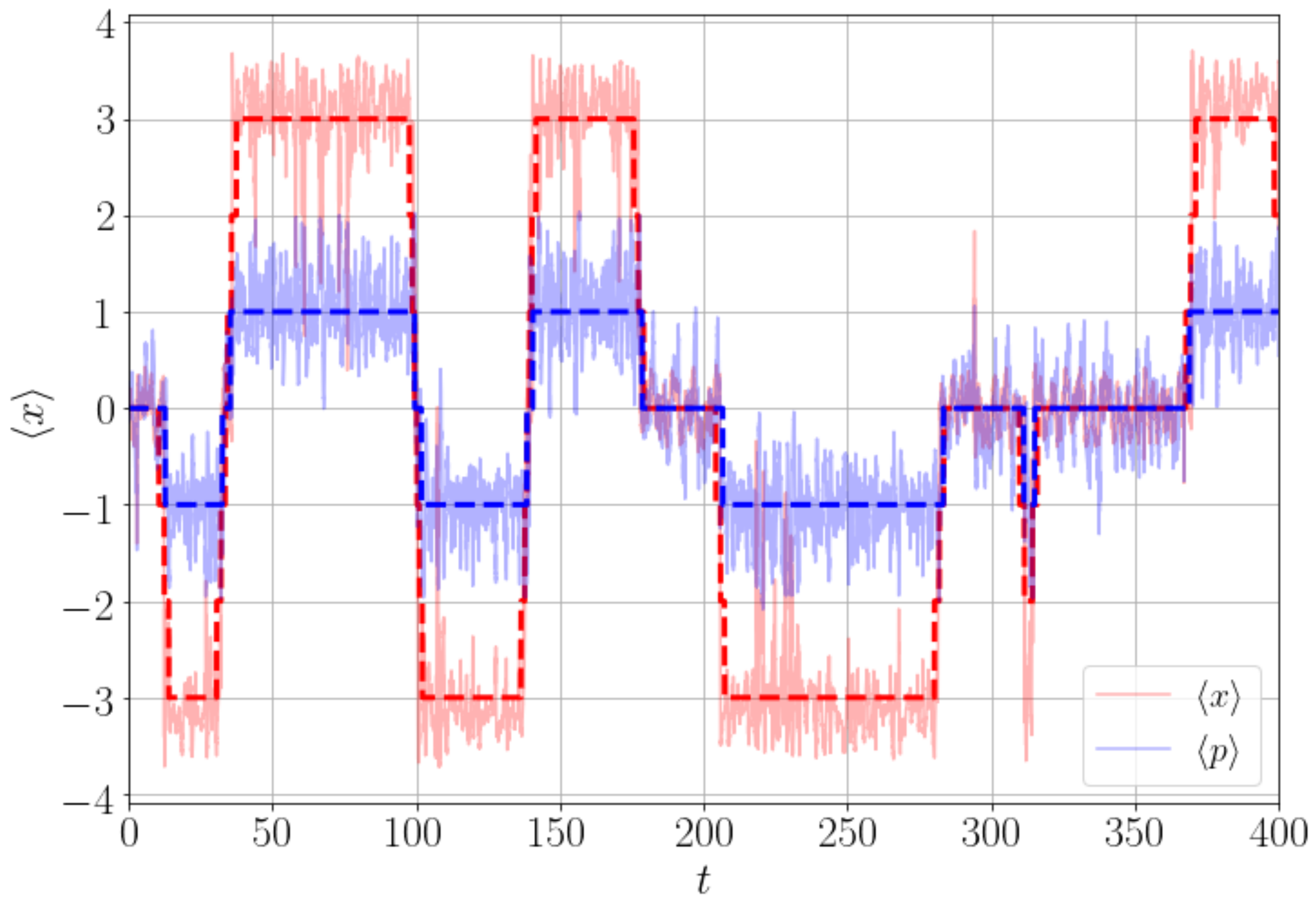
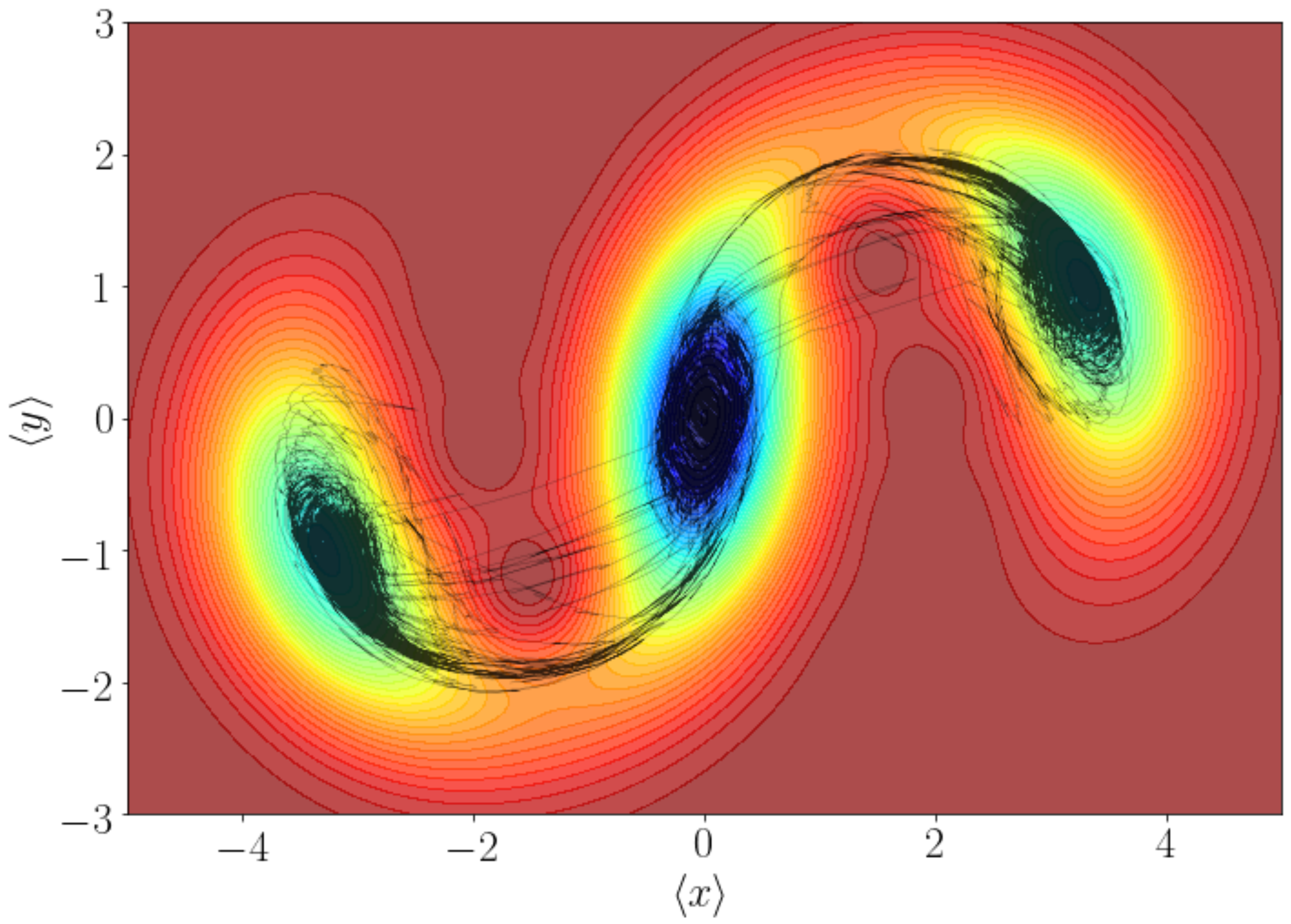




Current fluctuations in critical Kerr resonators

Gabriel T. Landi

University of São Paulo / University of Rochester

Nov. 2022

www.fmt.if.usp.br/~gtlandi

Current fluctuations & two-time correlations

C. E. Fiore, Pedro E. Harunari, C. E. Fernandez Noa, and GTL
Current fluctuations in nonequilibrium discontinuous phase transitions
ArXiv 2109.00385

M. Kewming, M. Mitchison & GTL,
“**Diverging current fluctuations in critical Kerr resonators**”,
arXiv 2205.02622.

GTL, M. Kewming, M. Mitchison, P. Potts,
“**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics.**”
Tutorial, in preparation


Pedro Harunari


Carlos Fiore


Patrick Potts

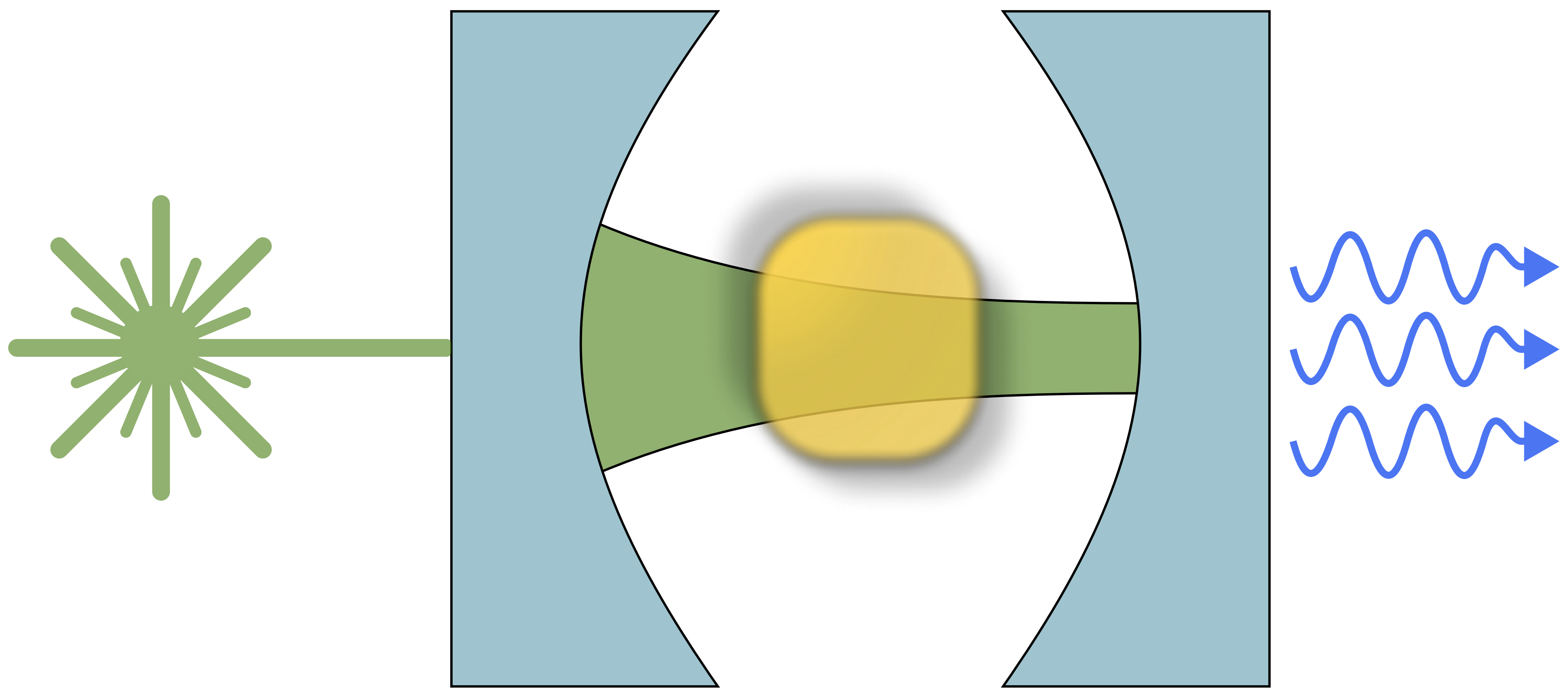

Michael Kewming


Mark Mitchison

Overview

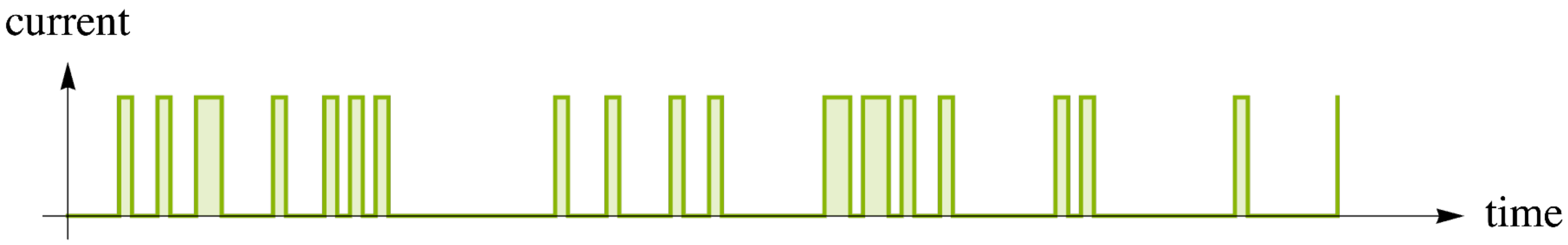
- Continuously measured quantum system.
- Ex: optical cavity
  - Monitor the photons that leak out.

