Non-Abelian Quantum Transport and Thermosqueezing Effects

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Overview

- Classical Onsager theory of transport
- Non-Abelian transport
- Collision models
- Linear response theory
- Application: Thermosqueezing
- Spin S dynamics.

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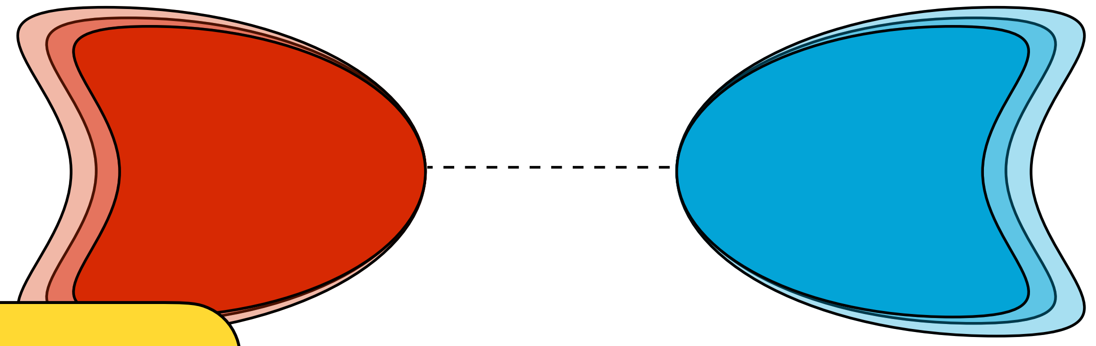
Gonzalo Manzano,[1,2,*] Juan M.R. Parrondo,[3] and Gabriel T. Landi[4]

Onsager theory



Entropy production rate

$$\dot{\Sigma} = \sum_k \delta\lambda_k J_k = \sum_{k\ell} L_{k\ell}\delta\lambda_k\delta\lambda_\ell \quad \text{(fluxes} \times \text{forces)}$$

Onsager's main results:

- L is symmetric: Peltier & Seebeck are equal.
- L is positive semi-definite: $\dot{\Sigma} \geqslant 0$

L_{NN}: Fick's law of diffusion

Particles flow due to gradient of concentration.

L_{EE}: Fourier's law

Heat flows due to gradient of temperature.

L_{NE}: Seebeck effect

Gradient of temperature generates a flow of particles/electrons.

