Non-Abelian Quantum Transport and Thermosqueezing Effects

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Overview

• Classical Onsager theory of transport
• Non-Abelian transport
• Collision models
• Linear response theory
• Application: Thermosqueezing
• Spin $S$ dynamics.

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Non-Abelian Quantum Transport and Thermosqueezing Effects

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Onsager theory

Entropy production rate

\[ \dot{\Sigma} = \sum_k \delta \lambda_k J_k = \sum_{k \ell} L_{k \ell} \delta \lambda_k \delta \lambda_{\ell} \quad \text{(fluxes } \times \text{ forces)} \]

Onsager's main results:
- $L$ is symmetric: Peltier & Seebeck are equal.
- $L$ is positive semi-definite: $\dot{\Sigma} \geq 0$

- Fluxes: $J_k = d\langle Q_k \rangle/dt$. Generated by gradients of affinities.
  \[ \delta \beta = \beta_L - \beta_R \quad \text{and} \quad -\delta \beta \mu = \beta_R \mu_R - \beta_L \mu_L \]

- Linear response: if the gradients are small
  \[
  \begin{pmatrix}
  J_E \\
  J_N
  \end{pmatrix} =
  \begin{pmatrix}
  L_{EE} & L_{EN} \\
  L_{NE} & L_{NN}
  \end{pmatrix}
  \begin{pmatrix}
  \delta \beta \\
  -\delta \beta \mu
  \end{pmatrix}
  \]

$L_{NN}$: Fick's law of diffusion
Particles flow due to gradient of concentration.

$L_{EE}$: Fourier's law
Heat flows due to gradient of temperature.

$L_{NE}$: Seebeck effect
Gradient of temperature generates a flow of particles/electrons.

$L_{EN}$: Peltier effect
Gradient of concentration generates heat flow.

Thermoelectric plates in our laptops.
Onsager theory in the quantum regime

Non-Abelian (non-commuting) charges

- In the quantum domain we can also have transport of charges that do not commute.

\[ \rho = \frac{1}{Z} \exp \left\{ - \sum_k \lambda_k Q_k \right\} \quad [Q_k, Q_{\ell}] \neq 0 \]

(non-Abelian thermal states - NATS)

- Ex: spin transport \( \rho = \frac{1}{Z} \exp \left\{ - \lambda_x \sigma_x - \lambda_y \sigma_y - \lambda_z \sigma_z \right\} \)

- Ex: Energy & radiation squeezing.

\[ H_{\text{Heis}} := \sum_{j < k} \frac{J_0}{3 |j - k|^\alpha} \left( \sigma_x^{(j)} \sigma_x^{(k)} + \sigma_y^{(j)} \sigma_y^{(k)} + \sigma_z^{(j)} \sigma_z^{(k)} \right) \]
Collision model approach

• We study non-Abelian transport in a collision model approach.

• Sequence of individual collisions between small ancillas of each bath.

• Two systems, A and B, each prepared in states

\[
\rho^x_{\lambda_k} = \frac{1}{Z_x} \exp\left\{ - \sum_k \lambda^x_k Q^x_k \right\}, \quad x = A, B
\]

with \( \lambda^A_k \neq \lambda^B_k \)

• Interaction map: \( \rho'_{AB} = U \left( \rho^A_{\lambda_A} \otimes \rho^B_{\lambda_B} \right) U^\dagger \)

When can we talk about transport?

- Transport means the thing leaving one system must equal that entering the other.

- Condition for strict charge conservation (SCC):

\[
[U, Q^A_k + Q^B_k] = 0, \quad \forall k
\]

- Define unique current operator

\[
\mathcal{J}_k = U^\dagger Q^{(A)}_k U - Q^{(A)}_k = -U^\dagger Q^{(B)}_k U + Q^{(B)}_k
\]

- Average current: \( J_k = \text{tr}\left( \mathcal{J}_k (\pi_A \otimes \pi_B) \right) \)
Entropy production

- Entropy production can be written in a fully information-theoretic way as

\[ \Sigma = I'(A:B) + D(\rho'_A \mid \mid \rho_A) + D(\rho'_B \mid \mid \rho_B) \geq 0 \]

- where

\[ I'(A : B) = S(\rho'_A) + S(\rho'_B) - S(\rho'_{AB}) \]

\[ D(\rho \mid \mid \sigma) = \text{tr}\left\{ \rho \ln \rho - \rho \ln \sigma \right\} \]

- Fully operational: irreversibility due to loss of AB correlations + irreversible local changes in A and B.

- NATS: entropy production reduces to Onsager’s result: \( \dot{\Sigma} = \sum_k \delta \lambda_k J_k \).


