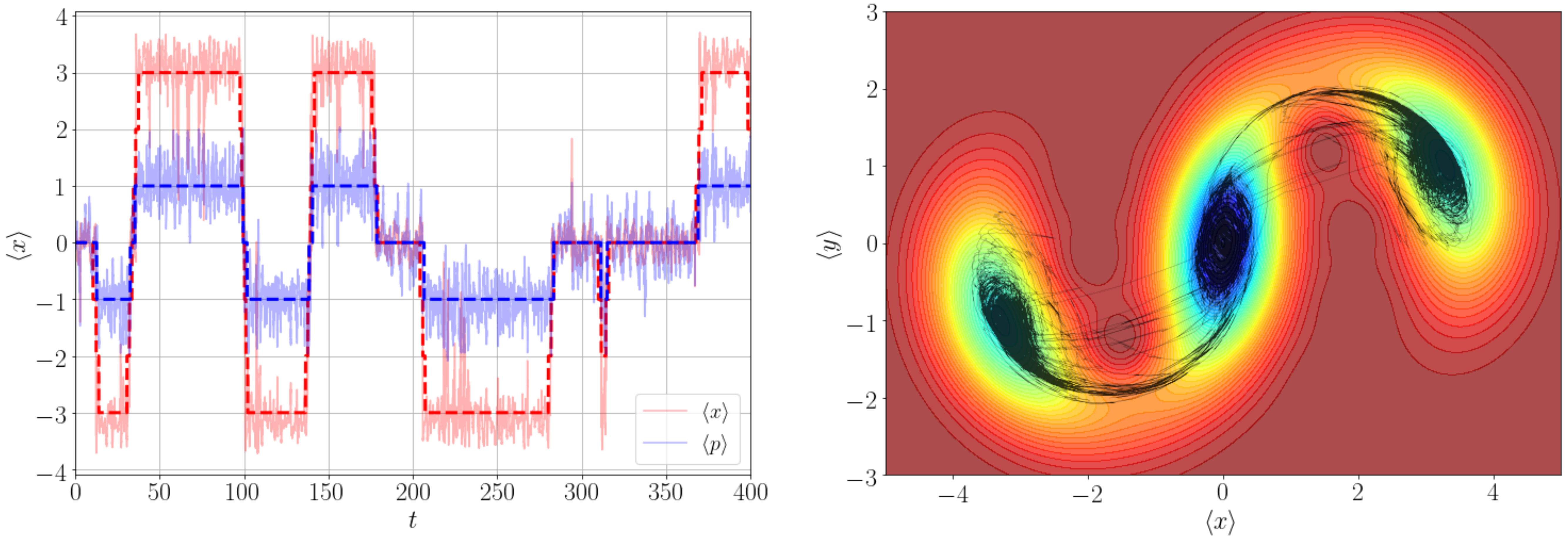


Information & fluctuations in continuously measured systems

Gabriel T. Landi

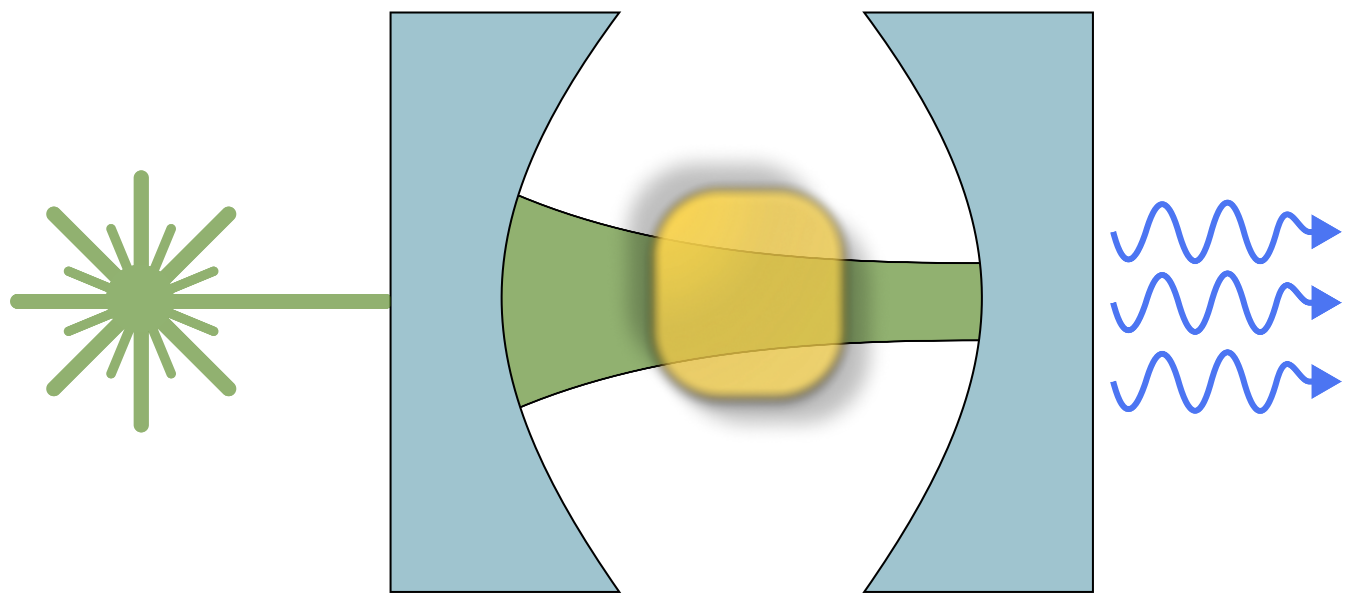
University of São Paulo / University of Rochester

September 2022 - Quantum Intelligence - Birr, Ireland

Quantum Thermodynamics
Quantum Transport

www.fmt.if.usp.br/~gtlandi

Overview



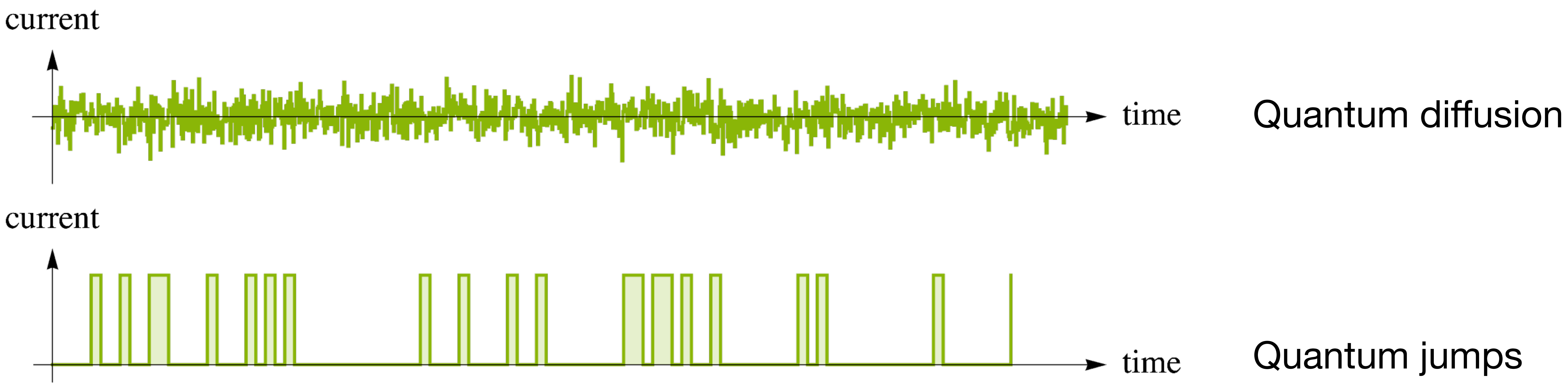
- Open Quantum Systems + continuous weak measurement.

  ➡ ***Classical (stochastic) currents (time-series)***

- **Motivation:**

  1. Current fluctuations & two-time correlations.

  2. What information the current conveys about the system.



Current fluctuations & two-time correlations

M. Kewming, M. Mitchison & GTL,
"Diverging current fluctuations in critical Kerr resonators",
arXiv 2205.02622. *Accepted in PRA.*

GTL, M. Kewming, M. Mitchison, P. Potts,
"Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurement and full counting statistics."
In preparation. Tutorial for PRX Quantum.


Patrick Potts


Michael Kewming


Mark Mitchison

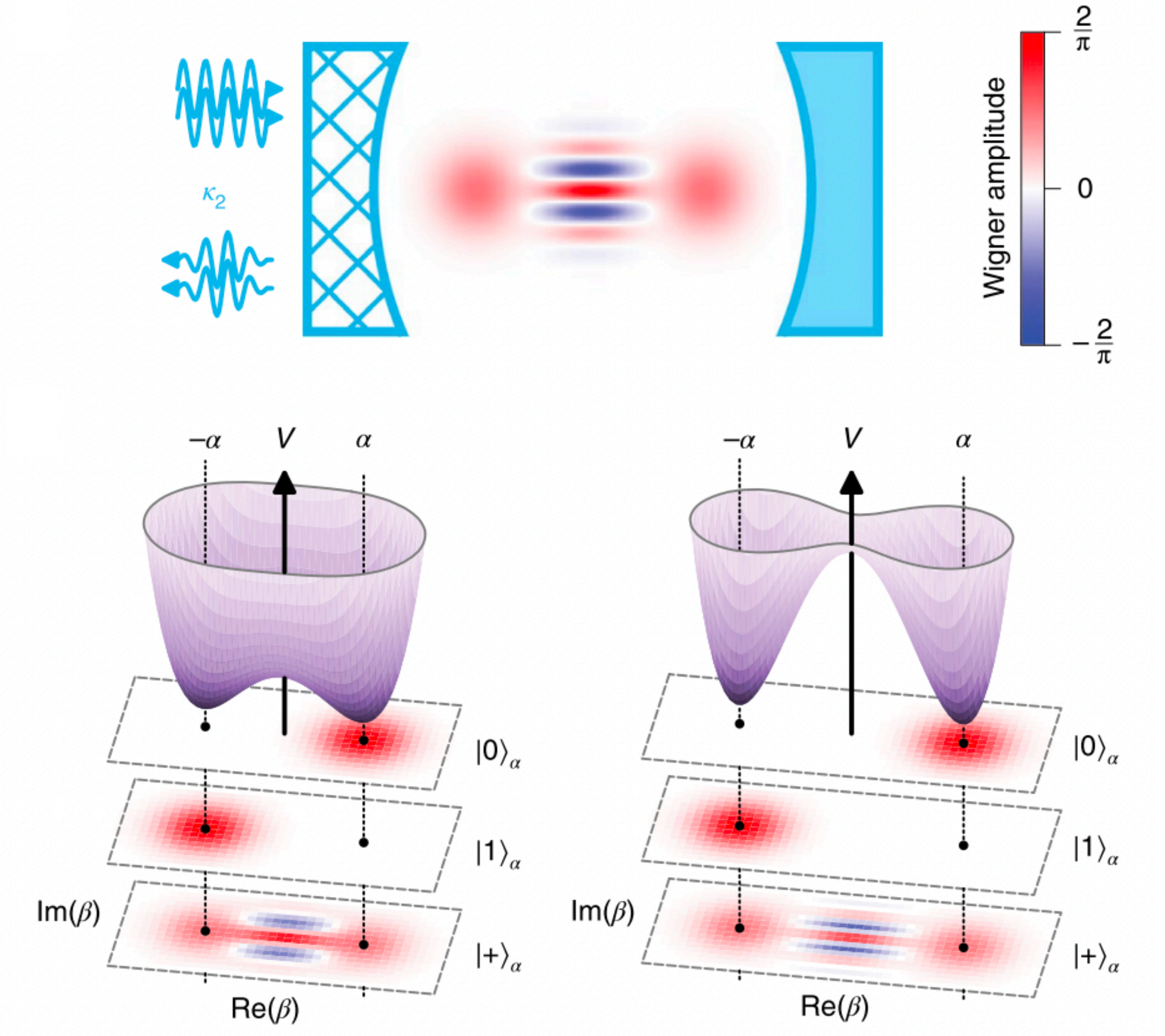
Example: Kerr model

- Quantum Master equation:

$$\frac{d\rho}{dt} = \mathscr{L}(\rho) = -i[H(t),\rho] + \kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right]$$

$$H = -\Delta a^\dagger a + \frac{U}{2}a^\dagger a^\dagger a a + \frac{G}{2}(a^{\dagger 2} + a^2)$$

