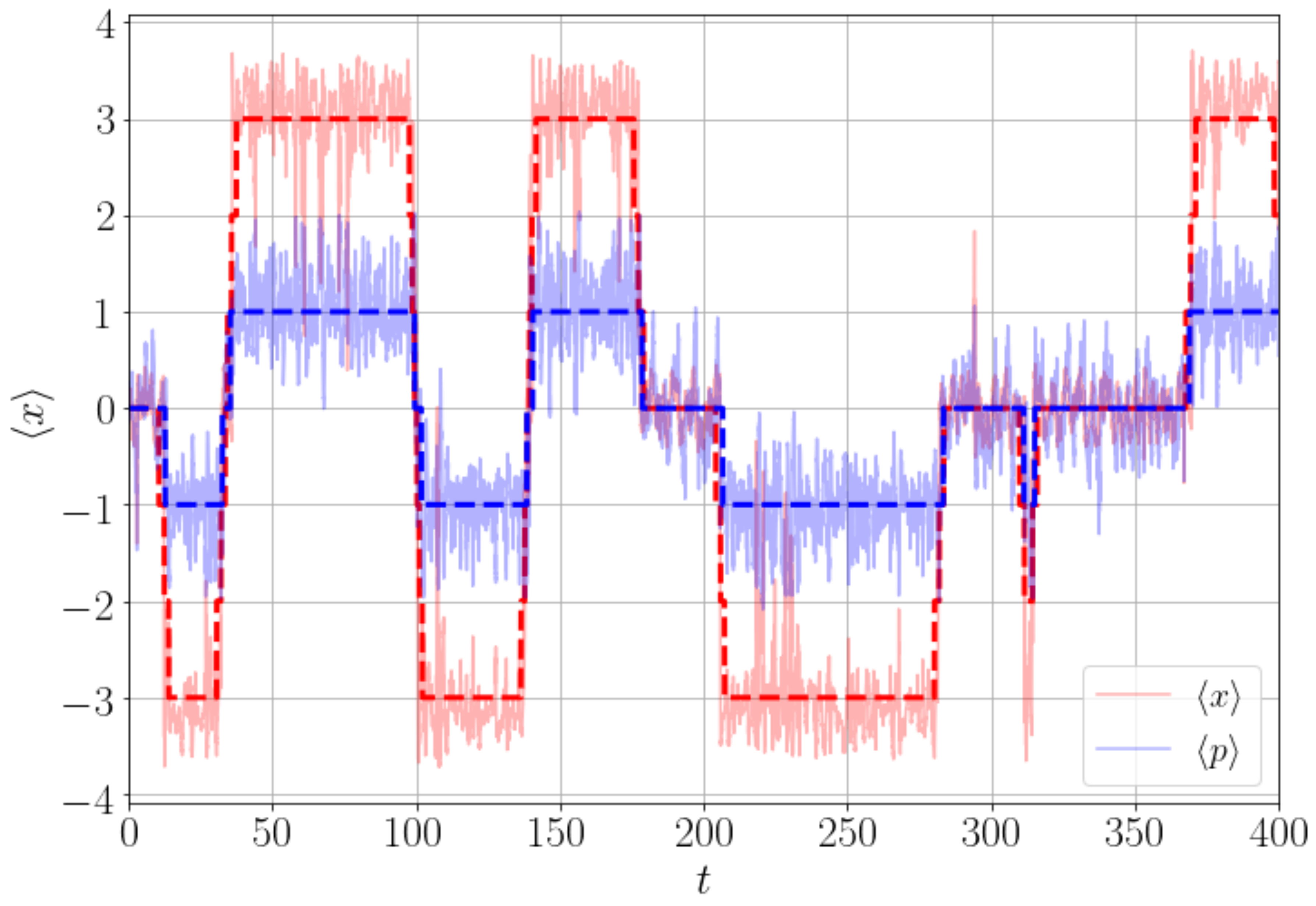

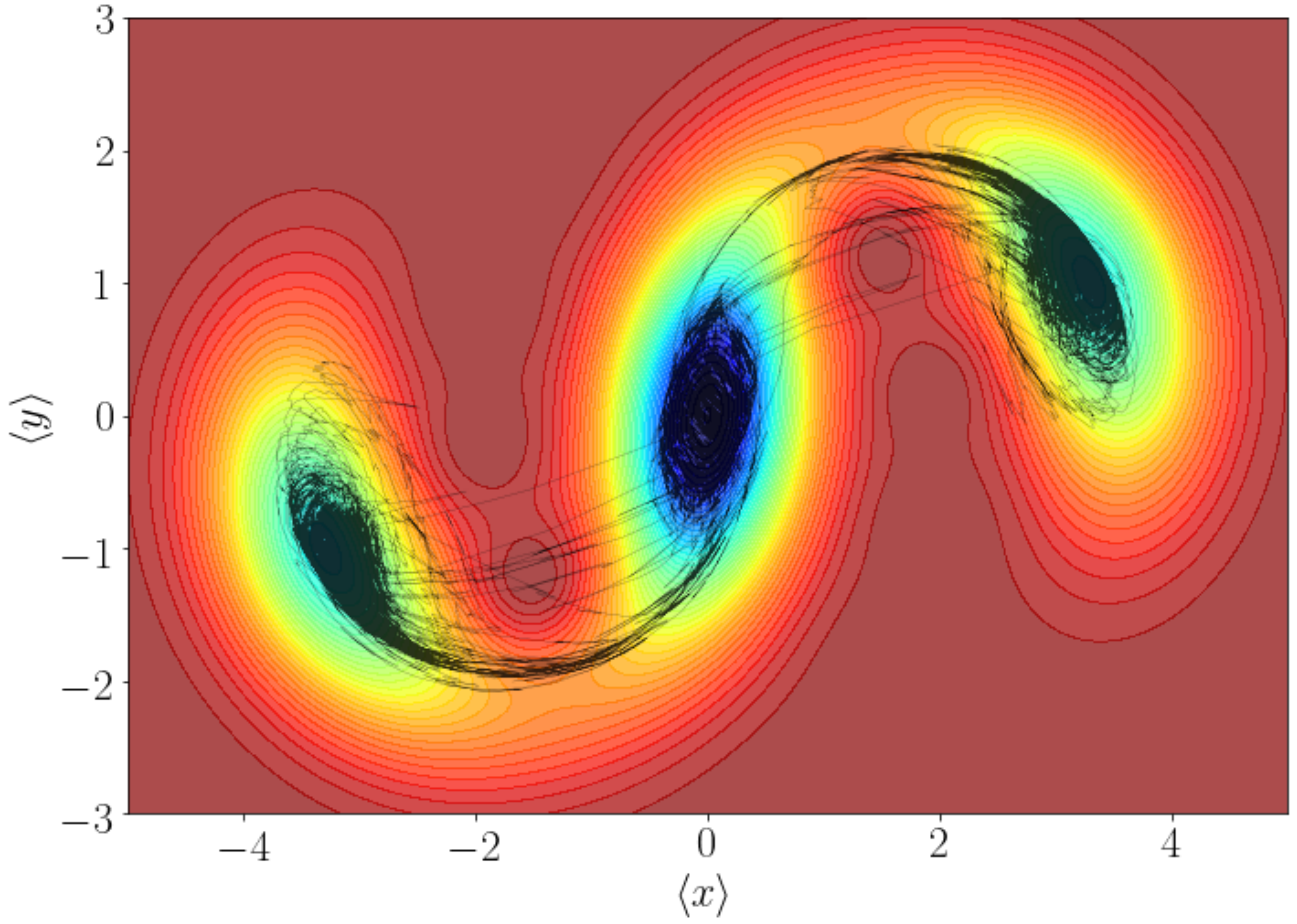

Current fluctuations in critical Kerr resonators

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Nov. 2022 - XVII Enceuntro SUF

Quantum Thermodynamics Quantum Transport

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Current fluctuations & two-time correlations

C. E. Fiore, Pedro E. Harunari, C. E. Fernandez Noa, and GTL
Current fluctuations in nonequilibrium discontinuous phase transitions
ArXiv 2109.00385

M. Kewming, M. Mitchison & GTL,
“**Diverging current fluctuations in critical Kerr resonators**”,
arXiv 2205.02622.

GTL, M. Kewming, M. Mitchison, P. Potts,
“**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics.**”
Tutorial, in preparation

Pedro Harunari

Carlos Fiore

Patrick Potts

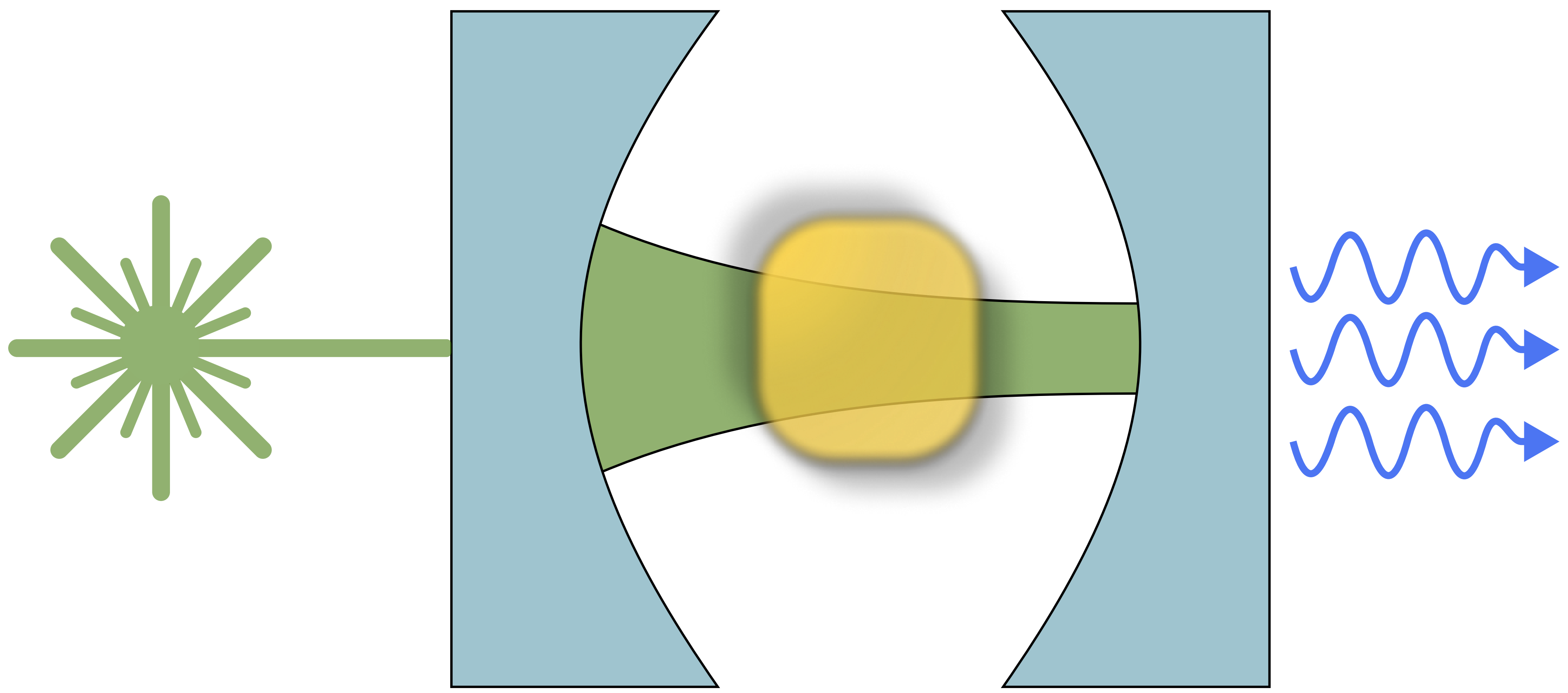
Michael Kewming

Mark Mitchison

Overview

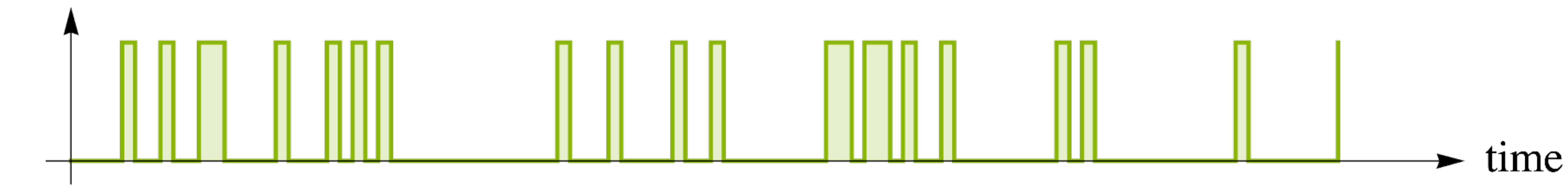
- Continuously measured quantum system.
- Ex: optical cavity
  - Monitor the photons that leak out.

    ➡ ***Classical stochastic currents (time-series)***

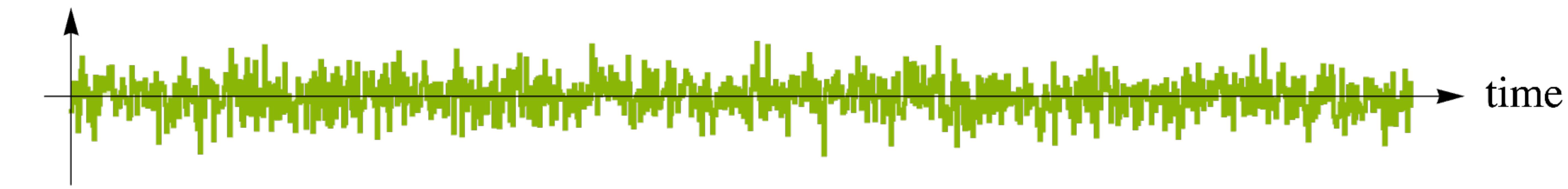


- Quantum jumps: discrete current → individual clicks in the detector.
- Quantum diffusion: continuous (noisy) current.