Thermocouples

L_{EN}: Peltier effect

Gradient of concentration generates heat flow

- Fluxes: $J_k = d\langle Q_k \rangle / dt$. Generated by gradients of af

$\delta_\beta = \beta_L - \beta_R$ and $-\delta$ ... $\beta_R \mu$

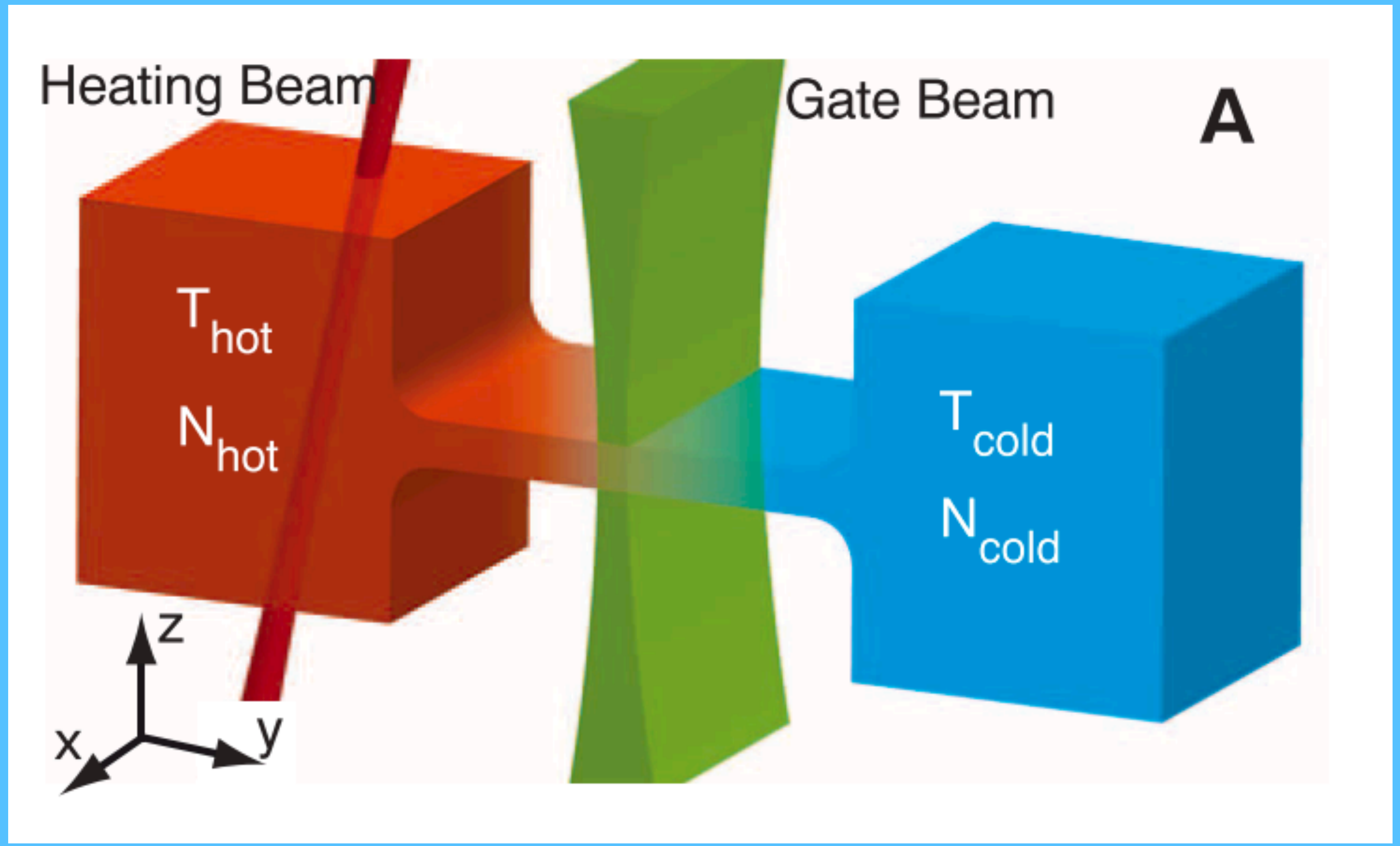
- **Linear response:** if the gradie

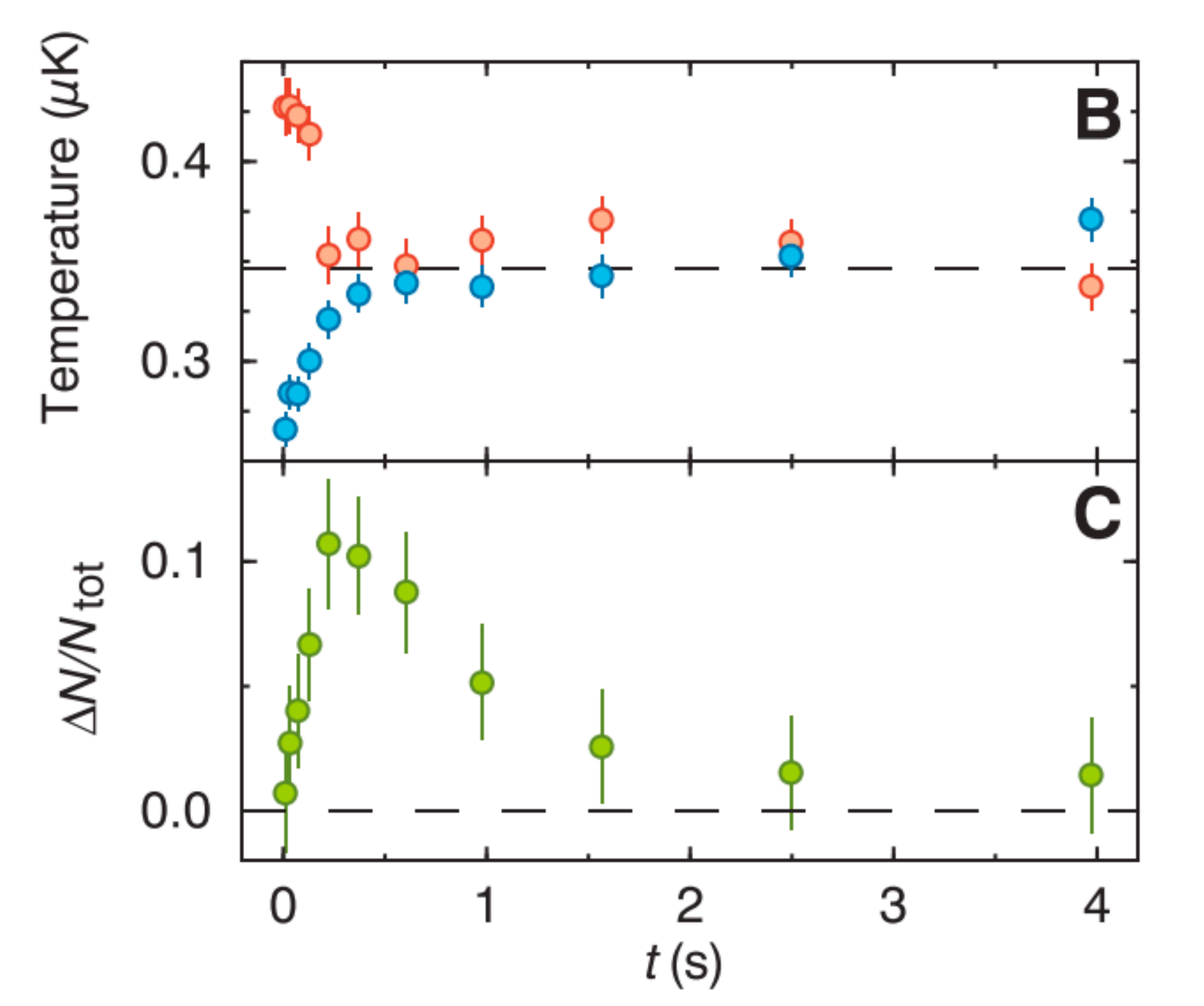
Thermoelectric plates in our laptops.

$$\begin{pmatrix} J_E \\ J_N \end{pmatrix} = \begin{pmatrix} L_{EE} & L_{EN} \\ L_{NE} & L_{NN} \end{pmatrix} \begin{pmatrix} \delta_\beta \\ -\delta_{\beta\mu} \end{pmatrix}$$

Onsager theory in the quantum regime

J. Brantut, et. al. **“A thermoelectric heat engine with ultracold atoms”**, Science 342, 6159 (2013)





Non-Abelian (non-commuting) charges

- In the quantum domain we can also have transport of charges that do not commute.

$$\rho = \frac{1}{Z}\exp\Big\{-\sum_k \lambda_k Q_k\Big\} \qquad [Q_k, Q_\ell] \neq 0$$

(non-Abelian thermal states - NATS)

- Ex: spin transport $\rho = \frac{1}{Z}\exp\Big\{-\lambda_x\sigma_x - \lambda_y\sigma_y - \lambda_z\sigma_z\Big\}$
- Ex: Energy & radiation squeezing.

ARTICLE

Received 22 Dec 2015 | Accepted 23 May 2016 | Published 7 Jul 2016 DOI: 10.1038/ncomms12051 OPEN

Microcanonical and resource-theoretic derivations of the thermal state of a quantum system with noncommuting charges

Nicole Yunger Halpern[1], Philippe Faist[2], Jonathan Oppenheim[3] & Andreas Winter[4,5]

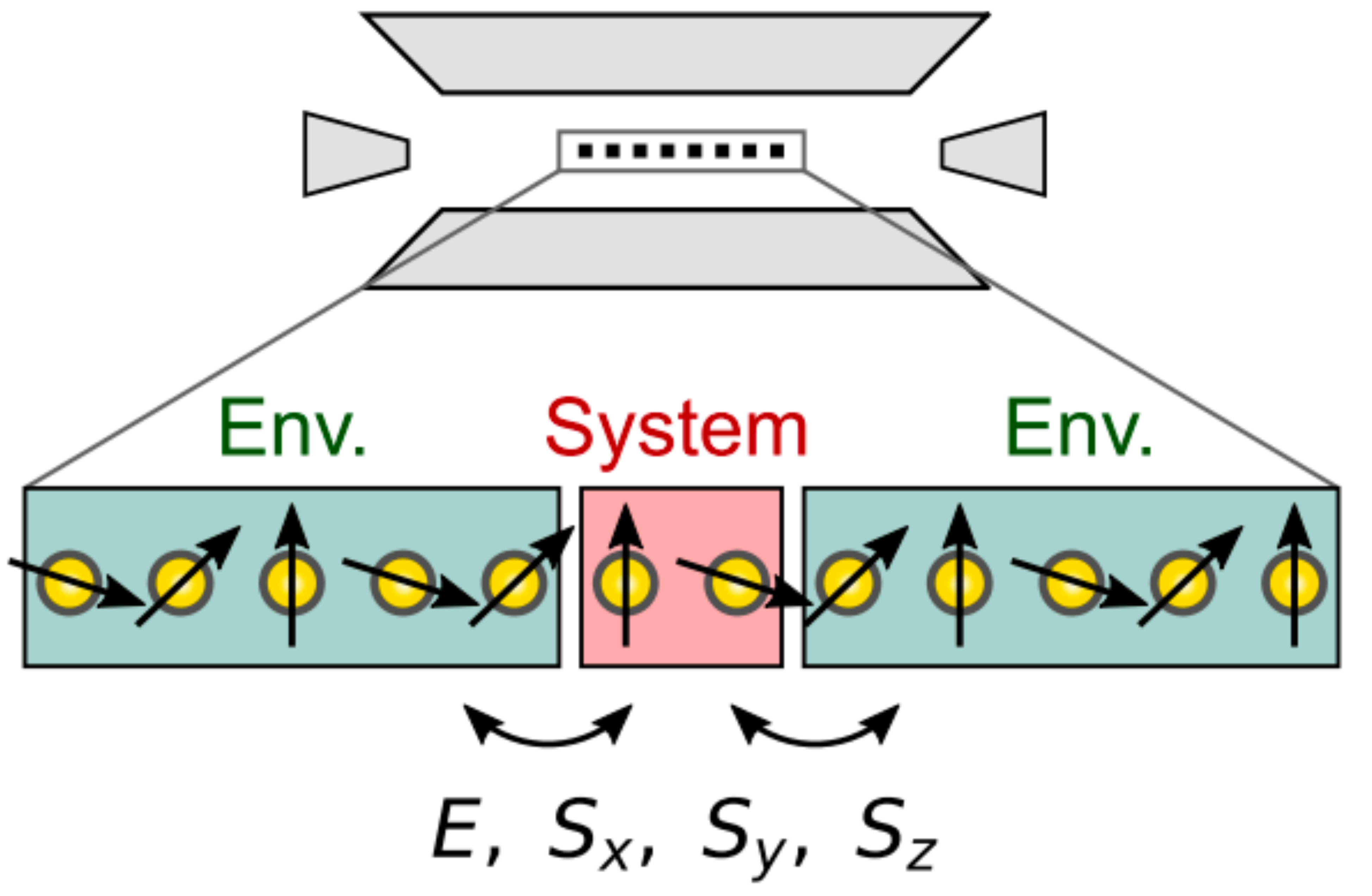
ARTICLE

Received 22 Dec 2015 | Accepted 23 May 2016 | Published 7 Jul 2016 DOI: 10.1038/ncomms12049 OPEN

