.

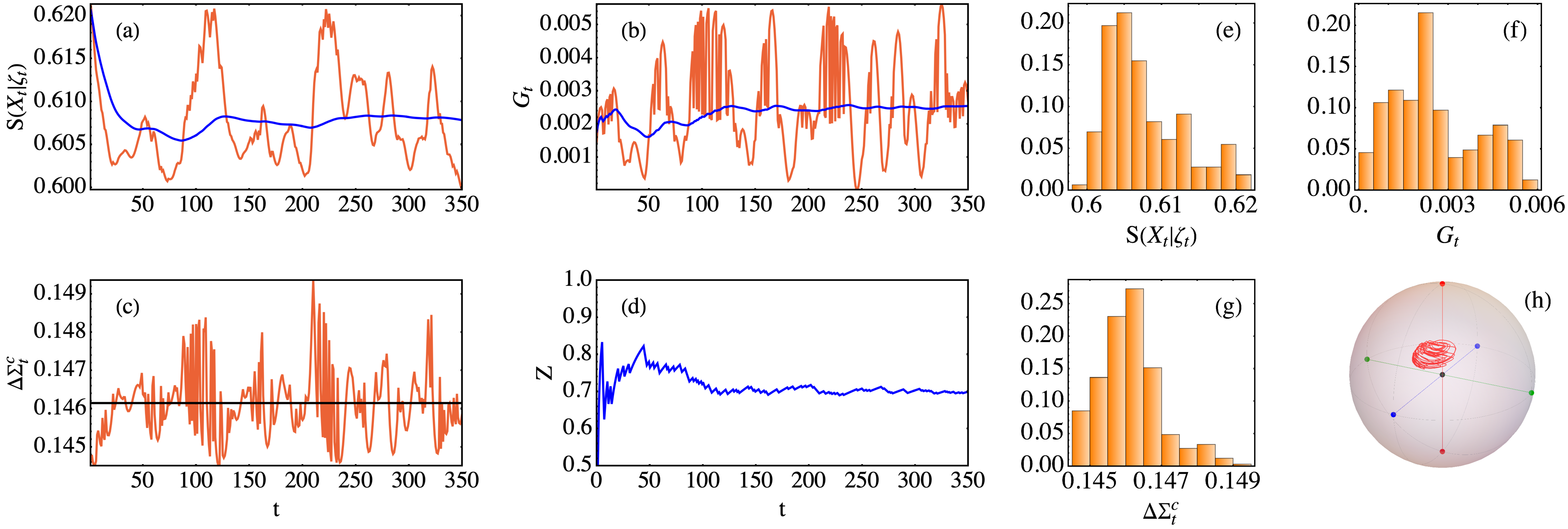
$$I(S_t : \zeta_t) = \sum_{\zeta_t} P(\zeta_t) D\left(\rho_{t|\zeta_t} || \rho_t\right)$$