Classical outcomes describing an underlying quantum system



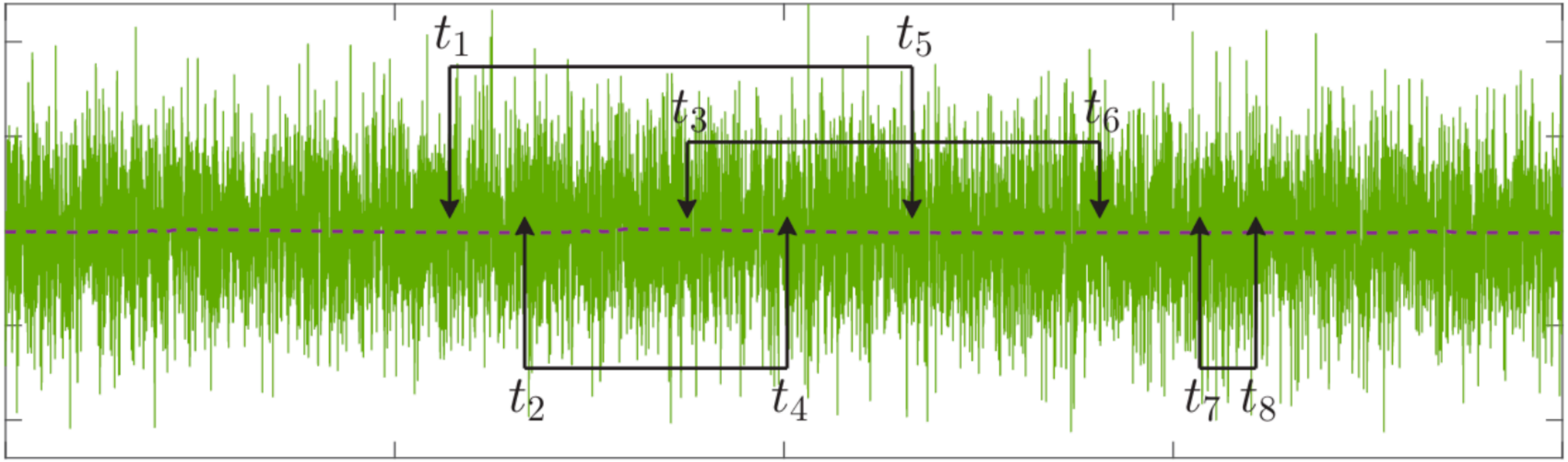
Quantum jumps



Quantum diffusion

Motivation

- In many experiments, this is the only way to probe it.
  - **What information does the current conveys about the system?**
- Basic quantity: average current $J = \langle I(t) \rangle$.
  - **But the current is stochastic:** *the full signal has a lot more information than the average.*
    - Current fluctuations & two-time correlations: outcomes are not independent.



Why study current fluctuations

- **Metrology:**
  - Quantum system as a sensor.
    - e.g. magnetic fields.
    - e.g. gravitational waves (LIGO)
      - LIGO in fact uses exactly the present setup, with optical cavities.

  * Some information is only contained in the correlations!
- **Thermo-kinetic uncertainty relations:**

Current fluctuations (noise) →

$$\frac{D}{J^2} \geqslant \frac{2}{\sigma}$$

← Entropy production (a measure of dissipation)

* Counterintuitive:

To reduce fluctuations, we must increase dissipation.

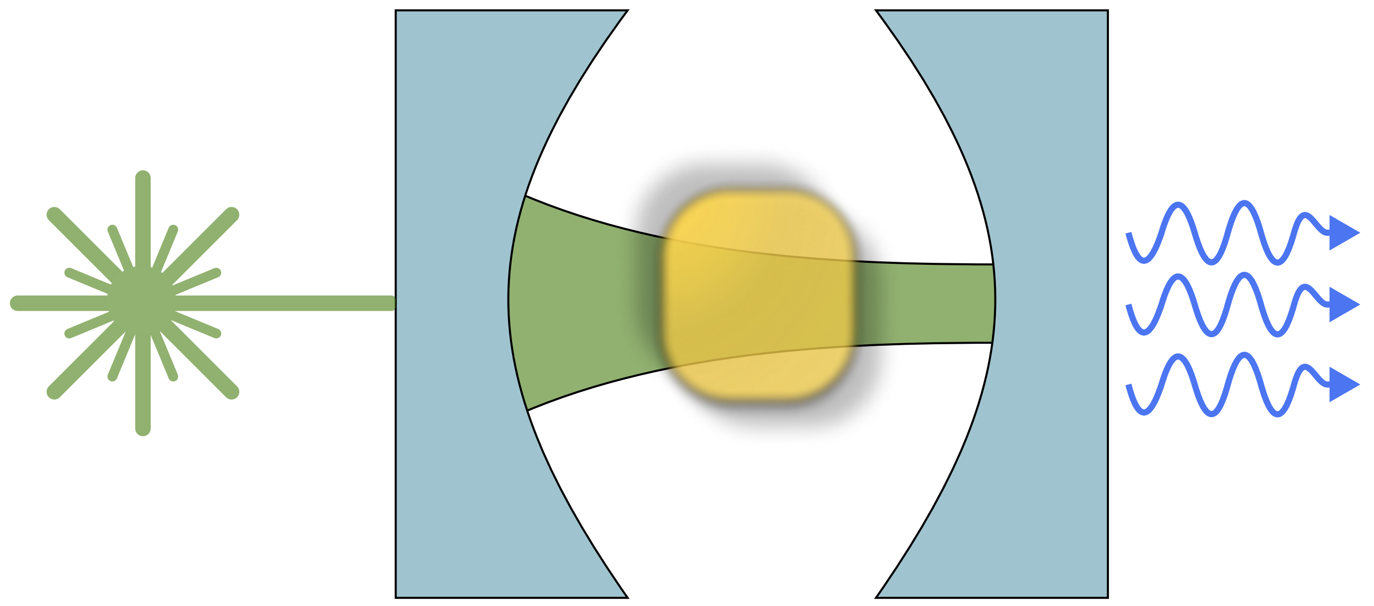
Parametric Kerr model

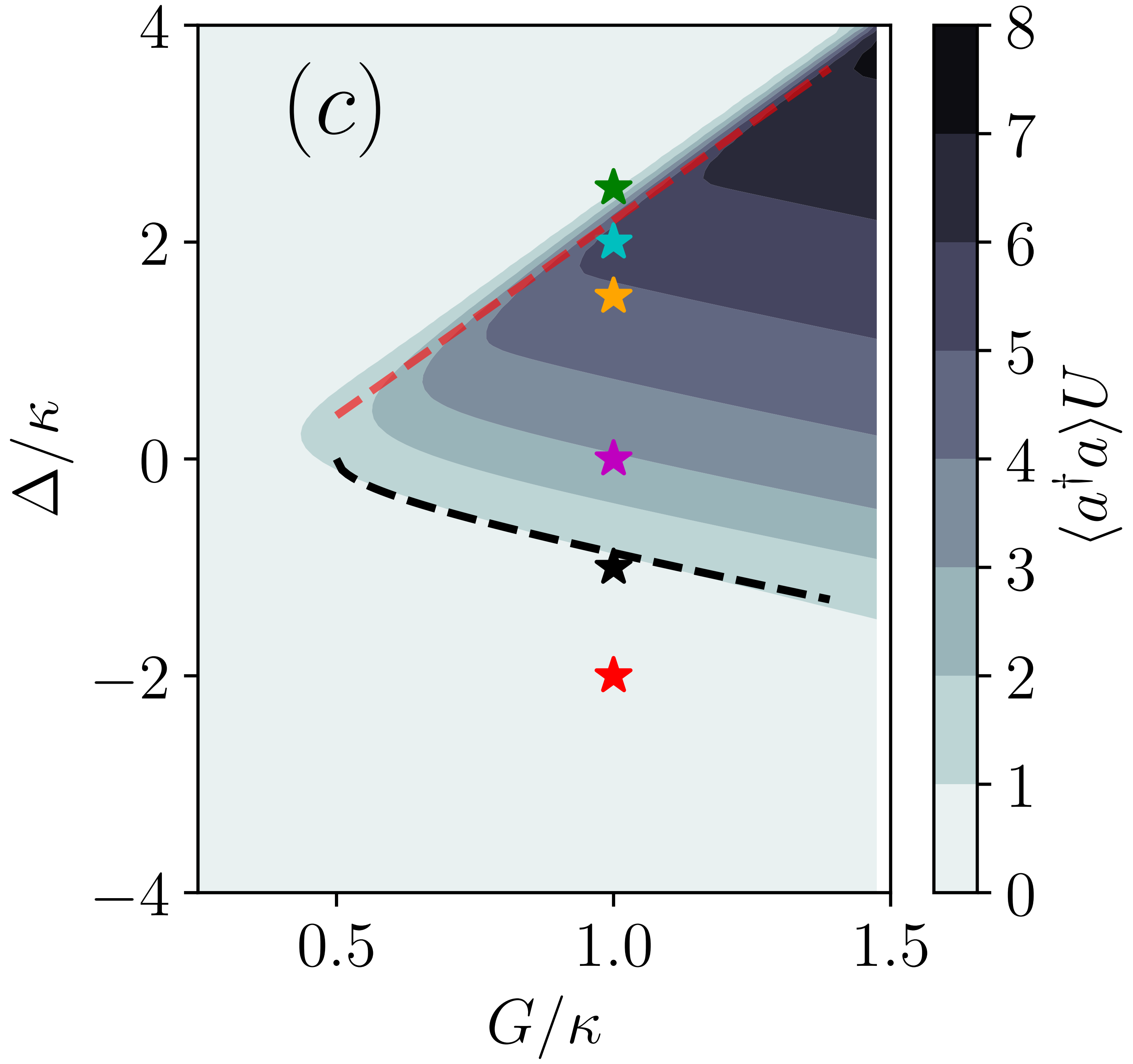
Parametric Kerr model

- Non-linear quantum harmonic oscillator:

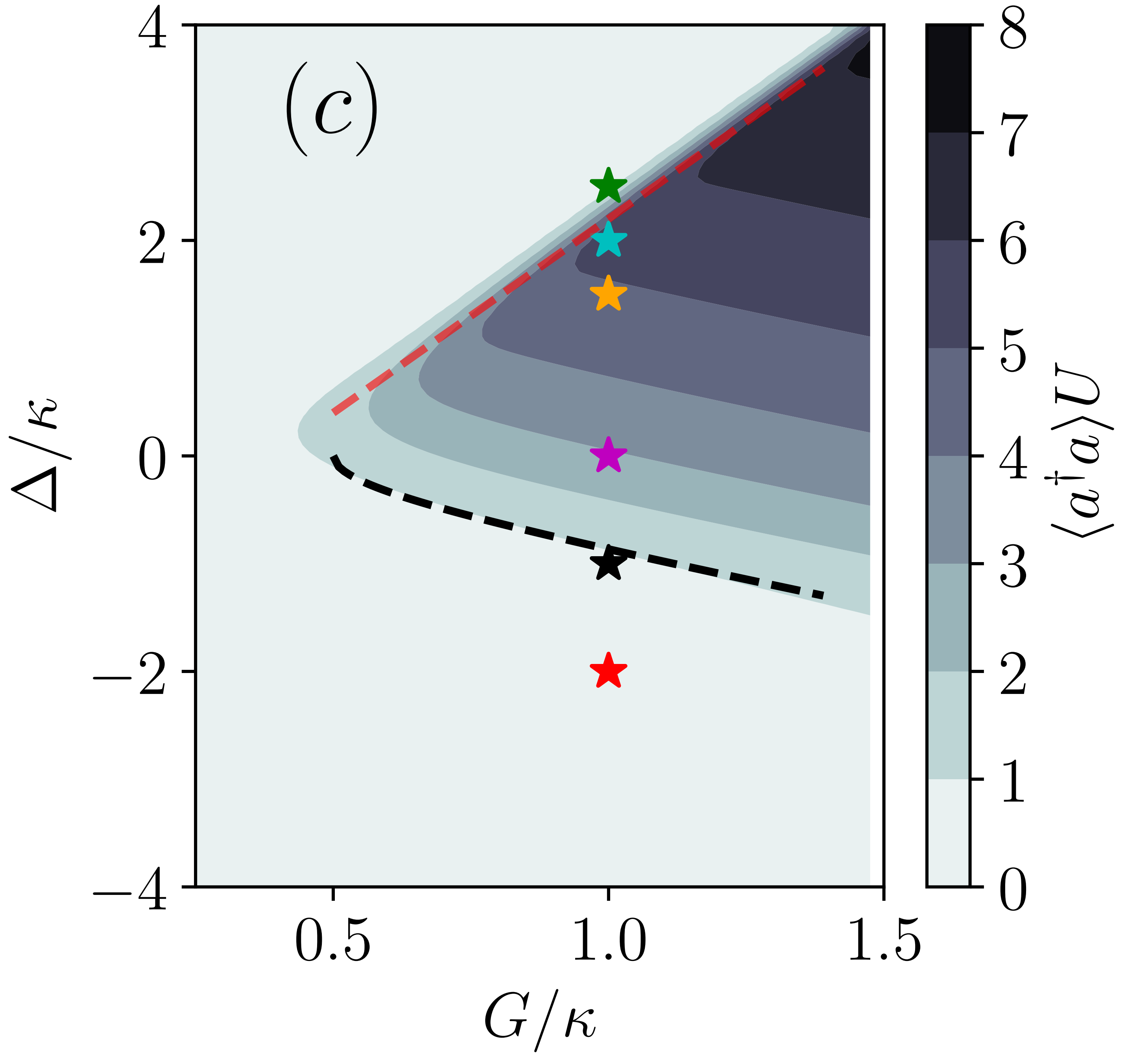
$$\frac{d\rho}{dt} = -i[H(t),\rho] + \kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right]$$