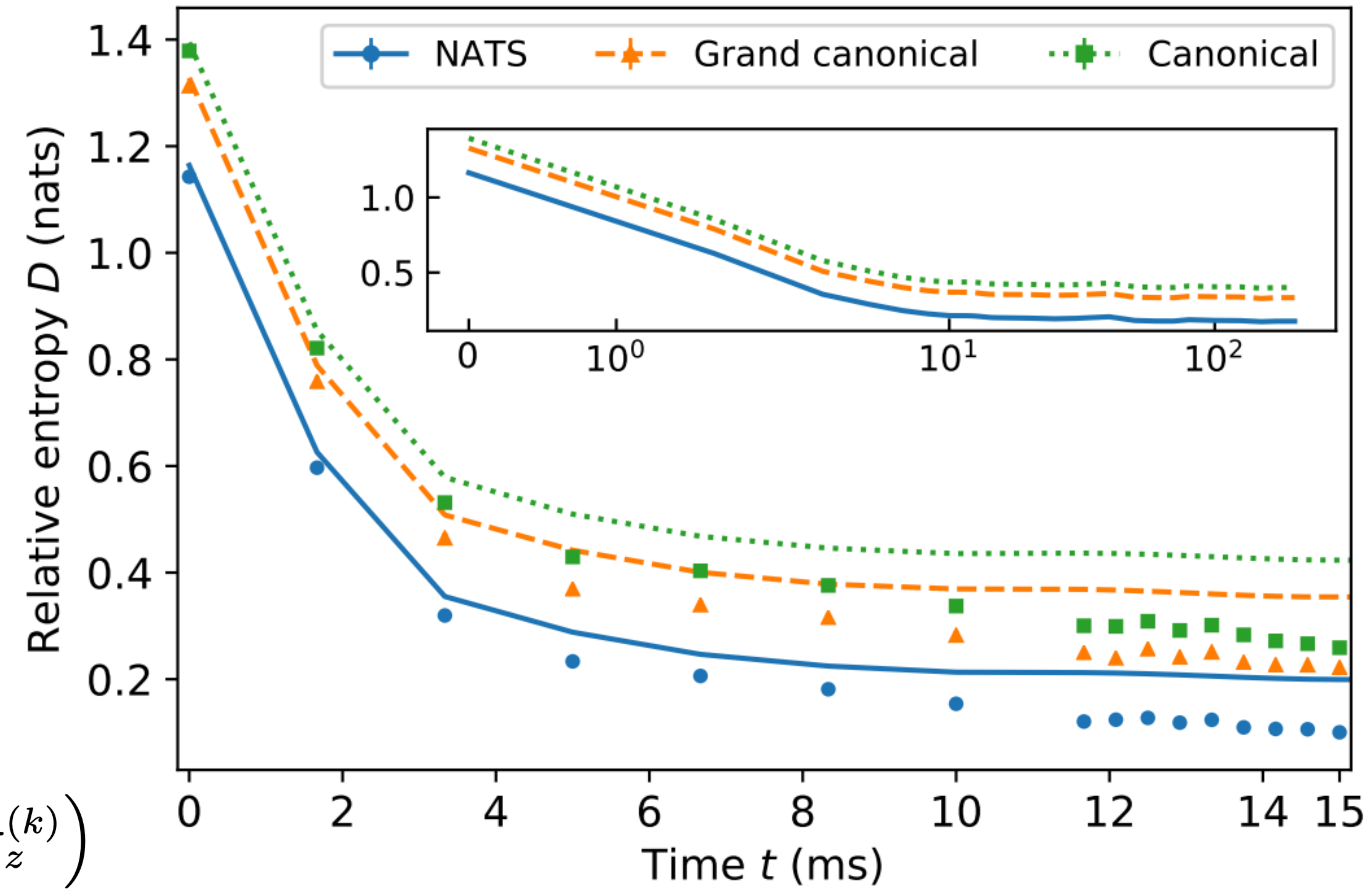
Thermodynamics of quantum systems with multiple conserved quantities

Yelena Guryanova[1], Sandu Popescu[1], Anthony J. Short[1], Ralph Silva[1,2] & Paul Skrzypczyk[1]

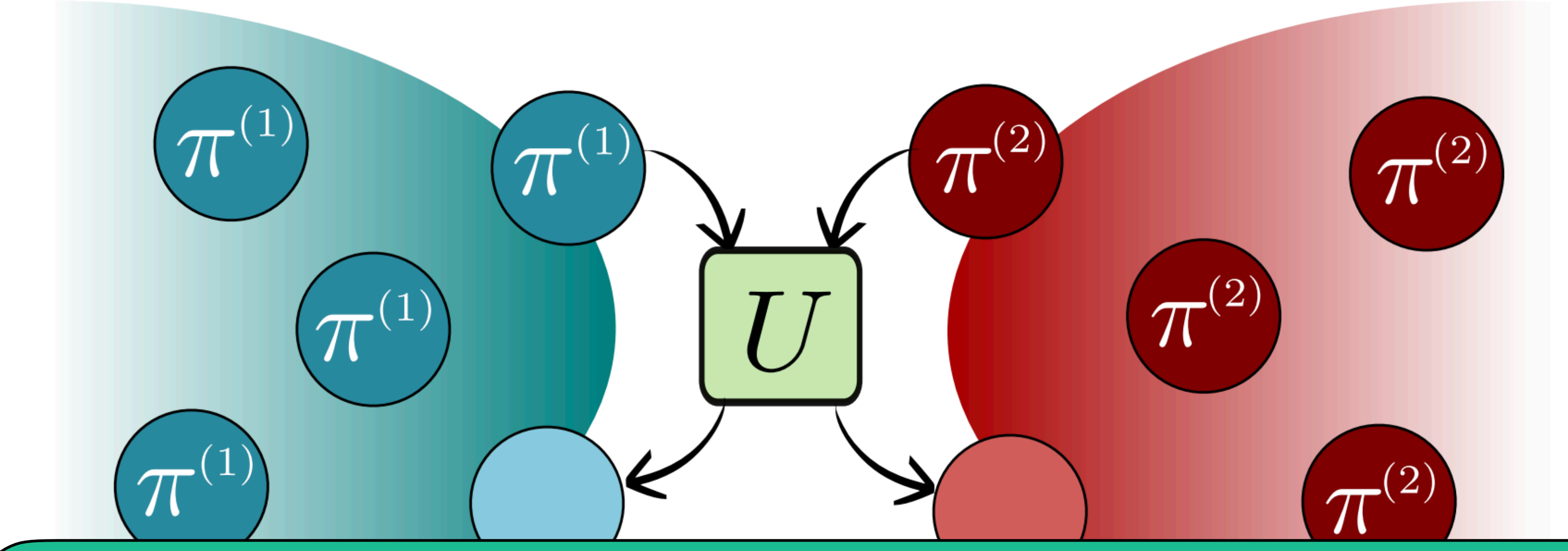
F. Kranzl, et. al. "Experimental observation of thermalisation with noncommuting charges", ArXiv 2202.04652.



$$H_{\text{Heis}} := \sum_{j<k} \frac{J_0}{3\,|j-k|^{\alpha}} \left(\sigma_x^{(j)} \sigma_x^{(k)} + \sigma_y^{(j)} \sigma_y^{(k)} + \sigma_z^{(j)} \sigma_z^{(k)} \right)$$



Collision model approach



- We study non-Abelian transport in a collision model approach.
  - Sequence of individual collisions between small ancillas of each bath.
- Two systems, A and B, each prepared in states

$$\rho^x_{\lambda_x} = \frac{1}{Z_x} \exp\Big\{ -\sum_k \lambda^x_k Q^x_k \Big\}, \qquad x = A, B$$

with $\lambda^A_k \neq \lambda^k_B$

- Interaction map: $\rho'_{AB} = U\left(\rho^A_{\lambda_A} \otimes \rho^B_{\lambda_B}\right)U^\dagger$

When can we talk about transport?

