Current fluctuations in critical Kerr resonators

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Nov. 2022 - XVII Enceunto SUF
Current fluctuations & two-time correlations

C. E. Fiore, Pedro E. Harunari, C. E. Fernandez Noa, and GTL

Current fluctuations in nonequilibrium discontinuous phase transitions
ArXiv 2109.00385

M. Kewming, M. Mitchison & GTL,
“Diverging current fluctuations in critical Kerr resonators”,
arXiv 2205.02622.

GTL, M. Kewming, M. Mitchison, P. Potts,
“Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics.”
Tutorial, in preparation
Overview

- Continuously measured quantum system.
- Ex: optical cavity
  - Monitor the photons that leak out.

  ➡ **Classical stochastic currents (time-series)**

- Quantum jumps: discrete current → individual clicks in the detector.
- Quantum diffusion: continuous (noisy) current.

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Diagram showing classical stochastic currents and quantum jumps and diffusion.
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Motivation

• In many experiments, this is the only way to probe it.
  • **What information does the current conveys about the system?**

• Basic quantity: average current $J = \langle I(t) \rangle$.
  • **But the current is stochastic:** *the full signal has a lot more information than the average.*

• Current fluctuations & two-time correlations: outcomes are not independent.
Why study current fluctuations

- **Metrology:**
  - Quantum system as a sensor.
    - e.g. magnetic fields.
    - e.g. gravitational waves (LIGO)
      - LIGO in fact uses exactly the present setup, with optical cavities.
  
  *Some information is only contained in the correlations!*

- **Thermo-kinetic uncertainty relations:**

\[
\frac{D}{J^2} \geq \frac{2}{\sigma}
\]

Current fluctuations (noise) → Entropy production (a measure of dissipation)

*Counterintuitive:*

To reduce fluctuations, we must increase dissipation.
Parametric Kerr model

- Non-linear quantum harmonic oscillator:

\[
\frac{d\rho}{dt} = -i[H(t), \rho] + \kappa \left[ a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\} \right]
\]

\[
H = -\Delta a^\dagger a + \frac{U}{2} a^\dagger a^\dagger a a + \frac{G}{2} (a^\dagger 2 + a^2)
\]

- \(a\) = annihilation operator
  photon operator for an optical cavity
- \(\Delta = \omega_p - \omega_c\) = detuning
- \(U\) = Kerr non-linearity.
  (requires a non-linear crystal inside the cavity)
- \(G\) = 2-photon pump
  (input laser produces photons in pairs)
- \(\kappa\) = loss rate
  rate at which photons leak out of the cavity
- 2 phase transitions, continuous and discontinuous
- Proper criticality occurs in the limit $U \to 0$ (“thermodynamic limit”)
Cat qubits

- Steady-state is a mixture of two Schrödinger cat states
  
  $|S\rangle = |\alpha\rangle + |-\alpha\rangle$

  $|A\rangle = |\alpha\rangle - |-\alpha\rangle$

- Use this to define cat qubits:

  $|0\rangle = |\alpha\rangle$

  $|1\rangle = |-\alpha\rangle$

- Cat qubits are more robust against errors.
  - Quantum computing with Kerr cats.

Semiclassical model

- Mean-field approximation: complex variable $\alpha(t)$

\[
\frac{d\alpha}{dt} = -(\kappa - i\Delta)\alpha - \frac{iU}{2}|\alpha|^2\alpha - iG\alpha^*
\]

- Pitchfork bifurcation at critical detuning $\Delta_c = -\sqrt{G^2 - \kappa^2/4}$

\[
\alpha^* = \pm \sqrt{n_0} e^{i\phi_0}
\]

\[
n_0 = \frac{\Delta}{U} + \sqrt{\frac{G^2 - \kappa^2/2}{U^2}}, \quad \phi_0 = \frac{1}{2}\arcsin\left(\frac{-\kappa}{2G}\right)
\]

* Can predict the continuous transition (bottom)

* Unable to predict the discontinuous transition (top).
Photo-detection current
How to simulate the stochastic photo-detections?

• Quantum jump unravelling.

• At each time step $dt$ a jump occurs with probability $\kappa dt \langle a^\dagger a \rangle$.
  
  • ($\propto dt$: very unlikely)

• If jump occurs: $|\psi\rangle \rightarrow a |\psi\rangle$

• If no jump occurs: $|\psi\rangle = e^{-iH_{\text{eff}} t} |\psi\rangle$

\[
H_{\text{eff}} = H - \frac{\kappa}{2} a^\dagger a
\]

(non-Hermitian Hamiltonian)

Every time a jump occurs, we count $dN = 1$

**Stochastic current:**

\[
I(t) = \frac{dN}{dt}
\]
**Photo-detection current**

- @ discontinuous transition: on/off (telegraph) behavior of the current.
- Photo-detection cannot resolve upper vs. lower blobs.

\[ J = \langle I \rangle = \text{"dynamical activity"} = \text{jumps/second} \]
Current fluctuations - Full Counting Statistics