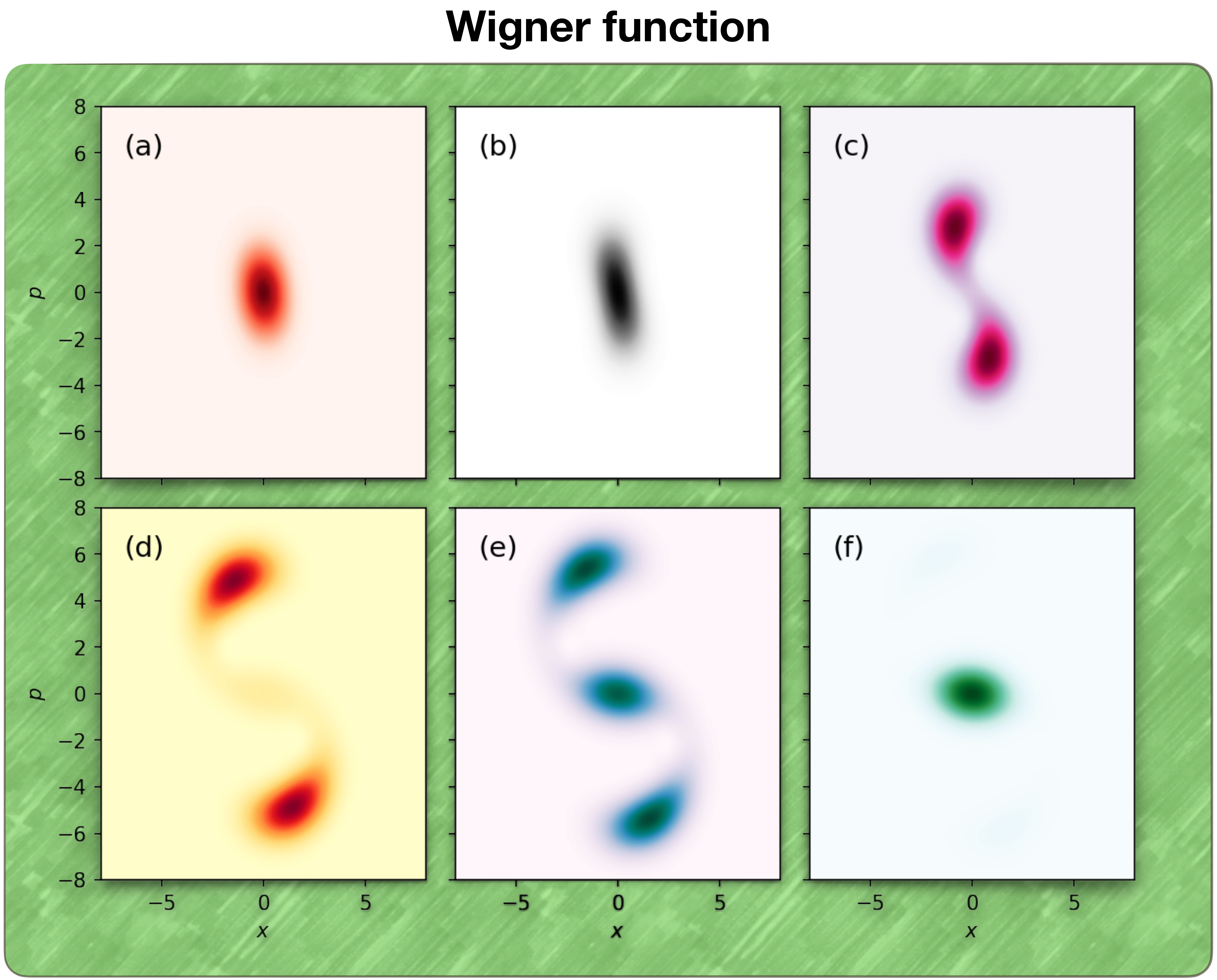
$$H = -\Delta a^\dagger a + \frac{U}{2}a^\dagger a^\dagger a a + \frac{G}{2}(a^{\dagger 2} + a^2)$$





- a = annihilation operator
  photon operator for an optical cavity
- $\Delta = \omega_p - \omega_c$ = detuning
- U = Kerr non-linearity.
  (requires a non-linear crystal inside the cavity)
- G = 2-photon pump
  (input laser produces photons in pairs)
- κ = loss rate
  rate at which photons leak out of the cavity





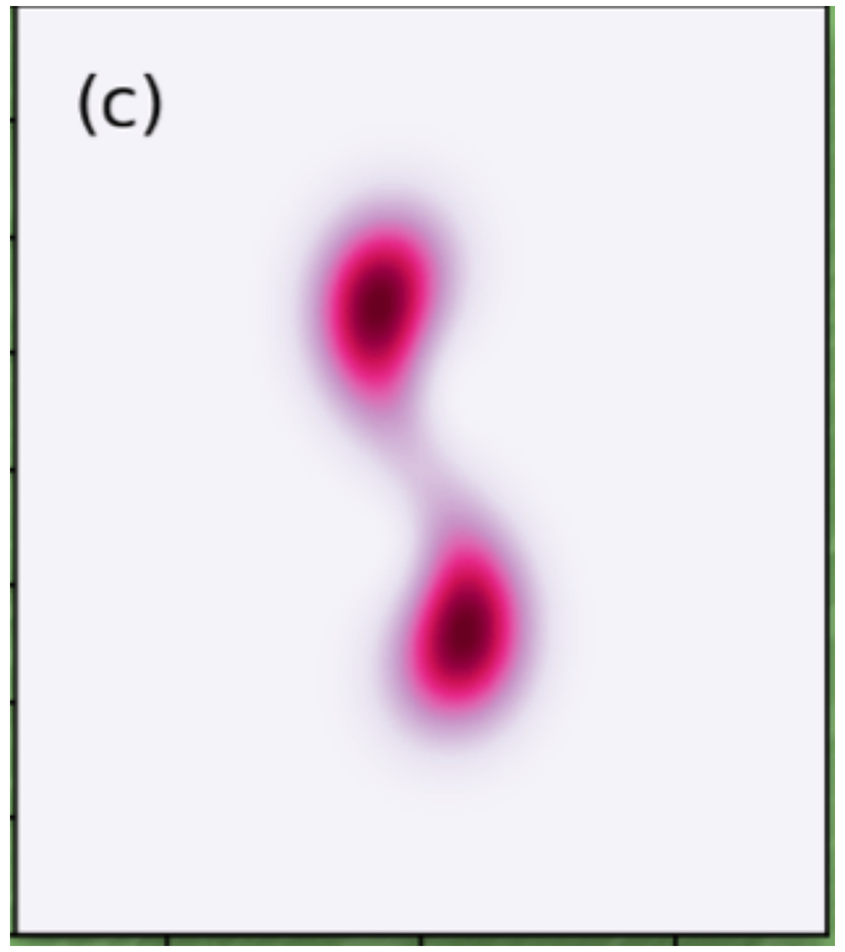
- 2 phase transitions, continuous and discontinuous
- Proper criticality occurs in the limit $U \rightarrow 0$ ("thermodynamic limit")

Cat qubits

- Steady-state is a mixture of two Schrödinger cat states

$$|S\rangle = |\alpha\rangle + |-\alpha\rangle$$

$$|A\rangle = |\alpha\rangle - |-\alpha\rangle$$

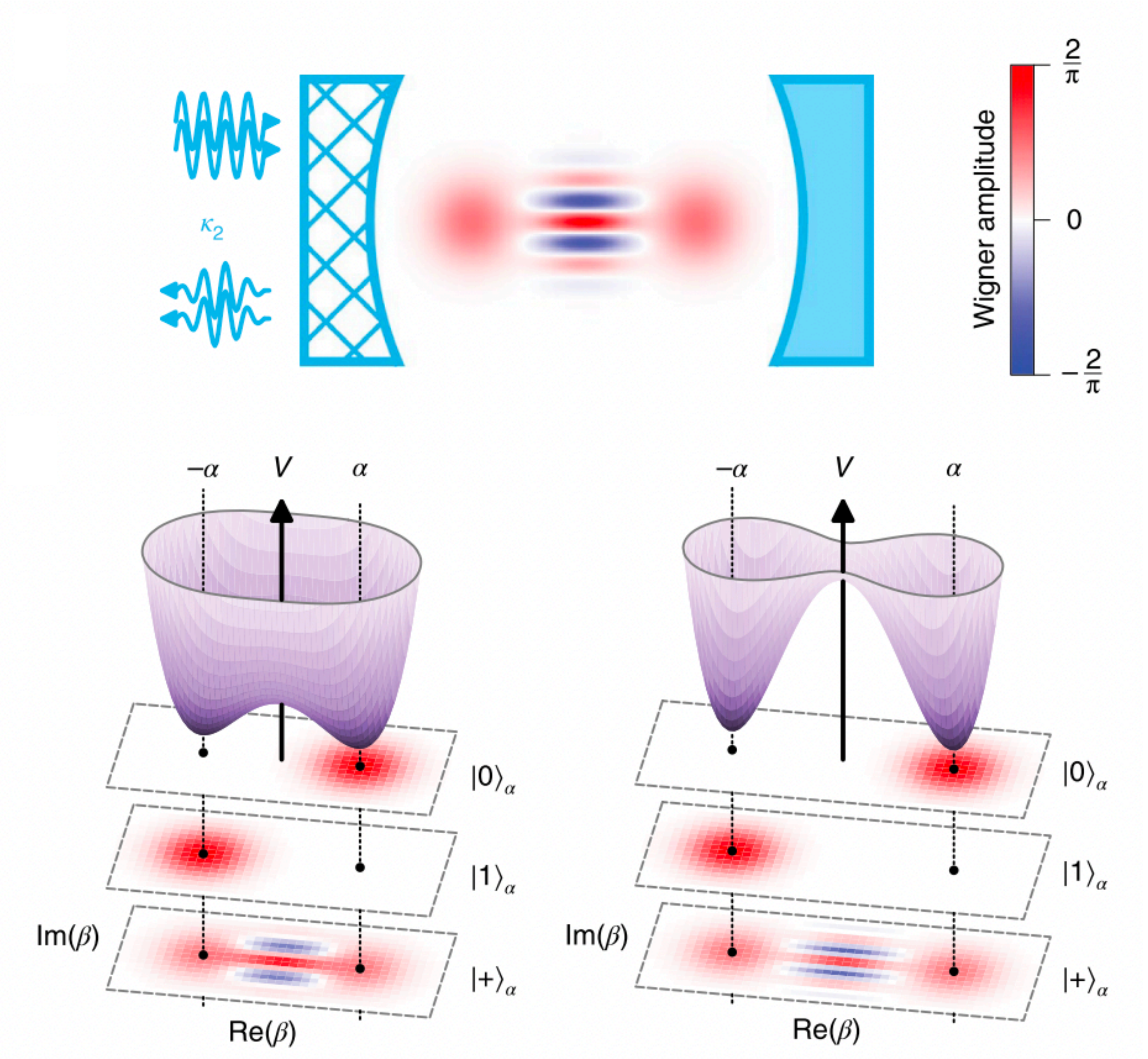


- Use this to define cat qubits:

$$|0\rangle = |\alpha\rangle$$

$$|1\rangle = |-\alpha\rangle$$

- Cat qubits are more robust against errors.
  - Quantum computing with Kerr cats.



Lescanne, *et. al.*, Nature, **16**, 509-513 (2020)

Semiclassical model

0D version of non-linear SEq. (Mininni)

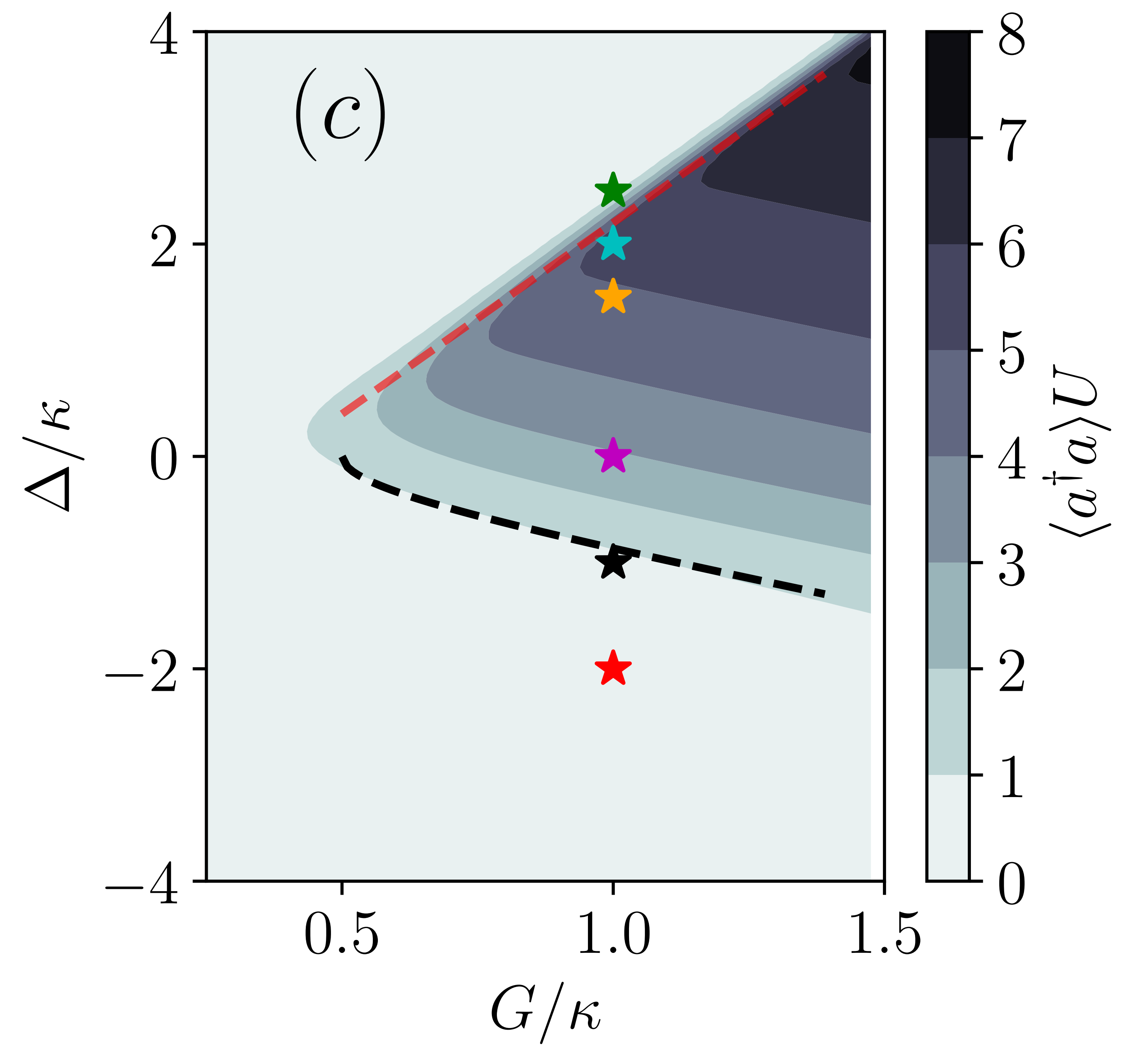
- Mean-field approximation: complex variable $\alpha(t)$

$$\frac{d\alpha}{dt} = -(\kappa - i\Delta)\alpha - \frac{iU}{2}|\alpha|^2\alpha - iG\alpha^*$$

- Pitchfork bifurcation at critical detuning $\Delta_c = -\sqrt{G^2 - \kappa^2/4}$

$$\alpha^* = \pm\sqrt{n_0}e^{i\phi_0}$$

$$n_0 = \frac{\Delta}{U} + \sqrt{\frac{G^2 - \kappa^2/2}{U^2}}, \qquad \phi_0 = \frac{1}{2}\arcsin\left(\frac{-\kappa}{2G}\right)$$



- Can predict the continuous transition (bottom)
- Unable to predict the discontinuous transition (top).