- Transport means the thing leaving one system must equal that entering the other.
- Condition for strict charge conservation (SCC):

$$[U, Q^A_k + Q^B_k] = 0, \qquad \forall k$$

- Define *unique* current operator

$$\mathcal{J}_k = U^\dagger Q^{(A)}_k U - Q^{(A)}_k$$
$$= -U^\dagger Q^{(B)}_k U + Q^{(B)}_k$$

- Average current: $J_k = \mathrm{tr}\Big(\mathcal{J}_k(\pi_A \otimes \pi_B)\Big)$

Entropy production

- Entropy production can be written in a fully information-theoretic way as

$$\Sigma = I'(A{:}B) + D(\rho'_A || \rho_A) + D(\rho'_B || \rho_B) \geqslant 0$$

- where

$$I'(A:B) = S(\rho'_A) + S(\rho'_B) - S(\rho'_{AB})$$

$$D(\rho || \sigma) = \text{tr}\Big\{\rho \ln \rho - \rho \ln \sigma\Big\}$$

- Fully operational: irreversibility due to loss of AB correlations + irreversible local changes in A and B.

- **NATS:** entropy production reduces to Onsager's result: $\dot{\Sigma} = \sum_k \delta\lambda_k J_k$.

M. Esposito, K. Lindenberg, C. Van den Broeck, "**Entropy production as correlation between system and reservoir**". *New Journal of Physics*, **12**, 013013 (2010).

Gabriel T. Landi and Mauro Paternostro, "**Irreversible entropy production, from quantum to classical**", *Review of Modern Physics*, **93**, 035008 (2021)

Linear response theory

Symmetric logarithmic derivative

The proof of our result uses concepts from quantum parameter estimation.

We define the SLD for each charge/affinity pair:

$$\Lambda_k \rho_\lambda + \rho_\lambda \Lambda_k = 2\frac{\partial \rho_\lambda}{\partial \lambda_k}$$

For commuting charges $\Lambda_k = \langle Q_k \rangle - Q_k$

The Onsager matrix can then be written as

$$L_{k\ell} = -\frac{1}{2}\langle\{\mathcal{J}_k, \Lambda_\ell\}\rangle$$

Onsager reciprocity follows from time-reversal invariance.

Main result

If the charges Q_k and the dynamics are time-reversal invariant, then the Onsager matrix can be written as

$$L_{k\ell} = \frac{1}{2}\int_0^1 dy \, \mathrm{cov}_y\left(\mathcal{J}_k, \mathcal{J}_\ell\right)$$

where $\mathcal{J}_k = U^\dagger Q_k^{(A)} U - Q_k^{(A)}$ and

$$\mathrm{cov}_y(A, B) = \mathrm{tr}\left(A\rho^y B\rho^{1-y}\right) - \mathrm{tr}(A\rho)\mathrm{tr}(B\rho)$$

is the y-covariance, with $\rho = \rho_\lambda^A \otimes \rho_\lambda^B$ being the equilibrium state.

For commuting charges we recover the *Kubo formula*

$$L_{k\ell} = \mathrm{cov}(\mathcal{J}_k, \mathcal{J}_\ell)$$

Consequence

The entropy production can be written as

$$\Sigma = \frac{1}{2}\int_0^1 dy\ \mathrm{cov}_y(D,D), \qquad D = \sum_k \delta\lambda_k \mathcal{J}_k$$

This can be further split as

$$\Sigma = \Sigma_{\mathrm{comm}} - I$$

where I is the Wigner-Yanase-Dyson skew information (a quantifier of coherence)

$$I(\pi, D) = \frac{1}{2}\int_0^1 dy\ \mathrm{tr}\big([\pi^y, D][\pi^{1-y}, D]\big) \geqslant 0$$

Reduction in the entropy production due to quantum coherence.

D. Petz, "**Covariance and Fisher information in quantum mechanics**" *J. Phys. A.*, **35**, 929–939 (2002)

Note that D is the operator associated to the entropy production:

$$\Sigma = \langle D \rangle_{\mathrm{AB}}$$

In the commuting case, we would have the Fluctuation-Dissipation relation

$$\langle D \rangle_{AB} = \frac{1}{2}\mathrm{Var}(D)_{\mathrm{eq}}$$

Non-commutativity breaks the FDR:

$$\langle D \rangle_{AB} = \frac{1}{2}\mathrm{Var}(D)_{\mathrm{eq}} - I$$

is the y-covariance, with $\rho = \rho_\lambda^A \otimes \rho_\lambda^B$ being the equilibrium state.

For commuting charges we recover the *Kubo formula*

$$L_{k\ell} = \mathrm{cov}(\mathcal{J}_k, \mathcal{J}_\ell)$$

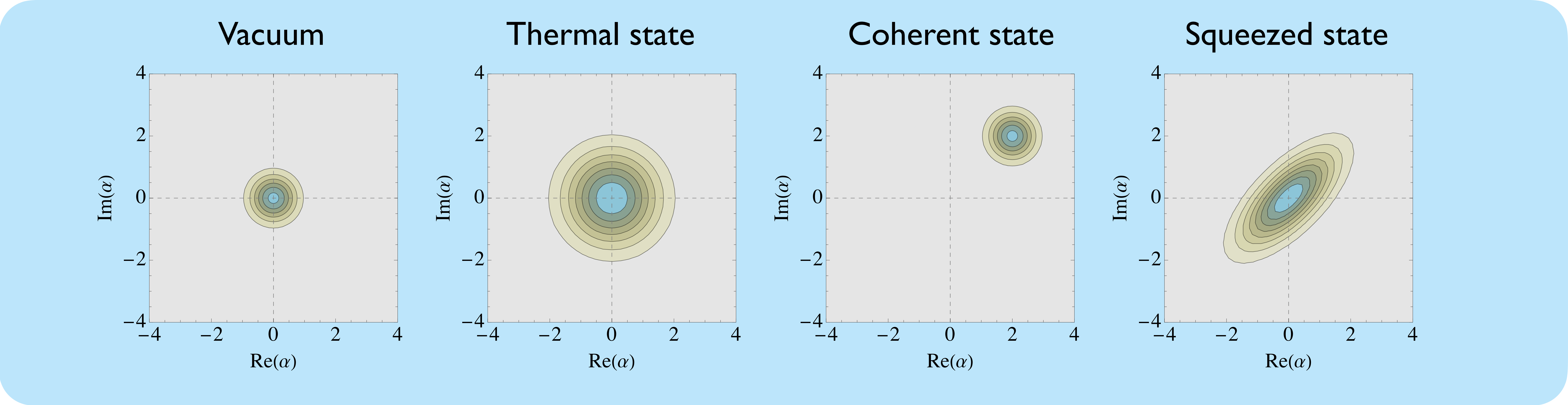
Thermosqueezing

Thermal Squeezed states

- Single QHO:

$$\rho = \frac{1}{Z}\exp\{-\beta H - \beta\mu A\}, \qquad H = \frac{\omega}{2}(p^2 + x^2), \qquad A = \frac{\omega}{2}(p^2 - x^2)$$

- Two charges, H (energy) and A (asymmetry).