- With each new detection

$$\Delta I_t = G_t - L_t = \text{gain} - \text{loss}$$

- Conditional entropy production

$$\Delta \Sigma^c = \Delta \Sigma^u - \Delta I$$

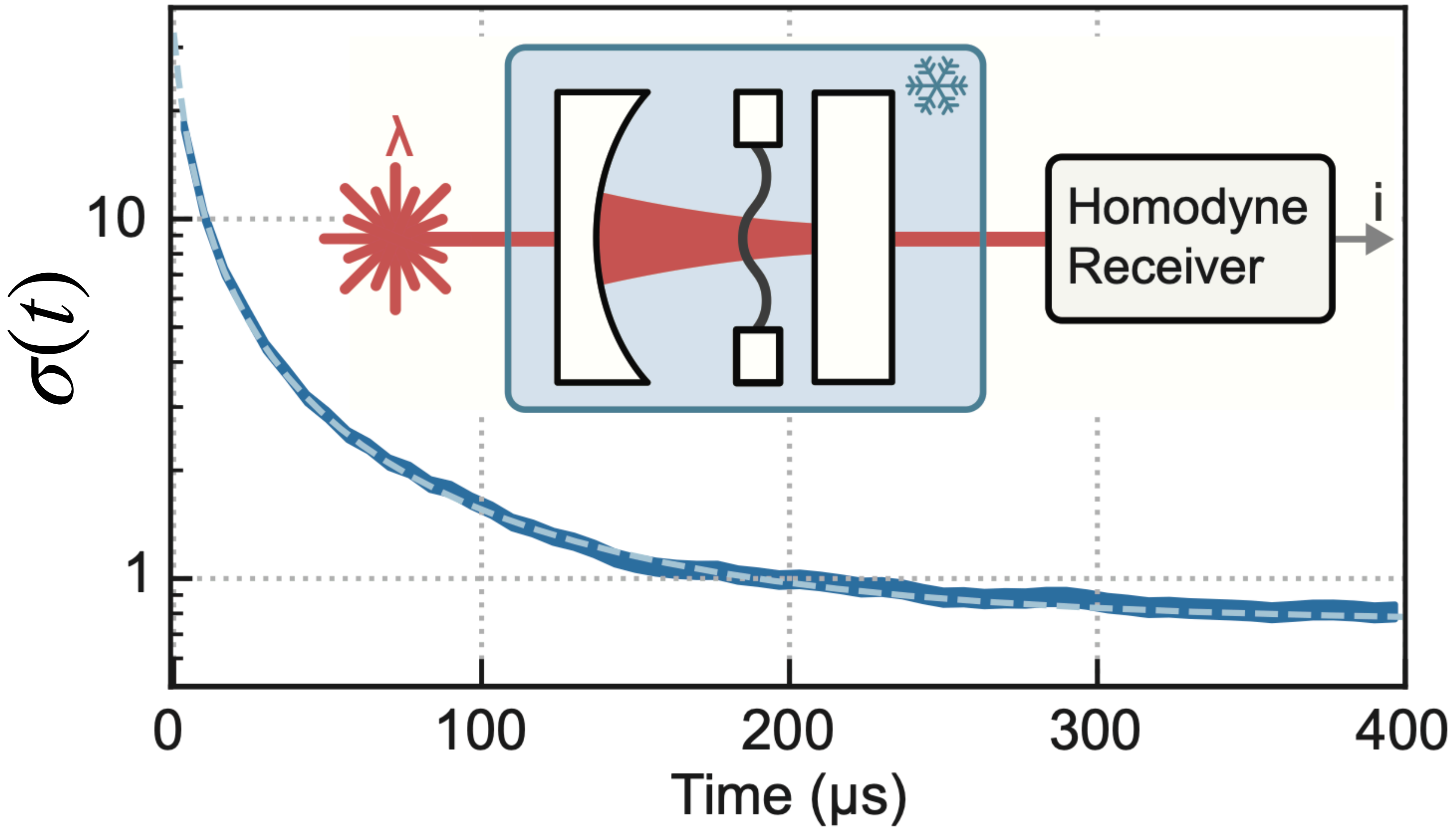