Photo-detection current

How to simulate the stochastic photo-detections?

- **Quantum jump unravelling.**
- At each time step dt a jump occurs with probability $\kappa dt\langle a^\dagger a\rangle$.
    - ($\propto dt$: very unlikely)
- If jump occurs: $|\psi\rangle \rightarrow a|\psi\rangle$
- If no jump occurs: $|\psi\rangle = e^{-iH_{\text{eff}}t}|\psi\rangle$

$$H_{\text{eff}} = H - \frac{\kappa}{2}a^\dagger a$$

(non-Hermitian Hamiltonian)

Every time a jump occurs, we count $dN = 1$

Stochastic current:

$$I(t) = \frac{dN}{dt}$$

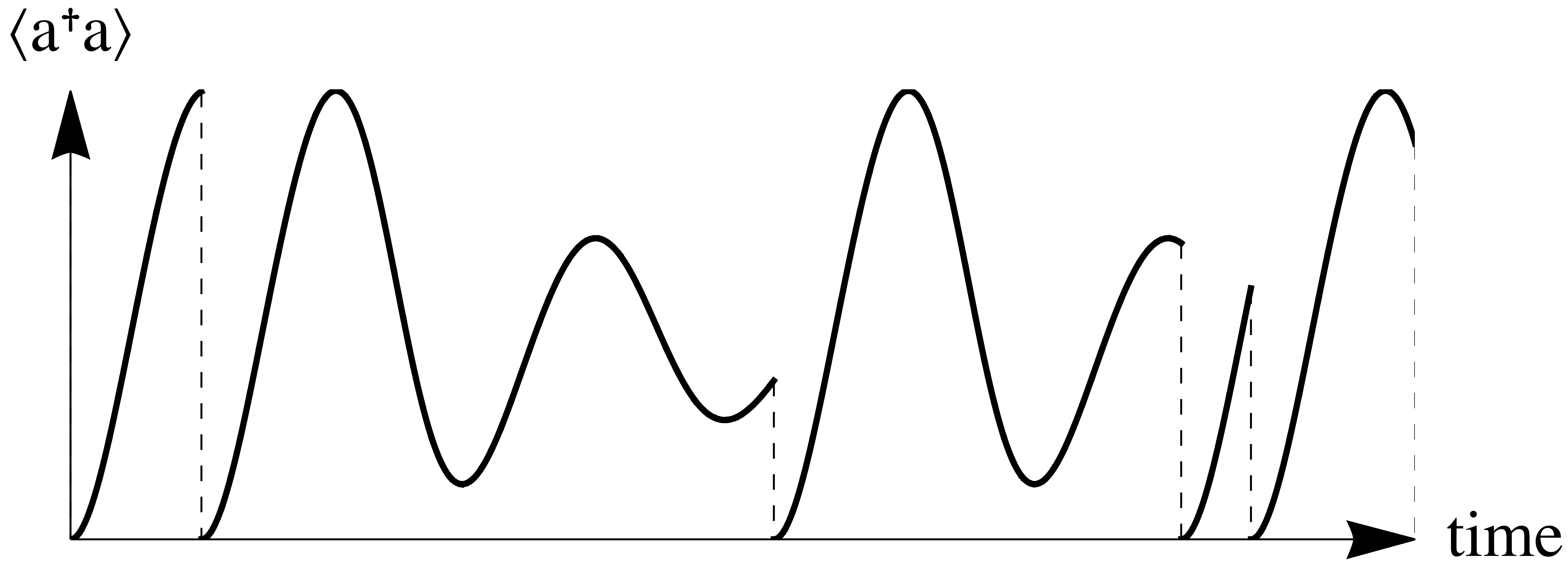
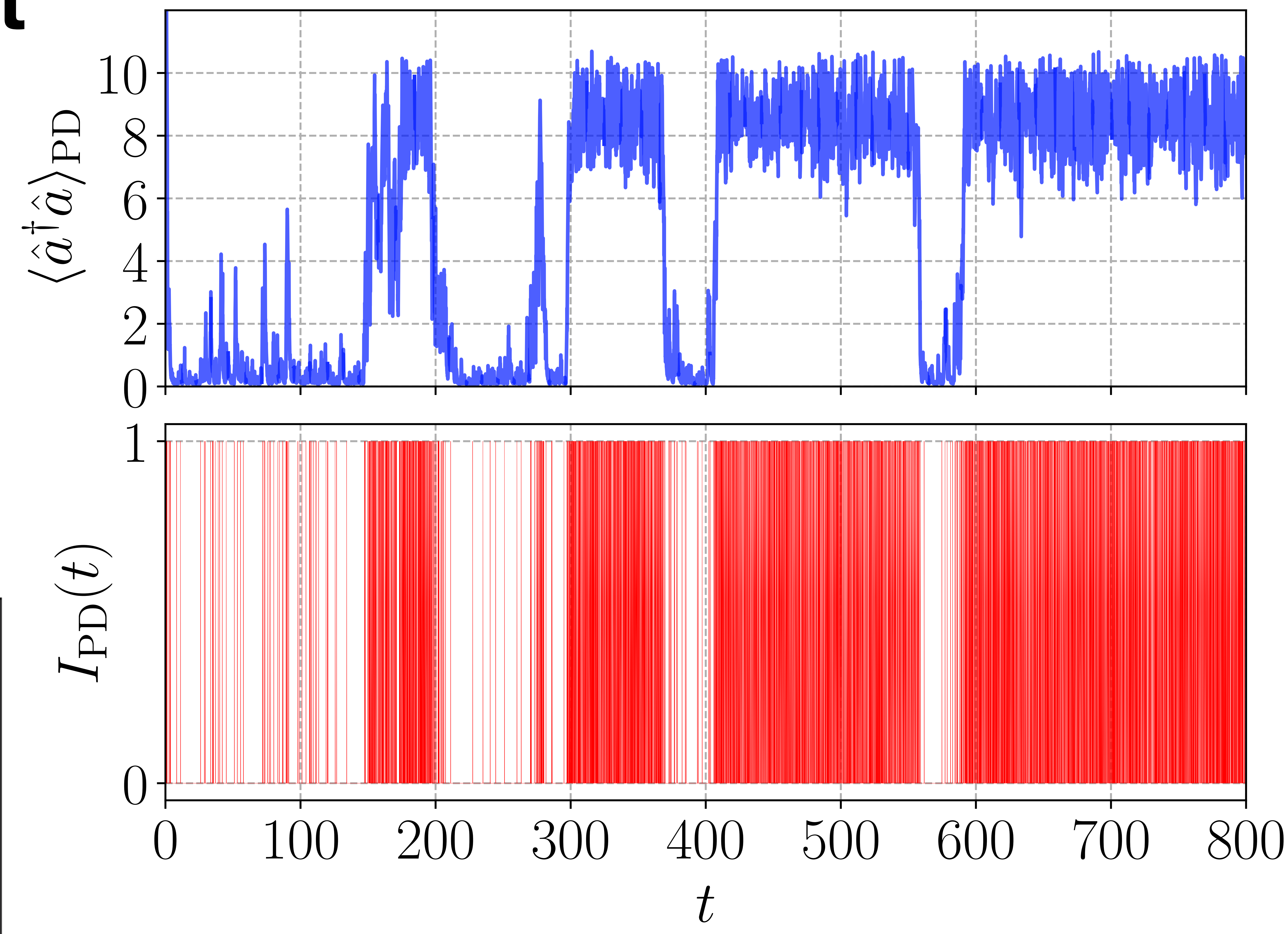
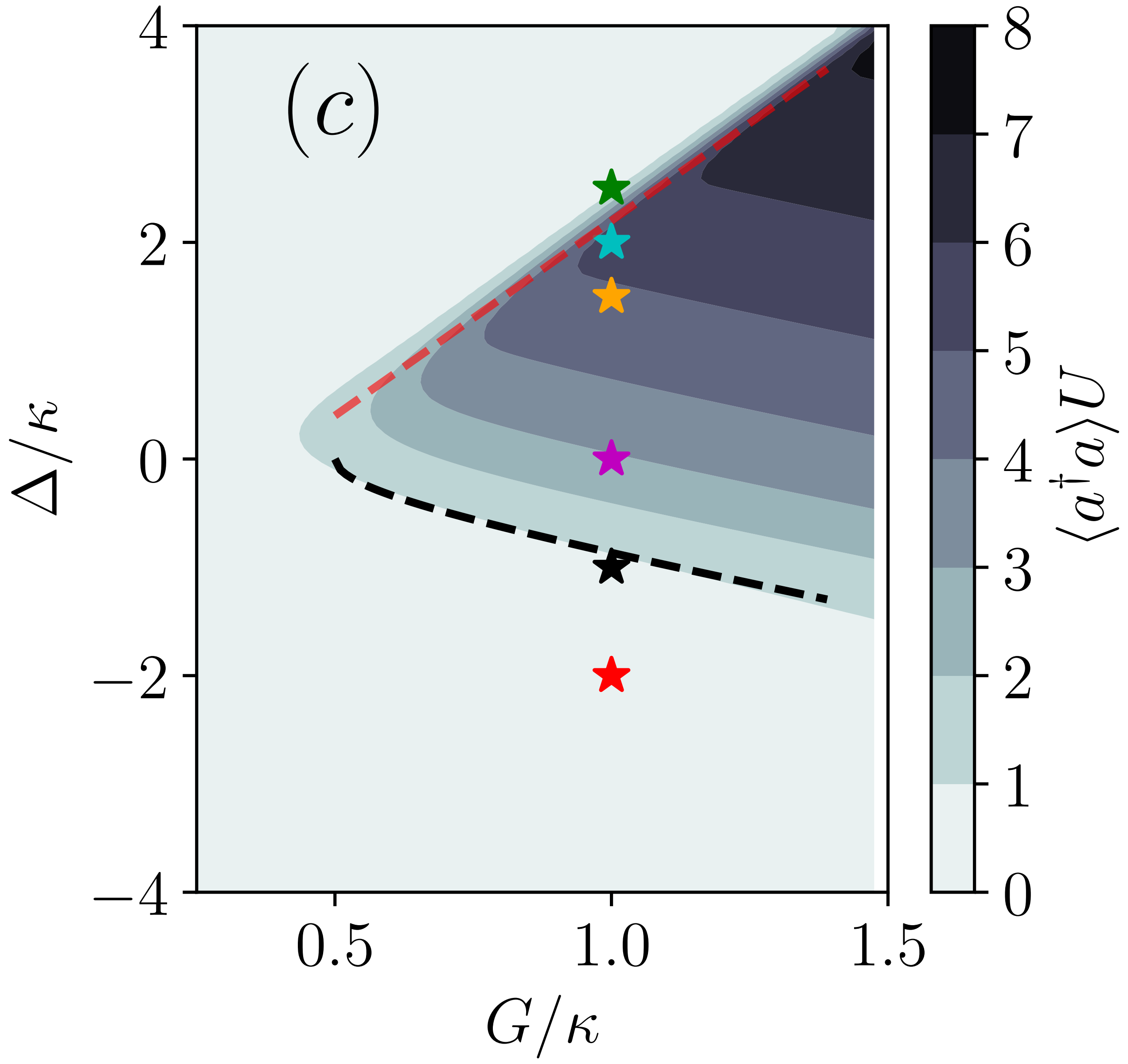
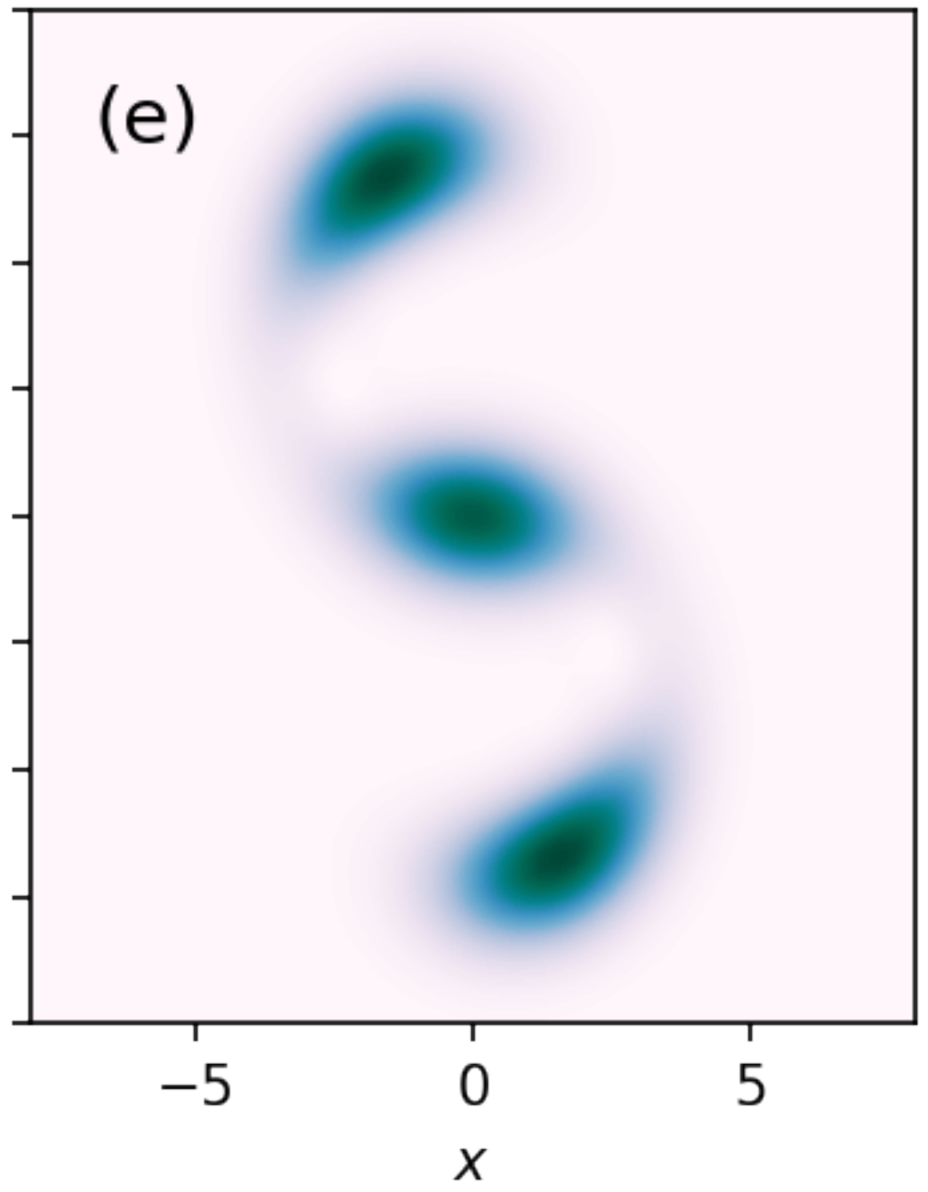