    ➡ ***Classical stochastic currents (time-series)***

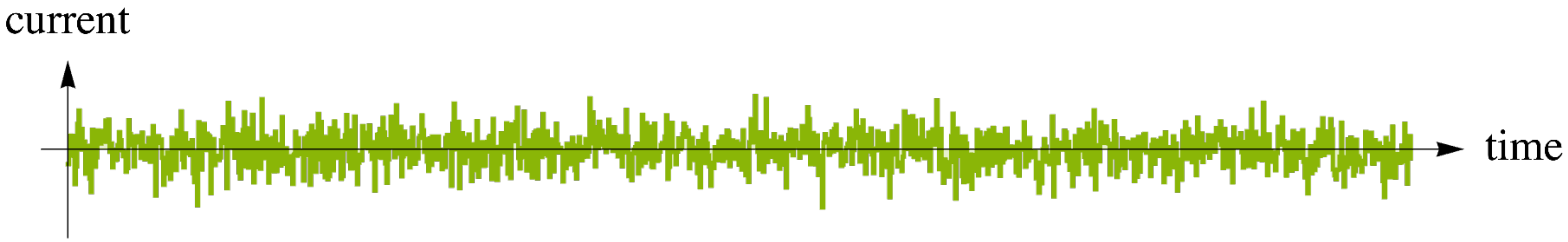


- Quantum jumps: discrete current → individual clicks in the detector.
- Quantum diffusion: continuous (noisy) current.

Classical outcomes describing an underlying quantum system



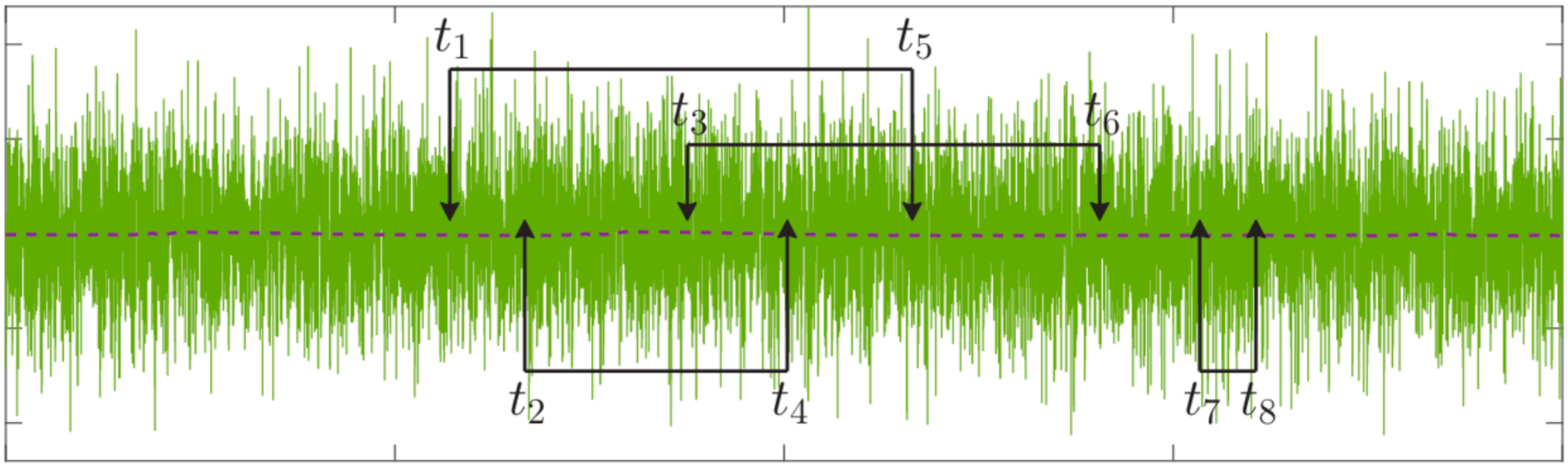
Quantum jumps



Quantum diffusion

Motivation

- In many experiments, this is the only way to probe it.
  - **What information does the current conveys about the system?**
- Basic quantity: average current $J = \langle I(t) \rangle$.
  - **But the current is stochastic:** *the full signal has a lot more information than the average.*
    - Current fluctuations & two-time correlations: outcomes are not independent.



Why study current fluctuations

- **Metology:**
    - Quantum system as a sensor.
        - e.g. magnetic fields.
        - e.g. gravitational waves (LIGO)
            - LIGO in fact uses exactly the present setup, with optical cavities.
    - ✱ Some information is only contained in the correlations!
- **Thermo-kinetic uncertainty relations:**

Current fluctuations (noise) ⟶

$$\frac{D}{J^2} \geqslant \frac{2}{\sigma}$$

Entropy production (a measure of dissipation)

✱ Counterintuitive:

To reduce fluctuations, we must increase dissipation.

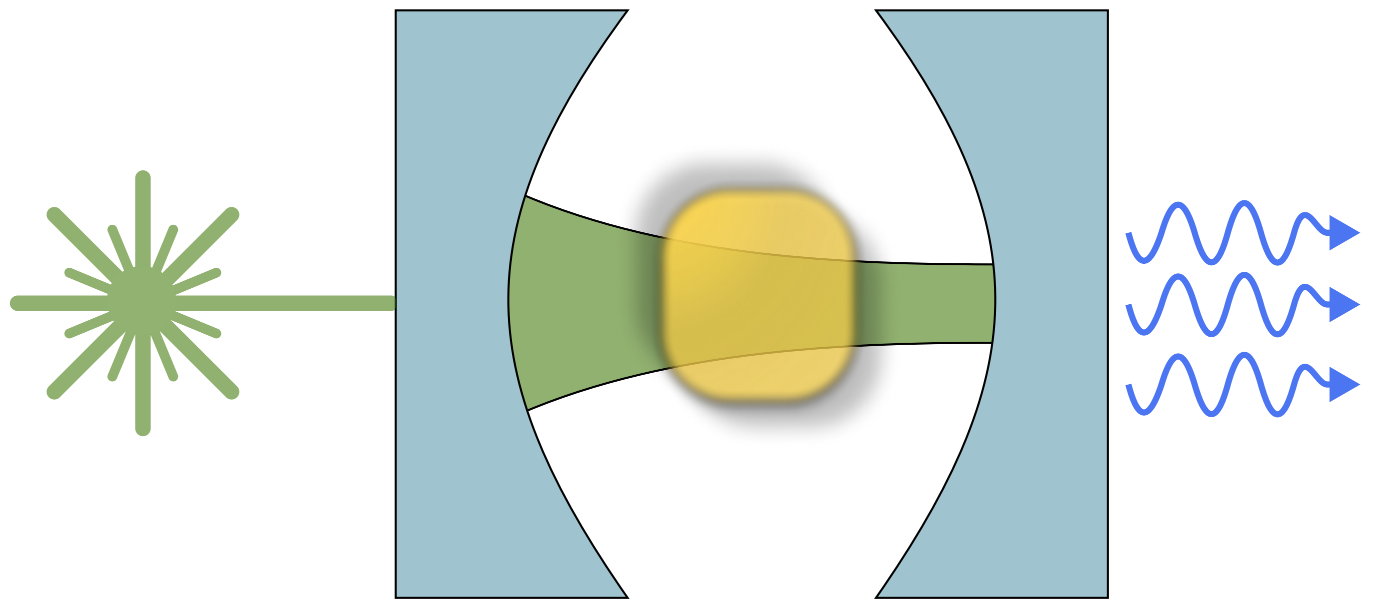
Parametric Kerr model

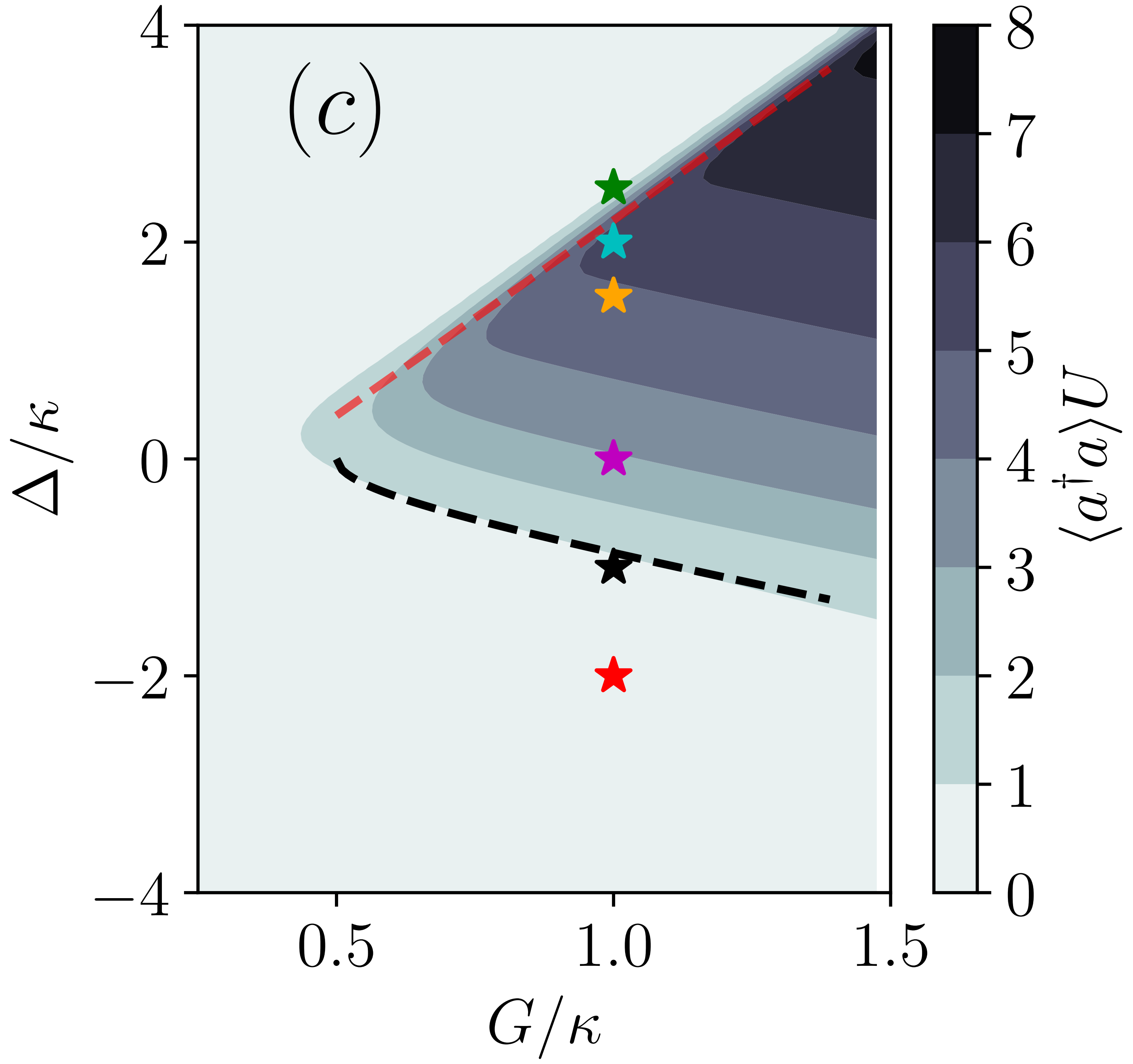
Parametric Kerr model

- Non-linear quantum harmonic oscillator:

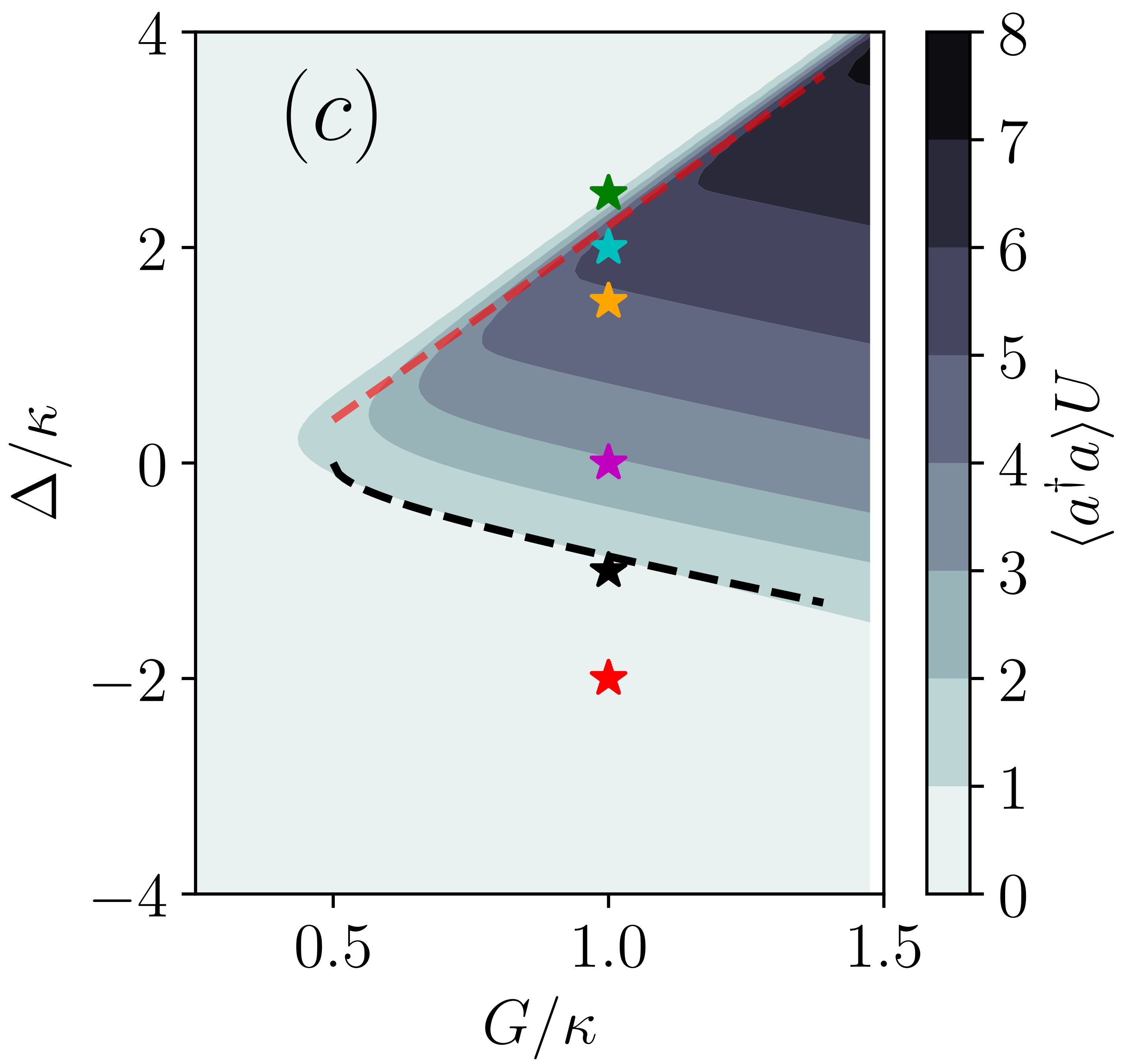
$$\frac{d\rho}{dt} = -i[H(t),\rho] + \kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right]$$

$$H = -\Delta a^\dagger a + \frac{U}{2}a^\dagger a^\dagger a a + \frac{G}{2}(a^{\dagger 2} + a^2)$$





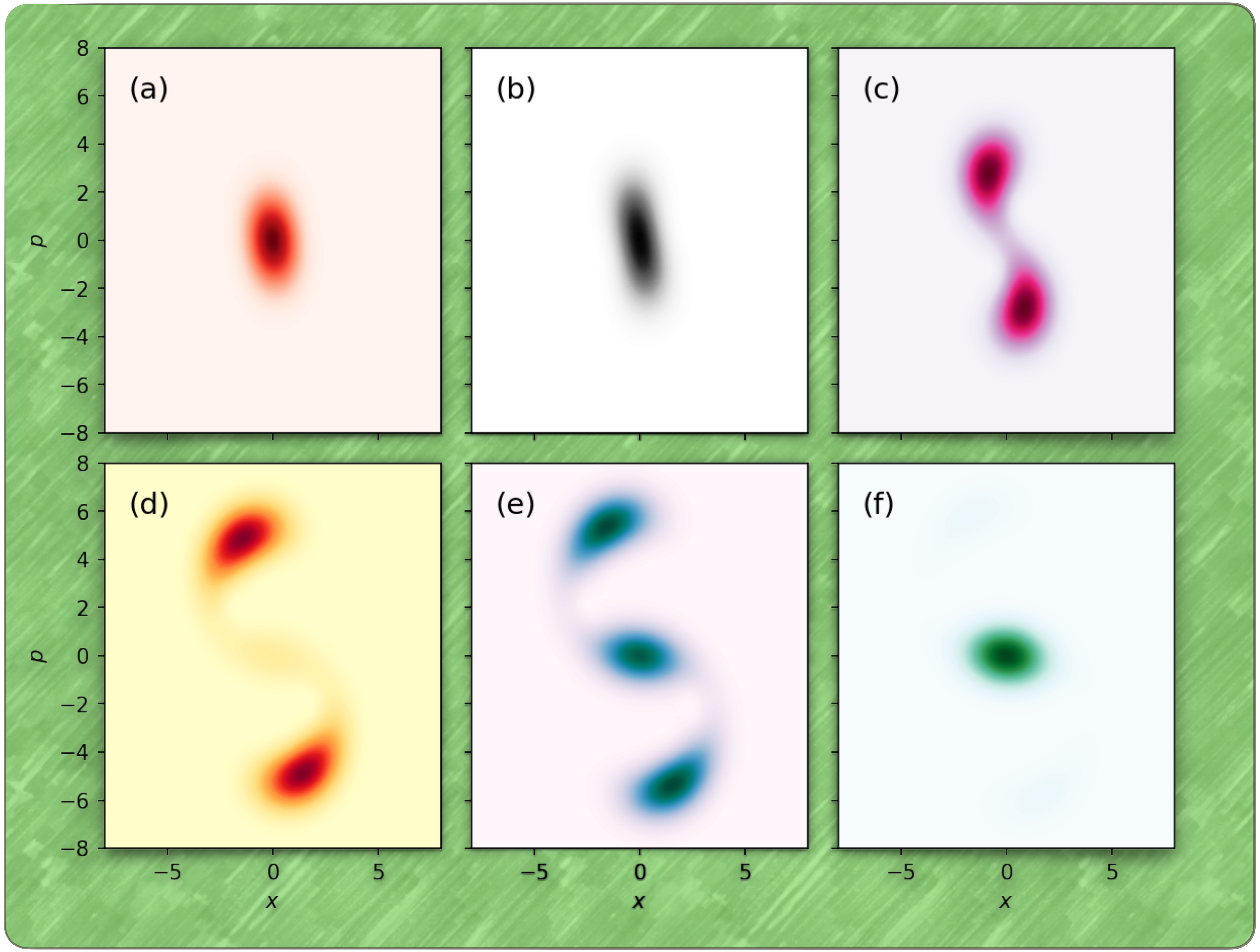
- a = annihilation operator photon operator for an optical cavity
- $\Delta = \omega_p - \omega_c$ = detuning
- U = Kerr non-linearity. (requires a non-linear crystal inside the cavity)
- G = 2-photon pump (input laser produces photons in pairs)
- κ = loss rate rate at which photons leak out of the cavity



- 2 phase transitions, continuous and discontinuous

- Proper criticality occurs in the limit $U \to 0$ ("thermodynamic limit")