Cat qubits:
useful for quantum error correction

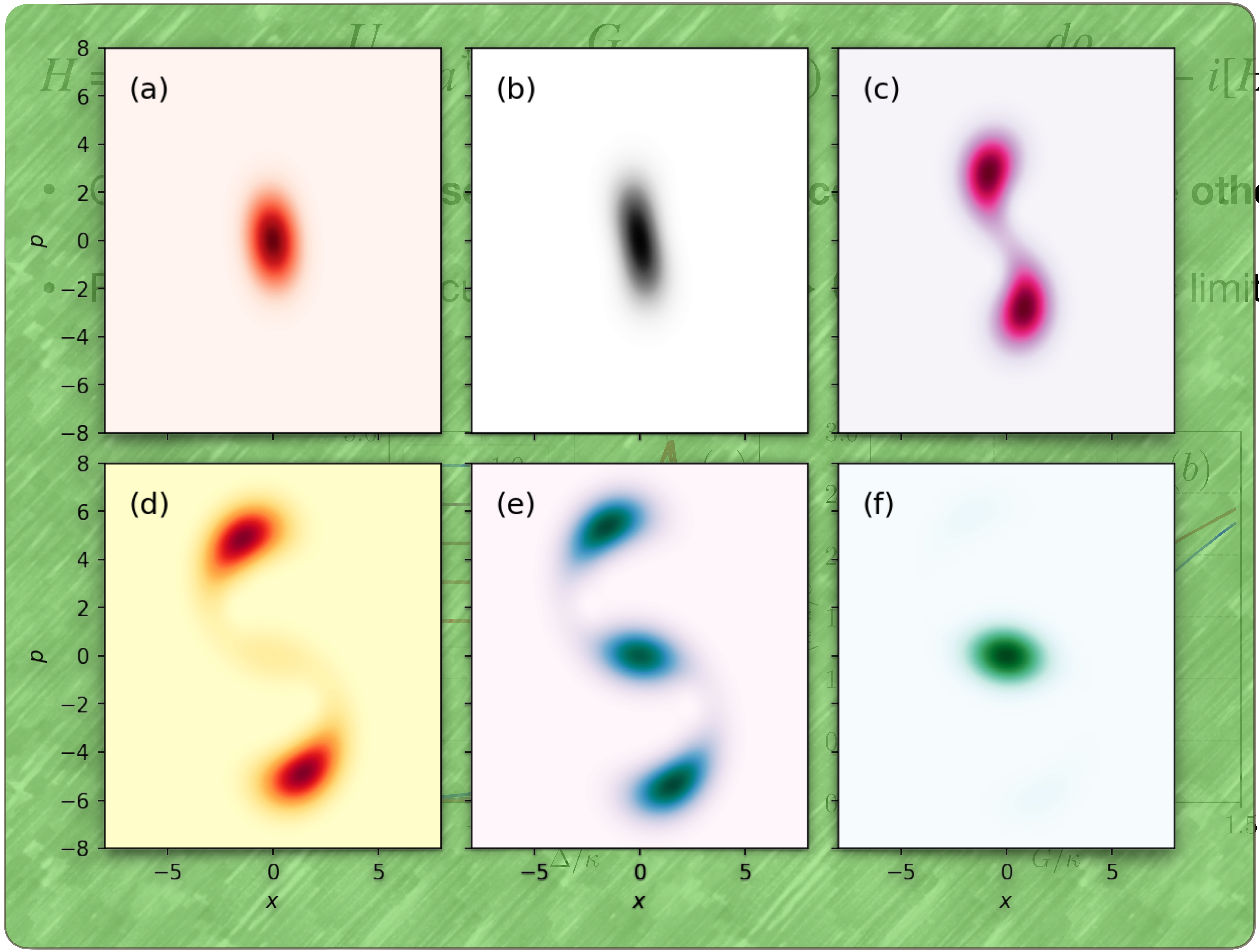


Lescanne, *et. al.*, Nature, **16**, 509-513 (2020)

$- i[H, \rho] + \kappa D[a]$

other discontinuous:

limit")



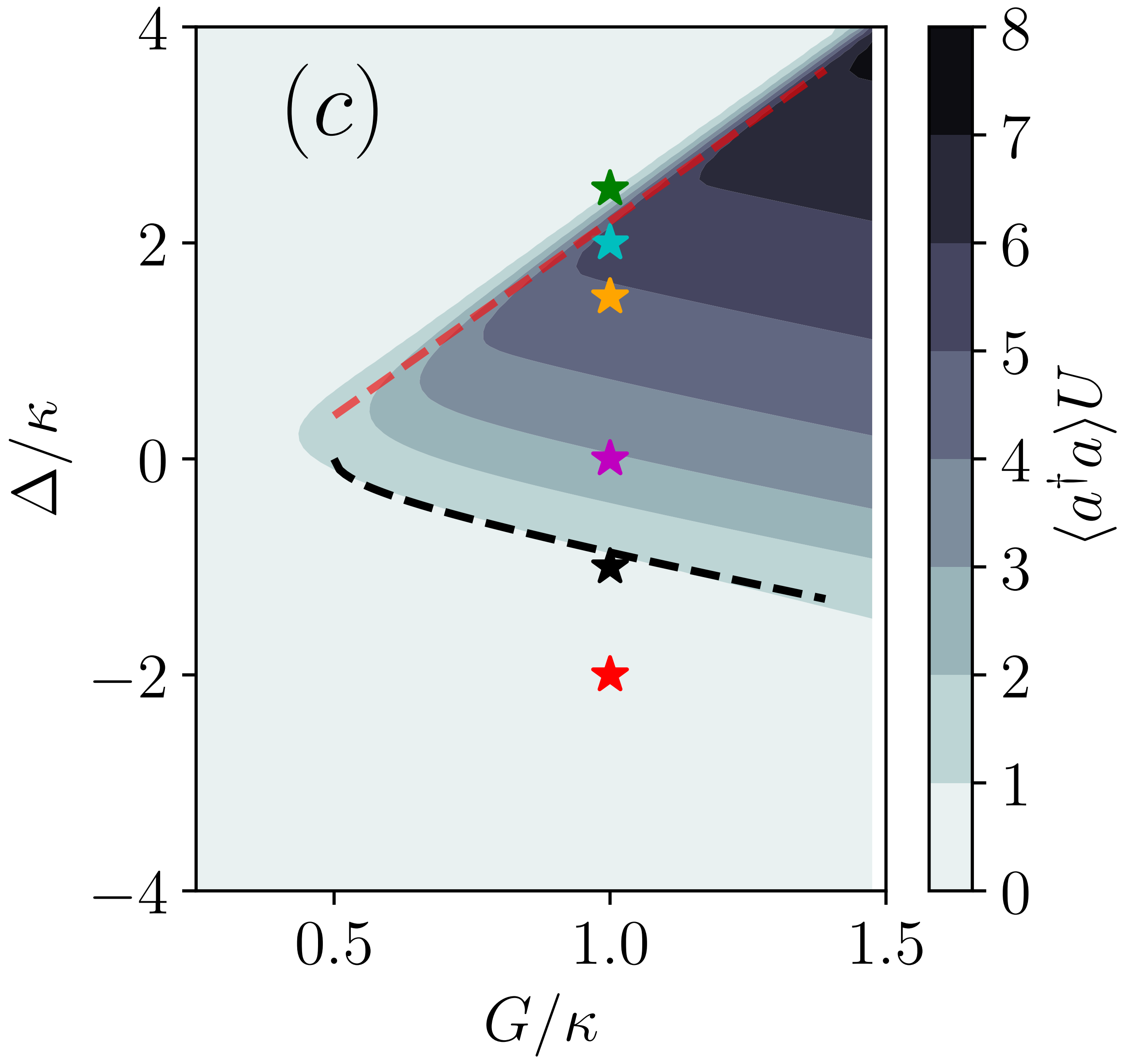


Photo-detection current

- Quantum Jump unravelling:

With prob. $p = dt\ \text{tr}(a\rho a^\dagger)$ the system jumps: $\rho \to \dfrac{a\rho a^\dagger}{\text{tr}(a\rho a^\dagger)}$

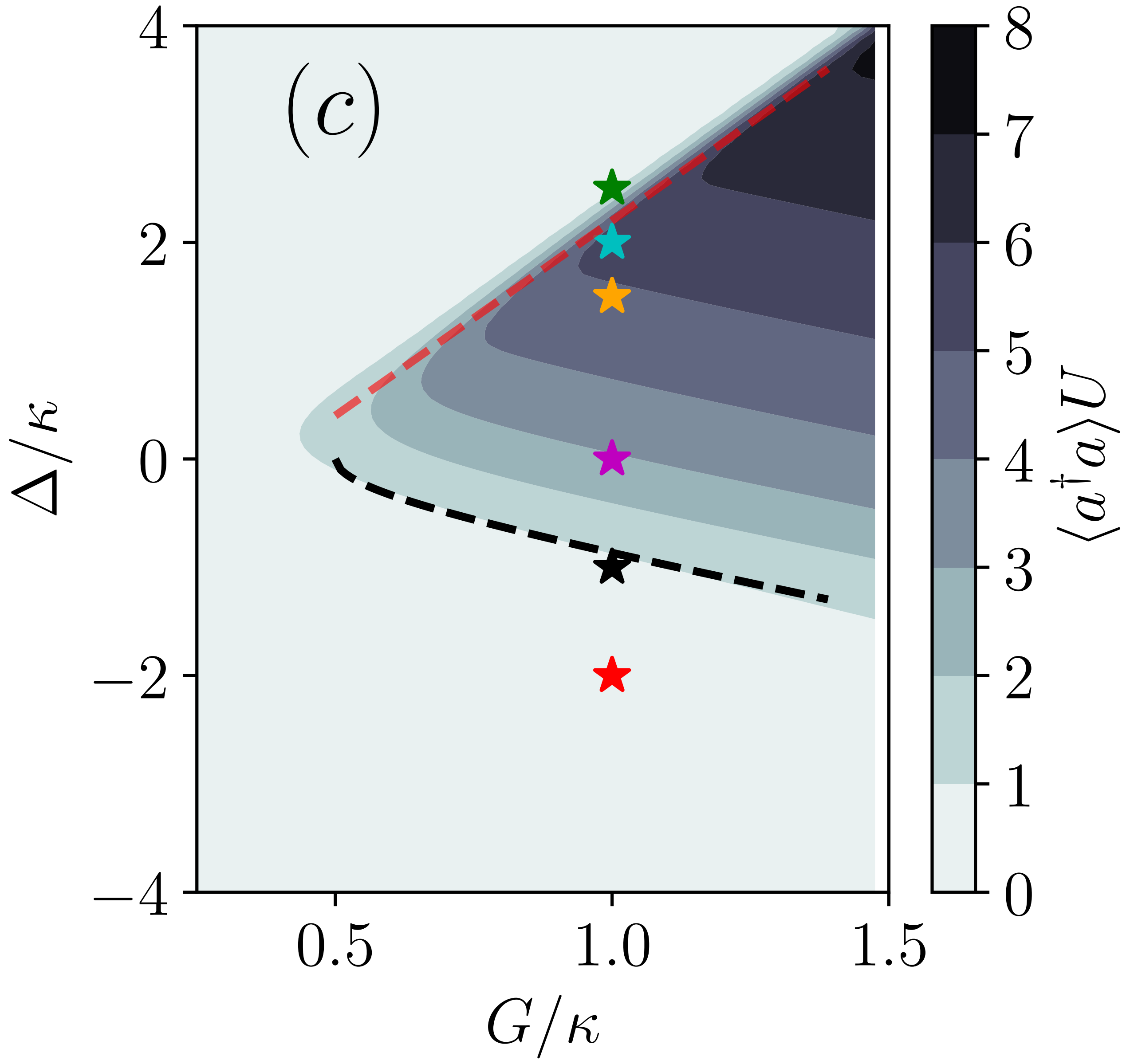
With prob. $1 - p$ it does not jump: $\rho \to \dfrac{M_0 \rho M_0^\dagger}{\text{tr}(M_0 \rho M_0^\dagger)}$