Photo-detection current

- @ discontinuous transition: on/off (telegraph) behavior of the current.
- Photo-detection cannot resolve upper vs. lower blobs.







$J = \langle I \rangle$ = **"dynamical activity" = jumps/second**

Current fluctuations - Full Counting Statistics

- Two-time correlation function:

$$F(\tau) := \big\langle I(t)I(t+\tau)\big\rangle - J^2$$

$$= J\ \delta(\tau) + J^2\big[g^{(2)}(\tau) - 1\big]$$

$g^{(2)}$ = Glauber's 2nd order coherence function

- Power spectrum:

$$S(\omega) = \int_{-\infty}^{\infty} e^{-i\omega\tau} F(\tau) d\tau$$

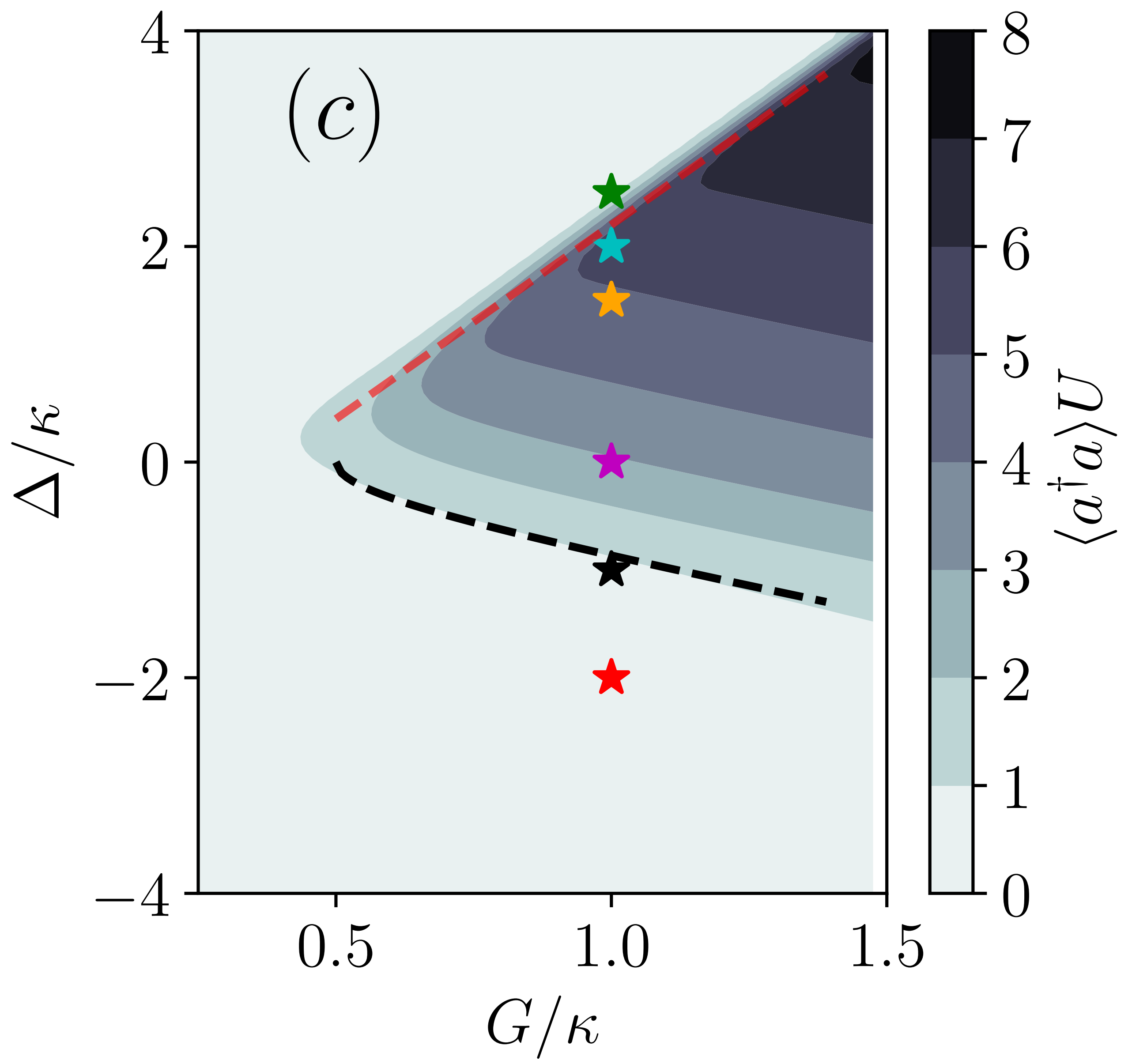
- Zero-frequency component of the power spectrum := "noise":

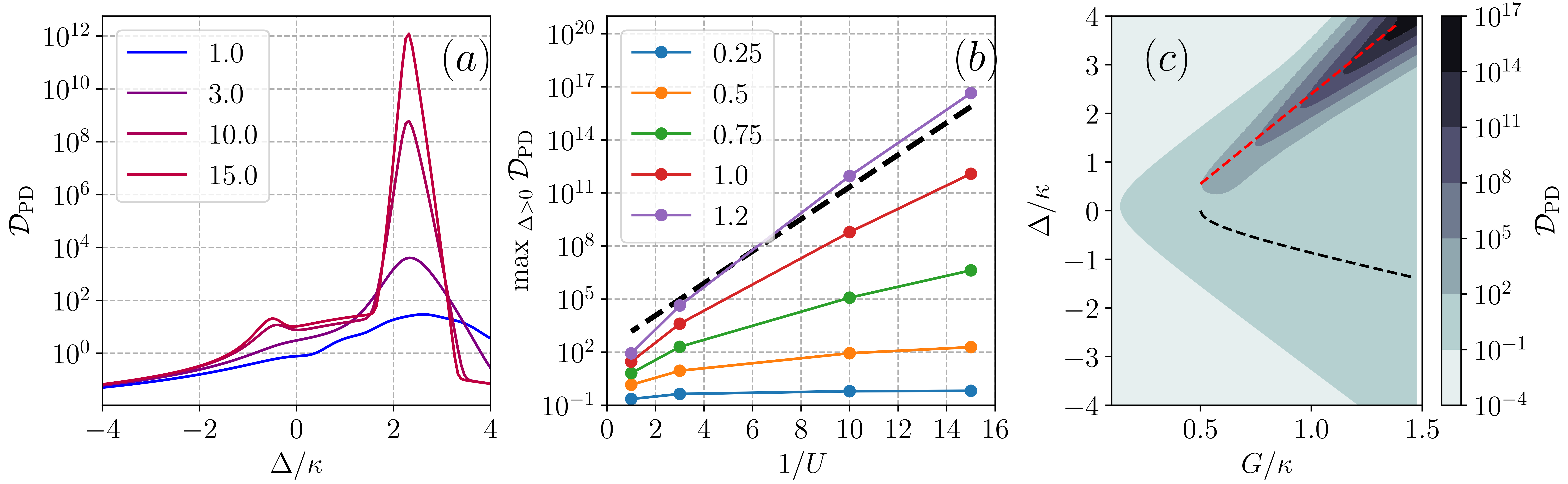
$$D = S(0) = \lim_{t\to\infty} \frac{d}{dt}\mathrm{Var}\big(N(t)\big)$$

Divergence of the diffusion coefficient

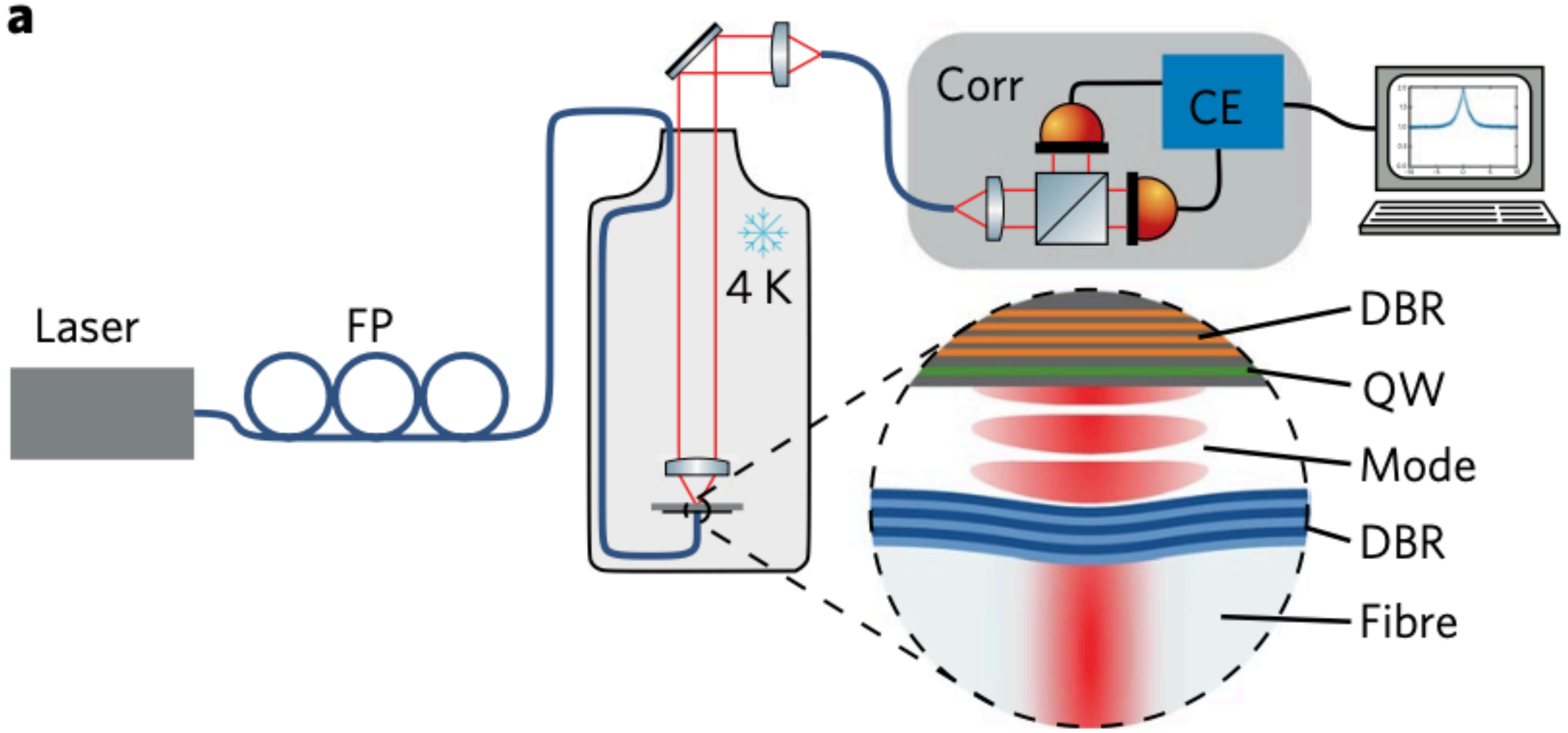
- "Thermodynamic limit:" $U \to 0$
- In the discontinuous transition ($\Delta > 0$)