Wigner function

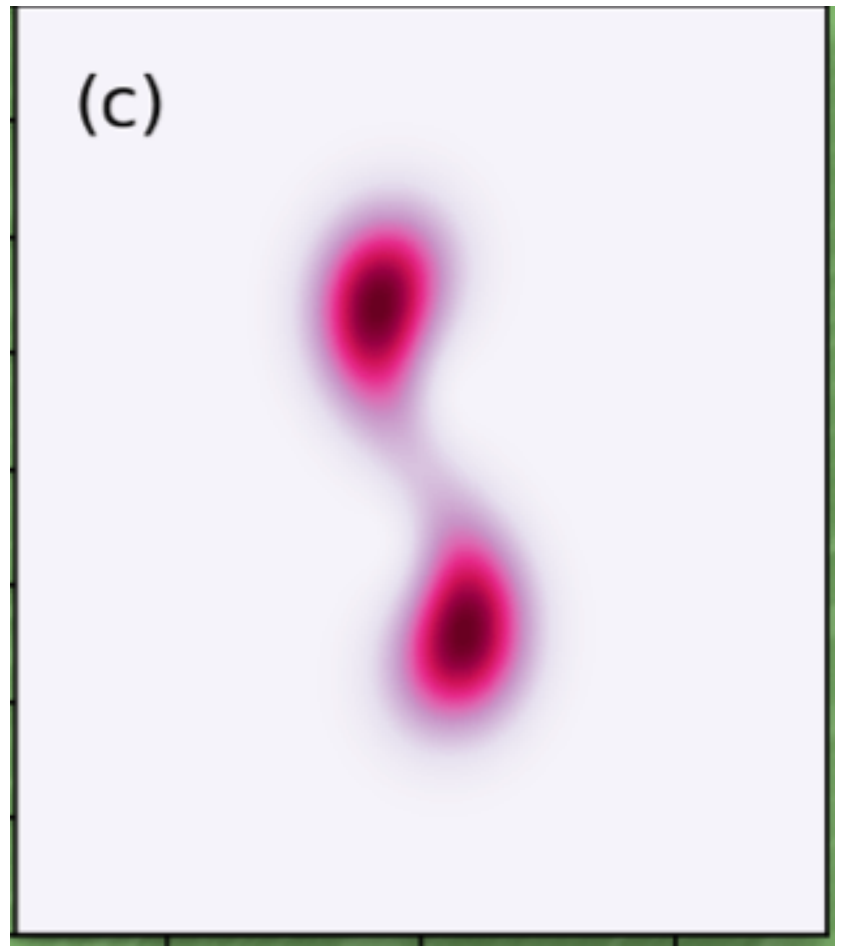


Cat qubits

- Steady-state is a mixture of two Schrödinger cat states

$$|S\rangle = |\alpha\rangle + |-\alpha\rangle$$

$$|A\rangle = |\alpha\rangle - |-\alpha\rangle$$

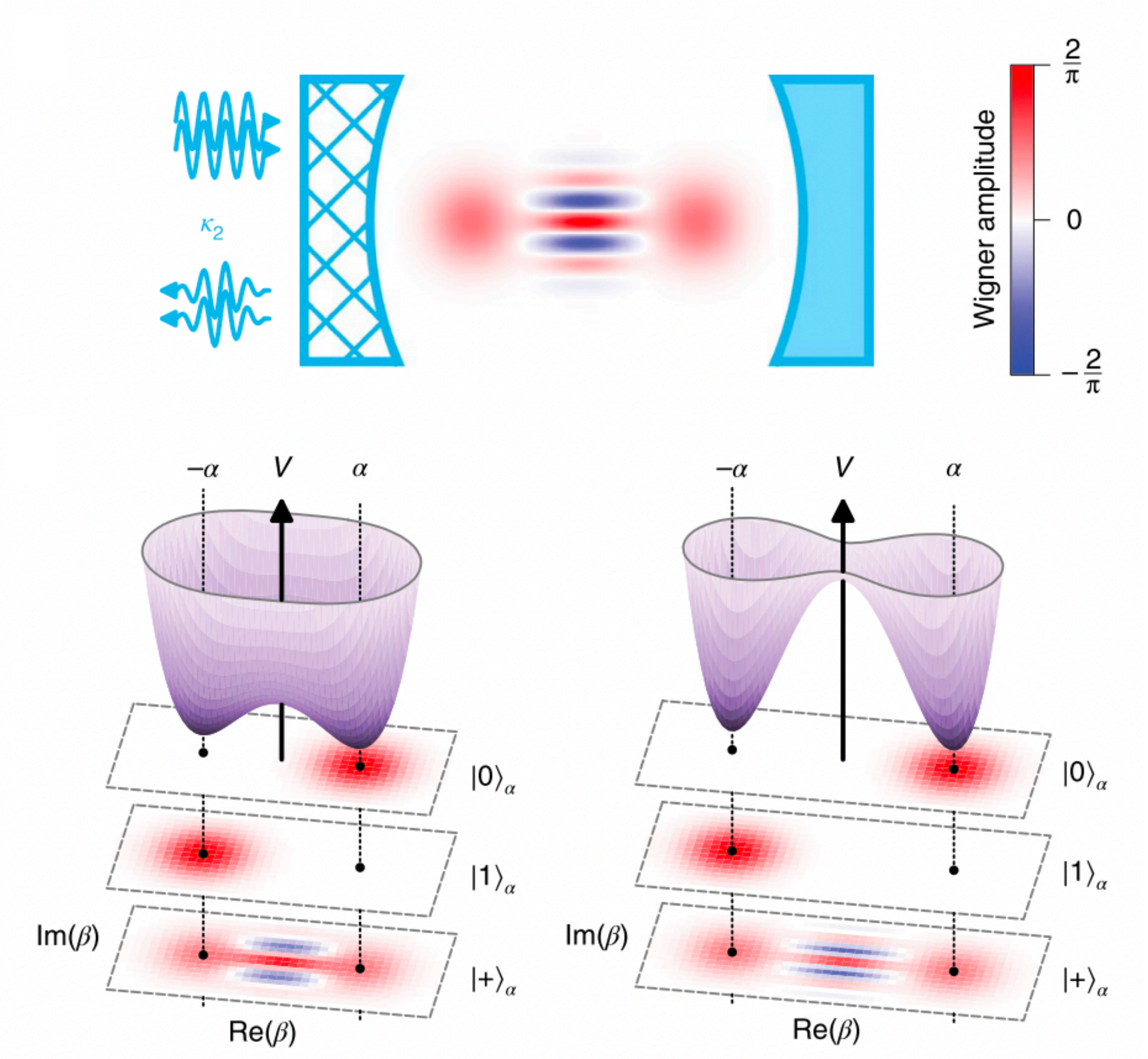


- Use this to define cat qubits:

$$|0\rangle = |\alpha\rangle$$

$$|1\rangle = |-\alpha\rangle$$

- Cat qubits are more robust against errors.
  - Quantum computing with Kerr cats.



Lescanne, *et. al.*, Nature, **16**, 509-513 (2020)

Semiclassical model

0D version of non-linear SEq. (Mininni)

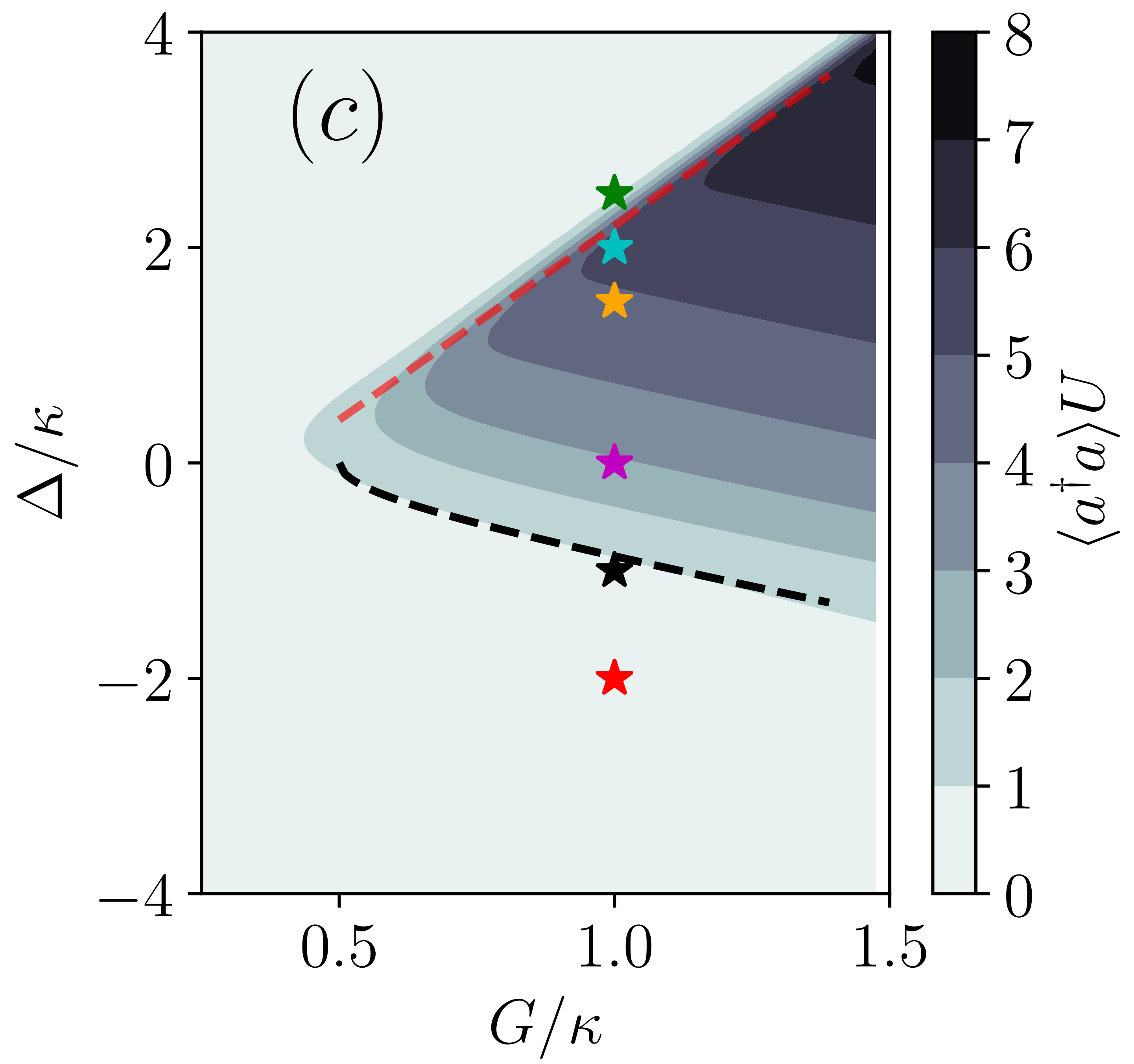
- Mean-field approximation: complex variable $\alpha(t)$

$$\frac{d\alpha}{dt} = -(\kappa - i\Delta)\alpha - \frac{iU}{2}|\alpha|^2\alpha - iG\alpha^*$$

- Pitchfork bifurcation at critical detuning $\Delta_c = -\sqrt{G^2 - \kappa^2/4}$

$$\alpha^* = \pm\sqrt{n_0}e^{i\phi_0}$$

$$n_0 = \frac{\Delta}{U} + \sqrt{\frac{G^2 - \kappa^2/2}{U^2}}, \qquad \phi_0 = \frac{1}{2}\arcsin\left(\frac{-\kappa}{2G}\right)$$



- Can predict the continuous transition (bottom)
- Unable to predict the discontinuous transition (top).

Photo-detection current

How to simulate the stochastic photo-detections?

- **Quantum jump unravelling.**
- At each time step dt a jump occurs with probability $\kappa dt\langle a^\dagger a\rangle$.
  - ($\propto dt$: very unlikely)
- If jump occurs: $|\psi\rangle \rightarrow a|\psi\rangle$
- If no jump occurs: $|\psi\rangle = e^{-iH_{\mathtt{eff}}t}|\psi\rangle$

Every time a jump occurs, we count $dN = 1$

Stochastic current:

$$I(t) = \frac{dN}{dt}$$

$$H_{\mathtt{eff}} = H - \frac{\kappa}{2}a^\dagger a$$

(non-Hermitian Hamiltonian)

