Irreversibility and current fluctuations in the quantum regime

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04/24/2023 - Iowa State University

https://www.pas.rochester.edu/~gtlandi/

Prof. George Porter
YouTube

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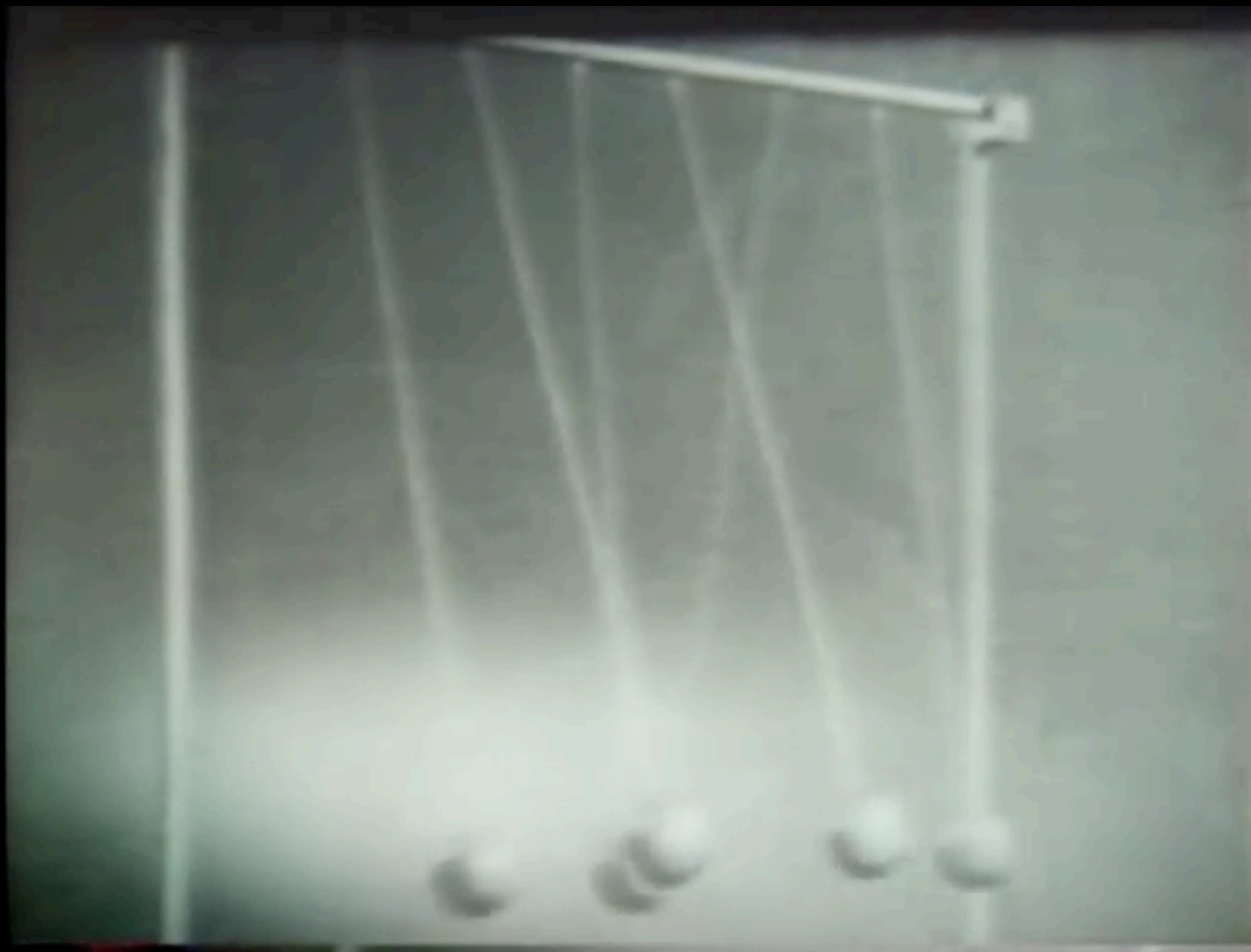

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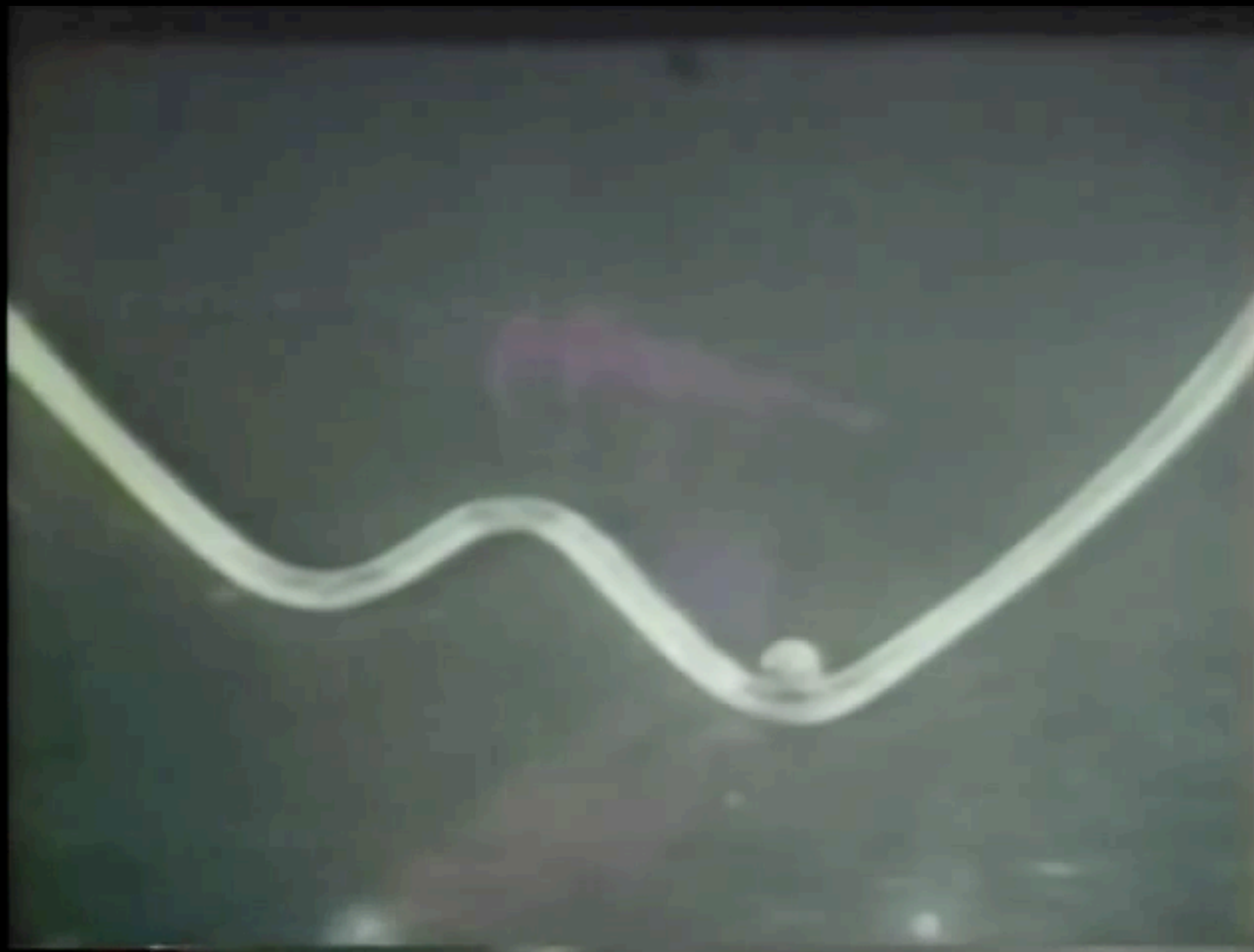
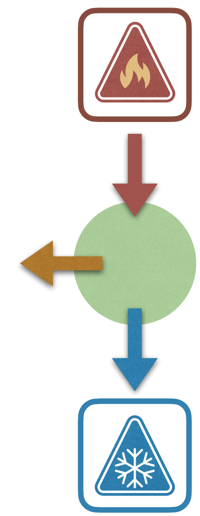


Irreversibility and the arrow of time

- Irreversibility: *how unlikely the backward process is, in comparison with the forward one.*
- But why does this happen?
  - The microscopic laws of the universe (Newton, Schrödinger, &c) are time-reversible.
- **Operational definition:** what is accessible and what is not.
  - **Dissipation:** heat lost to the environment cannot be recovered.
- Irreversible videos were those that involved a lot of dissipation.