Gabriel T. Landi and Mauro Paternostro, “Irreversible entropy production, from quantum to classical”, Review of Modern Physics, 93, 035008 (2021)
Linear response theory
Symmetric logarithmic derivative

The proof of our result uses concepts from quantum parameter estimation.

We define the SLD for each charge/affinity pair:

\[ \Lambda_k \rho_\lambda + \rho_\lambda \Lambda_k = 2 \frac{d \rho_\lambda}{d \lambda_k} \]

For commuting charges \( \Lambda_k = \langle Q_k \rangle - Q_k \)

The Onsager matrix can then be written as

\[ L_{k\ell} = -\frac{1}{2} \langle \{ \mathcal{F}_k, \Lambda_\ell \} \rangle \]

Onsager reciprocity follows from time-reversal invariance.

Main result

If the charges \( Q_k \) and the dynamics are time-reversal invariant, then the Onsager matrix can be written as

\[ L_{k\ell} = \frac{1}{2} \int_0^1 dy \ \text{cov}_y (\mathcal{F}_k, \mathcal{F}_\ell) \]

where \( \mathcal{F}_k = U_k^+ Q_k^{(A)} U_k - Q_k^{(A)} \) and

\[ \text{cov}_y (A, B) = \text{tr}(A \rho^y B \rho^{1-y}) - \text{tr}(A \rho) \text{tr}(B \rho) \]

is the y-covariance, with \( \rho = \rho_A^A \otimes \rho_B^B \) being the equilibrium state.

For commuting charges we recover the Kubo formula

\[ L_{k\ell} = \text{cov}(\mathcal{F}_k, \mathcal{F}_\ell) \]
The entropy production can be written as

\[ \Sigma = \frac{1}{2} \int_0^1 dy \ \text{cov}_y(D, D), \quad D = \sum_k \delta \lambda_k \mathcal{J}_k \]

This can be further split as

\[ \Sigma = \Sigma_{\text{comm}} - I \]

where \( I \) is the Wigner-Yanase-Dyson skew information (a quantifier of coherence)

\[ I(\pi, D) = \frac{1}{2} \int_0^1 dy \ \text{tr} \left( [\pi^y, D][\pi^{1-y}, D] \right) \geq 0 \]

Reduction in the entropy production due to quantum coherence.


Note that \( D \) is the operator associated to the entropy production:

\[ \Sigma = \langle D \rangle_{AB} \]

In the commuting case, we would have the Fluctuation-Dissipation relation

\[ \langle D \rangle_{AB} = \frac{1}{2} \text{Var}(D)_{\text{eq}} \]

Non-commutativity breaks the FDR:

\[ \langle D \rangle_{AB} = \frac{1}{2} \text{Var}(D)_{\text{eq}} - I \]

is the \( y \)-covariance, with \( \rho = \rho^A_\lambda \otimes \rho^B_\lambda \) being the equilibrium state.

For commuting charges we recover the Kubo formula

\[ L_{k\ell} = \text{cov}(\mathcal{J}_k, \mathcal{J}_\ell) \]
Thermosqueezing
Thermal Squeezed states

- Single QHO:
  \[
  \rho = \frac{1}{Z} \exp\{-\beta H - \beta \mu A\}, \quad H = \frac{\omega}{2} (p^2 + x^2), \quad A = \frac{\omega}{2} (p^2 - x^2)
  \]

- Two charges, \( H \) (energy) and \( A \) (asymmetry).
Squeezed Thermal Reservoirs as a Resource for a Nanomechanical Engine beyond the Carnot Limit

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LETTER

Efficiency of heat engines coupled to nonequilibrium reservoirs

Obinna Abah and Eric Lutz
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Entropy production and thermodynamic power of the squeezed thermal reservoir

Gonzalo Manzano, Fernando Galve, Roberta Zambrini, and Juan M. R. Parrondo
Phys. Rev. E 93, 052120 – Published 10 May 2016

Squeezed Thermal Reservoirs as a Resource for a Nanomechanical Engine beyond the Carnot Limit

Jan Klærs, Stefan Faelt, Atac Imamoglu, and Emre Togan
Phys. Rev. X 7, 031044 – Published 13 September 2017
Thermosqueezing

• Single QHO:

\[ \rho = \frac{1}{Z} \exp\{-\beta H - \beta \mu A\}, \quad H = \frac{\omega}{2}(p^2 + x^2), \quad A = \frac{\omega}{2}(p^2 - x^2) \]

• Two charges, \( H \) (energy) and \( A \) (asymmetry).

• Onsager coefficients:

  - Peltier: gradient of squeezing generates a flow of heat.
  - Seebeck: gradient of temperature generates a flow of asymmetry.