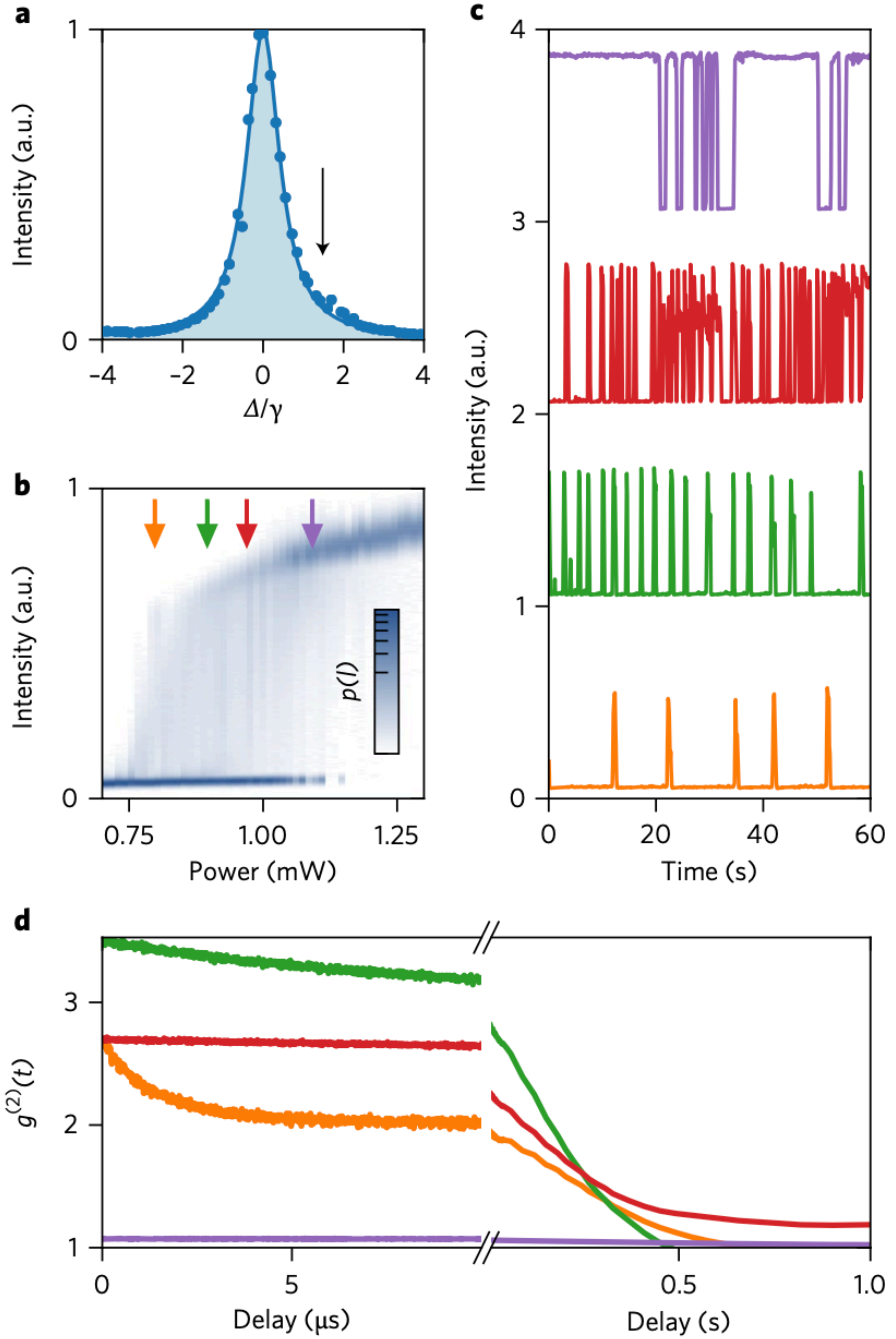
$$D \sim e^{1/U}$$





GaAs cavity polaritons.





T. Fink, *et. al.*, *Nature Physics,* **14**, 365 (2018)

Minimal 2-level model

Minimal 2-level model

- Let q_0 = prob. the system is in the middle blob.

  q_1 = prob. the system is in any of the outer blobs.

- Master equation:

$$\frac{d}{dt}\begin{pmatrix} q_0 \\ q_1 \end{pmatrix} = \begin{pmatrix} -a & b \\ a & -b \end{pmatrix}\begin{pmatrix} q_0 \\ q_1 \end{pmatrix}$$

a = rate to jump from $0 \to 1$

b = rate to jump from $1 \to 0$

- Dependence with U:

$$a \sim \exp\{(\Delta - \Delta_c)/U\}$$

$$b \sim \exp\{-(\Delta - \Delta_c)/U\}$$

Time between jumps

$$\tau_m \sim e^{1/U}$$

Prob. of being in the outer blobs

$$q_1 \sim (1 + e^{-(\Delta-\Delta_c)/U})^{-1}$$

Noise

- The minimal model gives for the noise:

$$D = q_1 D_1 + q_0 D_0 + q_1(1 - q_1)(\mu_1 - \mu_0)^2 \tau_m$$

- First 2 terms: fluctuations within each blob.
- Last term: fluctuations between blobs.
  - Depends on $\tau_m \sim e^{1/U}$: diverges exponentially.

Homodyne detection

Homodyne current

- Mix photon output with a strong laser source $\alpha = |\alpha| e^{i\phi}$.
  - Equivalent to measuring

$$\langle (a+\alpha)^\dagger (a+\alpha) \rangle = \left(|\alpha|^2 + |\alpha| \langle a e^{-i\phi} + a^\dagger e^{i\phi} \rangle + \langle a^\dagger a \rangle \right)$$

    - $|\alpha|^2$ is just a constant offset.
    - If α is large, then the current will predominantly

$$x := a e^{-i\phi} + a^\dagger e^{i\phi}$$

instead of $a^\dagger a$.

- Quantum diffusion unravelling:

$$d\rho = dt \mathscr{L} \rho + dW \left[\mathscr{H} \rho - \langle x \rangle \rho \right],$$

$$\mathscr{H} \rho = \kappa \left(a\rho + \rho a^\dagger \right)$$

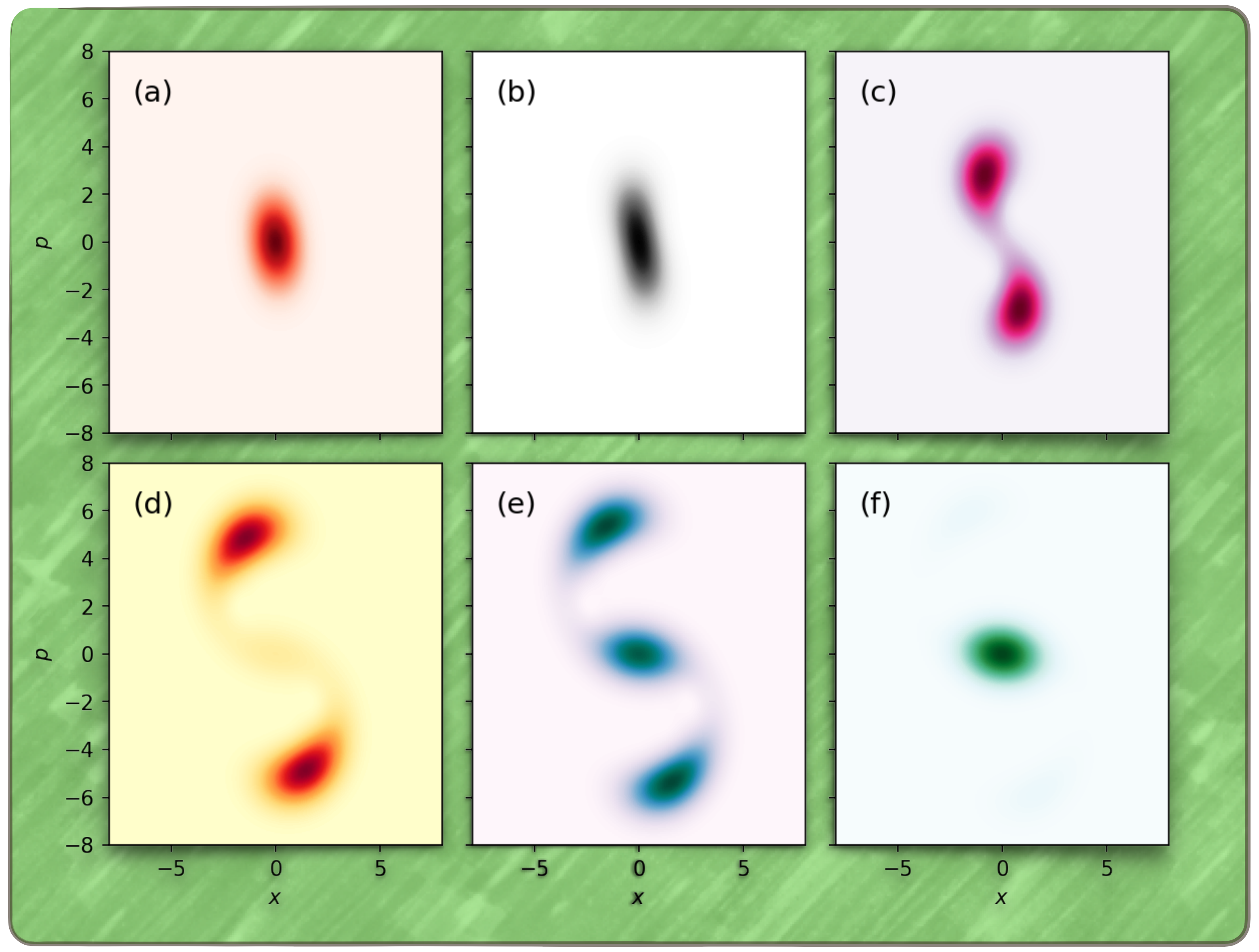
dW = Wiener increment:

$E(dW) = 0, \qquad dW^2 = dt$

Homodyne current (in $p = i(a^\dagger - a)$)

- The homodyne current switches between 3 values (+,0,-).
- Captures the tunneling between the 3 blobs.

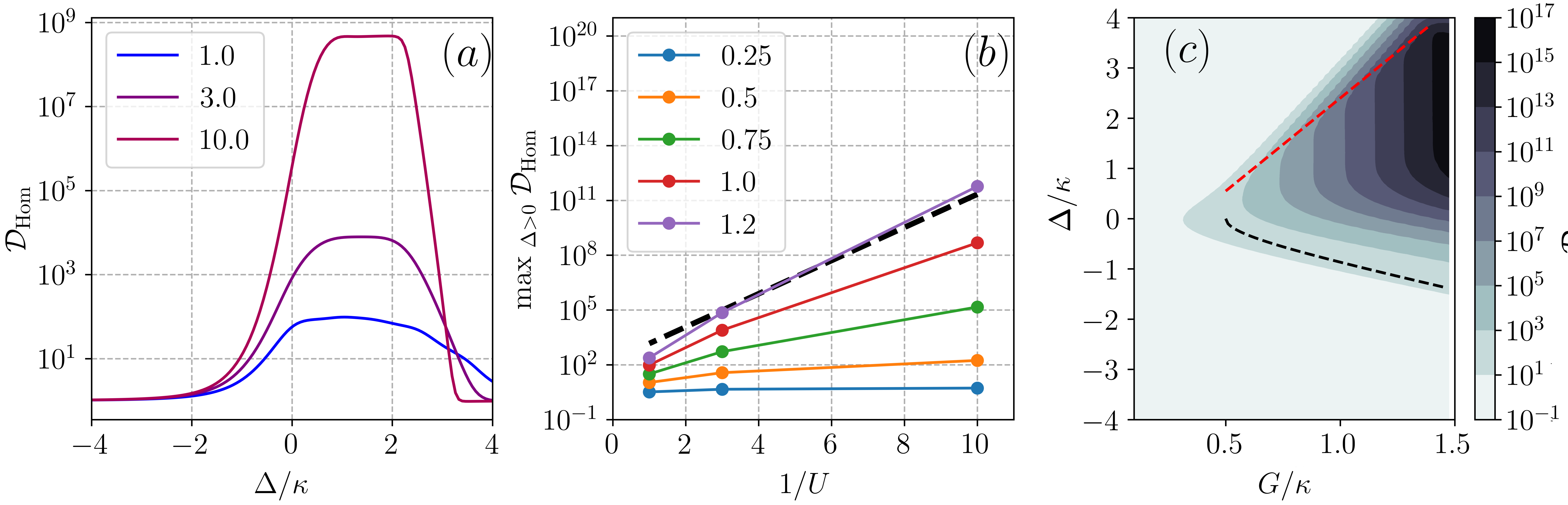
Divergence of the diffusion coefficient



- Homodyne current noise diverges exponentially in a much broader region.

