Current fluctuations in critical Kerr resonators

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Current fluctuations & two-time correlations

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Current fluctuations in nonequilibrium discontinuous phase transitions
ArXiv 2109.00385

M. Kewming, M. Mitchison & GTL,
“Diverging current fluctuations in critical Kerr resonators”,
arXiv 2205.02622.

GTL, M. Kewming, M. Mitchison, P. Potts,
“Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics.”
Tutorial, in preparation

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Overview

• Continuously measured quantum system.
• Ex: optical cavity
  • Monitor the photons that leak out.
    ➡ **Classical stochastic currents (time-series)**

• Quantum jumps: discrete current → individual clicks in the detector.
• Quantum diffusion: continuous (noisy) current.

![Diagram showing quantum jumps and quantum diffusion](image)
Motivation

• In many experiments, this is the only way to probe it.
  • What information does the current convey about the system?

• Basic quantity: average current $J = \langle I(t) \rangle$.
  • But the current is stochastic: the full signal has a lot more information than the average.
    • Current fluctuations & two-time correlations: outcomes are not independent.
Why study current fluctuations

• Metrology:
  • Quantum system as a sensor.
    • e.g. magnetic fields.
    • e.g. gravitational waves (LIGO)
      • LIGO in fact uses exactly the present setup, with optical cavities.
  * Some information is only contained in the correlations!

• Thermo-kinetic uncertainty relations:

\[ \frac{D}{J^2} \geq \frac{2}{\sigma} \]

Current fluctuations (noise) \( \rightarrow \) Entropy production (a measure of dissipation)

* Counterintuitive:
To reduce fluctuations, we must increase dissipation.
Parametric Kerr model
Parametric Kerr model

- Non-linear quantum harmonic oscillator:

\[
\frac{d\rho}{dt} = -i[H(t), \rho] + \kappa \left[ a \rho a^\dagger - \frac{1}{2} \{ a^\dagger a, \rho \} \right]
\]

\[
H = -\Delta a^\dagger a + \frac{U}{2} a^\dagger a a a + \frac{G}{2} (a^\dagger 2 + a^2)
\]

- \(a\) = annihilation operator
- photon operator for an optical cavity
- \(\Delta = \omega_p - \omega_c\) = detuning
- \(U\) = Kerr non-linearity.
  (requires a non-linear crystal inside the cavity)
- \(G\) = 2-photon pump
  (input laser produces photons in pairs)
- \(\kappa\) = loss rate
  rate at which photons leak out of the cavity
• 2 phase transitions, continuous and discontinuous

• Proper criticality occurs in the limit $U \to 0$ ("thermodynamic limit")
Cat qubits

• Steady-state is a mixture of two Schrödinger cat states

\[ |S\rangle = |\alpha\rangle + | - \alpha\rangle \]

\[ |A\rangle = |\alpha\rangle - | - \alpha\rangle \]

• Use this to define cat qubits:

\[ |0\rangle = |\alpha\rangle \]

\[ |1\rangle = | - \alpha\rangle \]

• Cat qubits are more robust against errors.
  • Quantum computing with Kerr cats.

Semiclassical model

- Mean-field approximation: complex variable $\alpha(t)$

\[
\frac{d\alpha}{dt} = -(\kappa - i\Delta)\alpha - \frac{iU}{2}|\alpha|^2\alpha - iG\alpha^*
\]

- Pitchfork bifurcation at critical detuning $\Delta_c = -\sqrt{G^2 - \kappa^2/4}$

\[
\alpha^* = \pm \sqrt{n_0}e^{i\phi_0}
\]

\[
n_0 = \frac{\Delta}{U} + \sqrt{\frac{G^2 - \kappa^2/2}{U^2}}, \quad \phi_0 = \frac{1}{2}\arcsin\left(\frac{-\kappa}{2G}\right)
\]

Can predict the continuous transition (bottom)

Unable to predict the discontinuous transition (top).
Photo-detection current
How to simulate the stochastic photo-detections?