- Two-time correlation function:
  \[ F(\tau) := \langle I(t)I(t + \tau) \rangle - J^2 \]
  \[ = J \delta(\tau) + J^2 [g^{(2)}(\tau) - 1] \]

- Power spectrum:
  \[ S(\omega) = \int_{-\infty}^{\infty} e^{-i\omega\tau} F(\tau) d\tau \]

- Zero-frequency component of the power spectrum := “noise”:
  \[ D = S(0) = \lim_{t \to \infty} \frac{d}{dt} \text{Var}(N(t)) \]

\[ g^{(2)} \Rightarrow \text{Glauber's 2nd order coherence function} \]
Divergence of the diffusion coefficient

- "Thermodynamic limit:" $U \to 0$
- In the discontinuous transition ($\Delta > 0$)

$$D \sim e^{1/U}$$
GaAs cavity polaritons.

T. Fink, et. al., Nature Physics, 14, 365 (2018)
Minimal 2-level model
Minimal 2-level model

- Let $q_0$ = prob. the system is in the middle blob.
  
  $q_1$ = prob. the system is in any of the outer blobs.

- Master equation:
  
  $$\frac{d}{dt} \begin{pmatrix} q_0 \\ q_1 \end{pmatrix} = \begin{pmatrix} -a & b \\ a & -b \end{pmatrix} \begin{pmatrix} q_0 \\ q_1 \end{pmatrix}$$

- Dependence with $U$:
  
  $$a \sim \exp\left\{ (\Delta - \Delta_c)/U \right\}$$

  $$b \sim \exp\left\{ -(\Delta - \Delta_c)/U \right\}$$

Time between jumps

$$\tau_m \sim e^{1/U}$$

Prob. of being in the outer blobs

$$q_1 \sim (1 + e^{-(\Delta-\Delta_c)/U})^{-1}$$
Noise

• The minimal model gives for the noise:

\[ D = q_1 D_1 + q_0 D_0 + q_1 (1 - q_1)(\mu_1 - \mu_0)^2 \tau_m \]

• First 2 terms: fluctuations within each blob.

• Last term: fluctuations between blobs.

• Depends on \( \tau_m \sim e^{1/U} \): diverges exponentially.
Homodyne detection
Homodyne current

• Mix photon output with a strong laser source \( \alpha = |\alpha|e^{i\phi} \).

• Equivalent to measuring

\[
\langle (a + \alpha)\dagger(a + \alpha) \rangle = \left( |\alpha|^2 + |\alpha| \langle ae^{-i\phi} + a^\dagger e^{i\phi} \rangle + \langle a^\dagger a \rangle \right)
\]

• \(|\alpha|^2\) is just a constant offset.

• If \( \alpha \) is large, then the current will predominantly measure \( a^\dagger a \) instead of \( a^\dagger a \).

• Quantum diffusion unravelling:

\[
d\rho = dt\mathcal{L}\rho + dW [\mathcal{H}\rho - \langle x \rangle\rho],
\]

\[
\mathcal{H}\rho = \kappa (a\rho + \rho a^\dagger)
\]

\[
dW = \text{Wiener increment:} \quad E(dW) = 0, \quad dW^2 = dt
\]
**Homodyne current (in \( p = i(a^\dagger - a) \))**

- The homodyne current switches between 3 values (+, 0, -).
- Captures the tunneling between the 3 blobs.
Divergence of the diffusion coefficient

- Homodyne current noise diverges exponentially in a much broader region.

\[ D \sim e^{1/U} \]

- Reflects sensitivity to all 3 blobs.
Summary & next steps
Summary

• Fluctuations in the time domain:
  • Classical time series produced by a quantum system.
  • Connection between quantum optics and full counting statistics.
• Parametric Kerr model: critical properties of the fluctuations shed light on the nature of the transitions
  • Photo-detection: exponential divergence in the discontinuous transition.
  • Homodyne: exponential divergence in the entire critical region.
Metrology

• Phase transition makes current very sensitive to changes in the parameters.
• We can use this as a sensor.
• \( \Delta = \omega_p - \omega_c \)
  • Dependence on the cavity frequency \( \omega_c \): affected by multiple processes.
  • Dependence on the pump frequency \( \omega_p \): sensitive sensing of input frequency.
• Single-photon detector.
Waiting time metrology/thermometry

- The time between clicks encodes information on the system.
- We can use this as a thermometer.
- Ex: quantum dot continuously monitored using a quantum-point contact.

**Feedback control**

- Reintroduce the filtered signal back into the system:

\[
\frac{df}{dt} = -i[H + \tilde{\mathcal{I}}(\tilde{I}(t))] + \kappa \{ a \rho a^\dagger - \frac{1}{2} \{ a^\dagger a, \rho \} \}
\]

where \( \tilde{I}(t) \) is a filtered version of the current \( \tilde{I}(t) \).

- Up until now, the theory has been restricted to linear feedback.

- Very recently this has been extended to arbitrary feedback protocols.

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University of Rochester

My group is moving to UofR next year, and we have open positions to work on

- theory of quantum thermodynamics
- open quantum systems
- quantum information

For more information, visit:

www.fmt.if.usp.br/~gtlandi