Gabriel T. Landi and Mauro Paternostro, "**Irreversible entropy production, from quantum to classical**",
Review of Modern Physics, **93**, 035008 (2021)

Entropy production



- Clausius formulated the notion of irreversibility using entropy.
- Consider a thermodynamic process involving heat & work:

$$\Delta U = W + Q_h + Q_c \qquad \text{(1st law = balance equation)}$$

- According to Clausius, entropy does not satisfy a balance equation:

$$\Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma$$

$\sigma \geqslant 0$ is the entropy produced in the process.

- $\sigma \geqslant 0$ is the mathematical statement of the 2nd law.
- To explore the power of the 2nd law, let us consider a cyclic operation:

$$W + Q_h + Q_c = 0 \quad \text{and} \quad \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma = 0$$

$$W + Q_h + Q_c = 0 \quad \text{and} \quad \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma = 0$$

$$\sigma \geqslant 0$$

- **Operation as a heat engine:** efficiency

$$\eta = \frac{|W|}{|Q_h|} = \eta_c - \frac{T_c \sigma}{|Q_h|} \qquad \text{where} \qquad \eta_c = 1 - \frac{T_c}{T_h}$$

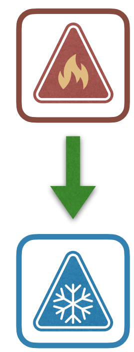
- The efficiency is always *lower* than Carnot's efficiency because entropy is produced (Carnot's statement of the 2nd law)

- **Heat flow** (no work): $Q_h = -Q_c$

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h \geqslant 0$$

Heat always flows from hot to cold

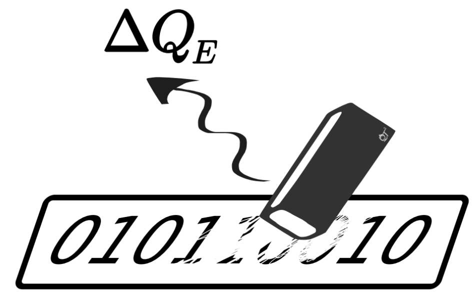
(Clausius' statement)



Otavio A. D. Molitor and Gabriel T. Landi, *Phys. Rev. A* **102**, 042217

- **Landauer's erasure:** Minimum cost to erase information

$$\Delta Q \geqslant k_B T \ln 2$$

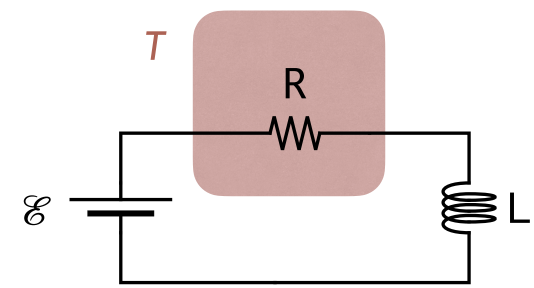


- What about $T \simeq 0$? Very relevant for quantum computation.
- If eraser is a waveguide:

$$\Delta Q \geqslant k_B T \ln 2 + \frac{3\hbar c}{\pi L} \ln^2(2)$$

- **Non-equilibrium steady-states:** not equilibrium.

$$\frac{dS}{dt} = \frac{\dot{Q}}{T} + \dot{\sigma} = 0 \qquad \text{so} \qquad \dot{\sigma} = -\frac{\dot{Q}}{T}$$



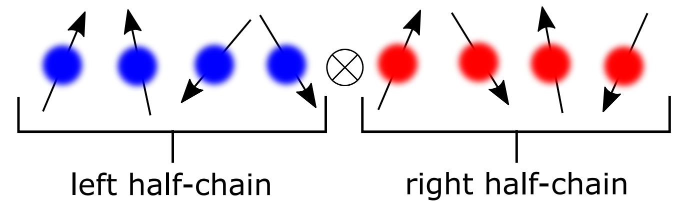
- Example: Joule heating.
  Continues as long as there is juice in the battery

$$\dot{\sigma} = \frac{\mathscr{E}^2}{RT}$$

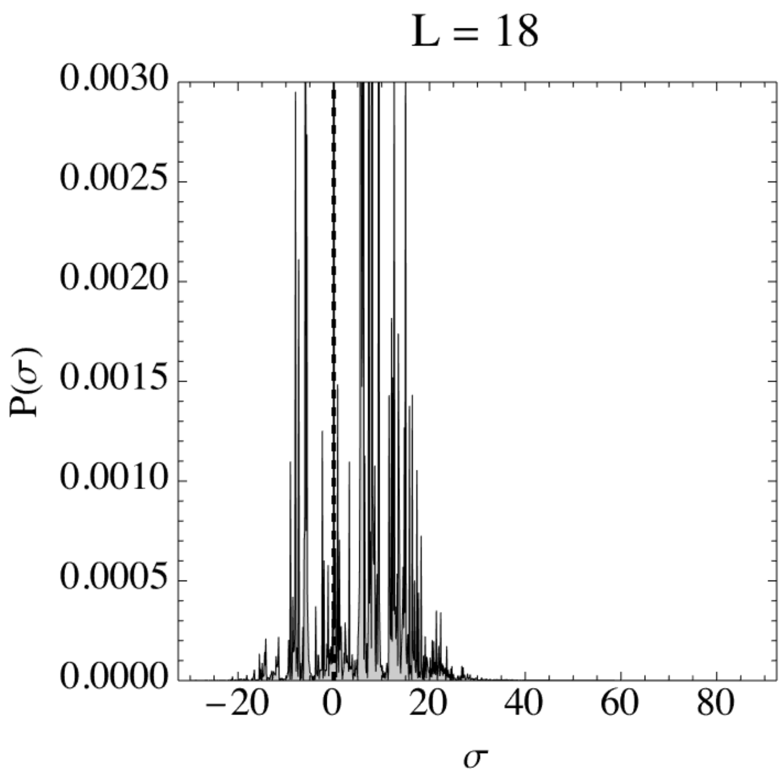
André M. Timpanaro, Jader P. Santos, and Gabriel T. Landi, *Phys. Rev. Lett.* **124**, 240601 (2020)

Fluctuations are significant in the micro-world

- Macro-world: heat flows from hot → cold.
- Micro-world: heat ***usually*** flows from hot → cold.



G. T. Landi and Dragi Karevski Phys. Rev. E **93**, 032122 (2015)



Heat Exchange Fluctuation Theorem

$$P(-\sigma) = e^{-\sigma} P(\sigma)$$

Implies 2nd law: $\langle \sigma \rangle \geqslant 0$

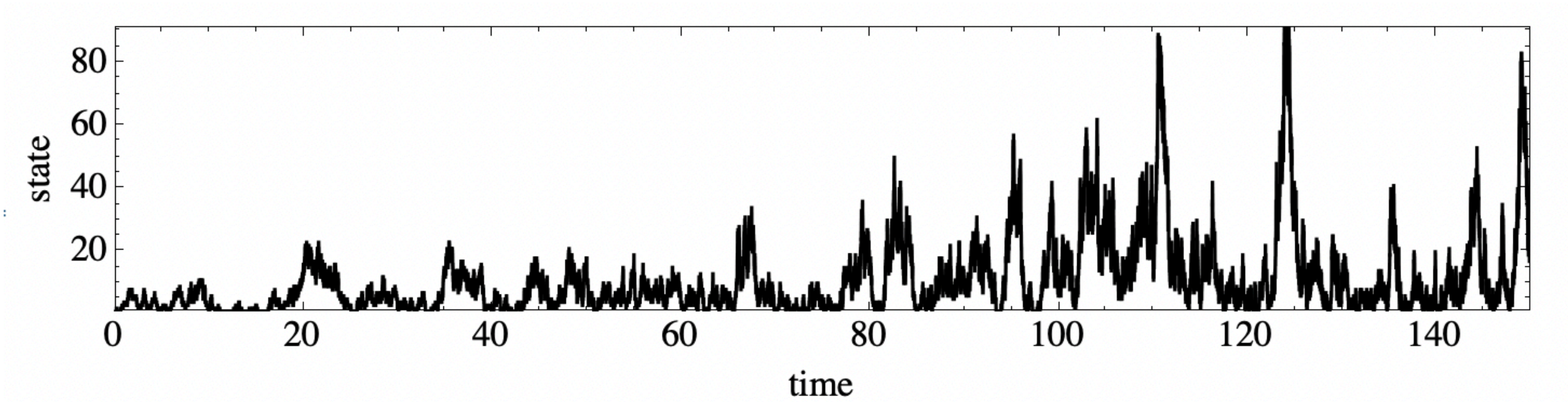
C. Jarzynski and D. Wójcik, Phys. Rev. Lett. 92, 230602 (2004)
G. E. Crooks, Journal of Statistical Physics, **90**, 1481–1487 (1998)

Entropy production and stochastic trajectories

- Fluctuations allow us to formulate the entropy production problem in terms of **trajectory probabilities**

$$\sigma[\gamma] = \ln \frac{P_F[\gamma]}{P_R[\gamma]}$$

- A process is reversible when the time-reversed process is as likely as the forward one.