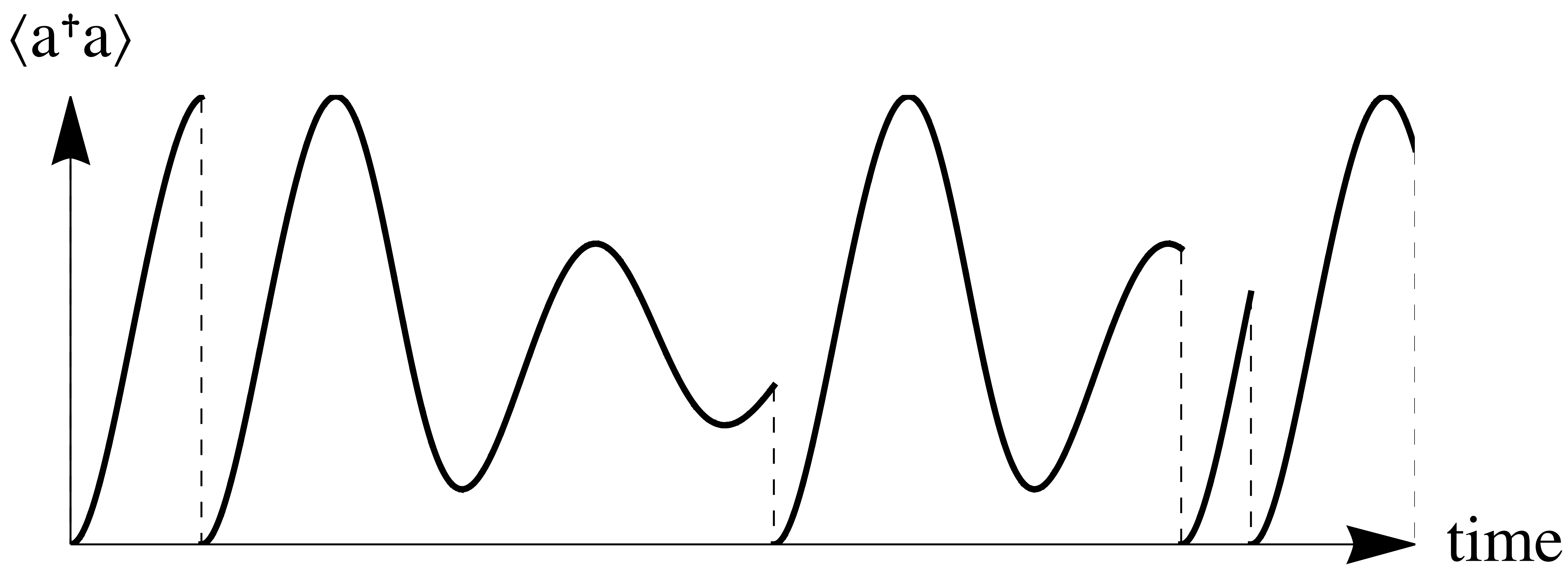
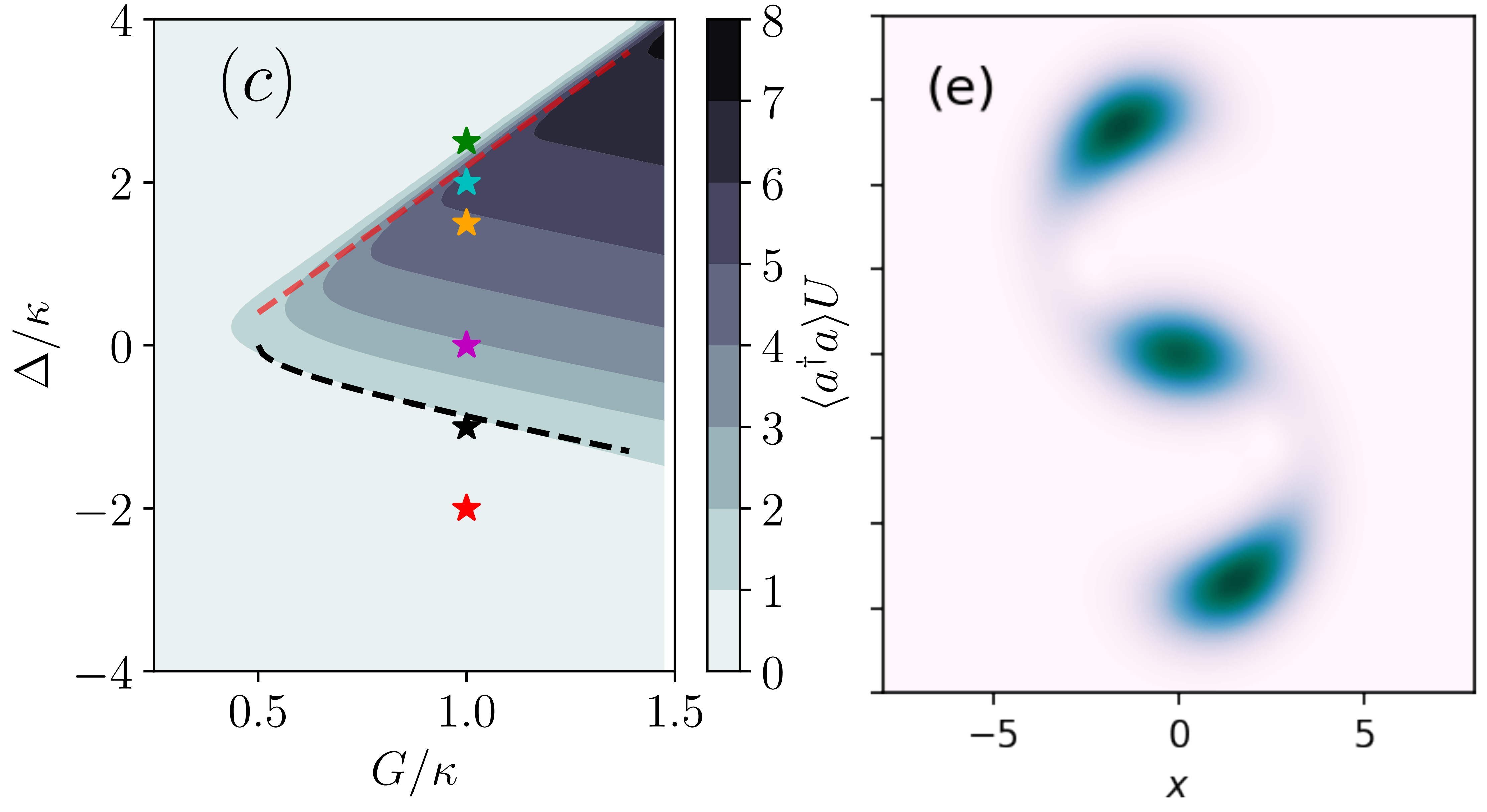
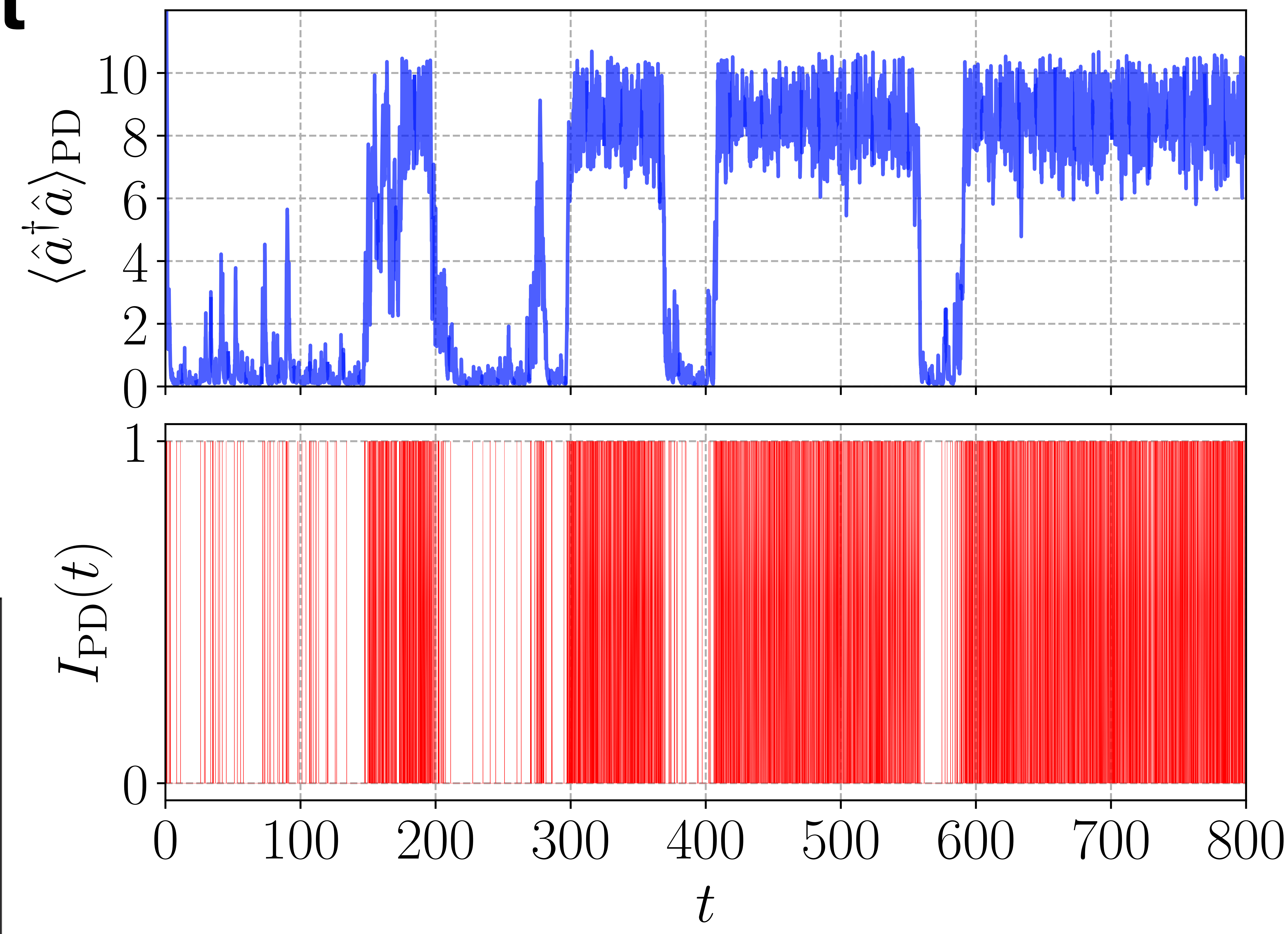


Photo-detection current

- @ discontinuous transition: on/off (telegraph) behavior of the current.
- Photo-detection cannot resolve upper vs. lower blobs.





$J = \langle I \rangle$ = "dynamical activity" = jumps/second

Current fluctuations - Full Counting Statistics

- Two-time correlation function:

$$F(\tau) := \left\langle I(t)I(t+\tau)\right\rangle - J^2$$

$$= J\,\delta(\tau) + J^2\left[g^{(2)}(\tau) - 1\right]$$

$g^{(2)}$ = Glauber's 2nd order coherence function

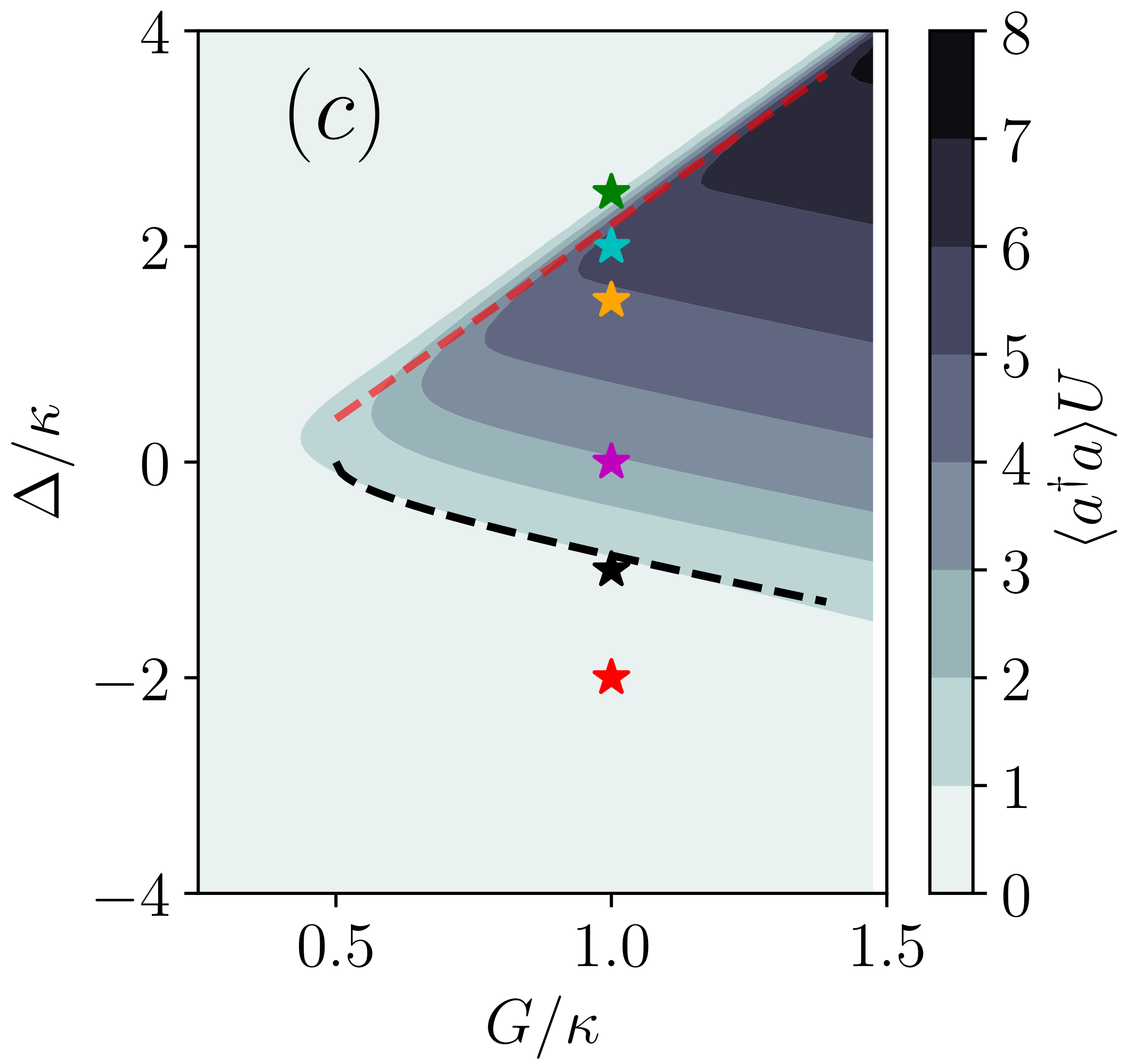
- Power spectrum:

$$S(\omega) = \int_{-\infty}^{\infty} e^{-i\omega\tau} F(\tau) d\tau$$

- Zero-frequency component of the power spectrum := "noise":

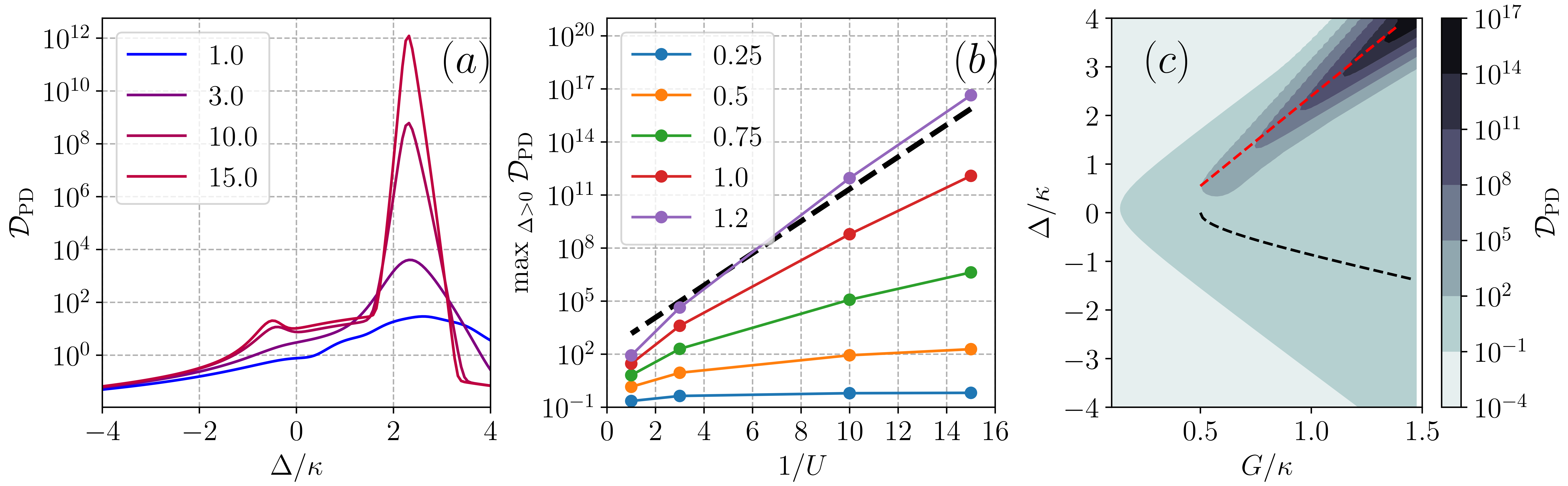
$$D = S(0) = \lim_{t\to\infty} \frac{d}{dt} \mathrm{Var}\big(N(t)\big)$$