Selected for a Viewpoint in *Physics*
PHYSICAL REVIEW X **7,** 031044 (2017)

Squeezed Thermal Reservoirs as a Resource for a Nanomechanical Engine beyond the Carnot Limit

Jan Klaers,* Stefan Faelt, Atac Imamoglu, and Emre Togan
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(Received 25 April 2017; revised manuscript received 25 July 2017; published 13 September 2017)

LETTER

Efficiency of heat engines coupled to nonequilibrium reservoirs

Obinna Abah[1] and Eric Lutz[1]

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EPL (Europhysics Letters), Volume 106, Number 2
Citation Obinna Abah and Eric Lutz 2014 *EPL* **106** 20001

Entropy production and thermodynamic power of the squeezed thermal reservoir

Gonzalo Manzano, Fernando Galve, Roberta Zambrini, and Juan M. R. Parrondo
Phys. Rev. E **93**, 052120 – Published 10 May 2016

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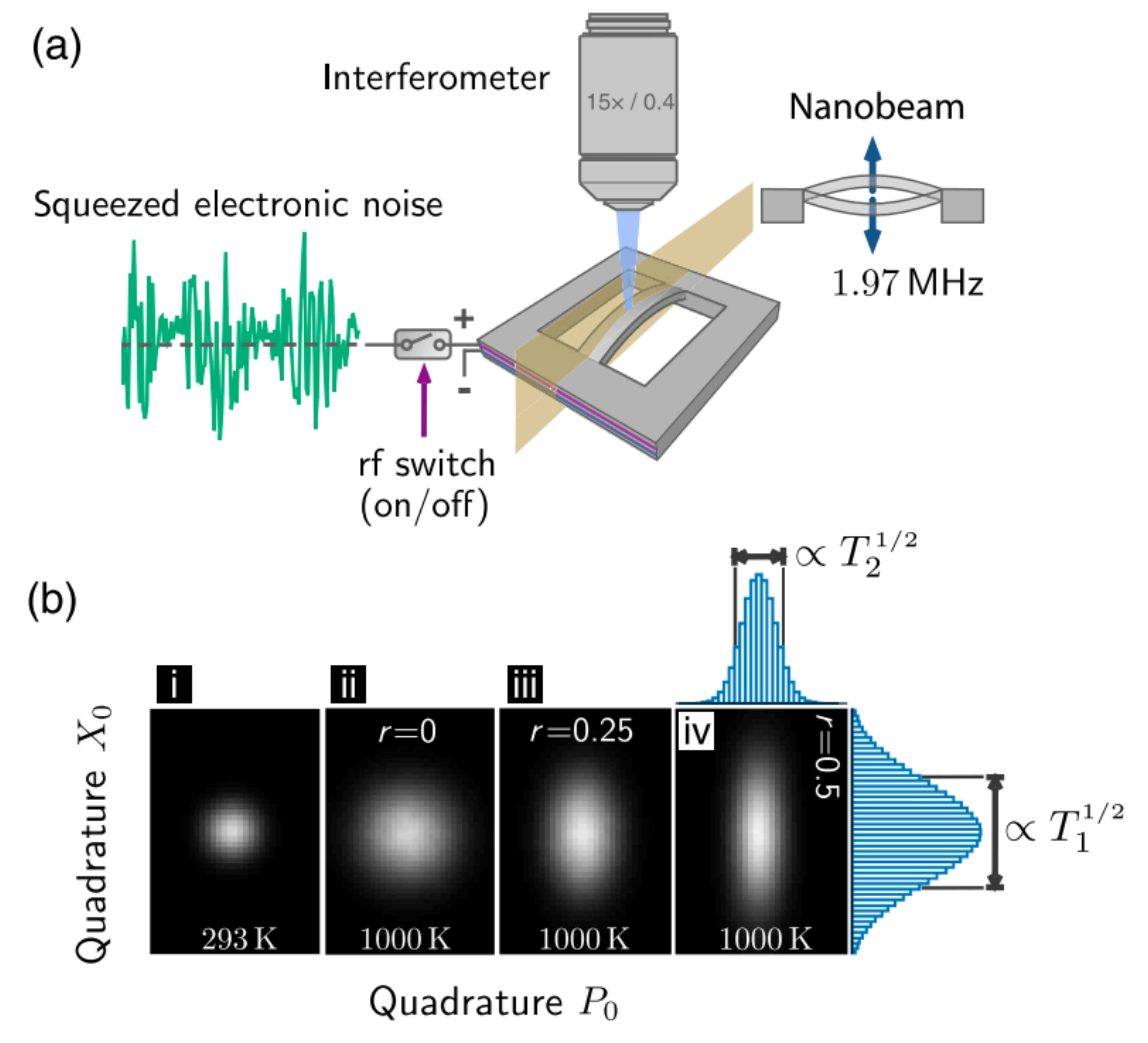
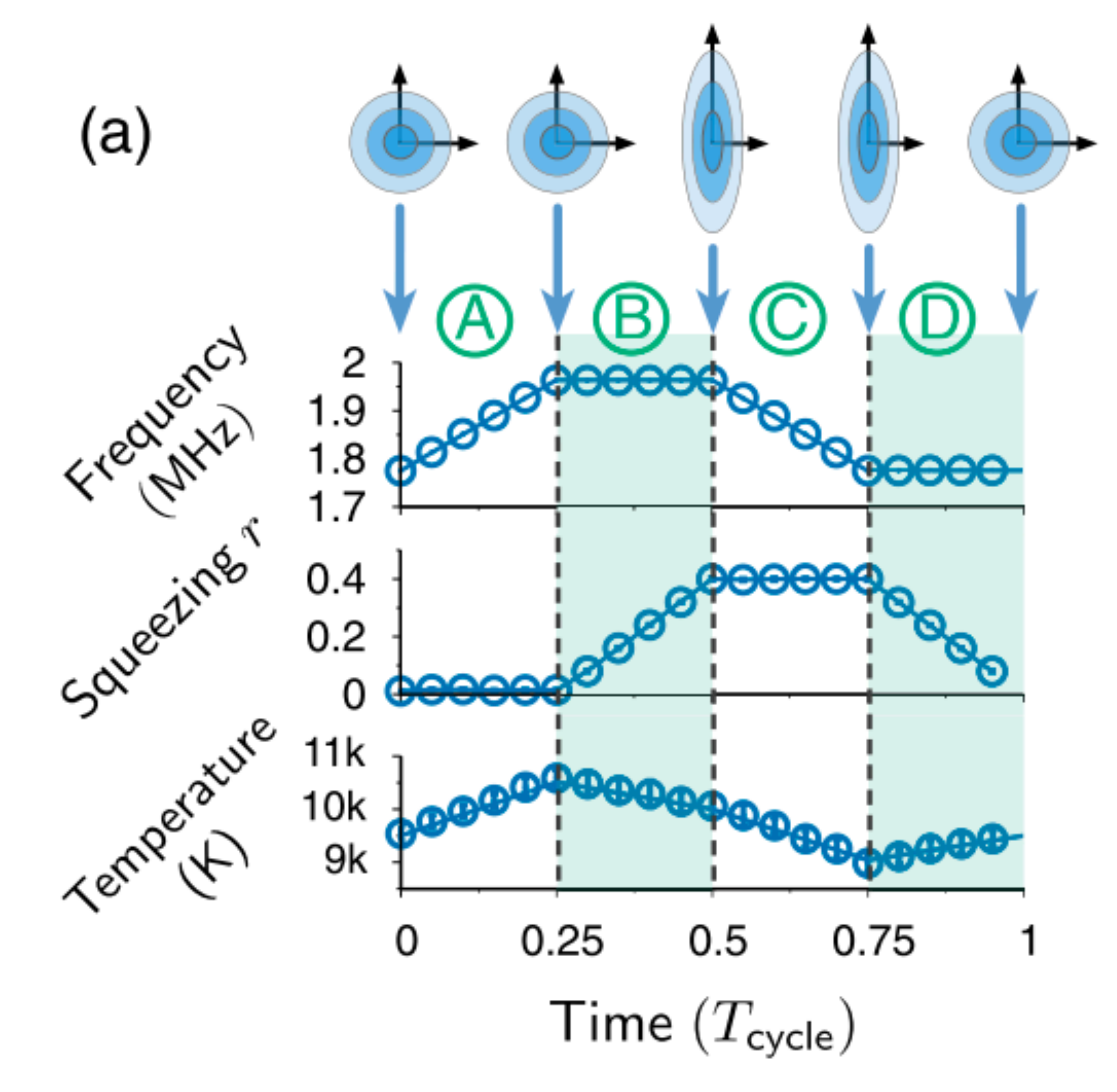
J. Klaers, et. al. "Squeezed thermal reservoirs as a resource for a nano-mechanical engine beyond the Carnot limit", Physical Review X, 7, 031044 (2014)





Thermosqueezing

- Single QHO:

$$\rho = \frac{1}{Z}\exp\{-\beta H - \beta\mu A\}, \qquad H = \frac{\omega}{2}(p^2 + x^2), \qquad A = \frac{\omega}{2}(p^2 - x^2)$$

- Two charges, H (energy) and A (asymmetry).
- Onsager coefficients:

Charge preserving Gaussian unitary

Unitary which preserves both energy and squeezing:

$$U = \exp\{-g\tau(a_1^\dagger a_2 - a_2^\dagger a_1)\}$$

Actually the only one which is also Gaussian (quadratic).