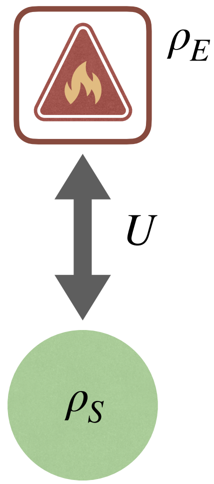


Entropy production for quantum systems

- Information-theoretic formulation: $\sigma = I(S:E) + D(\rho'_E || \rho_E)$
  - Operational interpretation: Characterizes irreversibility in terms of what you do not have access to:
    - System-environment correlations.
    - Changes in the environment.
- Tricky business: how to define heat currents for quantum master equations.

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

$$\rho'_{SE} = U(\rho_S \otimes \rho_E)U^\dagger$$

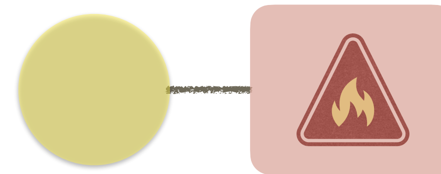



```
Describes an enormous
variety of processes!
(maybe a complicated U)
```

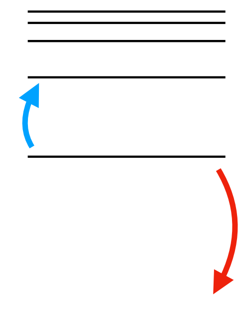
Gabriel T. Landi and Mauro Paternostro, "**Irreversible entropy production, from quantum to classical**", *Review of Modern Physics*, **93**, 035008 (2021)

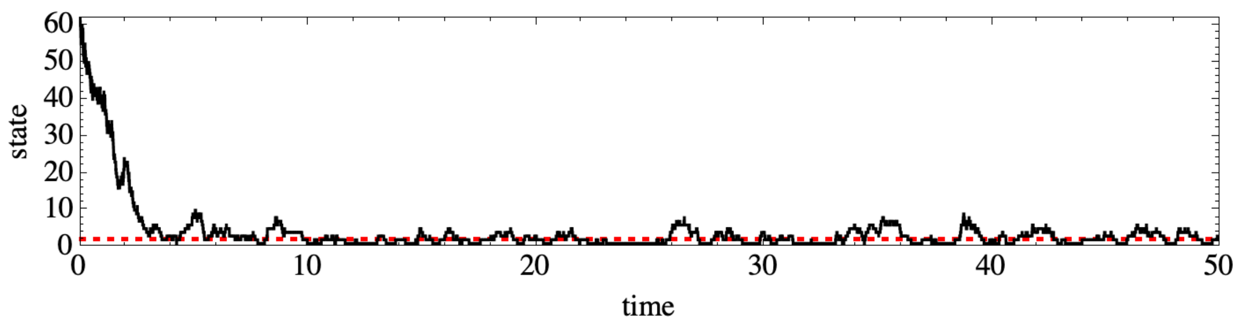
Gabriel T. Landi, Dario Poletti, Gernot Schaller, "**Nonequilibrium boundary-driven quantum systems: Models, methods, and properties.**" Reviews of Modern Physics, 94, (2022)

Relaxation towards equilibrium



- Imagine an atomic system relaxing towards equilibrium.
  - Population of energy eigenstates fluctuate until they reach thermal equilibrium.
- In addition: destroy any superpositions (**decoherence**).
- Entropy production rate can be split as $\sigma = \sigma_{\text{pop}} + \sigma_{\text{coh}}$





Additional entropy production due to coherence: Dissipation of information, without dissipation of energy.

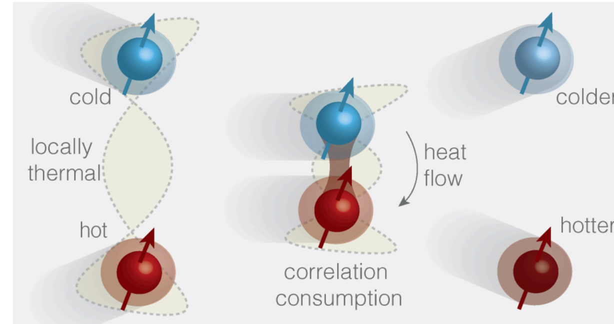
J. P. Santos, L. Céleri, GTL, M. Paternostro, npj Quantum Information **5**, 23 (2019)

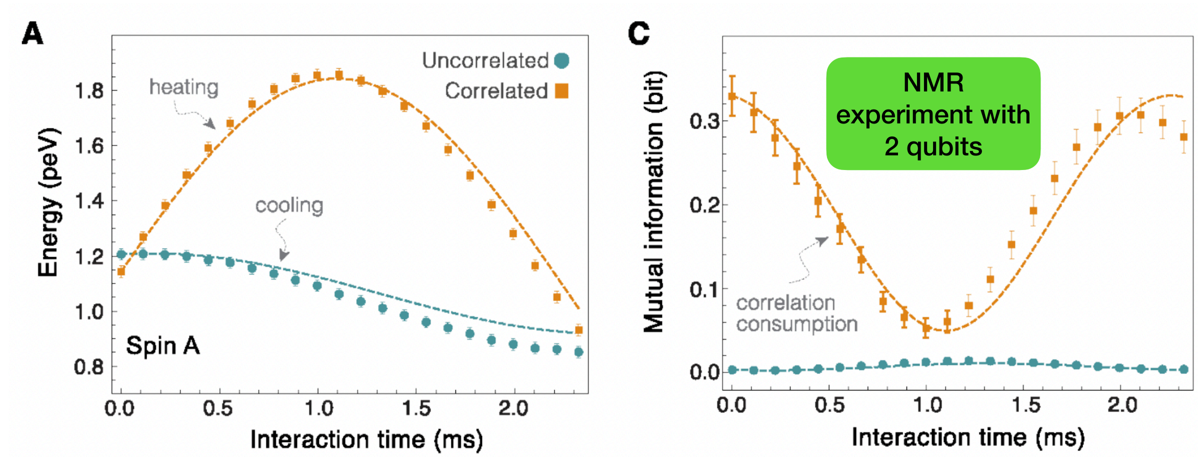
Information-thermodynamics

- In the presence of initial correlations the second law is modified to

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h \geqslant \Delta I(h:c)$$

- Heat can flow from cold to hot, provided we **consume** quantum correlations.





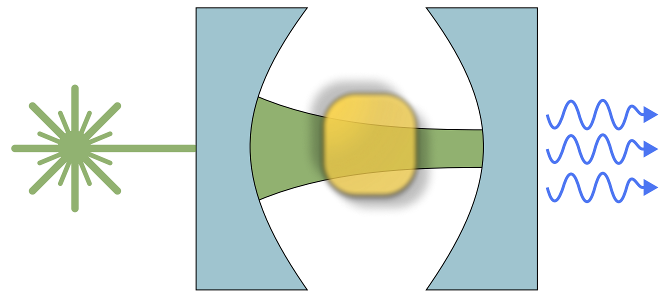
Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, "**Reversing the direction of heat flow using quantum correlations**", *Nature Communications*, **10**, 2456 (2019)

K. Micadei, G. T. Landi, E. Lutz, “Quantum fluctuation theorems beyond two-point measurements”, Phys. Rev. Lett. 124, 090602 (2020)

Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Roberto M. Serra, Eric Lutz, “Experimental validation of fully quantum fluctuation theorems”, Phys. Rev. Lett., 127, 180603 (2021).