Divergence of the diffusion coefficient

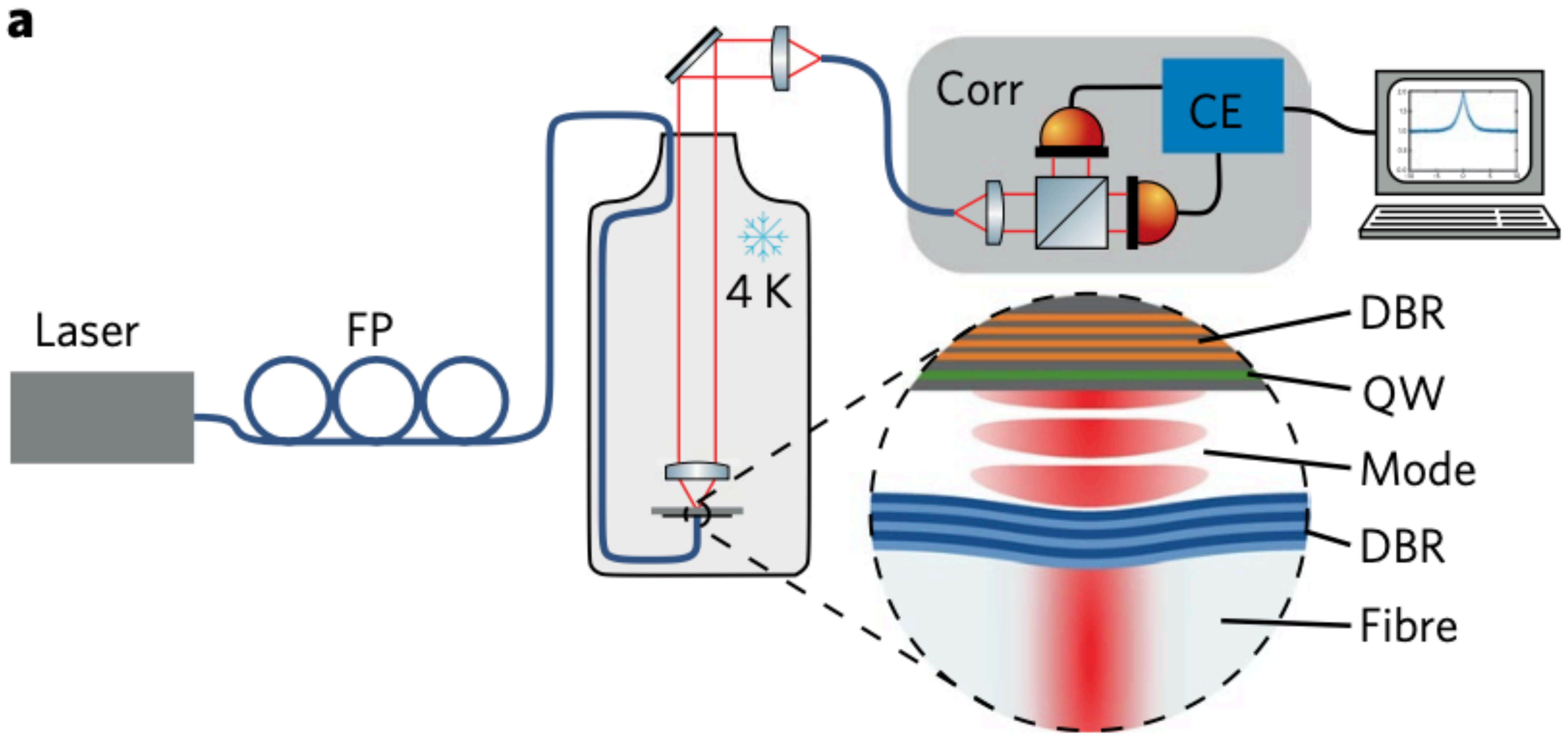


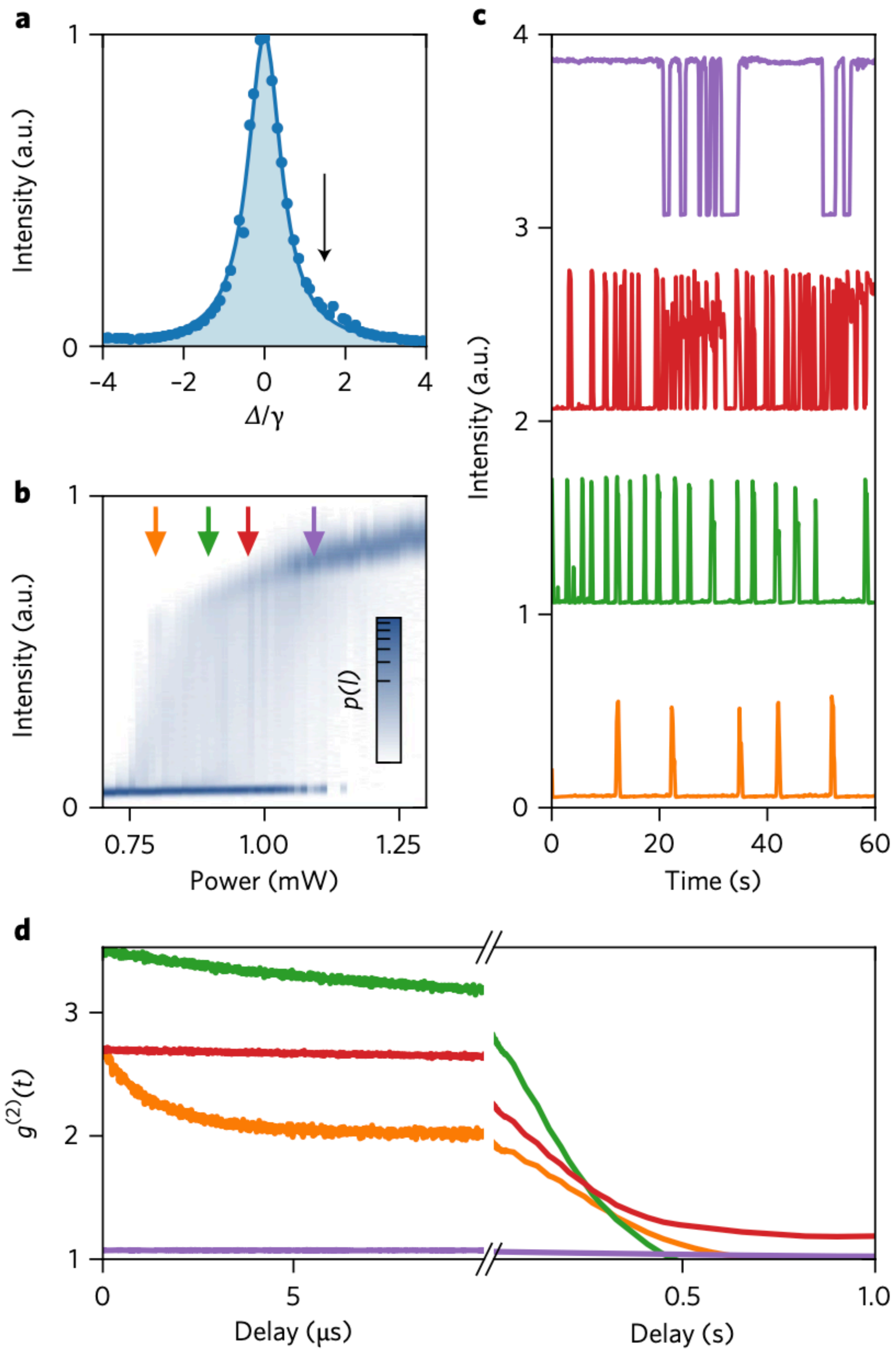
- "Thermodynamic limit:" $U \to 0$
- In the discontinuous transition ($\Delta > 0$)

$$D \sim e^{1/U}$$



GaAs cavity polaritons.





T. Fink, *et. al.*, *Nature Physics*, **14**, 365 (2018)

Homodyne detection

Homodyne current

- Mix photon output with a strong laser source $\alpha = |\alpha| e^{i\phi}$.
  - Equivalent to measuring

$$\langle (a+\alpha)^\dagger (a+\alpha)\rangle = \left(|\alpha|^2 + |\alpha| \langle a e^{-i\phi} + a^\dagger e^{i\phi}\rangle + \langle a^\dagger a\rangle \right)$$

    - $|\alpha|^2$ is just a constant offset.
    - If α is large, then the current will predominantly

$$x := a e^{-i\phi} + a^\dagger e^{i\phi}$$

instead of $a^\dagger a$.

- Quantum diffusion unravelling:

$$d\rho = dt \mathscr{L}\rho + dW \left[\mathscr{H}\rho - \langle x\rangle \rho \right],$$

$$\mathscr{H}\rho = \kappa\left(a\rho + \rho a^\dagger \right)$$

dW = Wiener increment:

$E(dW) = 0, \qquad dW^2 = dt$

Homodyne current (in $p = i(a^\dagger - a)$)

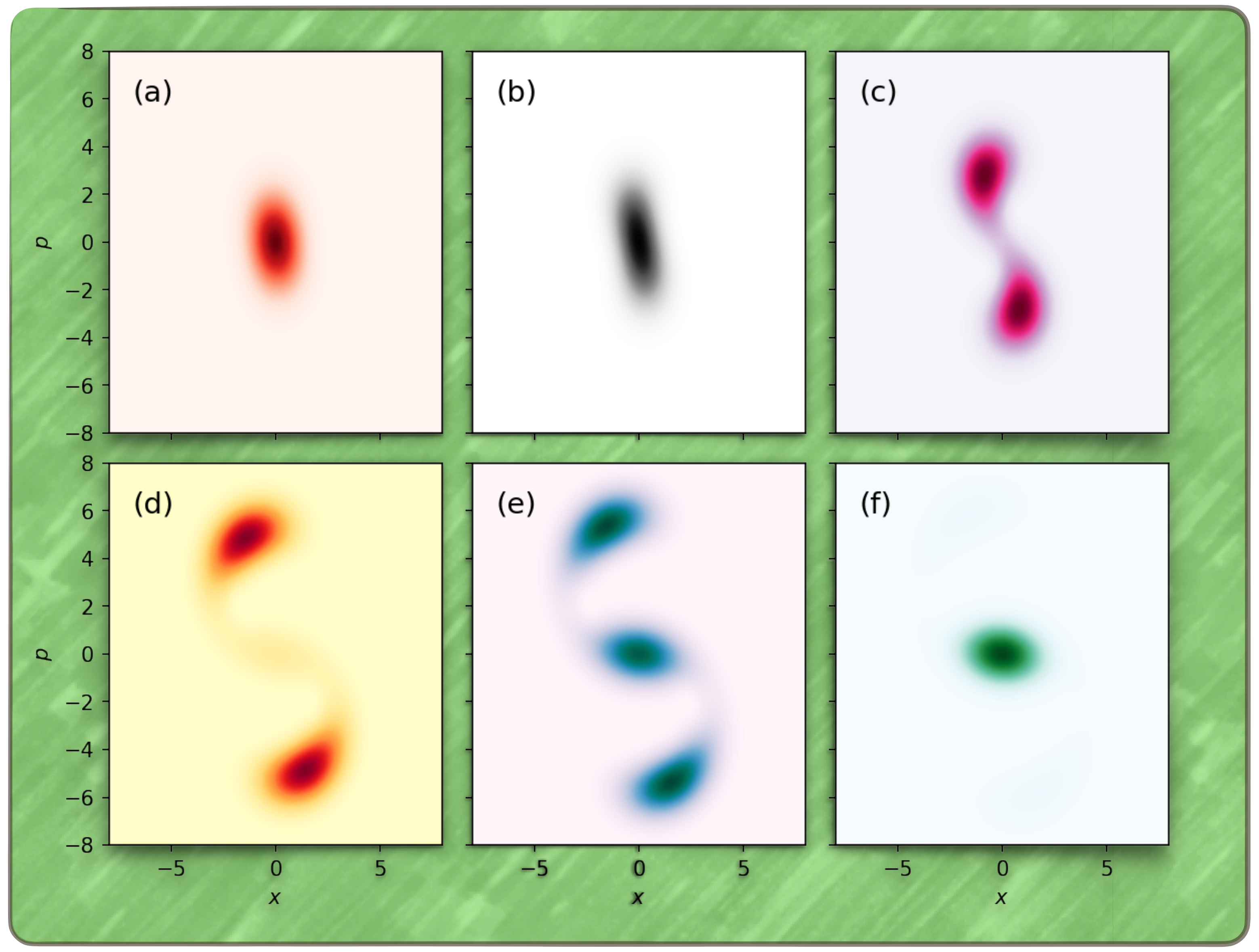
- The homodyne current switches between 3 values (+,0,-).
- Captures the tunneling between the 3 blobs.