**Charge preserving Gaussian unitary**

Unitary which preserves both energy and squeezing:

\[ U = \exp\{-g\tau(a_1^\dagger a_2 - a_2^\dagger a_1)\} \]

Actually the only one which is also Gaussian (quadratic).

**SU(1,1) algebra**

3 charges

\[ Q_1 = H = \frac{\omega}{2}(p^2 + x^2) \quad Q_2 = A = \frac{\omega}{2}(p^2 - x^2) \quad Q_3 = \frac{\omega}{2}\{x, p\} \]

The charges \( Q_1, Q_2, Q_3 \) form a non-Abelian group:

\[ [Q_1, Q_2] = 2iQ_3 \]

\[ [Q_3, Q_1] = 2iQ_2 \]

\[ [Q_2, Q_3] = -2iQ_1 \]
Transport coefficients

Thermal conductance: $\kappa = -\beta^2 L_{QQ}$

Squeezing conductance: $G = -\beta L_{AA}$

Entropy production/dissipated heat reads

$\dot{Q}_{\text{diss}} = \Sigma/\beta = \kappa \delta T^2/T + J_A G$

New Joule-like heating term due to squeezing.

Onsager matrix

Unitary which preserves both heat ($J_Q = J_H - \mu J_A$) and squeezing:

\[
J_Q = L_{QQ} \delta_\beta - L_{QA} \beta \delta_\mu \\
J_A = L_{AQ} \delta_\beta - L_{AA} \beta \delta_\mu
\]

with

\[
L_{QQ} = f_\tau (1 - \mu^2) \bar{n}(\bar{n} + 1) \\
L_{QA} = L_{AQ} = f_\tau \mu \bar{n}(\bar{n} + 1) \\
L_{AA} = f_\tau (1 - \mu^2)^{-1} \left[ \mu \bar{n}(\bar{n} + 1) + \frac{\tanh \alpha}{\alpha} (\bar{n}^2 + \bar{n}/2 + 1/2) \right]
\]

where

\[
\bar{n} = (e^{\beta \omega} - 1)^{-1}, \quad f_\tau = \omega^2 \sin^2(\varpi \tau), \quad \alpha = \beta \omega \sqrt{1 - \mu^2}
\]
FIG. 2. (a)–(c) Thermosqueezing Onsager coefficients $L_{11}, L_{12}, L_{22}$ on the log scale, computed from Eqs. (19), in units of $(\hbar\omega)^2 \sin^2(g\tau)$, as a function of the inverse temperature $\beta$ (in units of $\hbar\omega/k_B$) and the adimensional squeezing parameter $r$. 
Entropy reduction

Recall that

\[ \Sigma = \frac{1}{2} \text{var}(D) - \frac{1}{2} \int_{0}^{1} dy \ I_y(\pi, D) \]

Define the entropy reduction due to non-commutativity

\[ \mathcal{R} = \frac{1}{2\Sigma} \int_{0}^{1} dy I_y(\pi, D) \]

Classical case corresponds to \( \mathcal{R} = 0 \).
Cross coefficients

Thermopower, or Squeezing-Seebeck (*Squeebeck*) coefficient

\[ S = \frac{1}{T} \frac{L_{AQ}}{L_{AA}} \]

(flow of squeezing due to gradient of temperature)

Squeezing-Peltier (Squeetier (?)) coefficient:

\[ \Pi = \frac{L_{QA}}{L_{AA}} \]

(flow of heat due to gradient in squeezing)

The two are related by \( \Pi = TS \)
Spin $S$ Heisenberg dynamics
• Spin $S$ operators:

\[ S_z | m \rangle = m | m \rangle, \quad m = S, S-1, \ldots, -S \]

• Two spins in NATS:

\[ \rho_{\lambda A} = \frac{1}{Z} \exp\left\{ -\lambda A x S_A x -\lambda A z S_A z \right\} \]

• Interact with Heisenberg unitary:

\[ U = \exp\left\{ -it S_A \cdot S_B \right\} = \exp\left\{ -it \left( S_A x S_B x + S_A y S_B y + S_A z S_B z \right) \right\} \]
How do we actually measure these currents?

- Thermodynamics deals with *transformations*.
  - Require **two-point measurements (TPM)**
- Measurements in quantum mechanics are invasive.
  - *First measurement is the problem:*
    - Destroys initial quantum coherences.
    - (Can be overcome using identical copies)
  - **Next step:** operational definition based on specific experimental platforms.
Conclusions & outlook

• Quantum mechanics opens up the way for performing transport of non-commuting charges.

• We put forth a framework suitable for describing this in the linear response regime.

Perspectives:

• If the charges do not commute, how can we actually measure them?

• Current fluctuations and Thermodynamic Uncertainty Relations.

• Concrete applications of thermosqueezing.

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Extra slides