Irreversibility, current fluctuations and thermodynamic uncertainty relations in the quantum regime

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02/08/2023 - University of Rochester

https://www.pas.rochester.edu/~gtlandi/

Prof. George Porter
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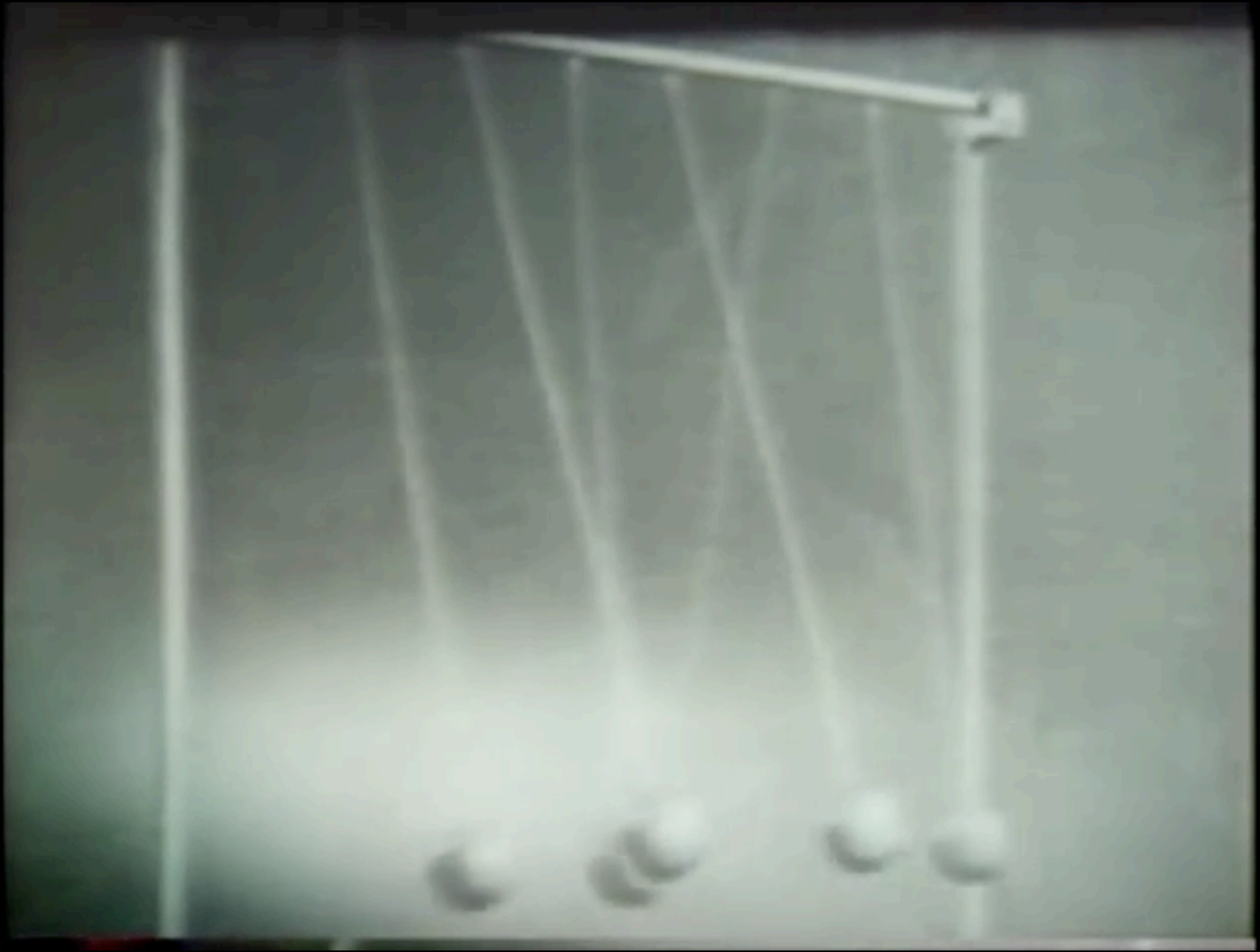