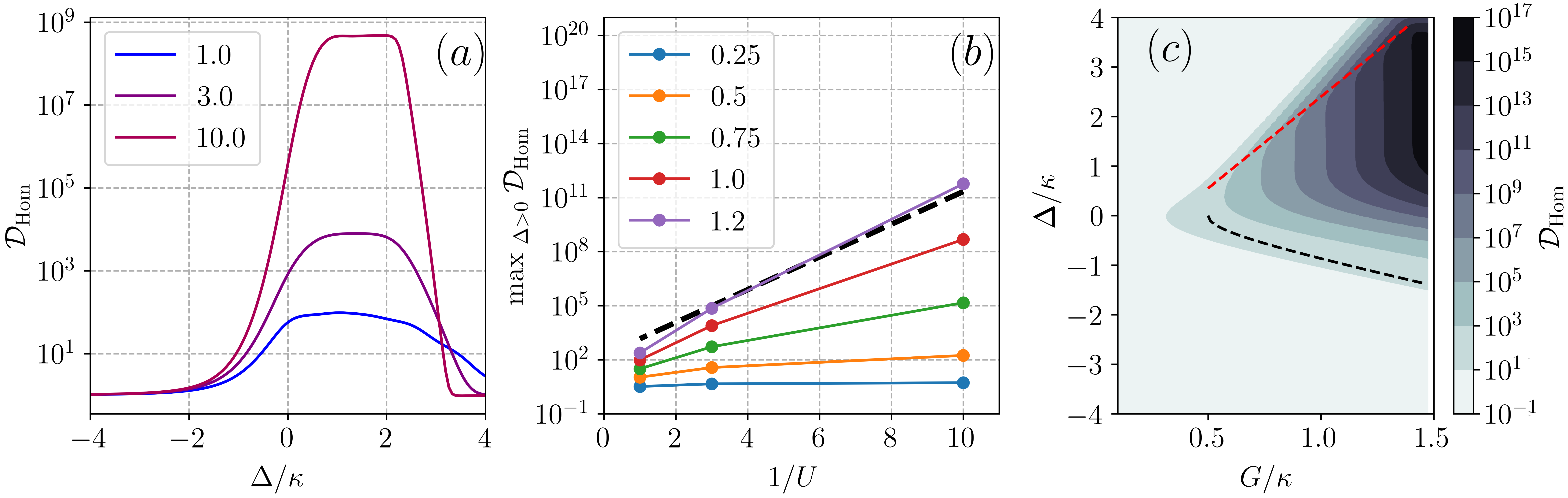
Divergence of the diffusion coefficient

- Homodyne current noise diverges exponentially in a much broader region.

$$D \sim e^{1/U}$$

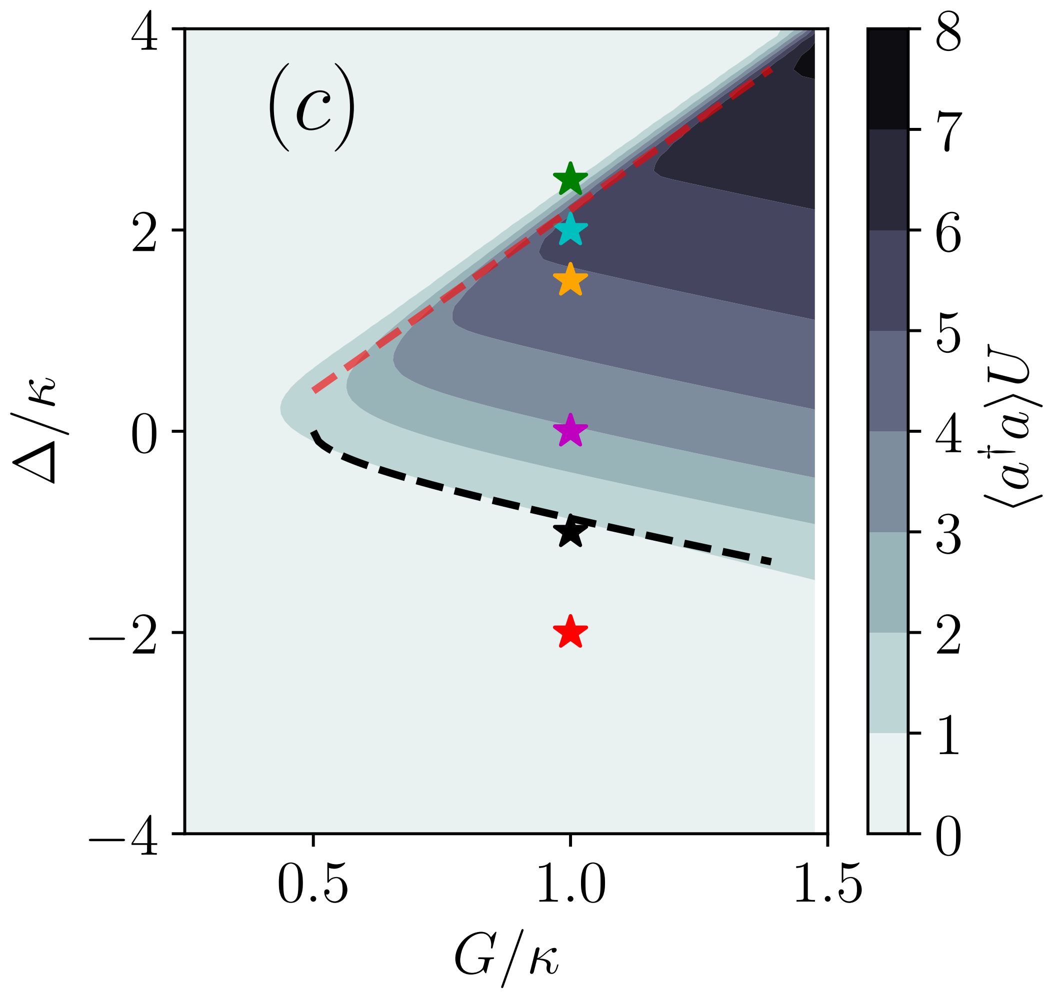
- Reflects sensitivity to all 3 blobs.

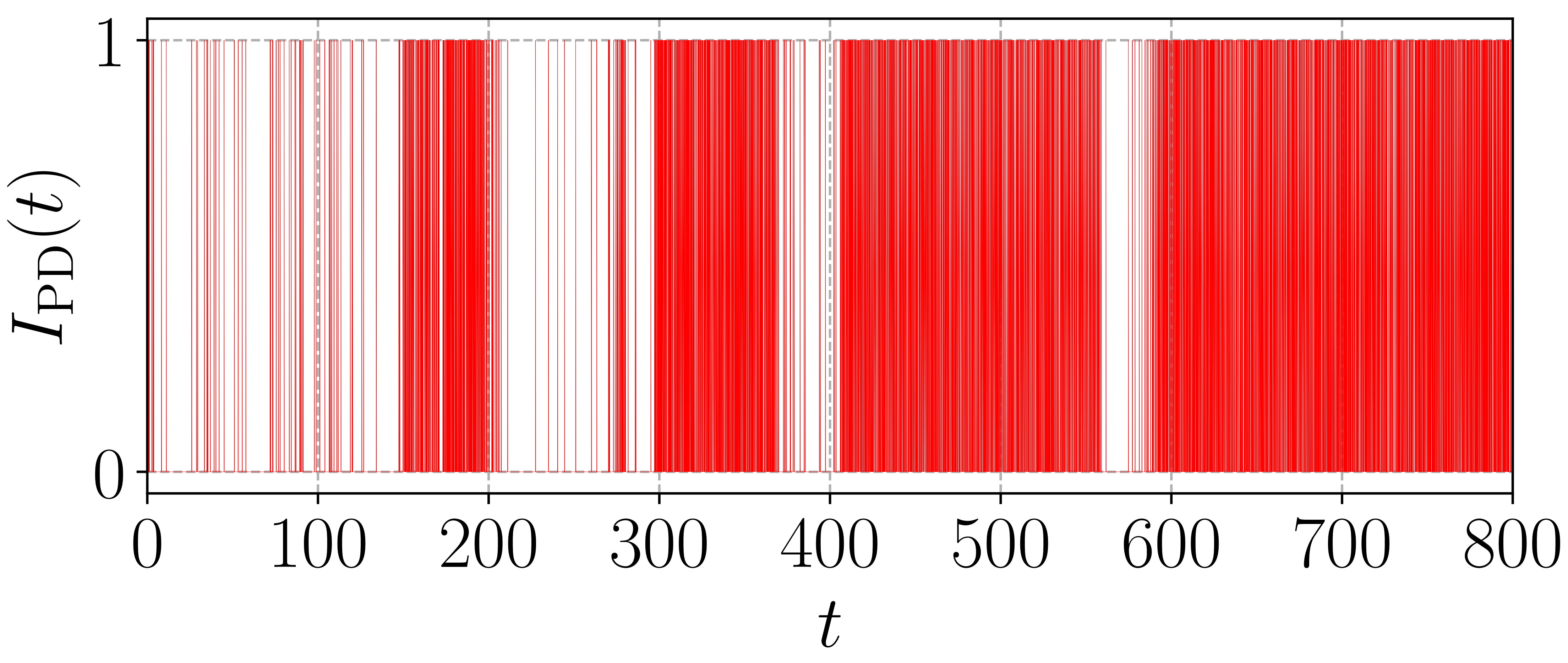




Metrology

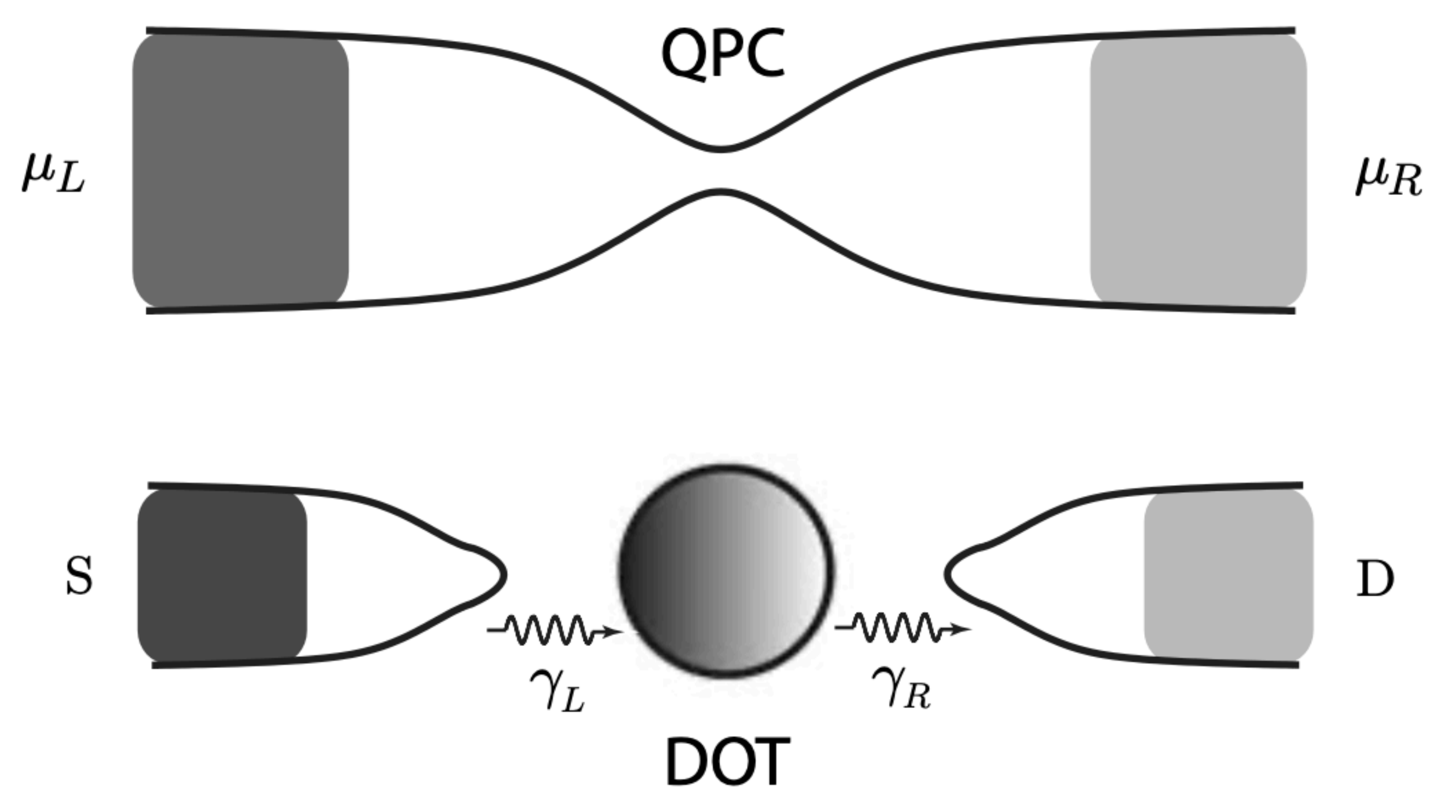
- Phase transition makes current very sensitive to changes in the parameters.
- We can use this as a sensor.
- $\Delta = \omega_p - \omega_c$
  - Dependence on the cavity frequency ω_c: affected by multiple processes.
  - Dependence on the pump frequency ω_p: sensitive sensing of input frequency.
  - Single-photon detector.