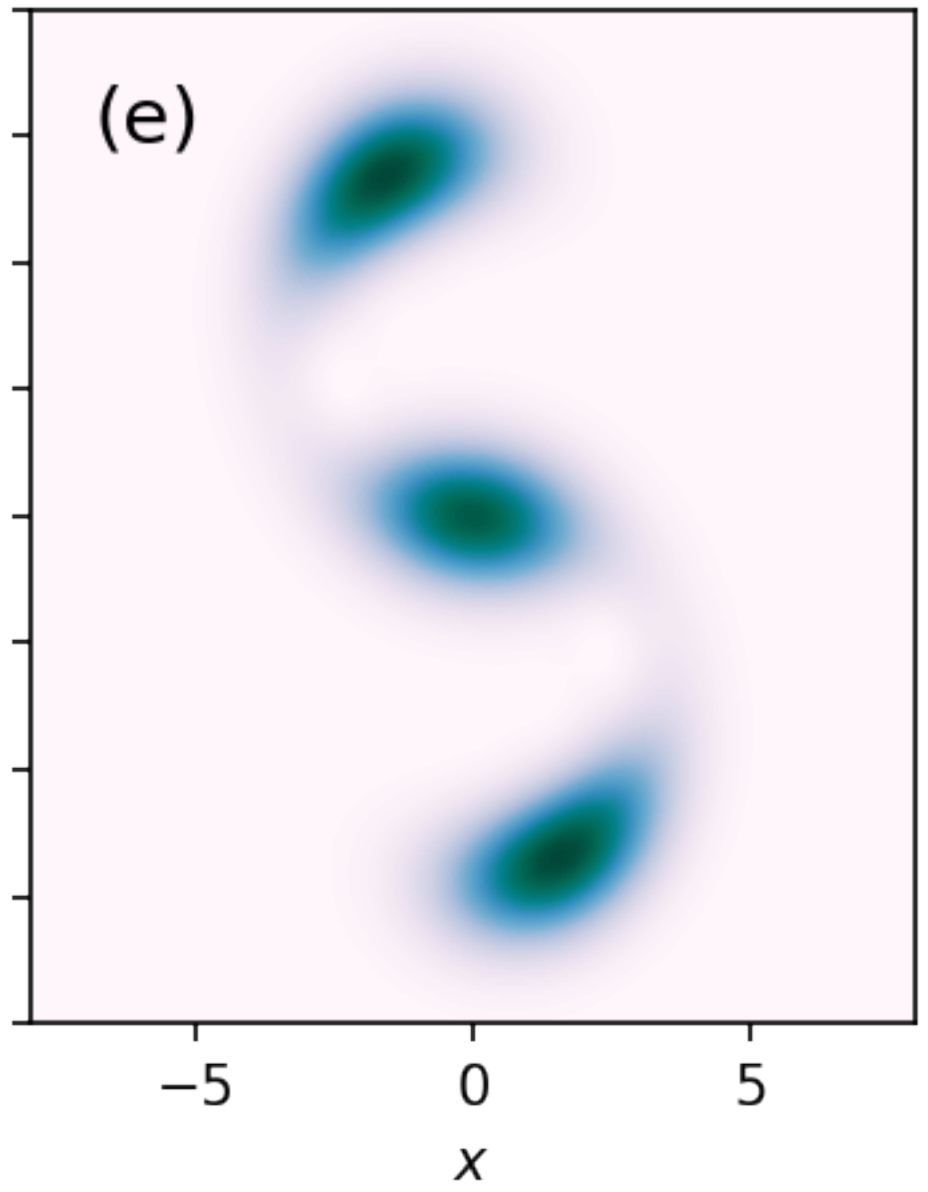
$$M_0 = 1 - dt\left(iH + \frac{\kappa}{2} a^\dagger a\right)$$

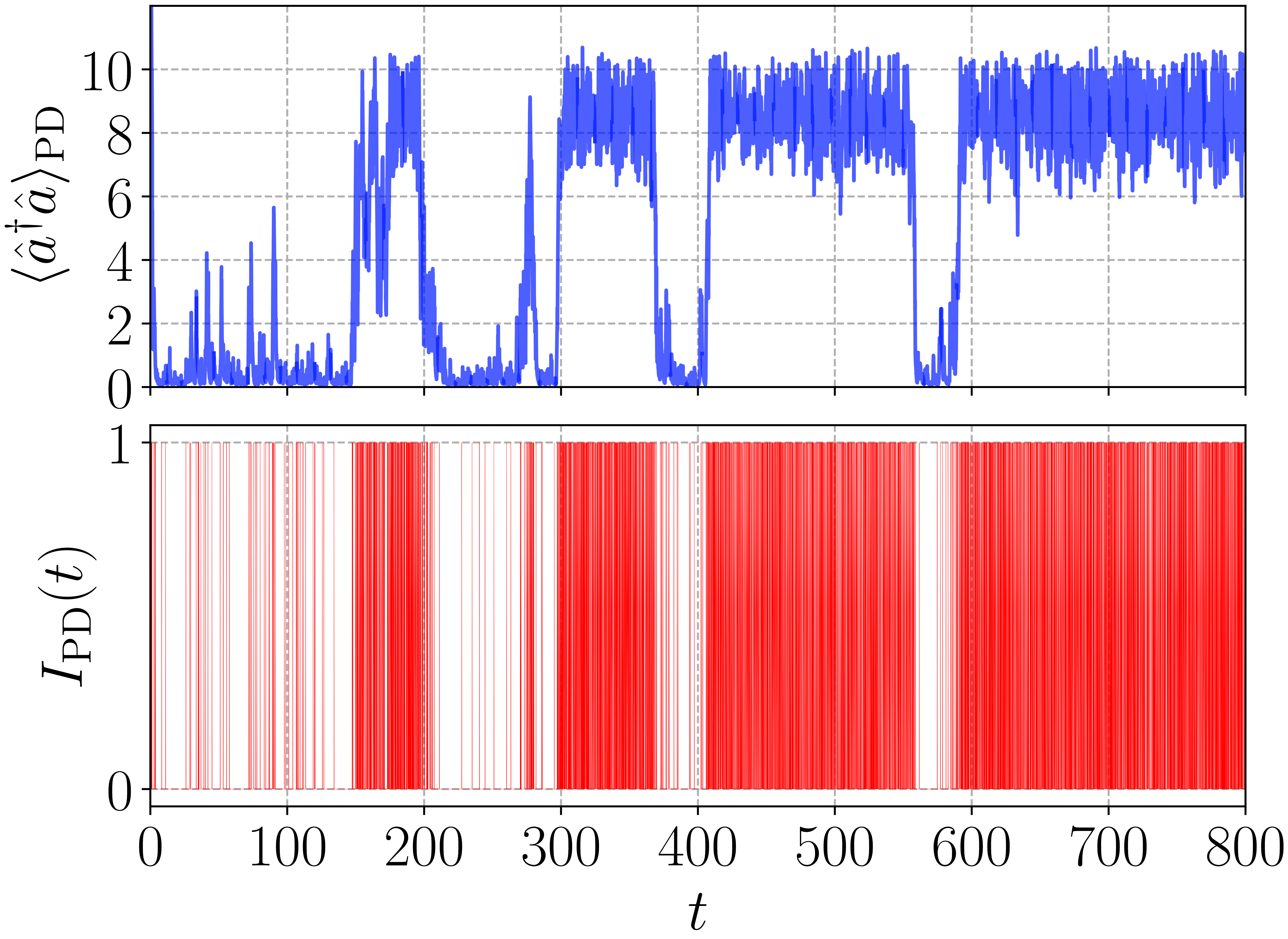
- Jump: $dN = 1$
- (stochastic) current: $I(t) = \frac{dN}{dt}$
- Integrated current (net charge): $N(t) = \int_0^t dt'\ I(t')$

Photo-detection current

- @ discontinuous transition: on/off (telegraph) behavior of the current.
- Photo-detection cannot resolve upper vs. lower blobs.







Full Counting Statistics

- Average photon current: $J = \mathrm{tr}\big(\mathcal{J}\rho\big) = \kappa\langle a^\dagger a\rangle$ where

$$\mathcal{J}\rho = \kappa a \rho a^\dagger$$

- Two-time correlation function:

$$F(\tau) := \mathrm{Cov}\big(I(t)I(t+\tau)\big) = K\delta(\tau) + \mathrm{tr}\big\{\mathcal{J}e^{\mathcal{L}|\tau|}\mathcal{J}\rho\big\} - J^2$$