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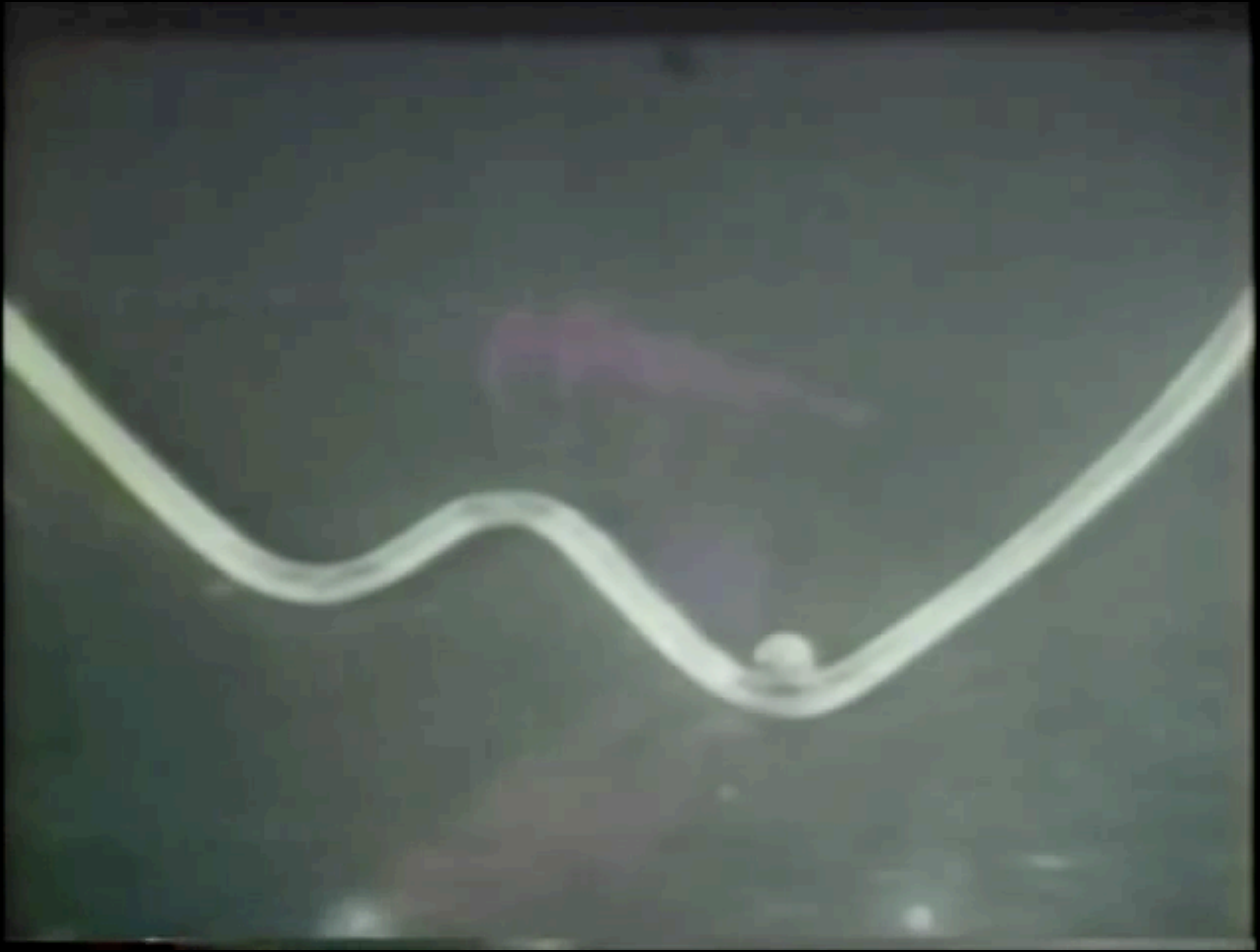


Irreversibility and the arrow of time

- Irreversibility: *how unlikely the backward process is, in comparison with the forward one.*
- But why does this happen?
  - The microscopic laws of the universe (Newton, Schrödinger, &c) are time-reversible.
- **Operational definition:** what is accessible and what is not.
  - **Dissipation:** heat lost to the environment cannot be recovered.
- Irreversible videos were those that involved a lot of dissipation.

Gabriel T. Landi and Mauro Paternostro, "**Irreversible entropy production, from quantum to classical**",
Review of Modern Physics, **93**, 035008 (2021)

Entropy production

- Clausius formulated the notion of irreversibility using entropy.
- Consider a thermodynamic process involving heat & work:

$$\Delta U = W + Q_h + Q_c \qquad \text{(1st law = balance equation)}$$

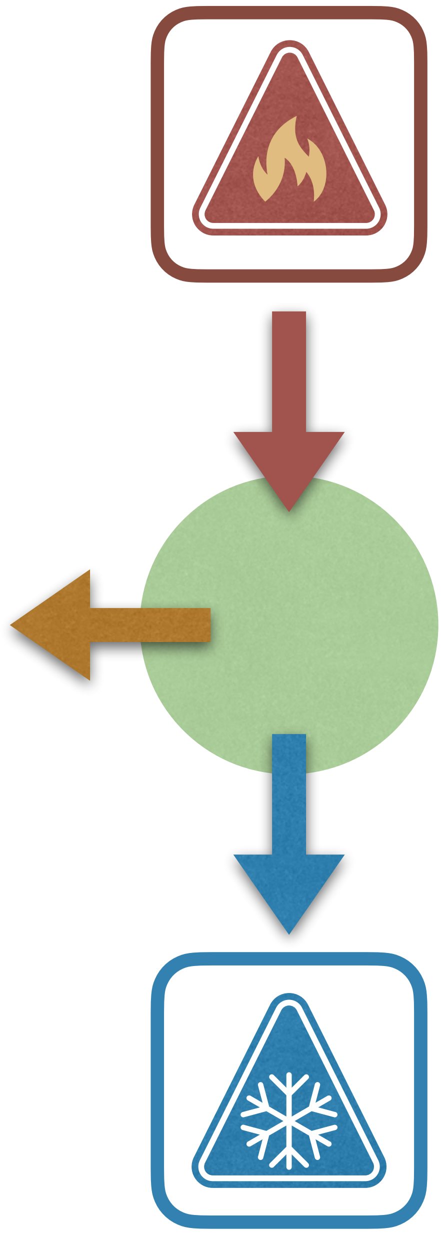
- According to Clausius, entropy does not satisfy a balance equation:

$$\Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma$$

$\sigma \geqslant 0$ is the entropy produced in the process.

- $\sigma \geqslant 0$ is the mathematical statement of the 2nd law.
- To explore the power of the 2nd law, let us consider a cyclic operation:

$$W + Q_h + Q_c = 0 \quad \text{and} \quad \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma = 0$$



$$W + Q_h + Q_c = 0 \qquad \text{and} \qquad \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma = 0$$

$$\sigma \geqslant 0$$

- **Operation as a heat engine:** efficiency

$$\eta = \frac{|W|}{|Q_h|} = \eta_c - \frac{T_c \sigma}{|Q_h|} \qquad \text{where} \qquad \eta_c = 1 - \frac{T_c}{T_h}$$

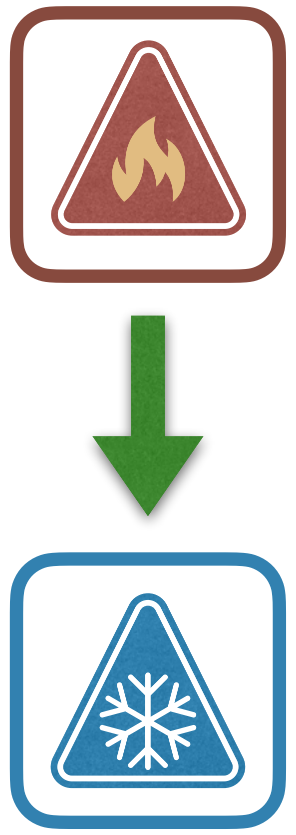
- The efficiency is always *lower* than Carnot's efficiency because entropy is produced (Carnot's statement of the 2nd law)

- **Heat flow** (no work): $Q_h = -Q_c$

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h \geqslant 0$$

Heat always flows from hot to cold

(Clausius' statement)