Quantum phase space

- Many quantum experiments are done using optical cavities with semi-transparent mirrors.
- Photons leaking out $\simeq$ zero temperature bath.
  - Spontaneous emission: excitations can leave, but not return.
- 2nd law is buggy @ $T = 0$: $\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h.$
  - Does not include vacuum fluctuations (*present in every measurement*).
- We reformulated the entropy production problem in terms of quantum phase space & the *Wigner function.*



$$\sigma = \left(\frac{1}{T_c^{\text{eff}}} - \frac{1}{T_h^{\text{eff}}}\right) Q_h$$

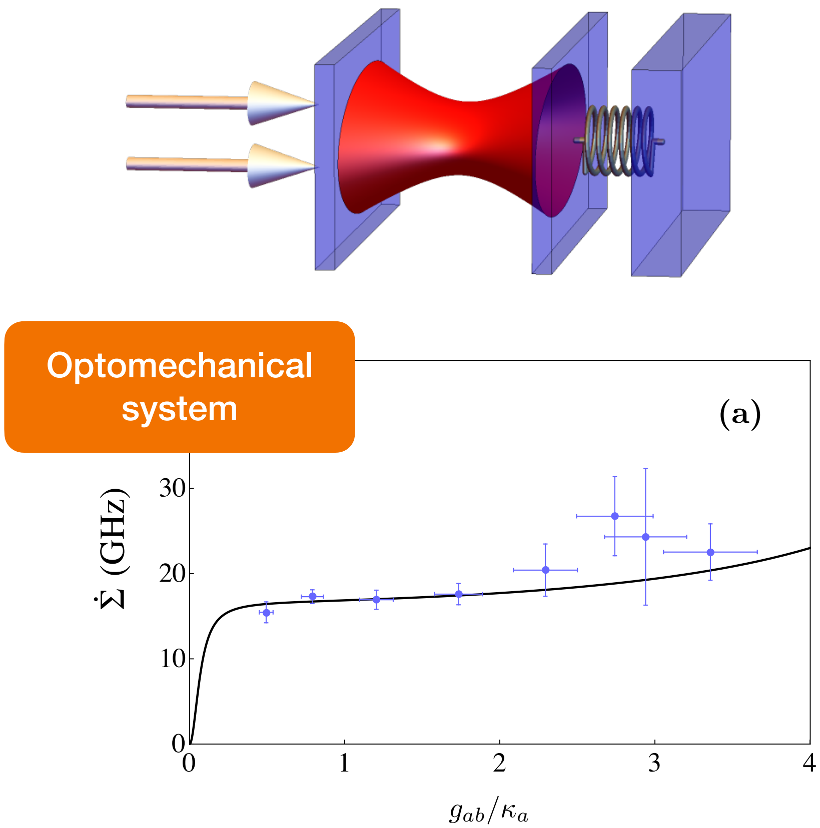
$$T^{\text{eff}} = \omega(\bar{n} + 1/2), \qquad \bar{n} = \frac{1}{e^{\beta\omega} - 1}$$

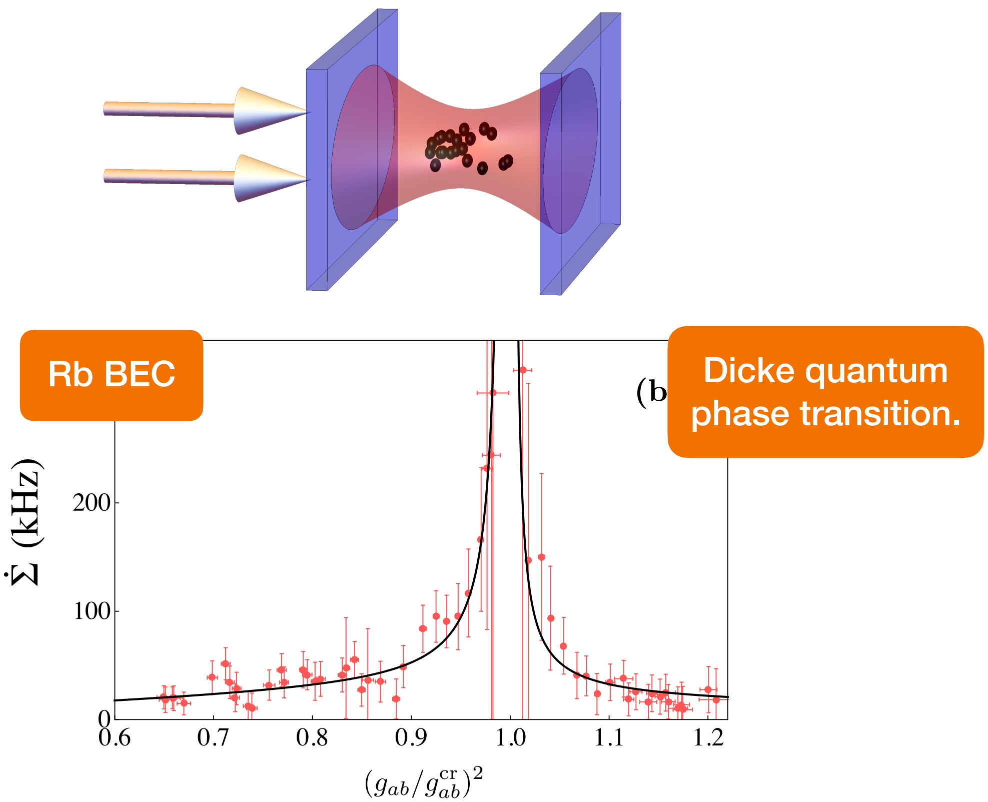
High temperatures: $\omega(\bar{n} + 1/2) \simeq T.$

Zero temperature: $\omega(\bar{n} + 1/2) = \omega/2.$

Jader P. Santos, G.T. Landi and Mauro Paternostro, Phys. Rev. Lett, **118**, 220601 (2017),

Experiments





M. Brunelli, L. Fusco, R. Landig, W. Wieczorek, J. Hoelscher-Obermaier, GTL, F Semião, A. Ferraro, N. Kiesel, T. Donner, G. De Chiara, and M. Paternostro. *Phys. Rev. Lett.*, **121**, 160604 (2018)

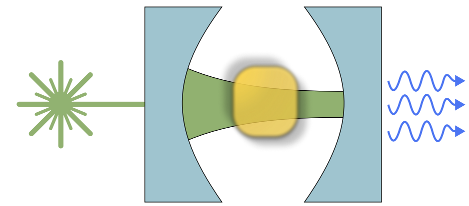
Continuously monitored quantum systems

Alessio Belenchia, Luca Mancino, Gabriel T. Landi and Mauro Paternostro, "**Entropy production in continuously measured quantum systems**", npj Quantum Information, **6**, 97 (2020).

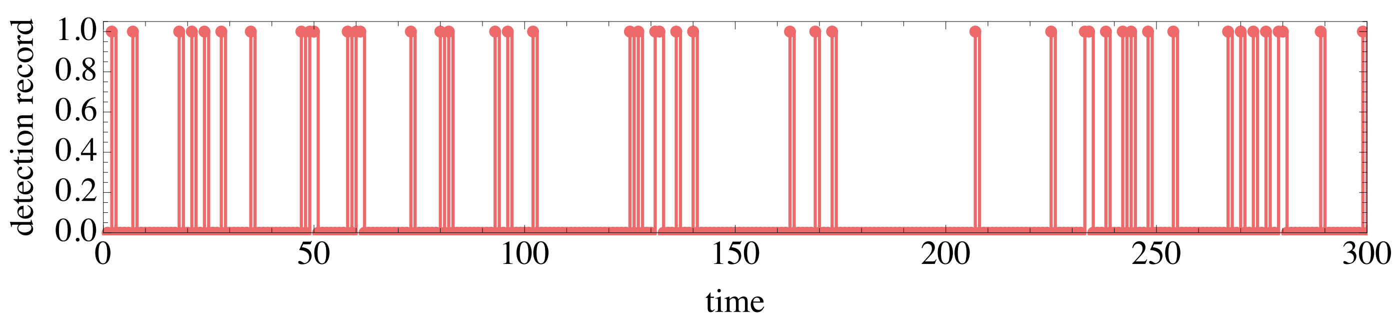
Gabriel T. Landi, Mauro Paternostro and Alessio Belenchia, "**Informational steady-states and conditional entropy production in continuously monitored systems**", PRX Quantum **3**, 010303, (2020).