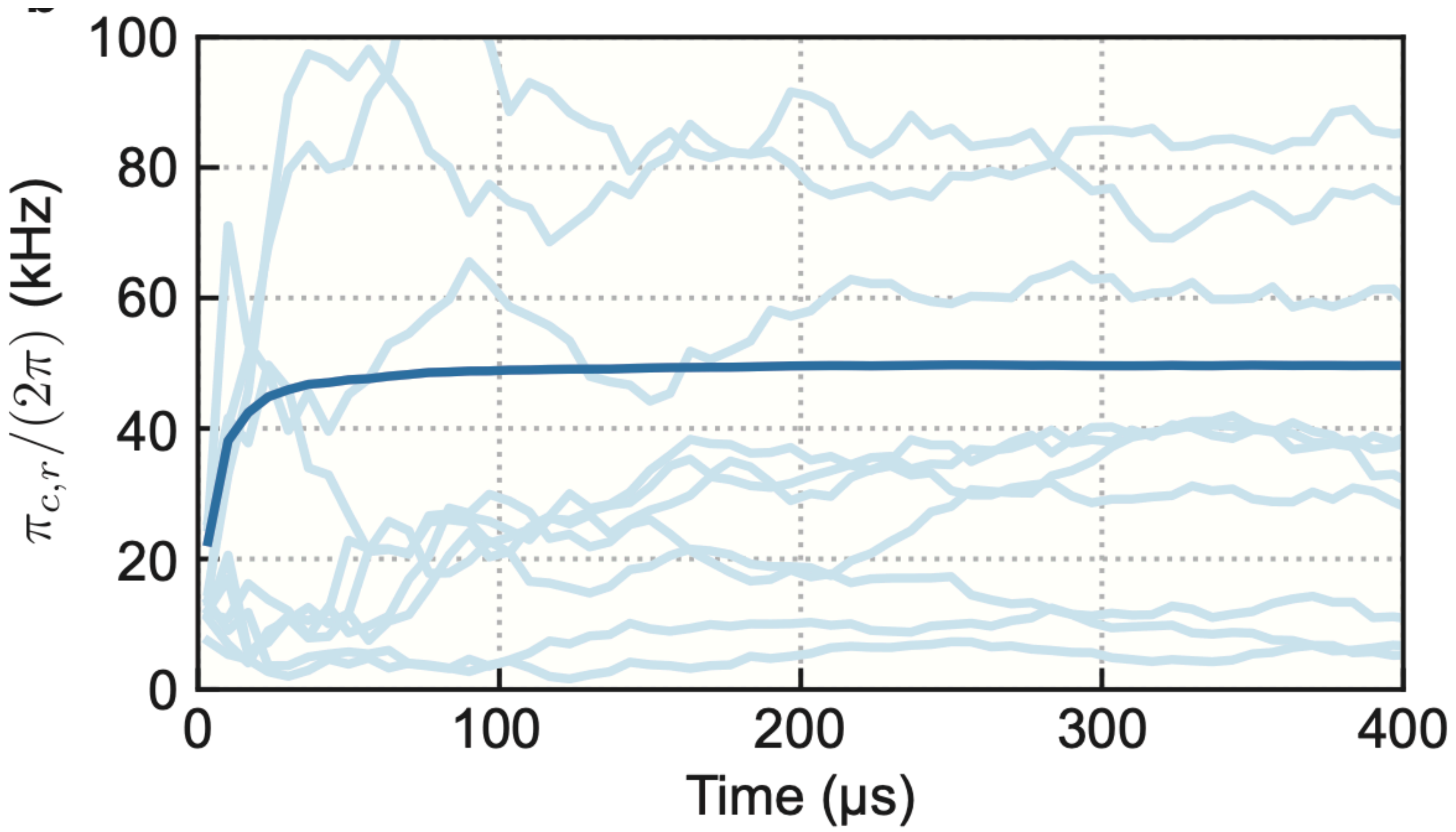

Alessio Belenchia, Luca Mancino, Gabriel T. Landi and Mauro Paternostro, "**Entropy production in continuously measured quantum systems**", npj Quantum Information, **6**, 97 (2020).