$K = J$ (in this case) is the **dynamical activity:** *jumps/second.*

- Power spectrum:

$$S(\omega) = \int_{-\infty}^{\infty} e^{-i\omega\tau}F(\tau)d\tau$$

- Zero-frequency component of the power spectrum := "noise":

$$D = S(0) = \lim_{t\to\infty}\frac{d}{dt}\mathrm{Var}\big(N(t)\big)$$

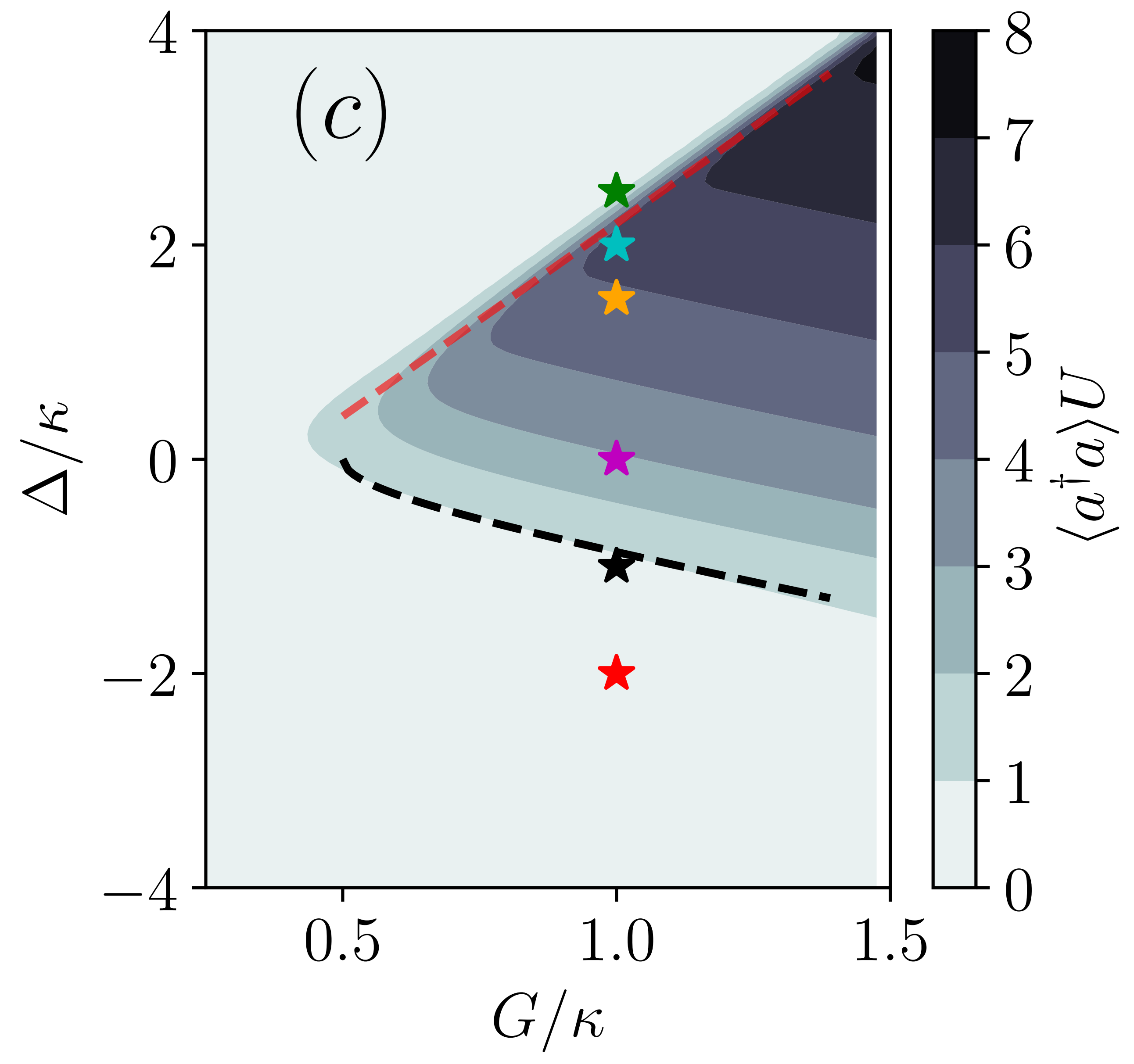
D is the quantity in TURs and KURs

TUR: $\dfrac{D}{J^2} \geqslant \dfrac{2}{\dot{\sigma}}$

KUR: $\dfrac{D}{J^2} \geqslant \dfrac{1}{K}$

Both can be violated in the quantum regime.

Divergence of the diffusion coefficient

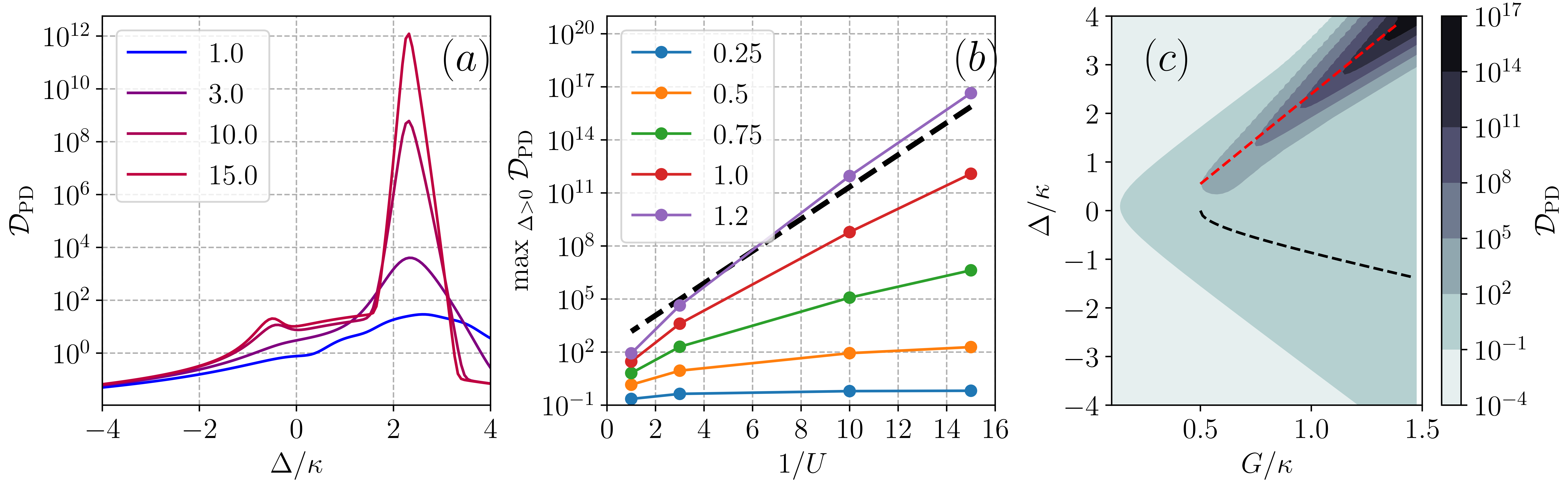


- "Thermodynamic limit:" $U \to 0$
- In the continuous transition ($\Delta < 0$)

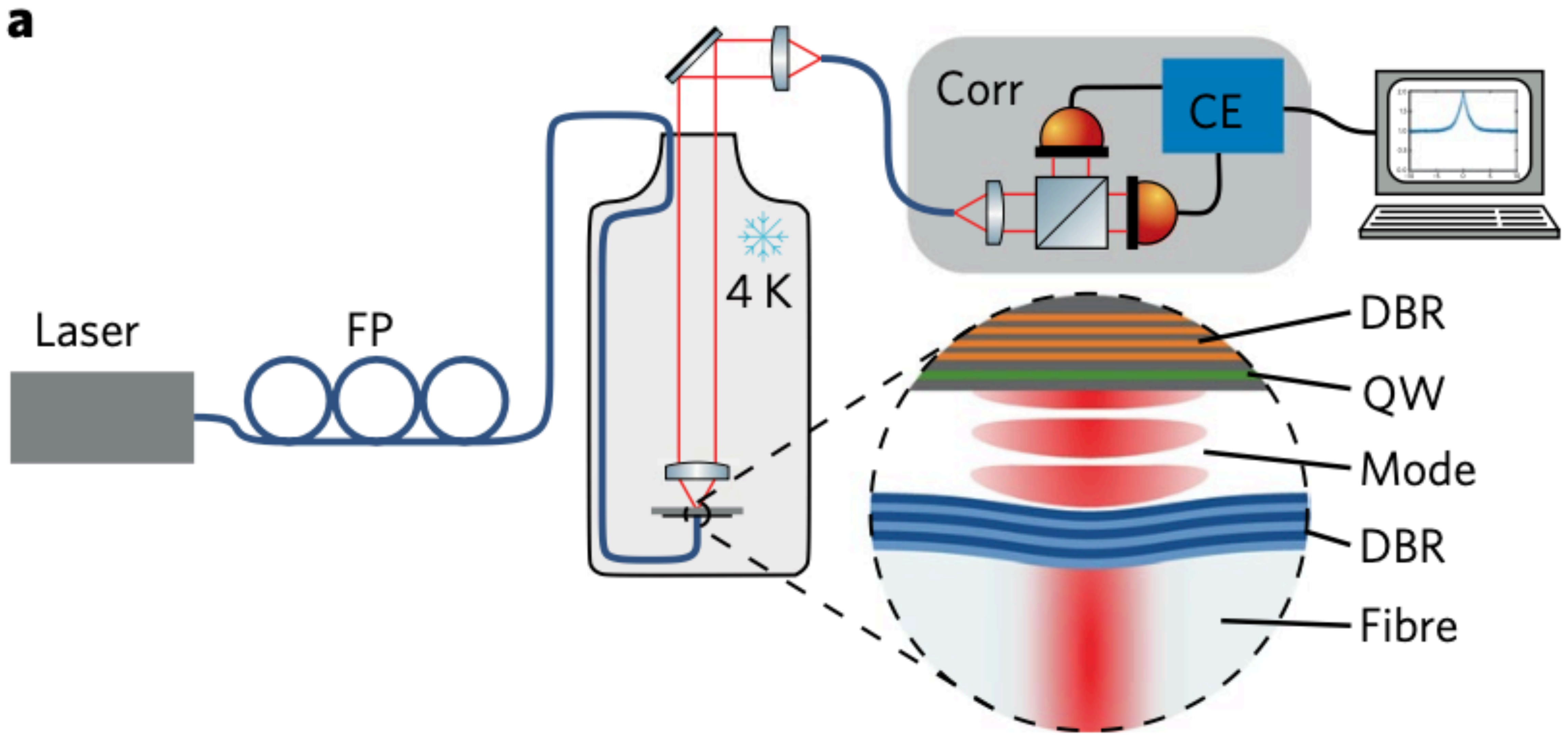
$$D \sim (1/U)^2$$

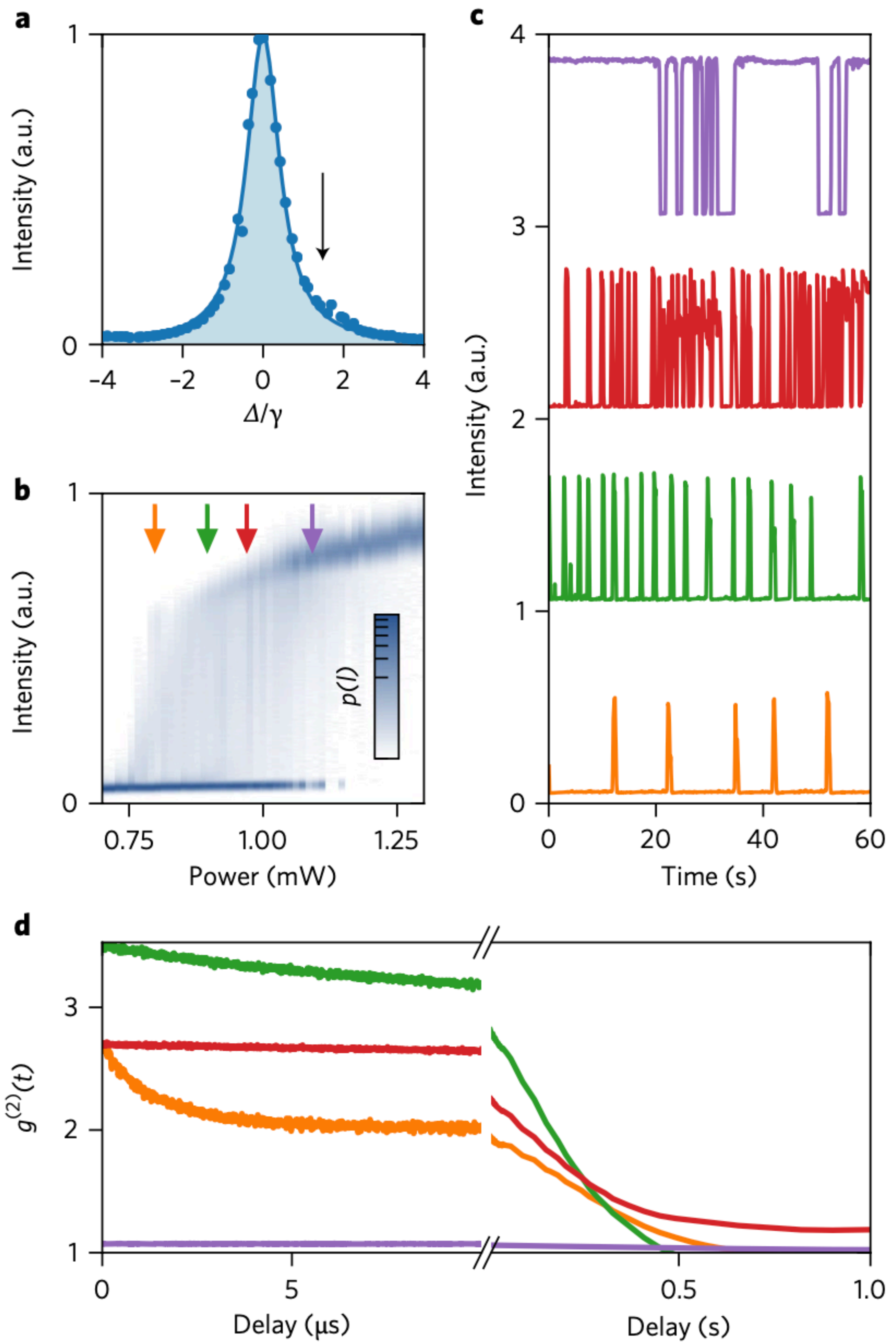
- In the discontinuous transition ($\Delta > 0$)

$$D \sim e^{1/U}$$



GaAs cavity polaritons.