$$D \sim e^{1/U}$$

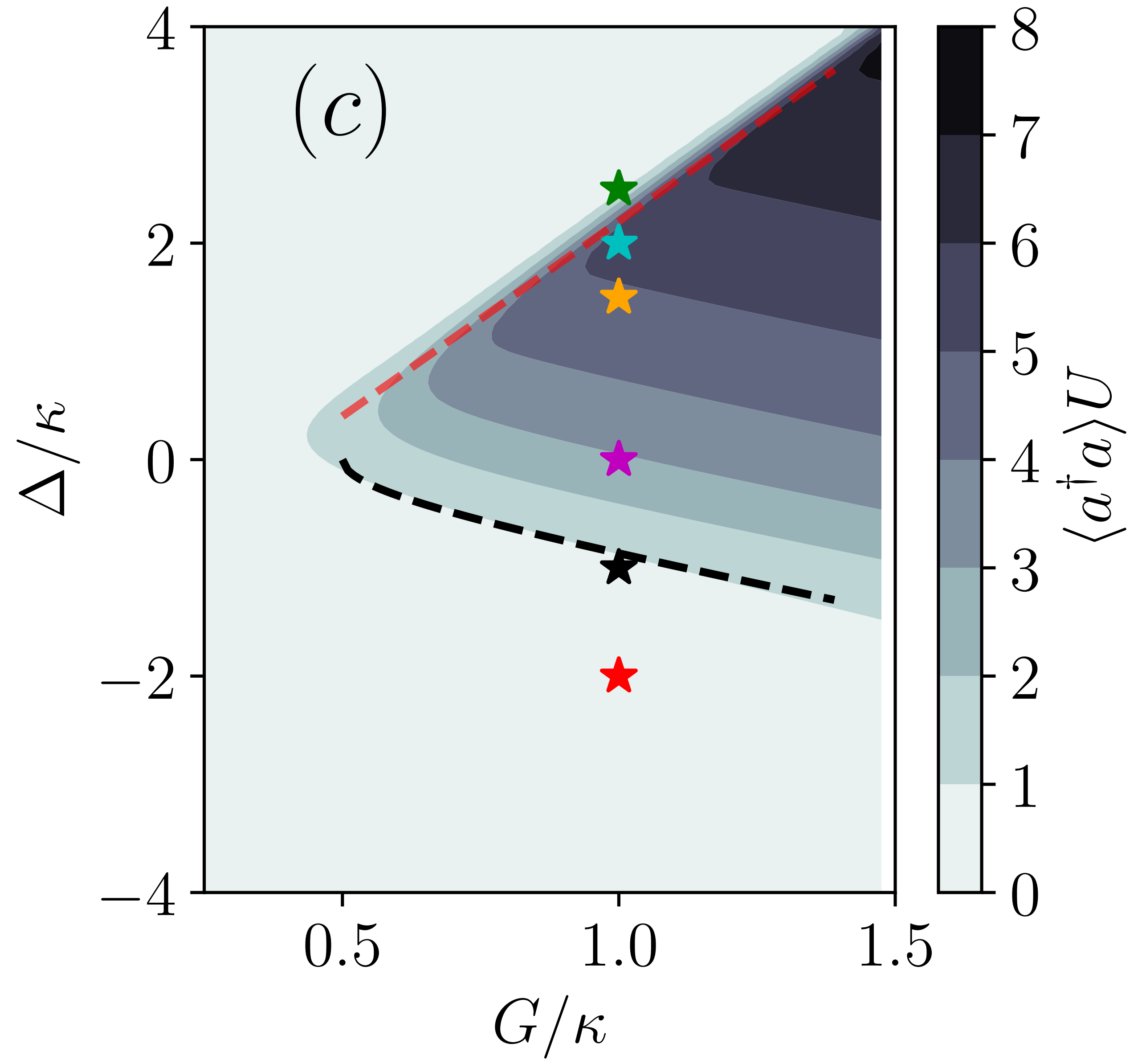
- Reflects sensitivity to all 3 blobs.

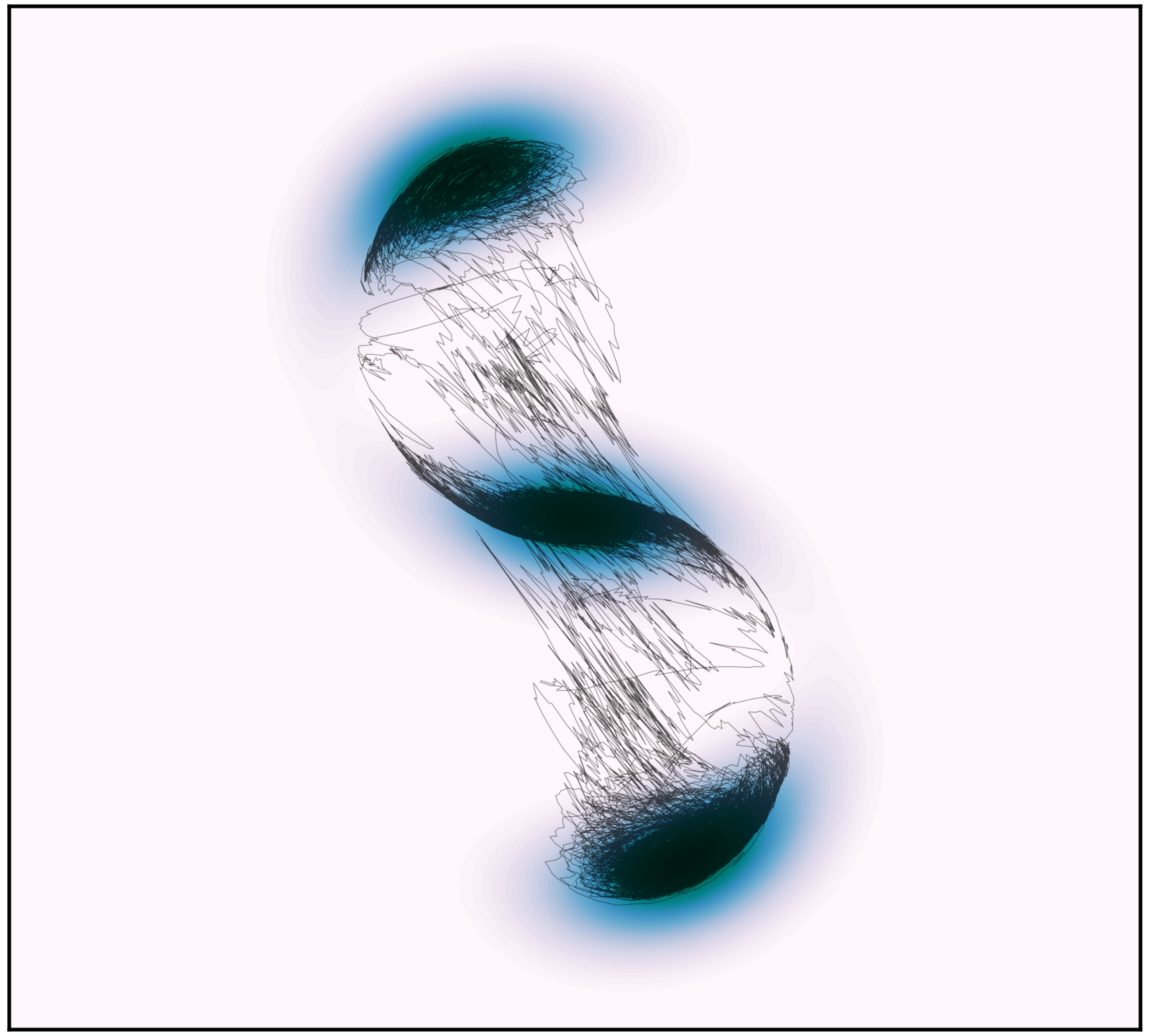


Summary & next steps

Summary

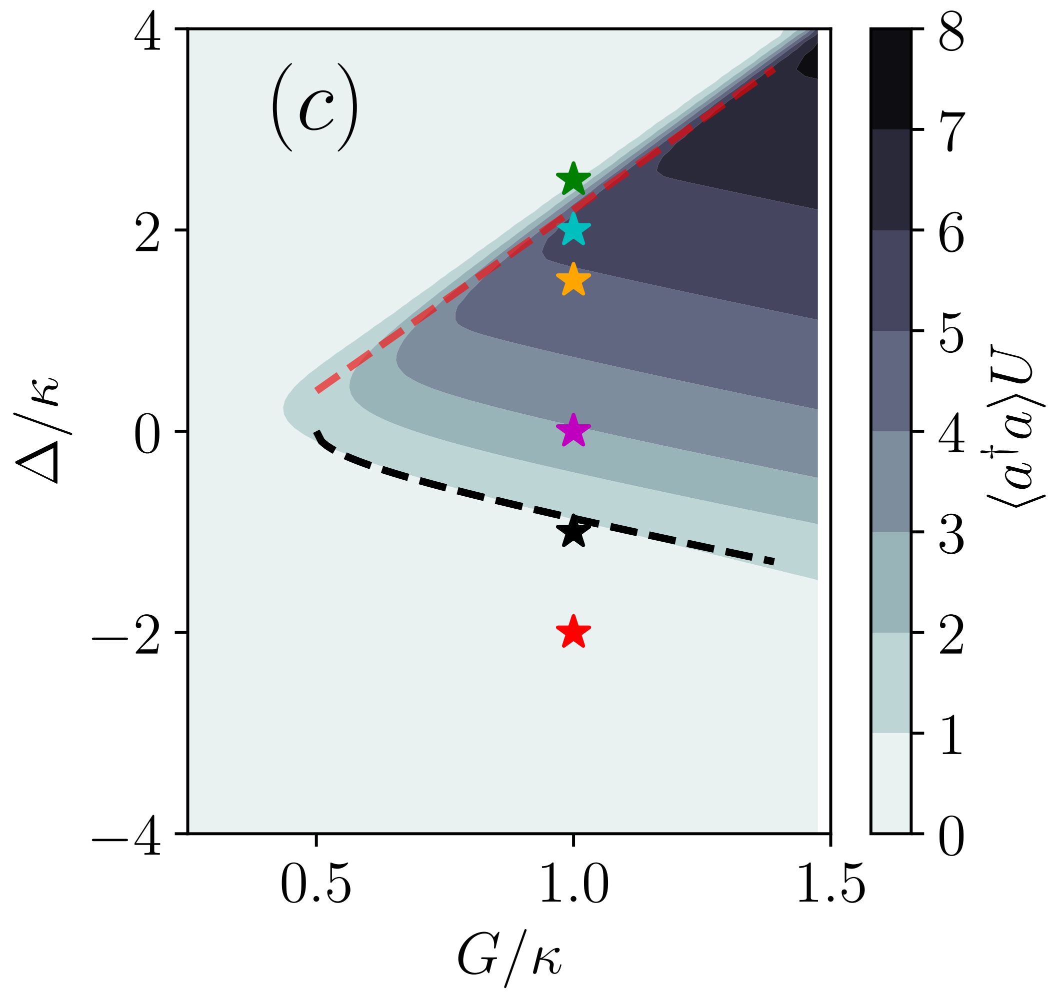
- Fluctuations in the time domain:
    - Classical time series produced by a quantum system.
- Connection between quantum optics and full counting statistics.
- Parametric Kerr model: critical properties of the fluctuations shed light on the nature of the transitions
    - Photo-detection: exponential divergence in the discontinuous transition.
    - Homodyne: exponential divergence in the entire critical region.