SU(1,1) algebra

3 charges

$$Q_1 = H = \frac{\omega}{2}(p^2 + x^2) \qquad Q_2 = A = \frac{\omega}{2}(p^2 - x^2) \qquad Q_3 = \frac{\omega}{2}\{x, p\}$$

The charges Q_1, Q_2, Q_3 form a non-Abelian group:

$$[Q_1, Q_2] = 2iQ_3$$

$$[Q_3, Q_1] = 2iQ_2$$

$$[Q_2, Q_3] = -2iQ_1$$

Transport coefficients

Thermal conductance: $\kappa = -\beta^2 L_{QQ}$

Squeezing conductance: $G = -\beta L_{AA}$

Entropy production/dissipated heat reads

$$\dot{Q}_{\text{diss}} = \Sigma/\beta = \kappa \delta T^2/T + J_A G$$

New Joule-like heating term due to squeezing.

Onsager matrix

Unitary which preserves both heat ($J_Q = J_H - \mu J_A$) and squeezing:

$$J_Q = L_{QQ}\delta_\beta - L_{QA}\beta\delta_\mu \qquad J_A = L_{AQ}\delta_\beta - L_{AA}\beta\delta_\mu$$

with

$$L_{QQ} = f_\tau(1-\mu^2)\bar{n}(\bar{n}+1)$$

$$L_{QA} = L_{AQ} = f_\tau \mu\, \bar{n}(\bar{n}+1)$$

$$L_{AA} = f_\tau(1-\mu^2)^{-1}\left[\mu\bar{n}(\bar{n}+1) + \frac{\tanh\alpha}{\alpha}(\bar{n}^2 + \bar{n}/2 + 1/2)\right]$$

where

$$\bar{n} = (e^{\beta\omega}-1)^{-1}, \qquad f_\tau = \omega^2 \sin^2(g\tau), \qquad \alpha = \beta\omega\sqrt{1-\mu^2}$$

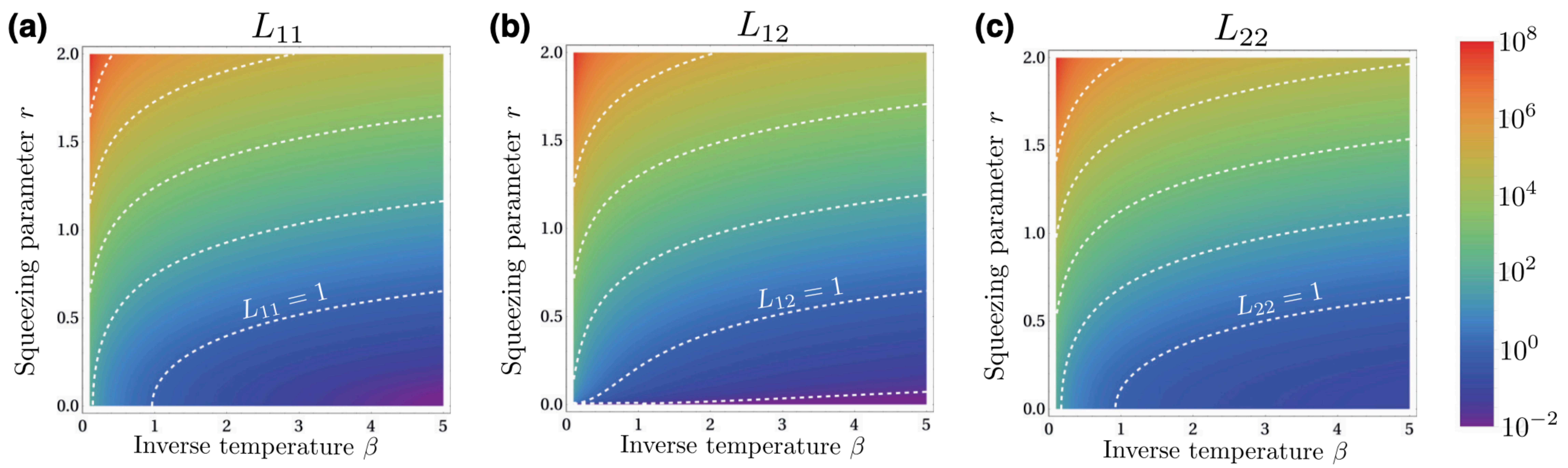


FIG. 2. (a)–(c) Thermosqueezing Onsager coefficients L_{11}, L_{12}, L_{22} on the log scale, computed from Eqs. (19), in units of $(\hbar\omega)^2 \sin^2(g\tau)$, as a function of the inverse temperature β (in units of $\hbar\omega/k_B$) and the adimensional squeezing parameter r.