T. Fink, *et. al.*, *Nature Physics,* **14**, 365 (2018)

Homodyne current

- Mix photon output with a strong laser source $\alpha = |\alpha| e^{i\phi}$.
  - Equivalent to measuring jumps of $(a + \alpha)\rho(a + \alpha)^\dagger$, where α is a large number.

$$J = \kappa\langle(a + \alpha)^\dagger(a + \alpha)\rangle = \kappa\Big(|\alpha|^2 + |\alpha|\langle ae^{-i\phi} + a^\dagger e^{i\phi}\rangle + \langle a^\dagger a\rangle \Big)$$

  - $|\alpha|^2$ is just a constant offset.
  - If α is large, then the current will predominantly

$$x := ae^{-i\phi} + a^\dagger e^{i\phi}$$

instead of $a^\dagger a$.

- Quantum diffusion unravelling:

$$d\rho = dt\mathscr{L}\rho + dW\left[\mathscr{H}\rho - \langle x\rangle\rho\right],$$

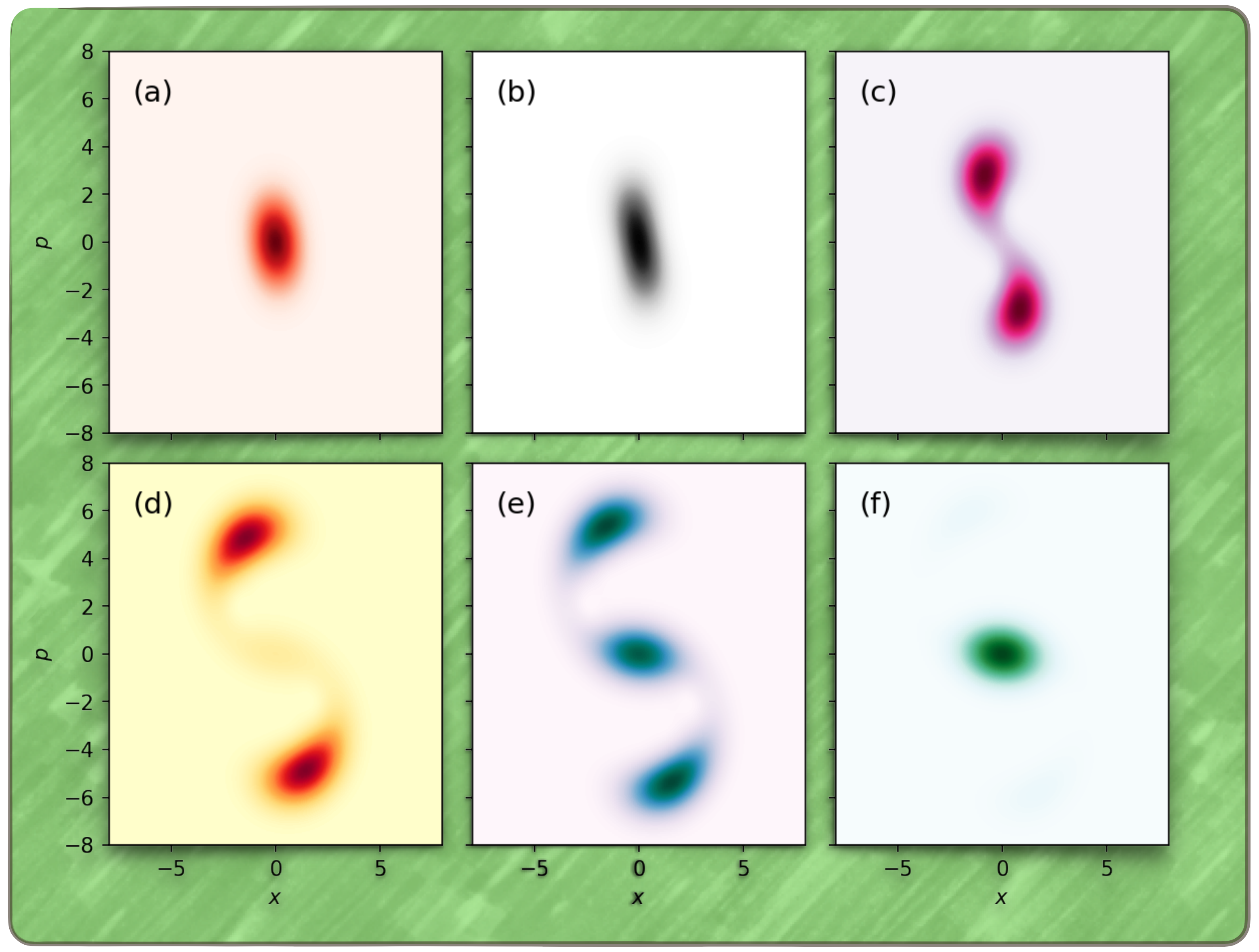
$$\mathscr{H}\rho = \kappa\big(a\rho + \rho a^\dagger\big)$$

dW = Wiener increment:
$E(dW) = 0, \qquad dW^2 = dt$

Homodyne current (in $p = i(a^\dagger - a)$)

- The homodyne current switches between 3 values (+,0,-).
- Captures the tunneling between the 3 blobs.

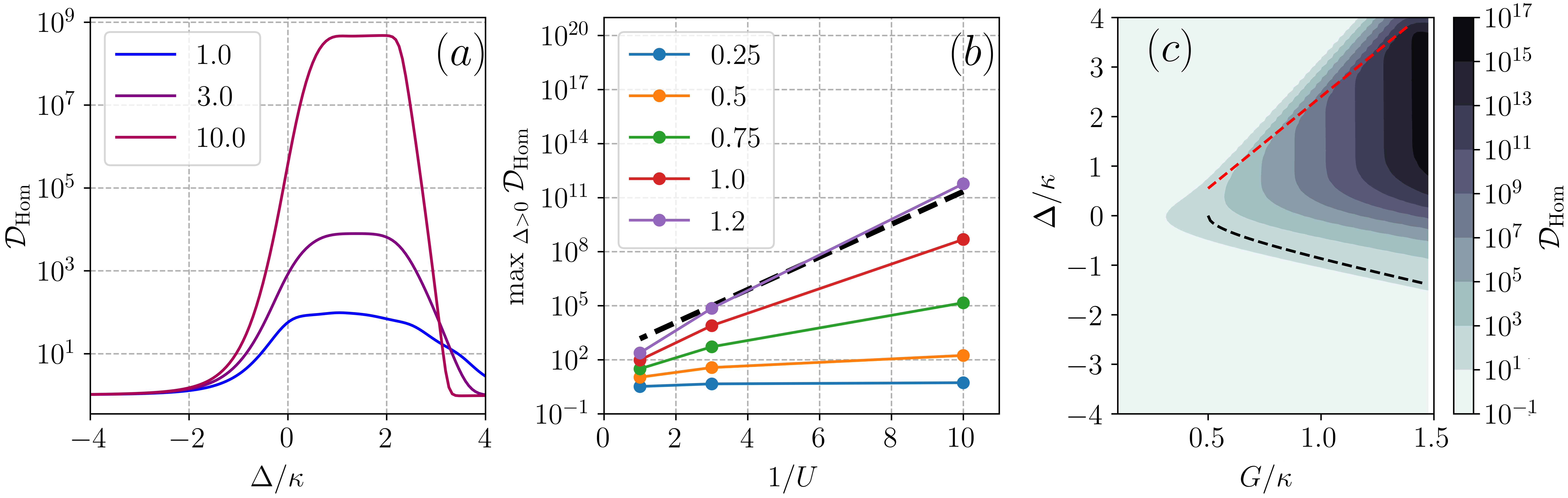
Divergence of the diffusion coefficient



- Homodyne current noise diverges exponentially in a much broader region.

$$D \sim e^{1/U}$$

- Reflects sensitivity to all 3 blobs.



What information the current conveys about the system?

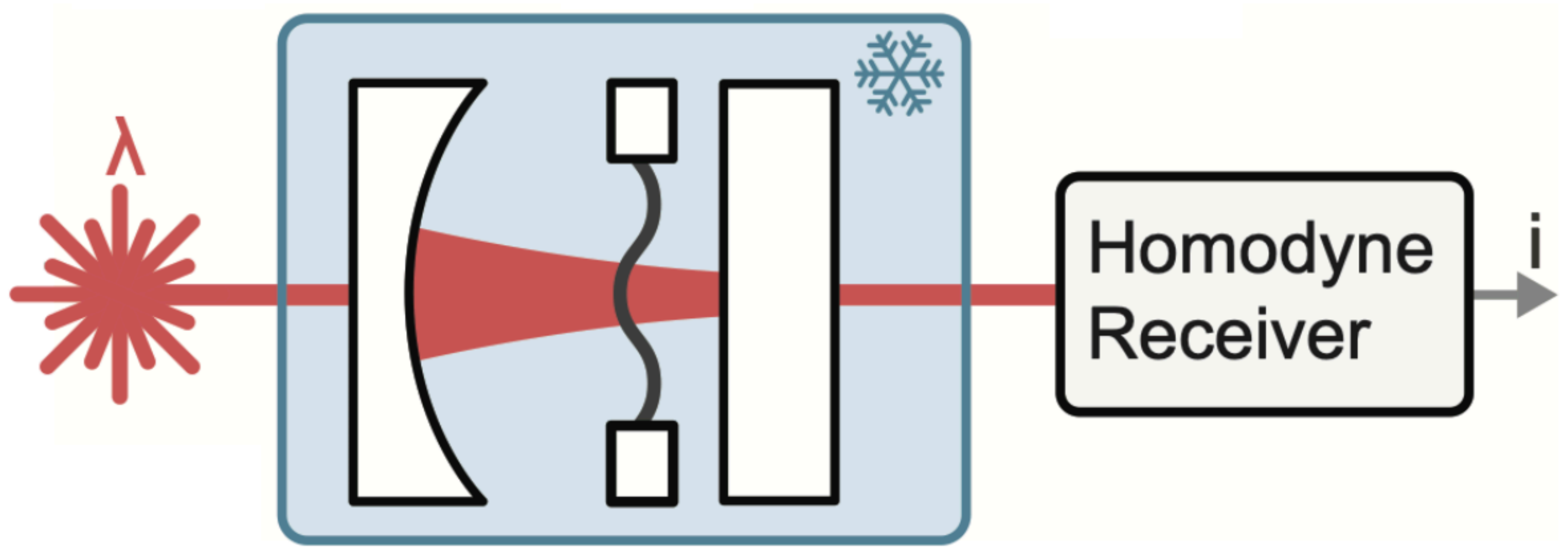
Gabriel T. Landi, Mauro Paternostro, and Alessio Belenchia,
“**Informational steady-states and conditional entropy production in continuously monitored systems**”
PRX Quantum 3, 010303 (2022)