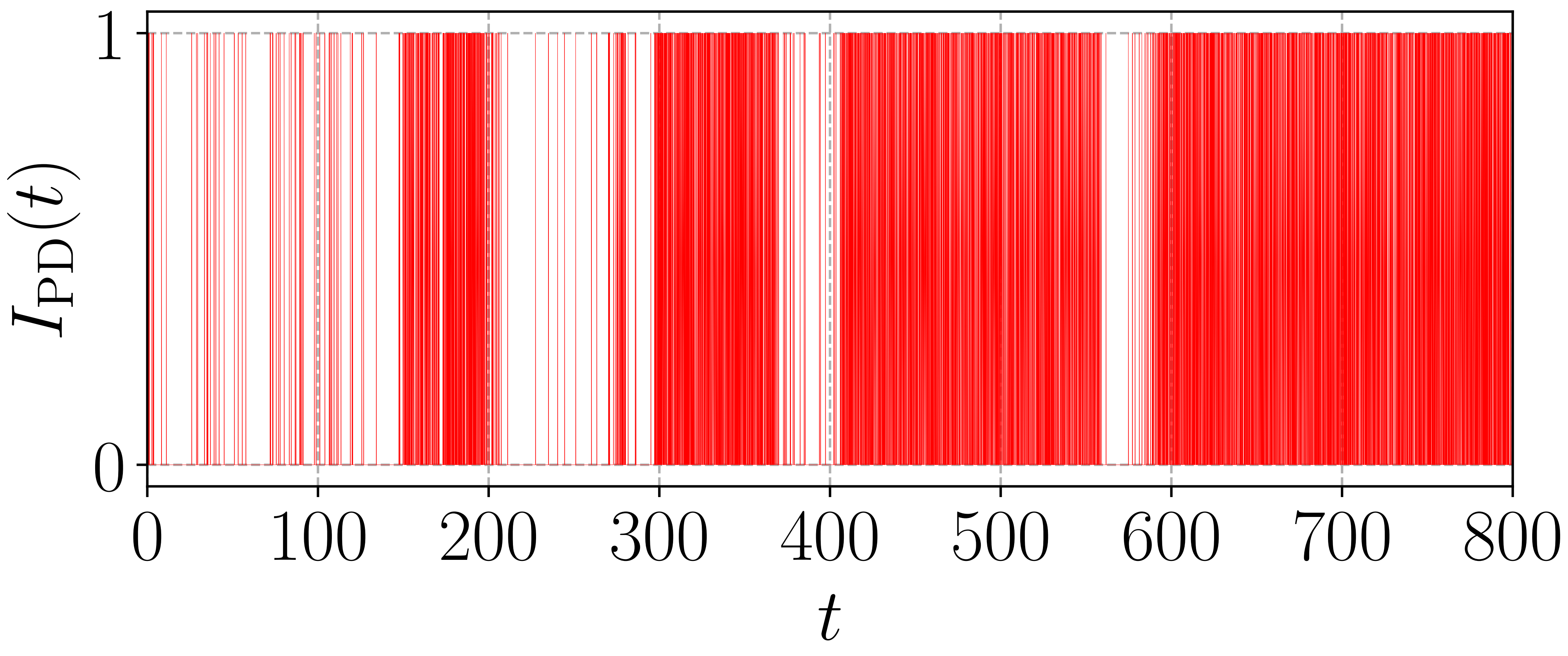




Metrology

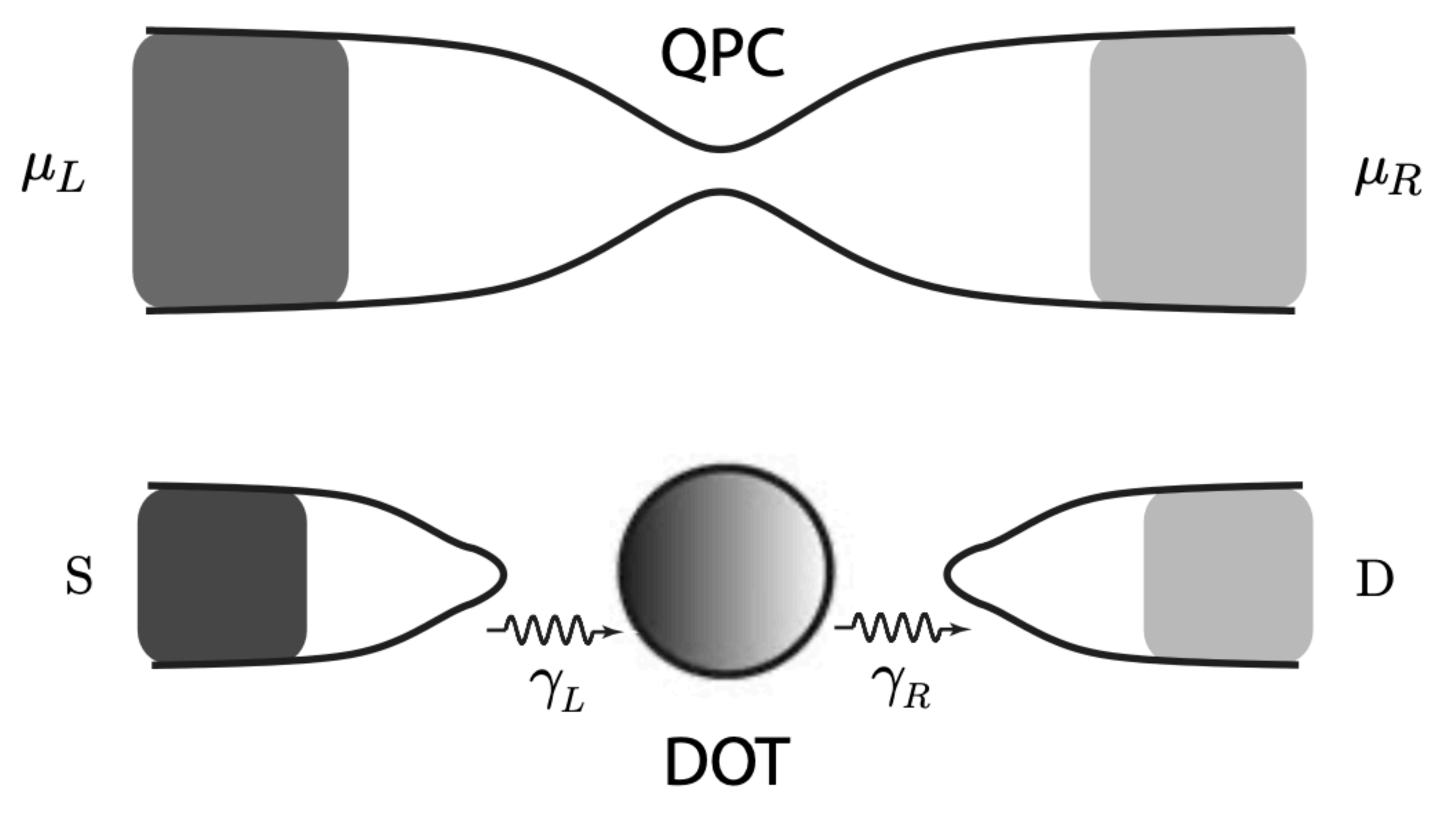
- Phase transition makes current very sensitive to changes in the parameters.
- We can use this as a sensor.
- $\Delta = \omega_p - \omega_c$
  - Dependence on the cavity frequency ω_c: affected by multiple processes.
  - Dependence on the pump frequency ω_p: sensitive sensing of input frequency.
  - Single-photon detector.

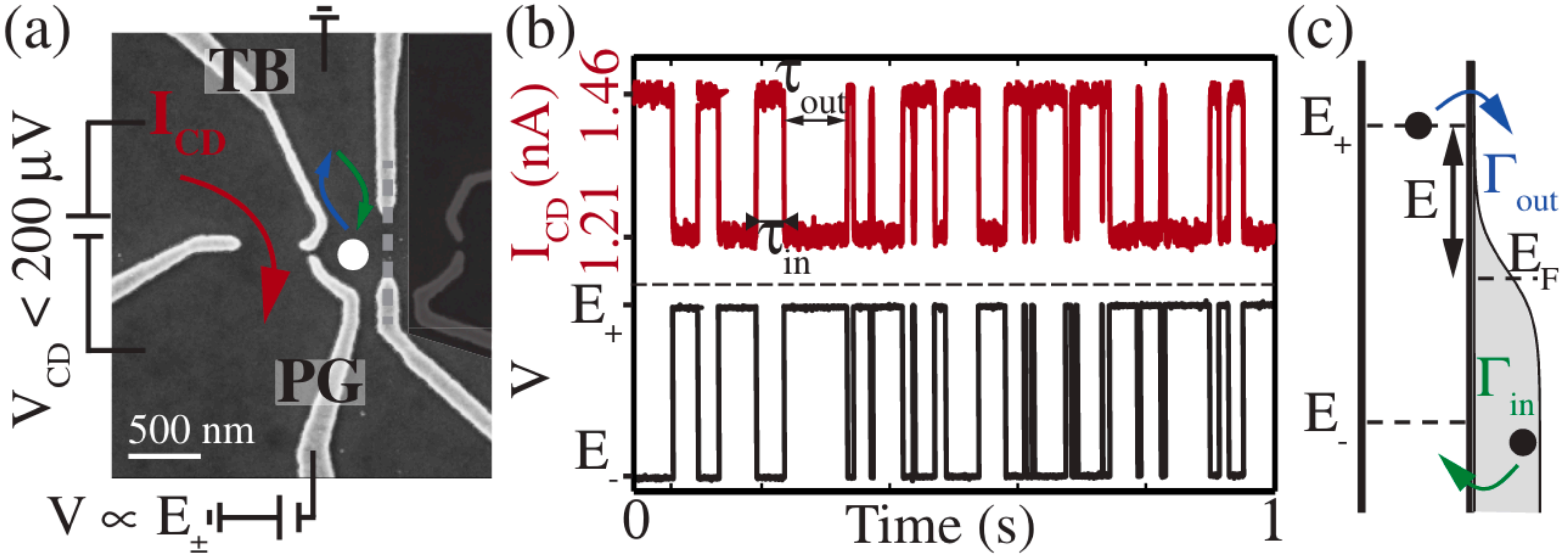




Waiting time metrology/thermometry

- The time between clicks encodes information on the system.
- We can use this as a thermometer.
- Ex: quantum dot continuously monitored using a quantum-point contact.



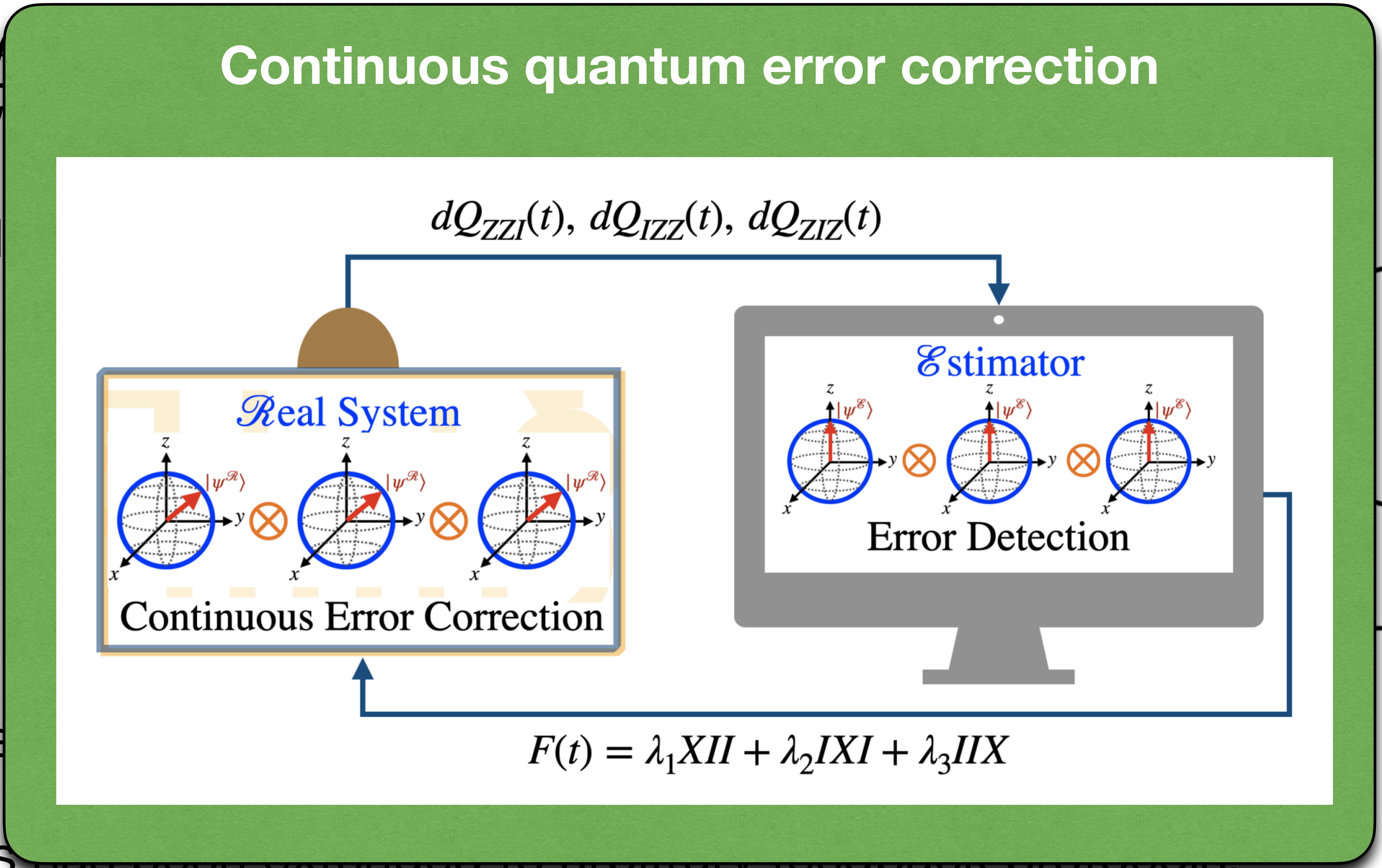


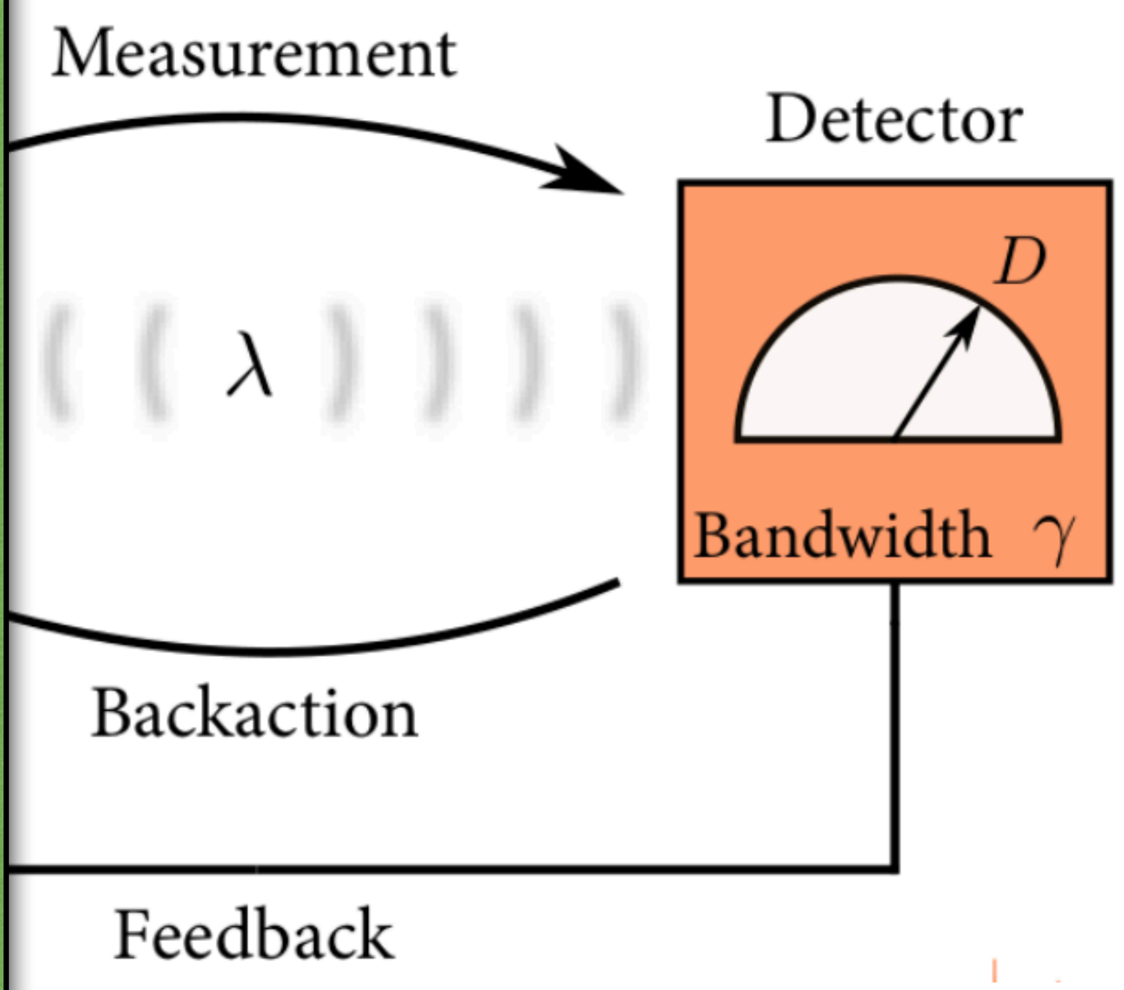
A. Hofmann, *et. al.*, PRL, 117, (2016)

Feedback control

- Reintroduce the filtered signal back into the system:

where $\tilde{I}(t)$ is a fil





- Up until now, the
- Very recently this has been extended to arbitrary feedback protocols.

B. Annby-Andersson, *et. al.*, "Quantum Fokker-Planck Master Equation for Continuous Feedback Control." Physical Review Letters, 129, (2022)

Thank you.

University of Rochester

My group is moving to UofR next year, and we have open positions to work on

- theory of quantum thermodynamics
- open quantum systems
- quantum information

For more information, visit:

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