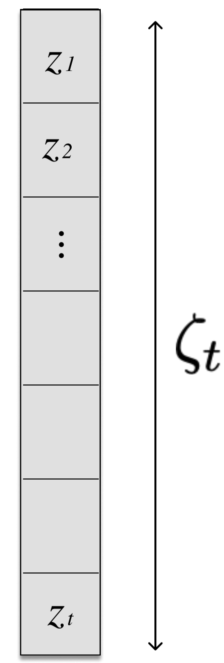
Massimiliano Rossi, Luca Mancino, Gabriel T. Landi, Mauro Paternostro, Albert Schliesser, Alessio Belenchia, "**Experimental assessment of entropy production in a continuously measured mechanical resonator**", Phys. Rev. Lett. **125**, 080601 (2020)

Continuously monitored quantum systems



- Continuous monitoring of photons that leak out of the cavity.
    - Individual clicks in the detector.
- Fundamental questions: what is entropy production *given* a detection record.
    - Operation: define thermodynamics in terms of what we can actually measure.
    - Includes *information* directly in the formulation.





Holevo information

- **Unconditional:** If we do not know the individual clicks: ρ_t
- **Conditional on the detection record:** $\rho_{t|\zeta_t}$
- **Holevo information:** accumulated information we learned from the detection.

$$I(S_t : \zeta_t) = \sum_{\zeta_t} P(\zeta_t) D\left(\rho_{t|\zeta_t} || \rho_t\right)$$

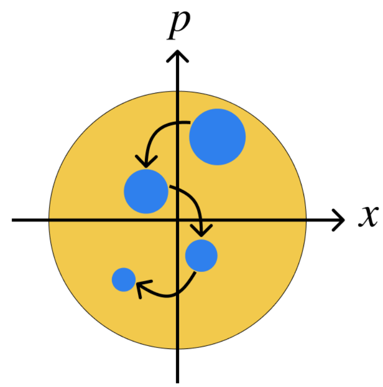
- With each new detection

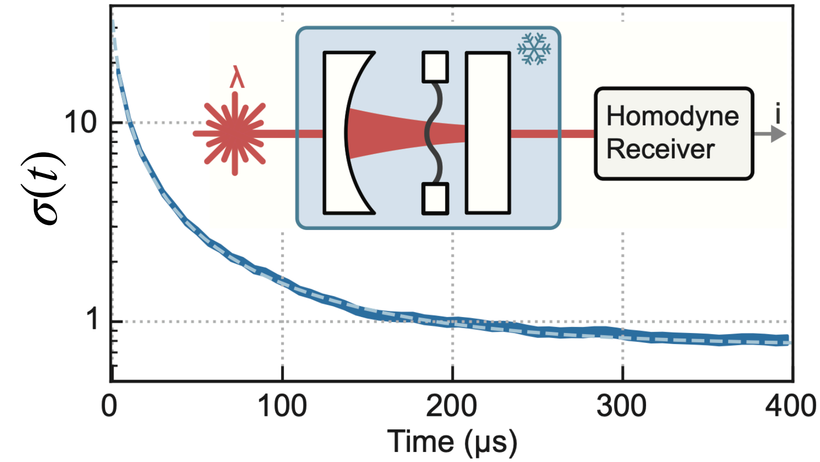
$$\Delta I_t = G_t - L_t = \text{gain} - \text{loss}$$

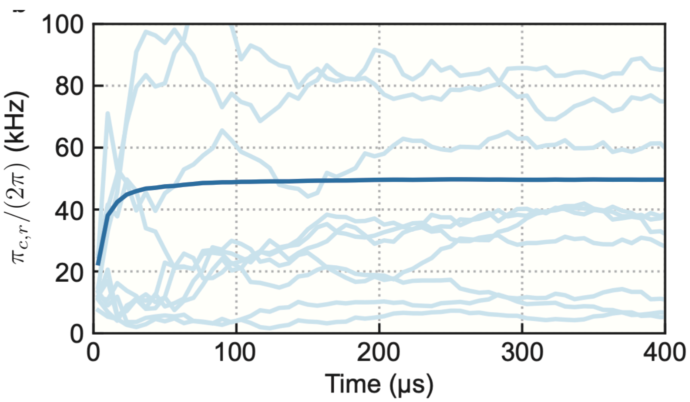
- Conditional entropy production

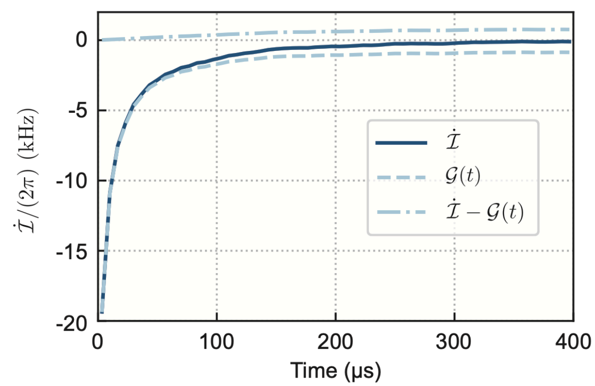
$$\Delta\Sigma^c = \Delta\Sigma^u - \Delta I$$

🇩🇰 Optomechanical setup









Informational steady-state:

Conditional dynamics relaxes to a colder state, which can only be maintained by continuously monitoring S.

Massimiliano Rossi, Luca Mancino, Gabriel T. Landi, Mauro Paternostro, Albert Schliesser, Alessio Belenchia, "**Experimental assessment of entropy production in a continuously measured mechanical resonator**", Phys. Rev. Lett. **125**, 080601 (2020)

Current fluctuations in the Parametric Kerr model

Michael J. Kewming, Mark T. Mitchison, Gabriel T. Landi, "**Diverging current fluctuations in critical Kerr resonators**."
Physical Review A, 106, (2022)

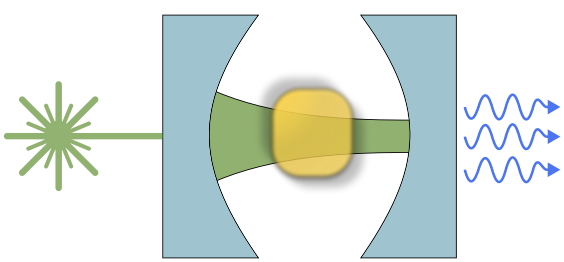
Gabriel T. Landi, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics**," 2023. arxiv 303.04270

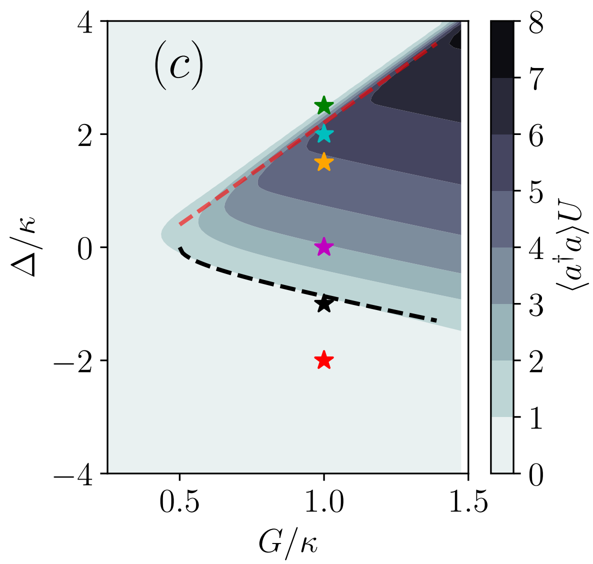
Parametric Kerr model

- Non-linear quantum harmonic oscillator:

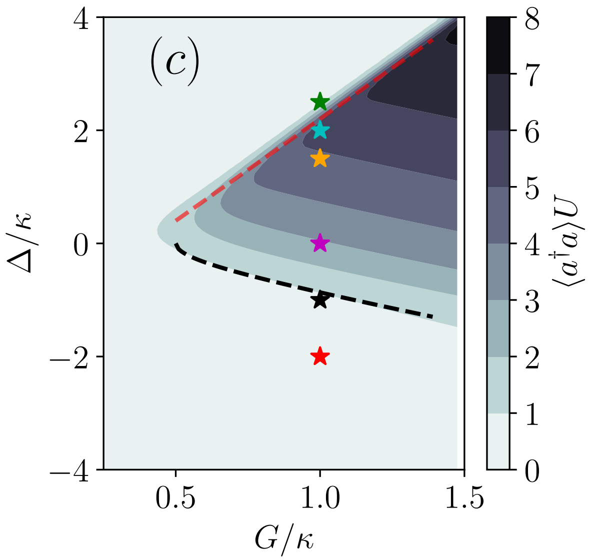
$$\frac{d\rho}{dt} = -i[H(t),\rho] + \kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right]$$

$$H = -\Delta a^\dagger a + \frac{U}{2}a^\dagger a^\dagger a a + \frac{G}{2}(a^{\dagger 2} + a^2)$$

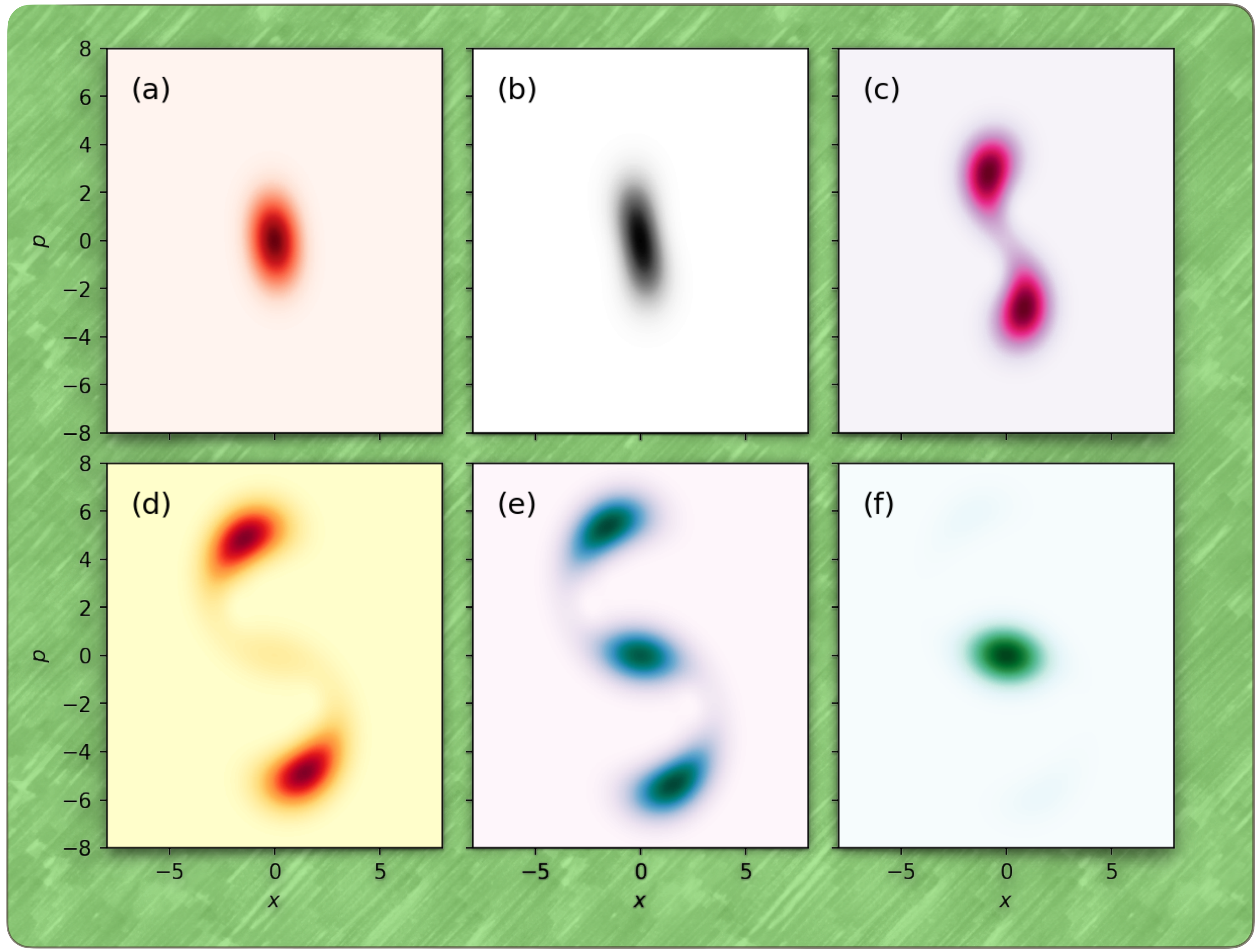




- a = annihilation operator
  photon operator for an optical cavity
- $\Delta = \omega_p - \omega_c$ = detuning
- U = Kerr non-linearity.
  (requires a non-linear crystal inside the cavity)
- G = 2-photon pump
  (input laser produces photons in pairs)
- κ = loss rate
  rate at which photons leak out of the cavity



Wigner function



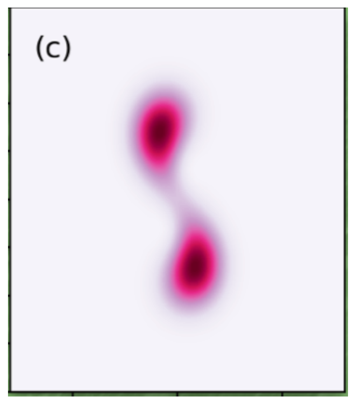
- 2 phase transitions, continuous and discontinuous
- Proper criticality occurs in the limit $U \to 0$ ("thermodynamic limit")

Cat qubits

- Steady-state is a mixture of two Schrödinger cat states

$$|S\rangle = |\alpha\rangle + |-\alpha\rangle$$

$$|A\rangle = |\alpha\rangle - |-\alpha\rangle$$

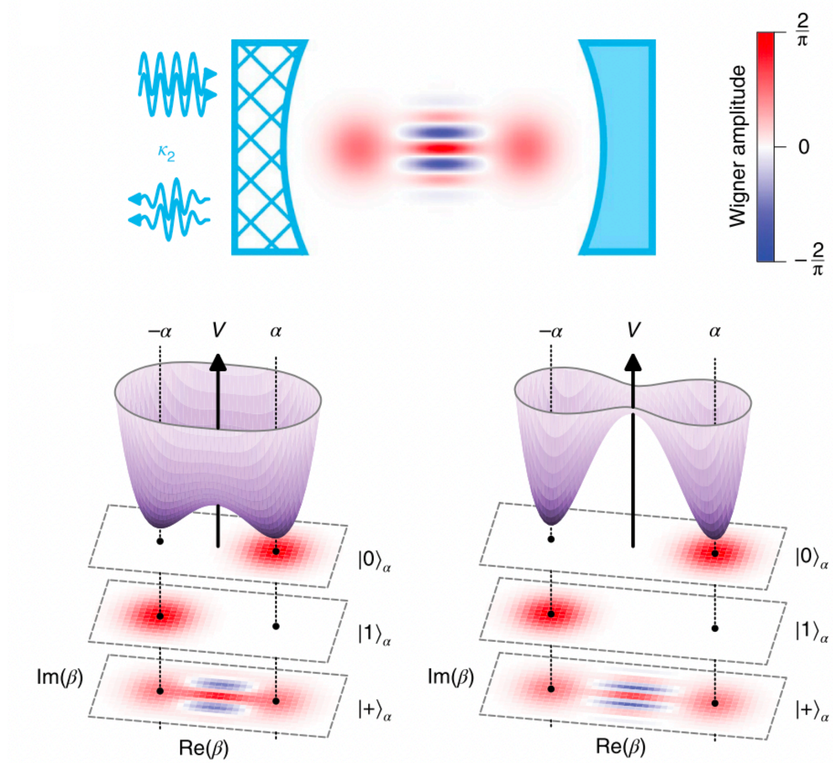


- Use this to define cat qubits:

$$|0\rangle = |\alpha\rangle$$

$$|1\rangle = |-\alpha\rangle$$

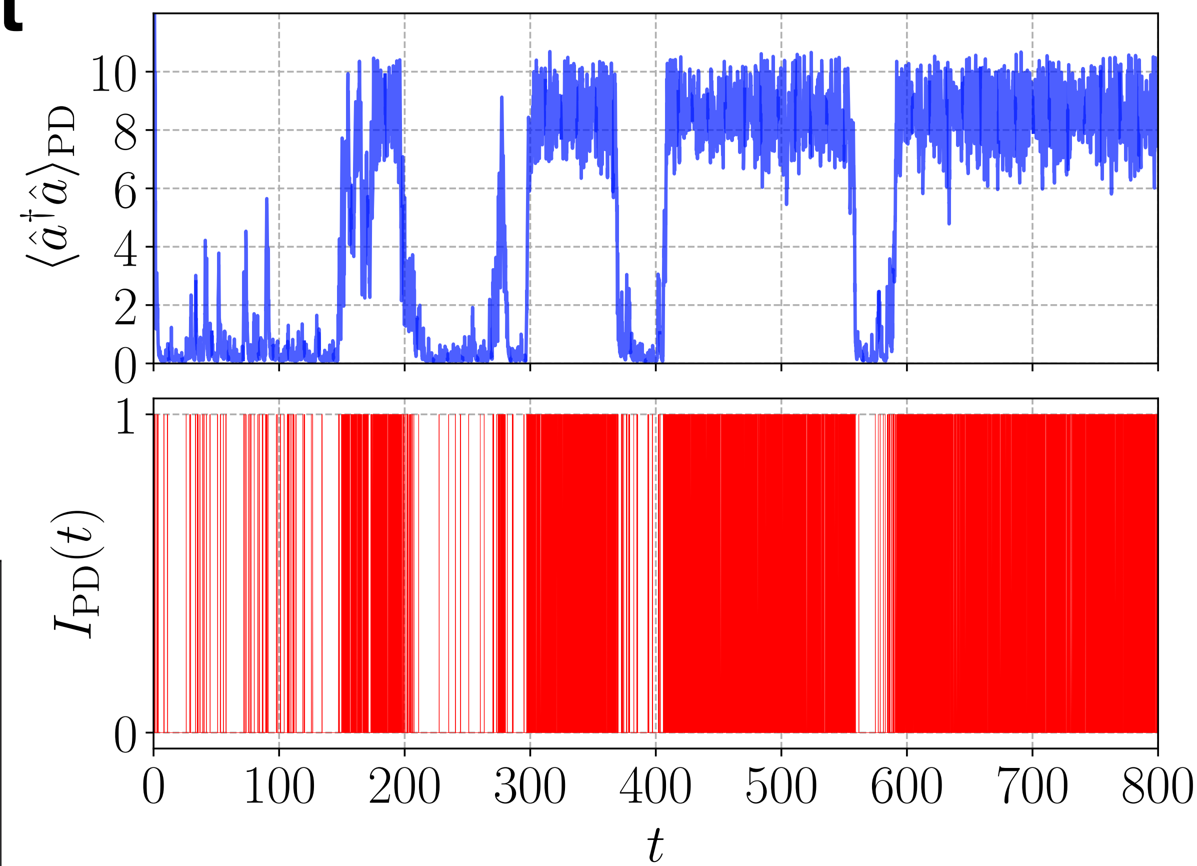
- Cat qubits are more robust against errors.
    - Quantum computing with Kerr cats.

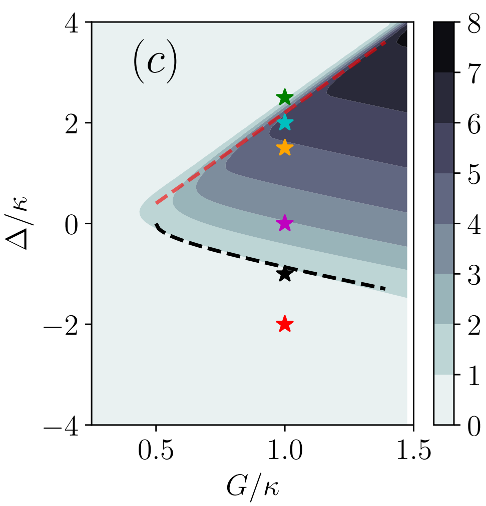


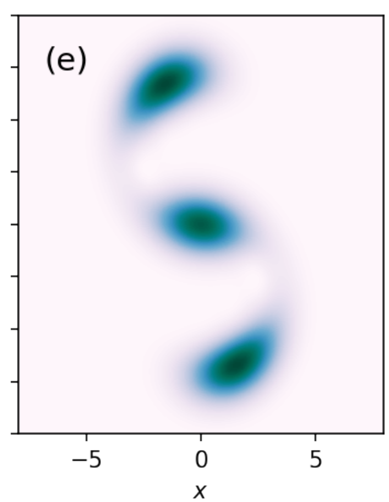
Lescanne, *et. al.*, Nature, **16**, 509-513 (2020)

Photo-detection current

- @ discontinuous transition: on/off (telegraph) behavior of the current.
- Photo-detection cannot resolve upper vs. lower blobs.





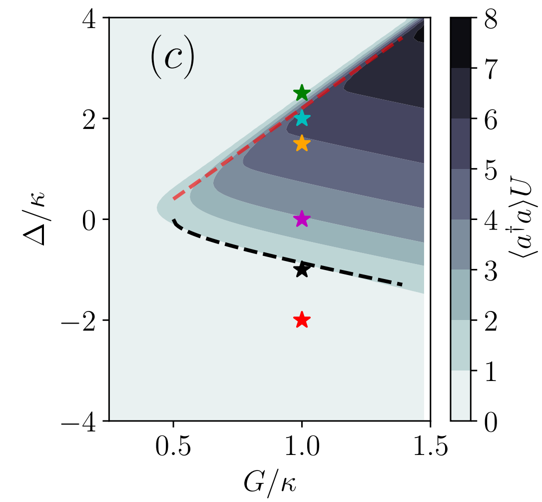


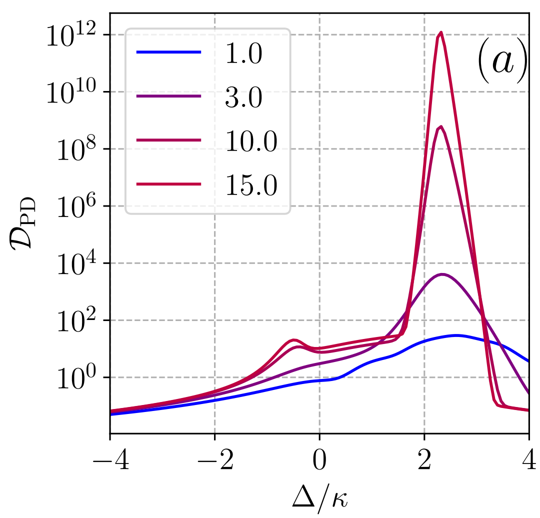
$J = \langle I \rangle$ = photon current = clicks/second

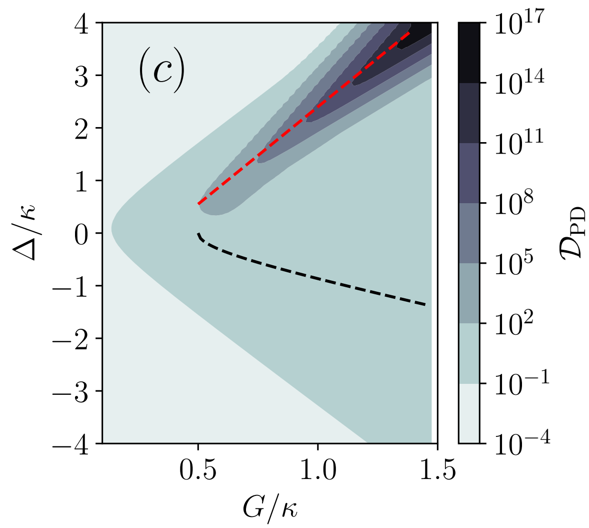
Exponential divergence of the noise

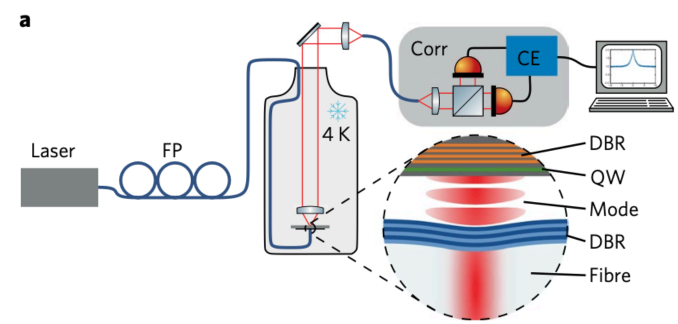
- "Thermodynamic limit:" $U \to 0$
- In the discontinuous transition ($\Delta > 0$)