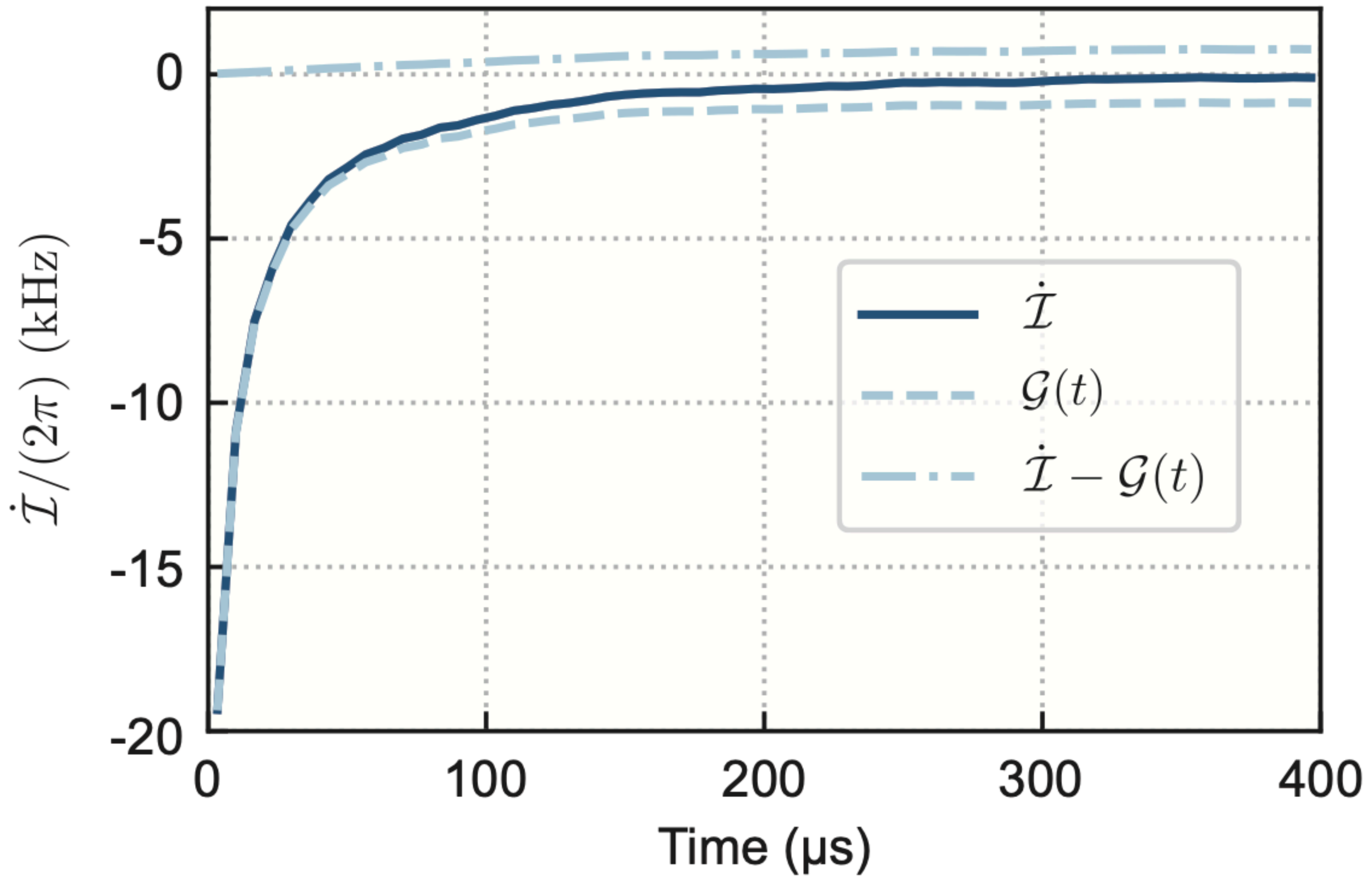
Gabriel T. Landi, Mauro Paternostro and Alessio Belenchia, "**Informational steady-states and conditional entropy production in continuously monitored systems**", PRX Quantum **3**, 010303, (2020).

# 🇩🇰 Copenhagen setup

- Optomechanical system continuously monitored by an
- Competition: Thermal bath vs. Measurement.
- Quadratures of the mechanical mode: $x = (q, p)$




**Informational steady-state:**

**Conditional dynamics relaxes to a colder state, which can only be maintained by continuously monitoring S.**

Massimiliano Rossi, Luca Mancino, Gabriel T. Landi, Mauro Paternostro, Albert Schliesser, Alessio Belenchia, "**Experimental assessment of entropy production in a continuously measured mechanical resonator**", Phys. Rev. Lett. **125**, 080601 (2020)

# Conclusions

- Information is a thermodynamic resource.
- How to incorporate information in the laws of thermodynamics is an open research question.
- I have focused on some recent developments, concerning the 2nd law (entropy production)
  - Quantum correlations.
  - Quantum coherence.
  - Non-Abelian charges.
  - Quantum phase space.
  - Continuous measurements.

Thank you!

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QT²
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