Massimiliano Rossi, Luca Mancino, Gabriel T. Landi, Mauro Paternostro, Albert Schliesser, Alessio Belenchia
“**Experimental assessment of entropy production in a continuously measured mechanical resonator**"
Phys. Rev. Lett. 125, 080601 (2020)

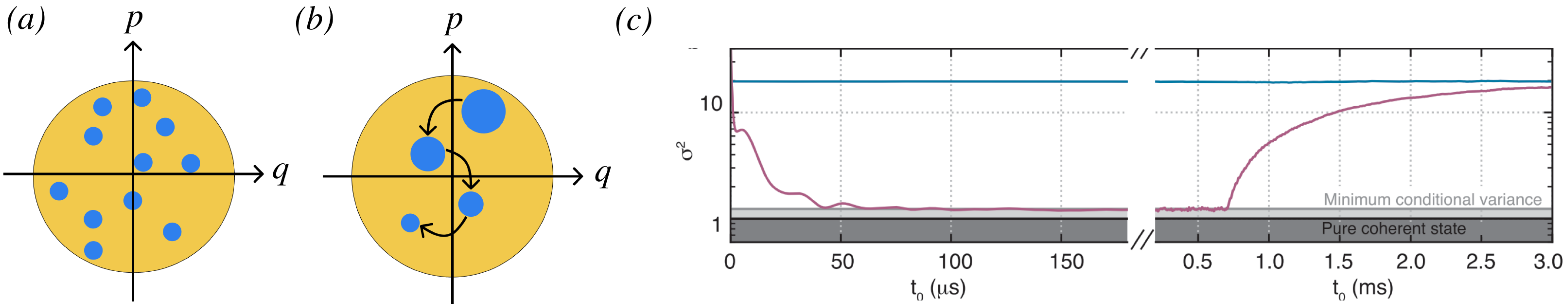
Mauro Paternostro

Alessio Belenchia

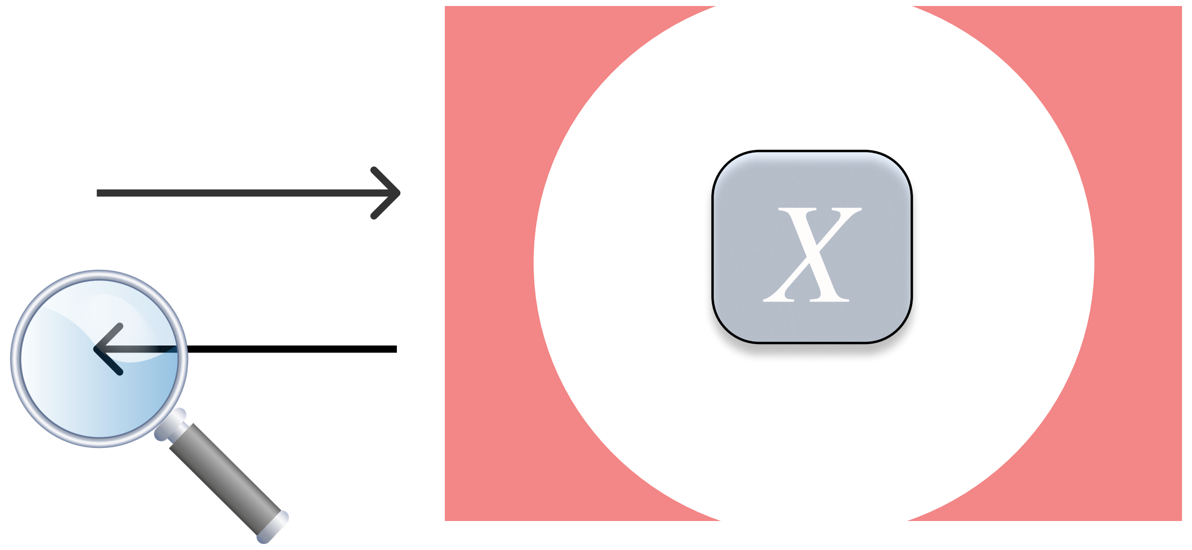
Optomechanical experiment 🇩🇰

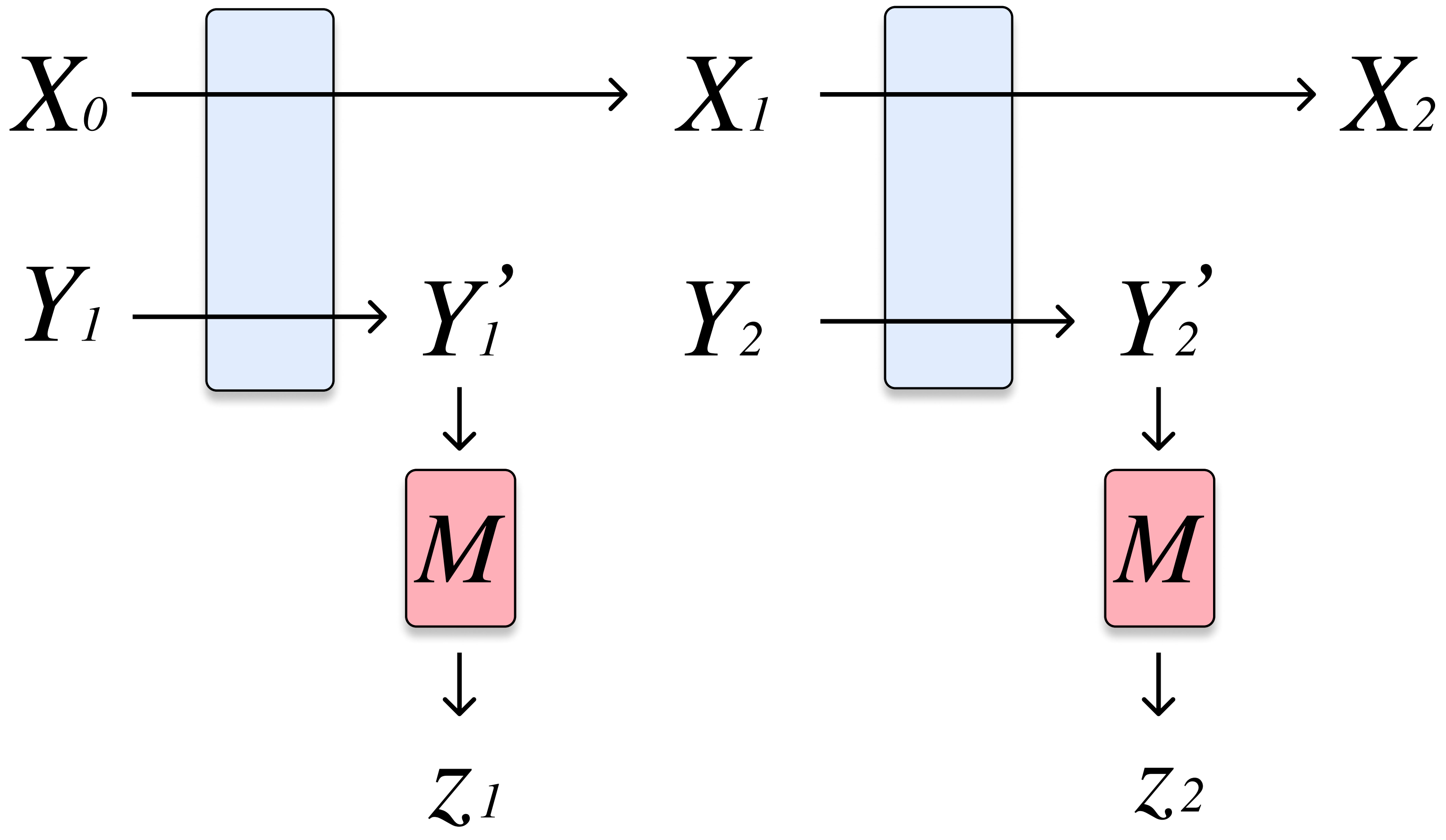


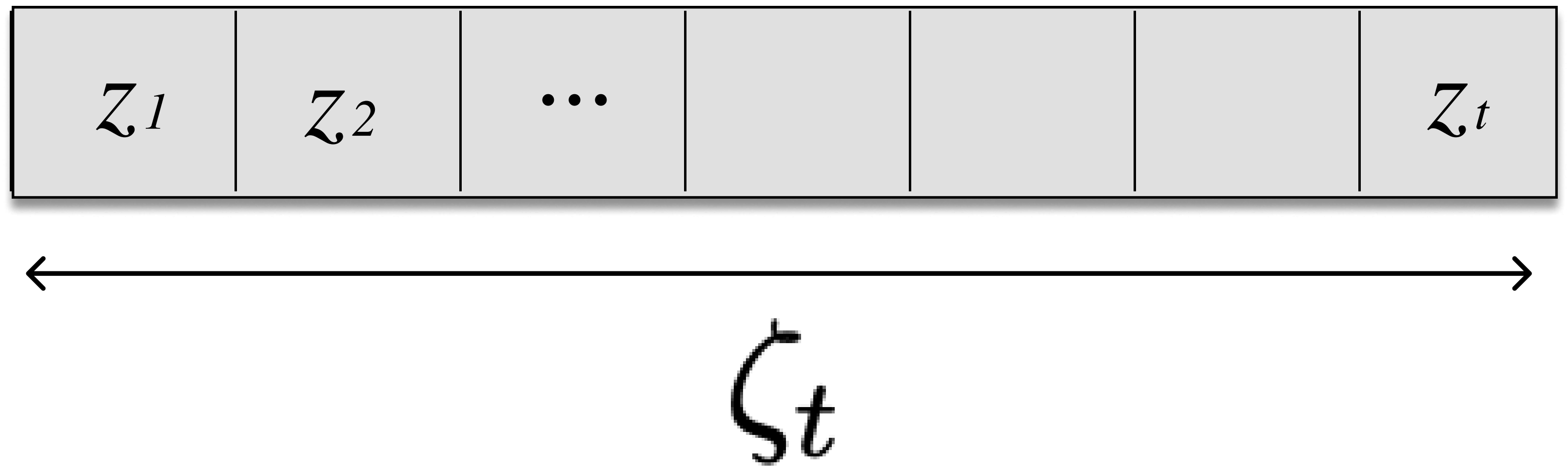
- $\sigma(t)$ = variance of the position of a mechanical membrane.
- Start measuring at $t = 0$.
  - Use information to push oscillator to the middle.
    - Reduce uncertainty.
      - *Informationally driven cooling.*



Collision model







- Unconditional dynamics (no measurement):

$$\rho_{X_t} = \mathscr{E}(\rho_{X_{t-1}}) = \mathrm{tr}_{Y_t}\left\{ U_t\left(\rho_{X_{t-1}} \otimes \rho_{Y_t}\right) U_t^\dagger \right\}$$