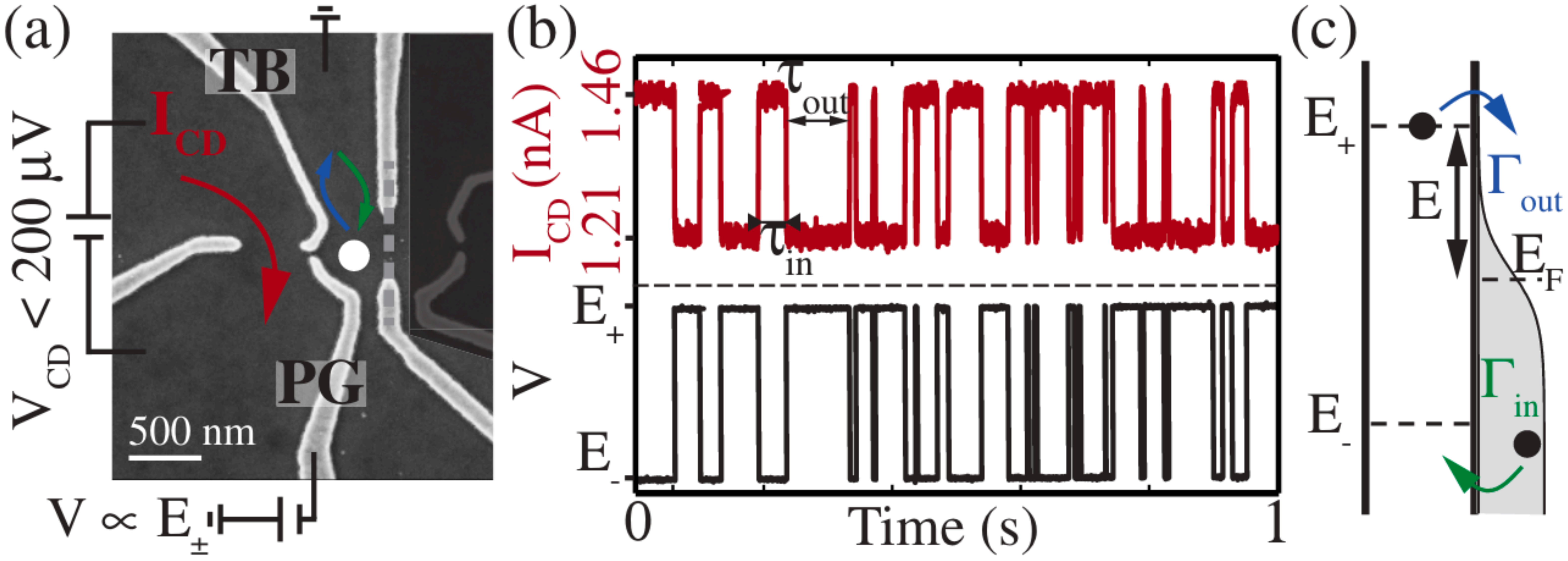




Waiting time metrology/thermometry

- The time between clicks encodes information on the system.
- We can use this as a thermometer.
- Ex: quantum dot continuously monitored using a quantum-point contact.

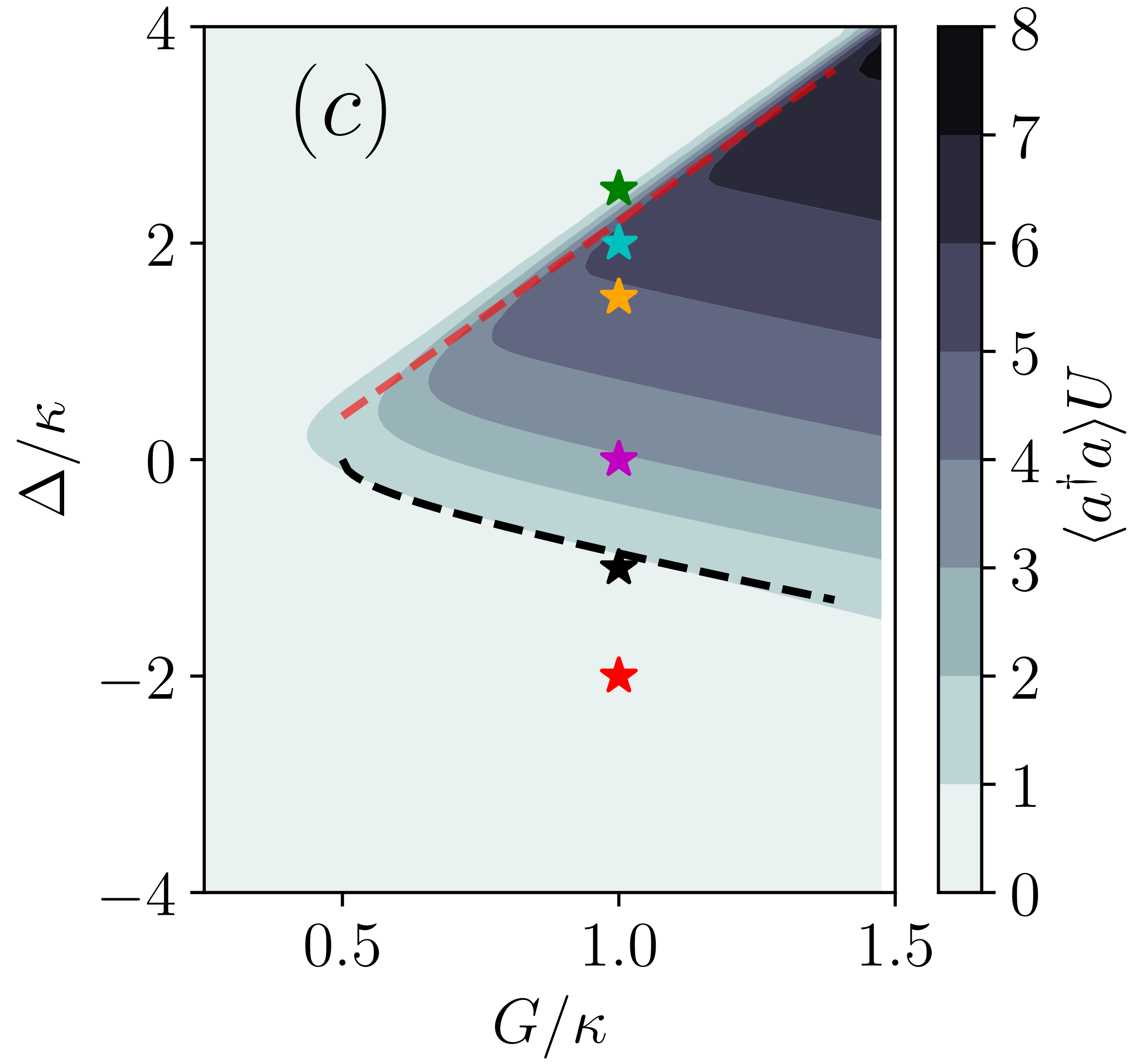


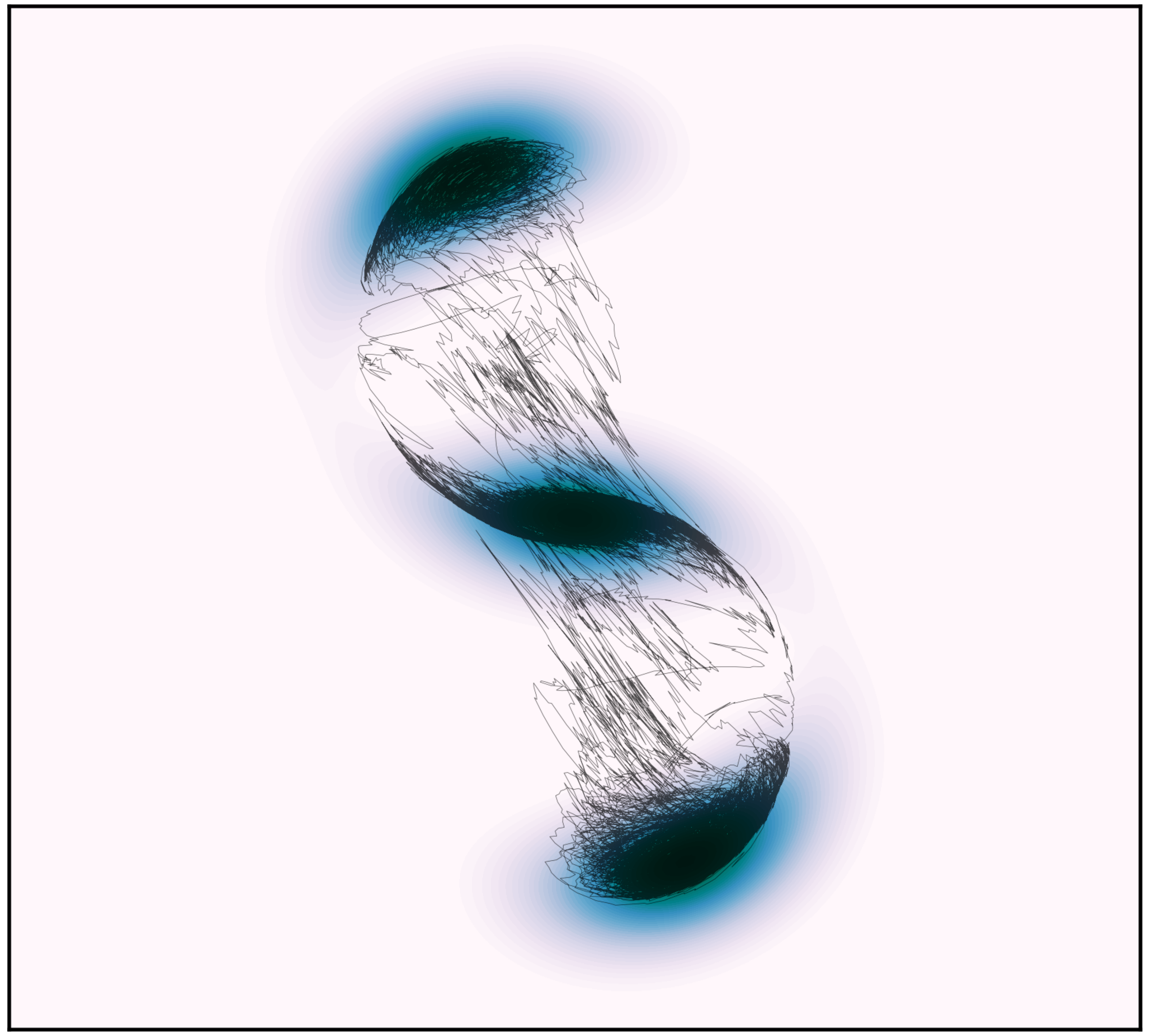


A. Hofmann, *et. al.*, PRL, 117, (2016)

Summary

- Fluctuations in the time domain:
    - Classical time series produced by a quantum system.
- Connection between quantum optics and full counting statistics.
- Parametric Kerr model: critical properties of the fluctuations shed light on the nature of the transitions
    - Photo-detection: exponential divergence in the discontinuous transition.
    - Homodyne: exponential divergence in the entire critical region.





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Thank you.