- Quantum jump unravelling.
- At each time step $dt$ a jump occurs with probability $\kappa dt \langle a^\dagger a \rangle$.
  - ($\propto dt$: very unlikely)
- If jump occurs: $|\psi\rangle \rightarrow a |\psi\rangle$
- If no jump occurs: $|\psi\rangle = e^{-iH_{\text{eff}}t} |\psi\rangle$

$H_{\text{eff}} = H - \frac{\kappa}{2} a^\dagger a$

(non-Hermitian Hamiltonian)

Every time a jump occurs, we count $dN = 1$

Stochastic current:

$$I(t) = \frac{dN}{dt}$$
Photo-detection current

- @ discontinuous transition: on/off (telegraph) behavior of the current.
- Photo-detection cannot resolve upper vs. lower blobs.

\[ J = \langle I \rangle \text{ = "dynamical activity" = jumps/second} \]
Current fluctuations - Full Counting Statistics

- Two-time correlation function:

\[ F(\tau) := \langle I(t)I(t+\tau) \rangle - J^2 \]
\[ = J \delta(\tau) + J^2 \left[ g^{(2)}(\tau) - 1 \right] \]

- Power spectrum:

\[ S(\omega) = \int_{-\infty}^{\infty} e^{-i\omega \tau} F(\tau) d\tau \]

- Zero-frequency component of the power spectrum := “noise”:

\[ D = S(0) = \lim_{t \to \infty} \frac{d}{dt} \text{Var}(N(t)) \]

\[ g^{(2)} = \text{Glauber's 2nd order coherence function} \]
Divergence of the diffusion coefficient

- “Thermodynamic limit:” $U \to 0$
- In the discontinuous transition ($\Delta > 0$)

$$D \sim e^{1/U}$$
GaAs cavity polaritons.

Homodyne detection
Homodyne current

- Mix photon output with a strong laser source $\alpha = |\alpha| e^{i\phi}$.

- Equivalent to measuring

$$\langle (a + \alpha)^\dagger (a + \alpha) \rangle = \left( |\alpha|^2 + |\alpha| \langle a e^{-i\phi} + a^\dagger e^{i\phi} \rangle + \langle a^\dagger a \rangle \right)$$

- $|\alpha|^2$ is just a constant offset.

- If $\alpha$ is large, then the current will predominantly measure $\alpha x$ instead of $a^\dagger a$.

- Quantum diffusion unravelling:

$$d\rho = dt \mathcal{L} \rho + dW \left[ \mathcal{H} \rho - \langle x \rangle \rho \right],$$

$$\mathcal{H} \rho = \kappa (a \rho + \rho a^\dagger)$$

$$dW = \text{Wiener increment:}$$

$$E(dW) = 0, \quad dW^2 = dt$$
Homodyne current (in $\mathbf{p} = i(\mathbf{a}^\dagger - \mathbf{a})$)

- The homodyne current switches between 3 values (+,0,-).
- Captures the tunneling between the 3 blobs.
Divergence of the diffusion coefficient

- Homodyne current noise diverges exponentially in a much broader region.

\[ D \sim e^{1/U} \]

- Reflects sensitivity to all 3 blobs.
Metrology

- Phase transition makes current very sensitive to changes in the parameters.
- We can use this as a sensor.
- \( \Delta = \omega_p - \omega_c \)
  - Dependence on the cavity frequency \( \omega_c \): affected by multiple processes.
  - Dependence on the pump frequency \( \omega_p \): sensitive sensing of input frequency.
- Single-photon detector.
Waiting time metrology/thermometry

- The time between clicks encodes information on the system.
- We can use this as a thermometer.
- Ex: quantum dot continuously monitored using a quantum-point contact.

Summary

- Fluctuations in the time domain:
  - Classical time series produced by a quantum system.

- Connection between quantum optics and full counting statistics.

- Parametric Kerr model: critical properties of the fluctuations shed light on the nature of the transitions
  - Photo-detection: exponential divergence in the discontinuous transition.
  - Homodyne: exponential divergence in the entire critical region.
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