✤ Entropy: $S(X_t) = -\,\mathrm{tr}\left\{\rho_{X_t} \ln \rho_{X_t}\right\}$

- Conditional dynamics, *given the measurement outcomes* (unnormalized)

$$\rho_{X_t|\zeta_t} = \mathscr{E}_{z_t}(\rho_{X_{t-1}|\zeta_{t-1}}) = \mathrm{tr}_{Y_t}\left\{ M_{z_t} U_t\left(\rho_{X_{t-1}|\zeta_{t-1}} \otimes \rho_{Y_t}\right) U_t^\dagger M_{z_t}^\dagger \right\}$$

$P(\zeta_t)$ = probability of a trajectory ζ_t.

$$\rho_{X_t} = \sum_{\zeta_t} P(\zeta_t)\ \rho_{X_t|\zeta_t}$$

✤ Quantum-classical conditional entropy:

$$S(X_t|\zeta_t) = \sum_{\zeta_t} P(\zeta_t)\ S(\rho_{X_t|\zeta_t})$$

Holevo information

$$I(X_t : \zeta_t) = S(X_t) - S(X_t | \zeta_t) = \sum_{\zeta_t} P(\zeta_t) D\left(\rho_{X_t|\zeta_t} || \rho_{X_t}\right) \geqslant 0$$

- Represents the *net* amount of information gained about X over the entire trajectory ζ_t.
- Change can have any sign: $\Delta I_t = I(X_t : \zeta_t) - I(X_{t-1}, \zeta_{t-1}) \lesseqgtr 0$.
- Further split into a Holevo information gain/loss:

$$\Delta I_t = G_t - L_t$$

$$G_t = I(X_t : z_t | \zeta_{t-1}) = I(X_t : \zeta_t) - I(X_t : \zeta_{t-1}) \geqslant 0$$

$$L_t = I(X_{t-1} : \zeta_{t-1}) - I(X_t : \zeta_{t-1}) \geqslant 0$$

Informational steady-state:

$$\Delta I_{ISS} = 0$$

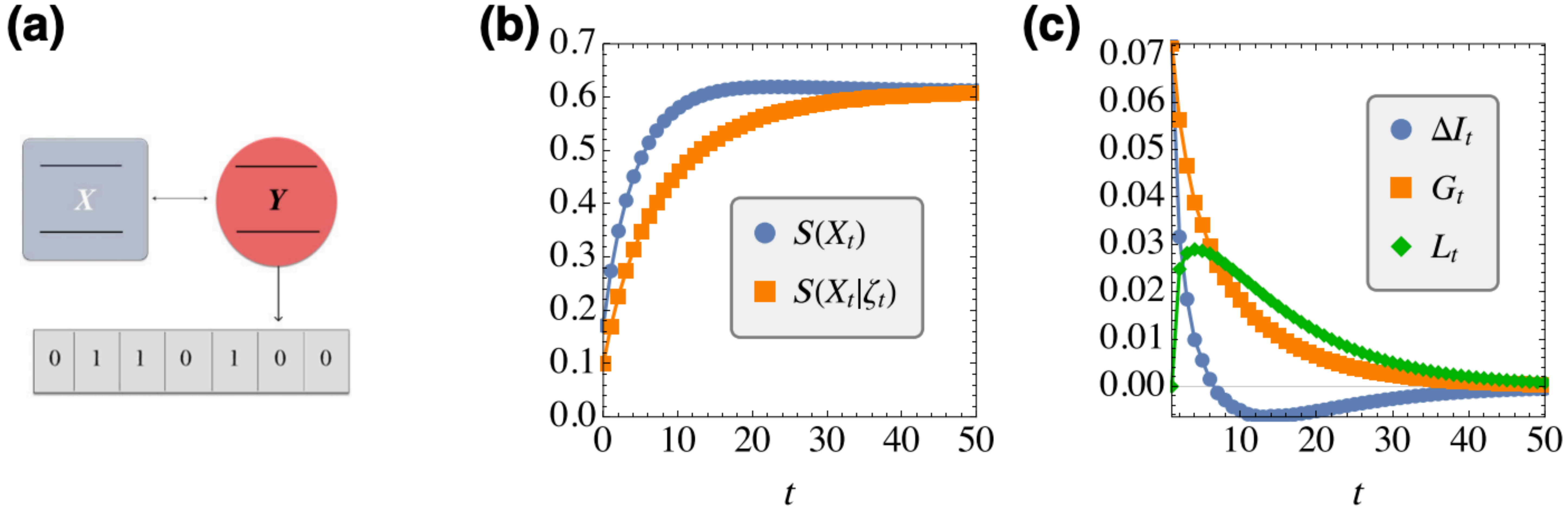
but

$$G_{SS} = L_{SS} \neq 0.$$

Examples

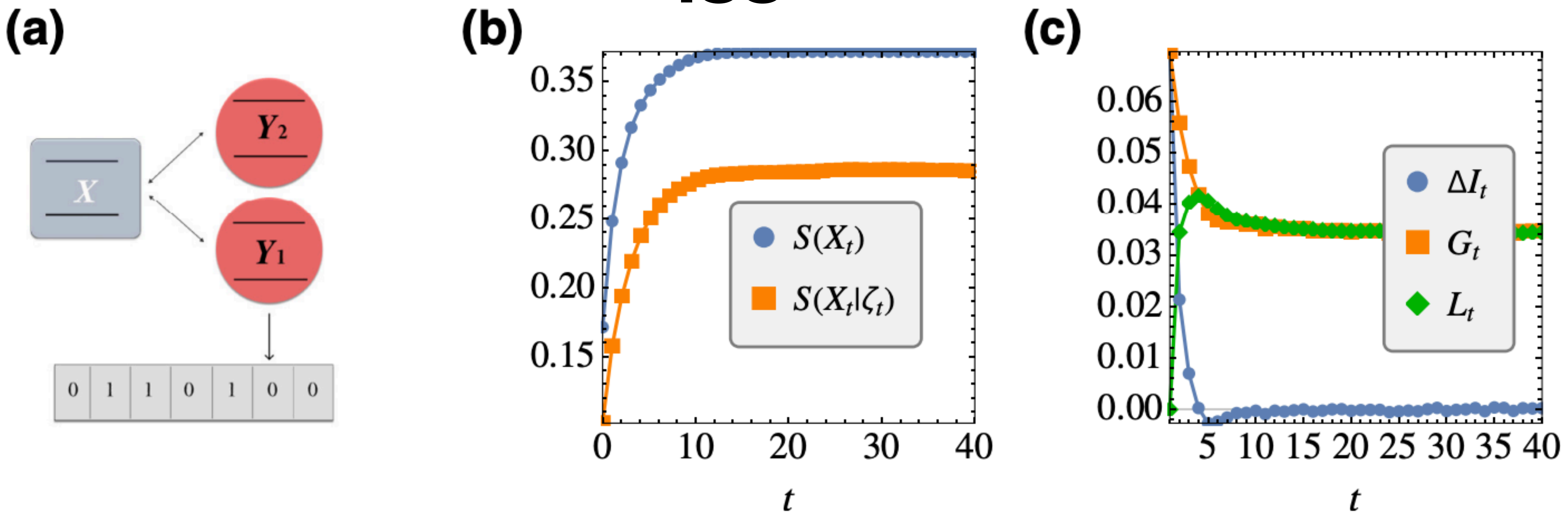
- Long-time limit no additional information is acquired.

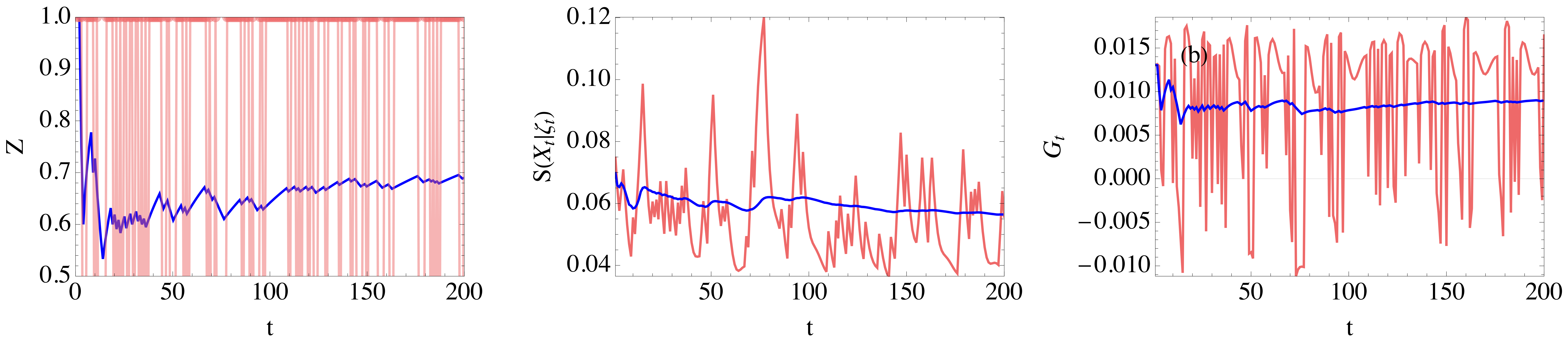
No ISS



- Measurements continue to gain information.
  - But gain & loss balance out.

ISS





- Blue = accumulated average.
- Z is the measurement outcome.
- System reaches an ISS because G_t tends to a finite value on average.

Optomechanical experiment 🇩🇰

Information gain/loss rates characterizing the information steady-state

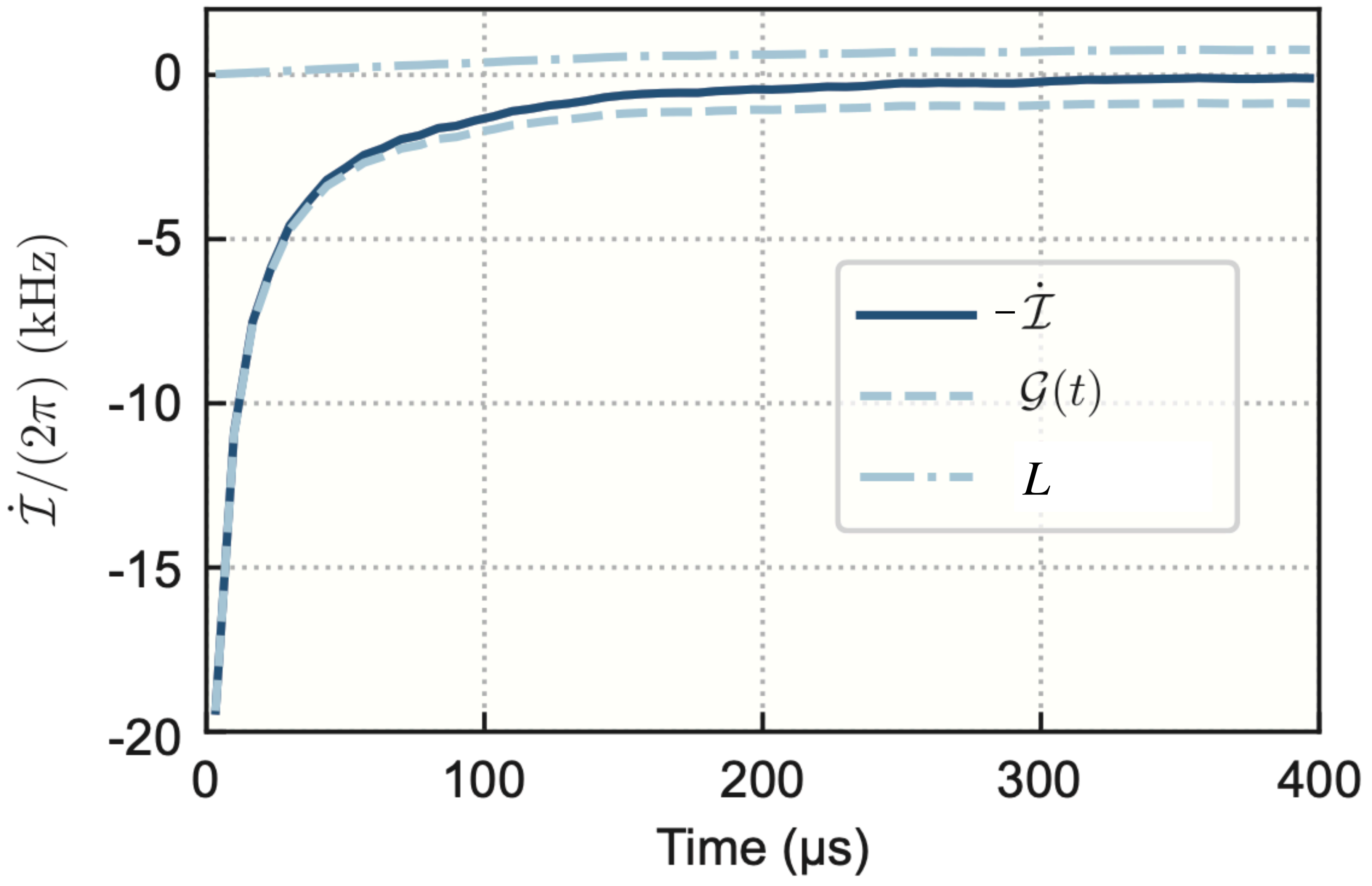
- Unconditional dynamics: interplay between measurement backaction and thermal bath:

$$\sigma_u := \langle \Delta x^2 \rangle = \bar{n} + 1/2 + \Gamma_{\mathtt{qba}}/\Gamma_m$$

- Stochastic evolution of 1st & 2nd moments:

$$\frac{dx_c}{dt} = -\frac{\Gamma_m}{2} x_c + \sqrt{4\eta\Gamma_{\mathtt{qba}}}\sigma_c(t)\xi(t)$$

$$\frac{d\sigma_c}{dt} = \Gamma_m(\sigma_u - \sigma_c) - 4\eta\Gamma_{\mathtt{qba}}\sigma_c^2$$



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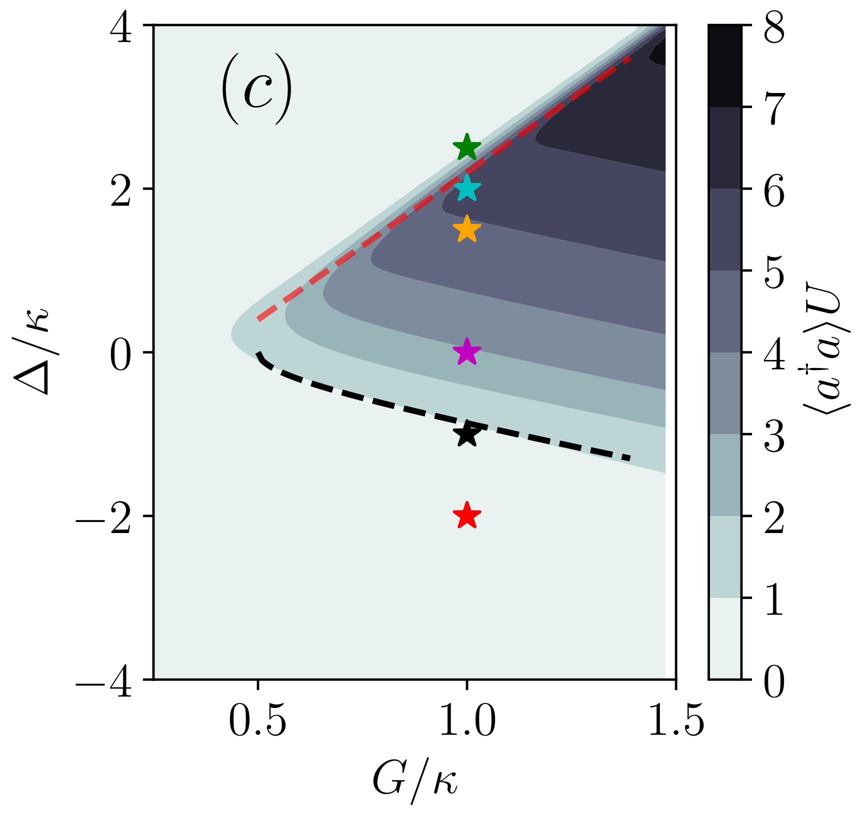
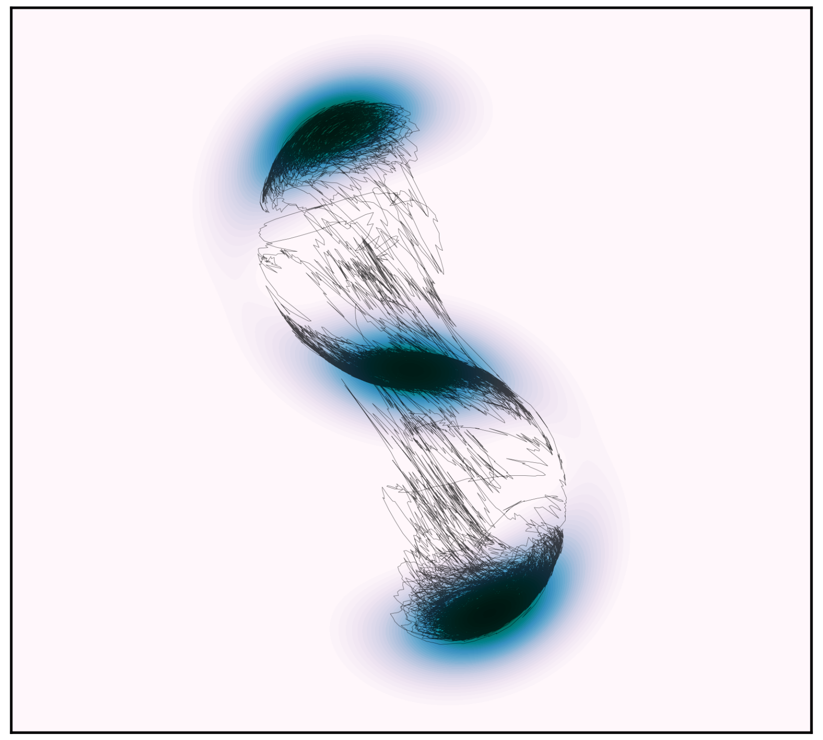
"**Experimental assessment of entropy production in a continuously measured mechanical resonator**"
Phys. Rev. Lett. 125, 080601 (2020)

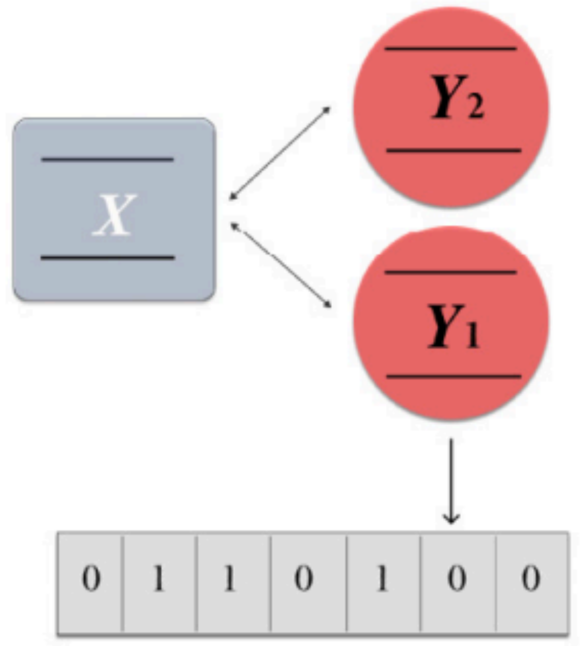
Conclusions

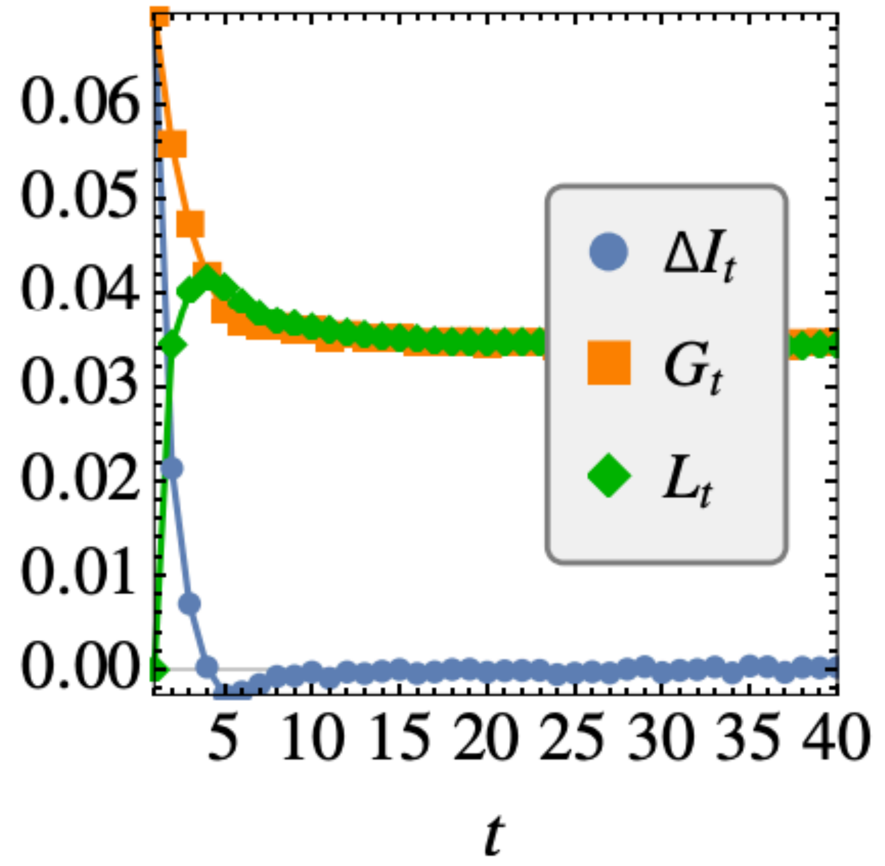
- Fluctuations in the time domain:
  - NESS beyond averages
- Connection between quantum optics and full counting statistics.
- Parametric Kerr model: critical properties of the fluctuations shed light on the Nature of the transitions
  - Photo-detection: exponential divergence in the discontinuous transition.
  - Homodyne: exponential divergence in the entire critical region.





- Information gain/loss in continuous measurements.
  - Modeled using a Collision Model.
- Holevo information: cumulative knowledge we acquired about the system.
- Decomposable into Gain - Loss.
- Optomechanical experiment.





University of Rochester

Post-doc positions 2023

My group is moving to UofR next year, and we have open post-doc positions to work on

- theory of quantum thermodynamics
- open quantum systems
- quantum information

For more information:

www.fmt.if.usp.br/~gtlandi

Thank you.

Quantum Thermodynamics
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Quantum Transport

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