$$D \sim e^{1/U}$$





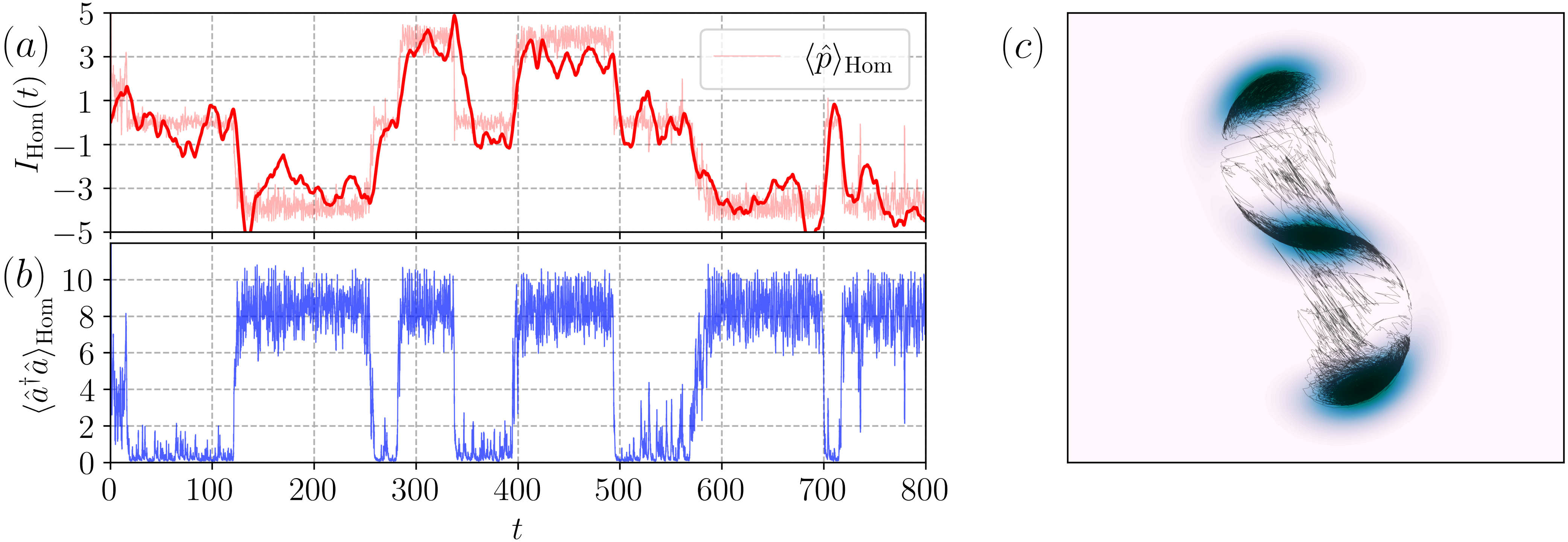




T. Fink, *et. al.*, *Nature Physics,* **14**, 365 (2018)

Homodyne current

- Observable is now $p = i(a^\dagger - a)$.
- The homodyne current switches between 3 values (+,0,-).
- Captures the tunneling between the 3 blobs.

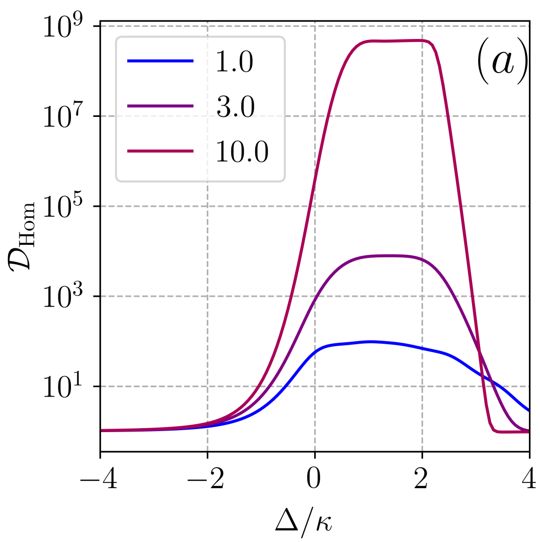


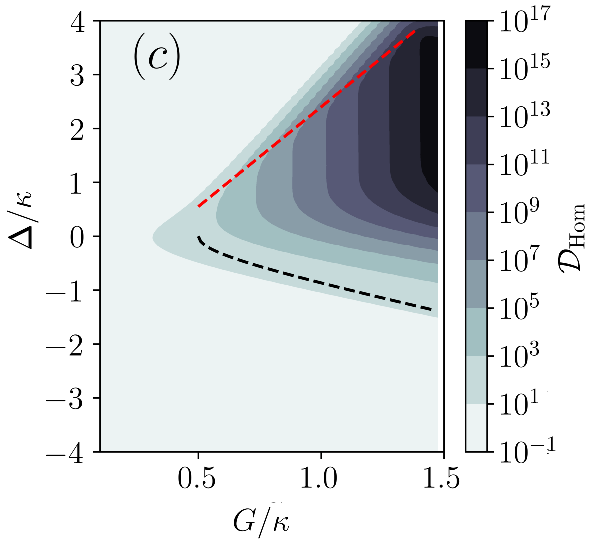
Divergence of the noise in the homodyne case

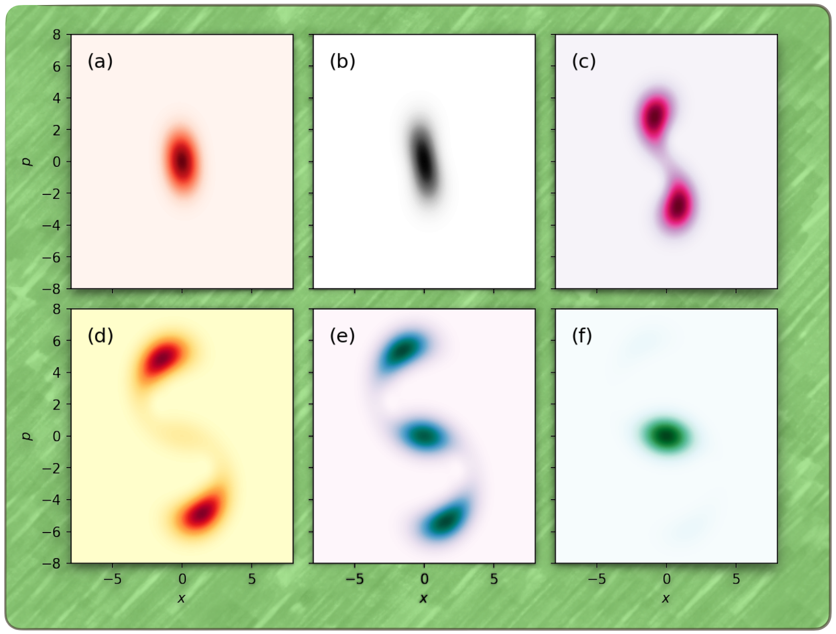
- Homodyne current noise diverges exponentially in a much broader region.

$$D \sim e^{1/U}$$

- Reflects sensitivity to all 3 blobs.







First study of Full Counting Statistics of the homodyne current

Conclusions

Thank you!

- **Entropy production:** quantifies dissipation/irreversibility.
- In the quantum realm:
  - How to define it.
  - What does it imply?
    - Quantum coherences and correlations.
- **Current fluctuations:**
  - 2nd law changes *given* information we learn about the system
  - Characterize correlations present in a signal.
    - Yields information on critical systems.

G.T. Landi, M. J. Kewming, M.T. Mitchison and P. Potts,

"**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics**"

arXiv 2303.04270
To appear soon as a tutorial in PRX Quantum.

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