Entropy reduction

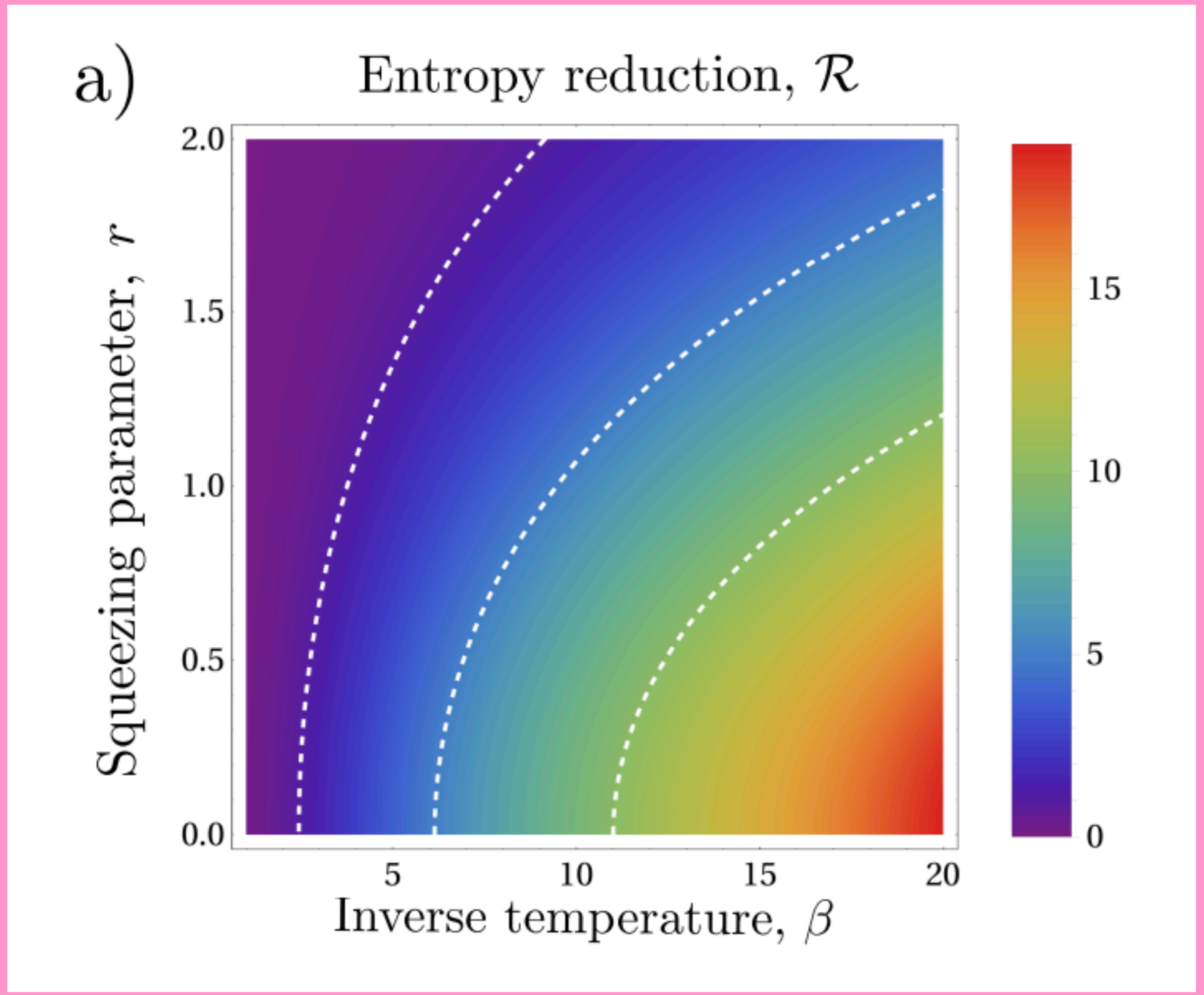
Recall that

$$\Sigma = \frac{1}{2}\mathrm{var}(D) - \frac{1}{2}\int_0^1 dy\ I_y(\pi, D)$$

Define the entropy reduction due to non-commutativity

$$\mathscr{R} = \frac{1}{2\Sigma}\int_0^1 dy I_y(\pi, D)$$

Classical case corresponds to $\mathscr{R} = 0$.



Cross coefficients

Thermopower, *or* Squeezing-Seebeck (*Squeebeck*) coefficient

$$S = \frac{1}{T}\frac{L_{AQ}}{L_{AA}}$$

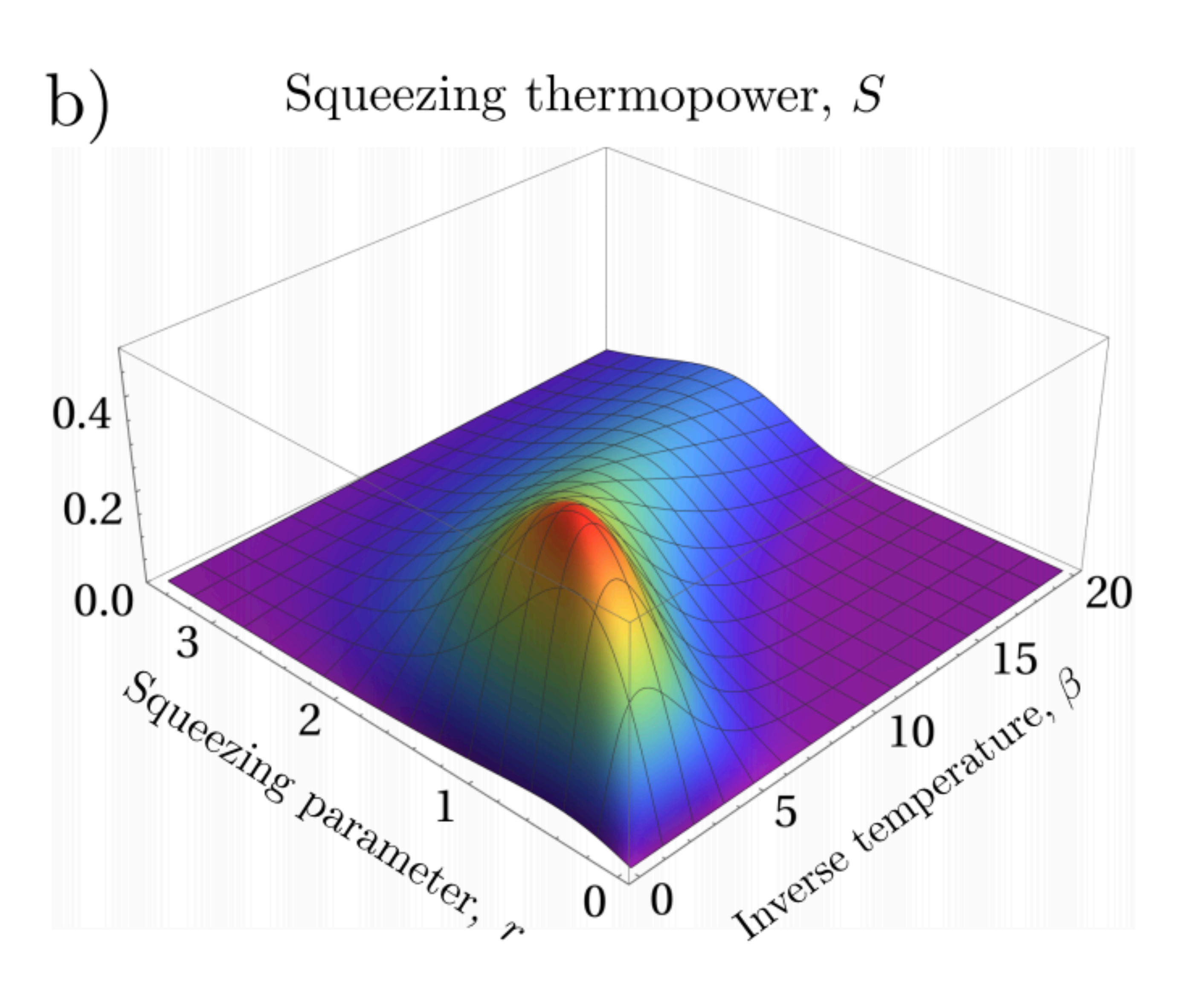
(flow of squeezing due to gradient of temperature)

Squeezing-Peltier (Squeetier (?)) coefficient:

$$\Pi = \frac{L_{QA}}{L_{AA}}$$

(flow of heat due to gradient in squeezing)

The two are related by $\Pi = TS$



Spin S Heisenberg dynamics

- Spin S c

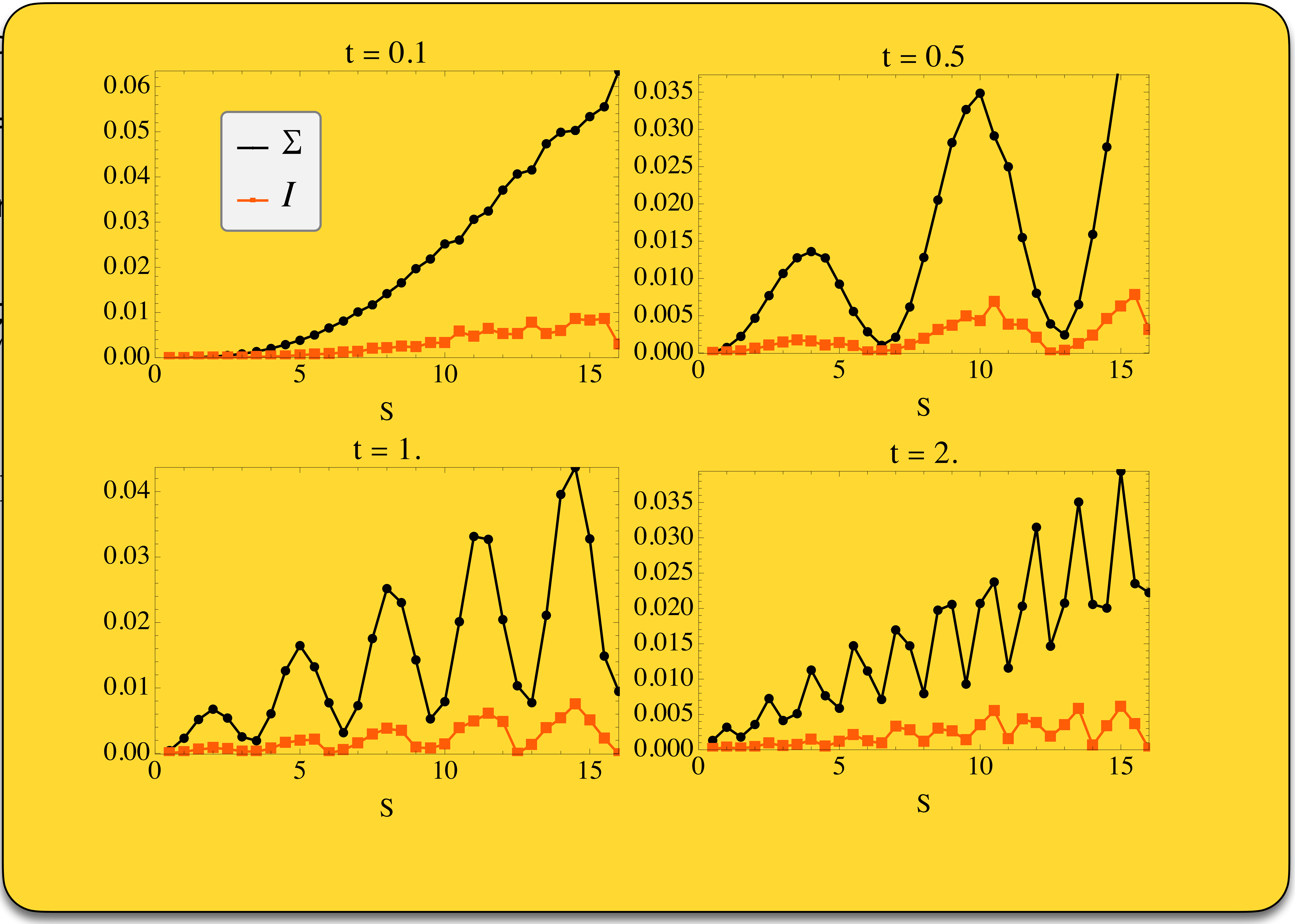
$S_z|m\rangle =$

- Two spi

$\rho^A_{\lambda_A} = \frac{1}{Z}$

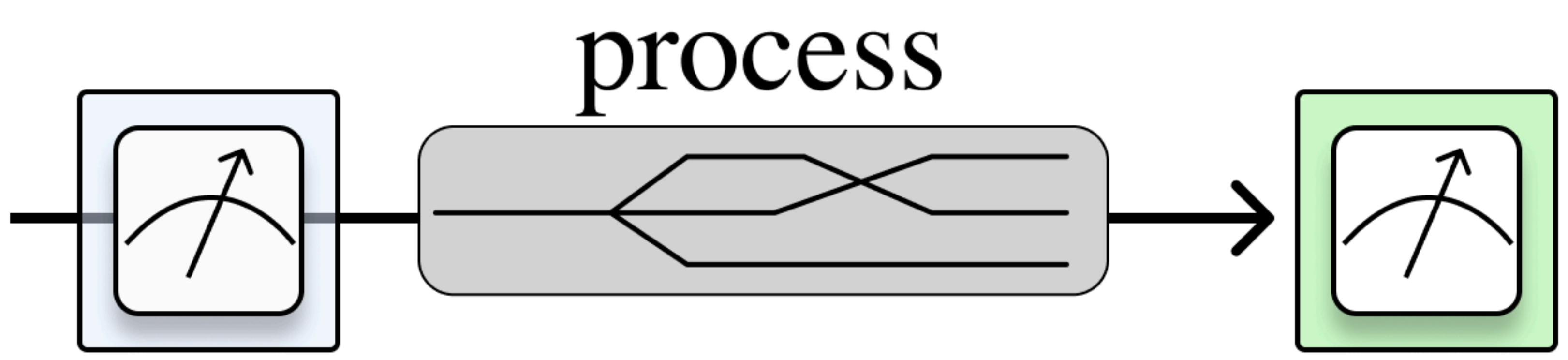
- Interact

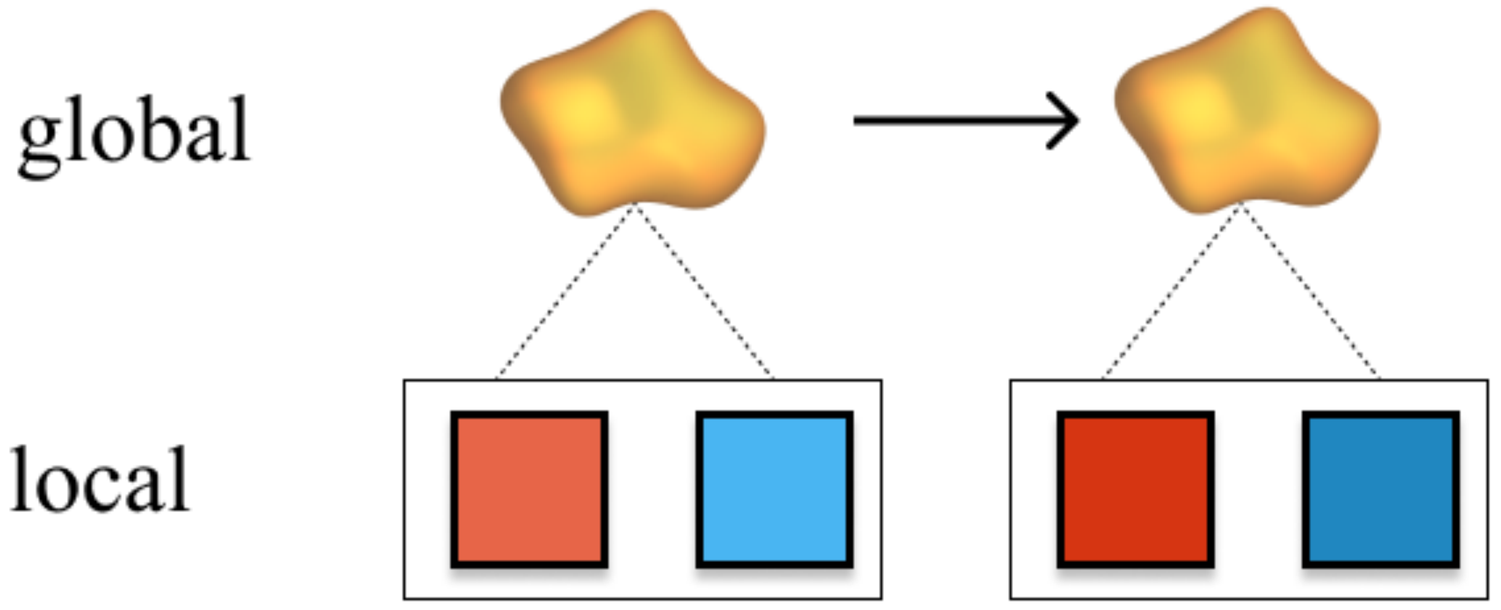
$U = \mathrm{ex}$



How do we actually measure these currents?

- Thermodynamics deals with *transformations.*
    - Require **two-point measurements (TPM)**
- Measurements in quantum mechanics are invasive.
- *First measurement is the problem:*
    - Destroys initial quantum coherences.
- (Can be overcome using identical copies)
- **Next step:** operational definition based on specific experimental platforms.





Kaonan Micadei, Gabriel T. Landi, Eric Lutz, arXiv:2103.14570

Conclusions & outlook

- Quantum mechanics opens up the way for performing transport of non-commuting charges.
- We put forth a framework suitable for describing this in the linear response regime.

Perspectives:

- If the charges do not commute, how can we actually measure them?
- Current fluctuations and Thermodynamic Uncertainty Relations.
- Concrete applications of thermosqueezing.

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THANK YOU!

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Extra slides