Otavio A. D. Molitor and Gabriel T. Landi, *Phys. Rev. A* **102**, 042217

- **Landauer's erasure:** Minimum cost to erase information

$$\Delta Q \geqslant k_B T \ln 2$$

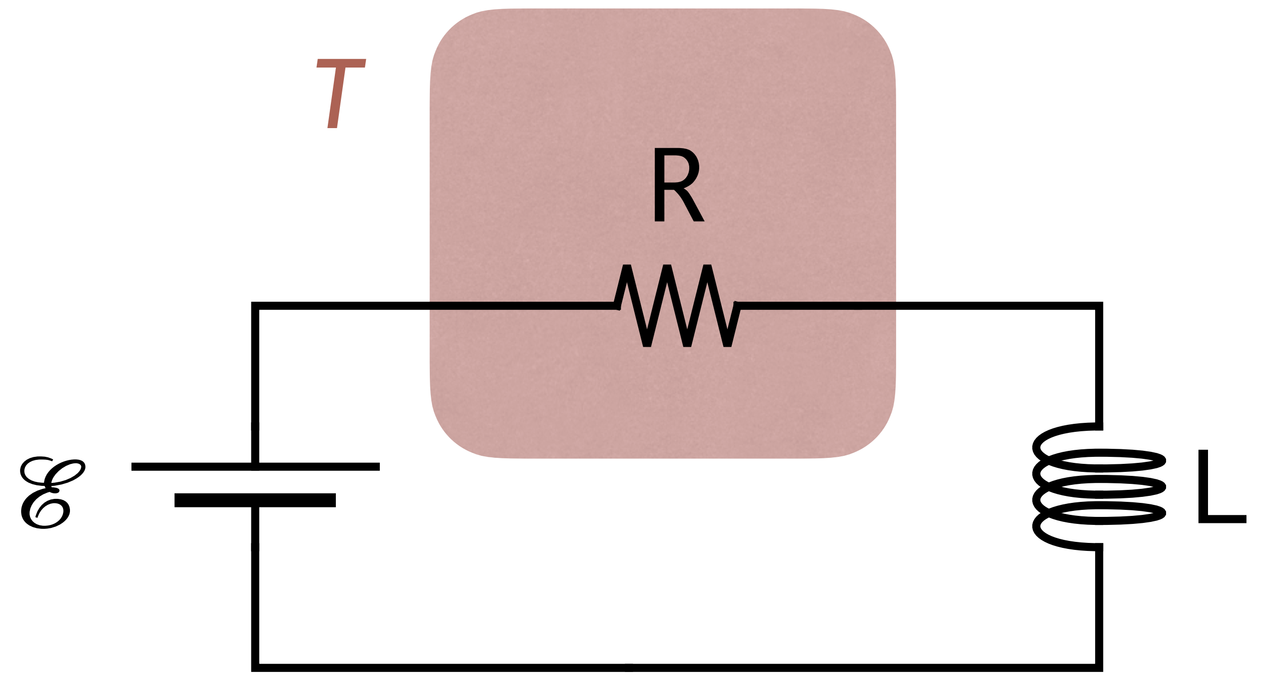
- What about $T \simeq 0$? Very relevant for quantum computation.

- If eraser is a waveguide:

$$\Delta Q \geqslant k_B T \ln 2 + \frac{3\hbar c}{\pi L} \ln^2(2)$$

- **Non-equilibrium steady-states:** not equilibrium.

$$\frac{dS}{dt} = \frac{\dot{Q}}{T} + \dot{\sigma} = 0 \qquad \text{so} \qquad \dot{\sigma} = -\frac{\dot{Q}}{T}$$



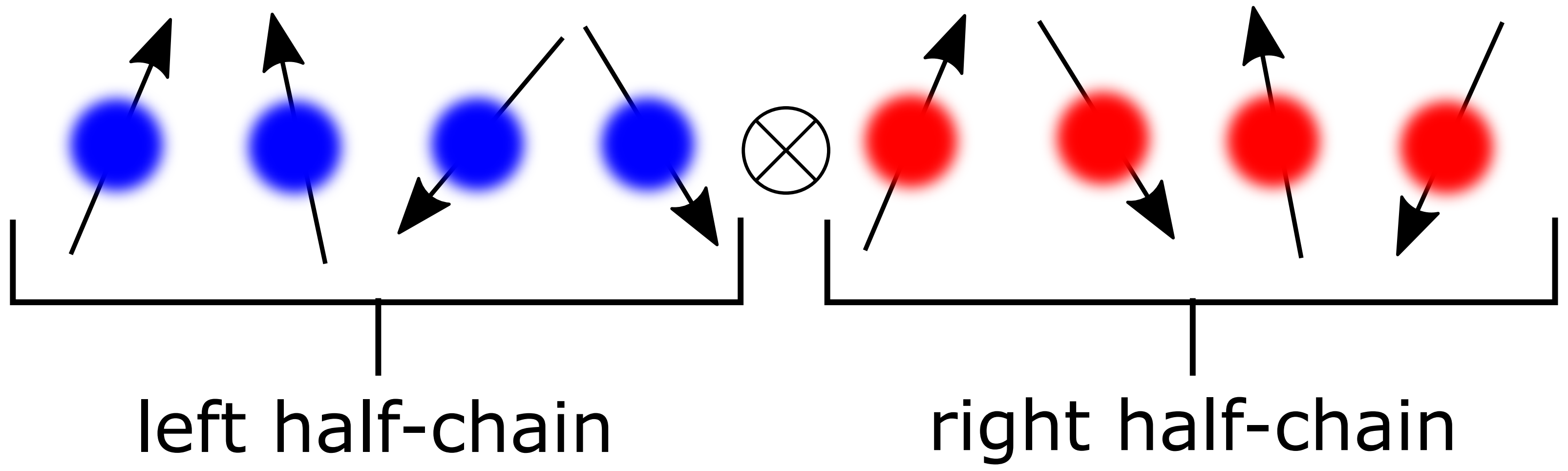
- Example: Joule heating.
  Continues as long as there is juice in the battery

$$\dot{\sigma} = \frac{\mathscr{E}^2}{RT}$$

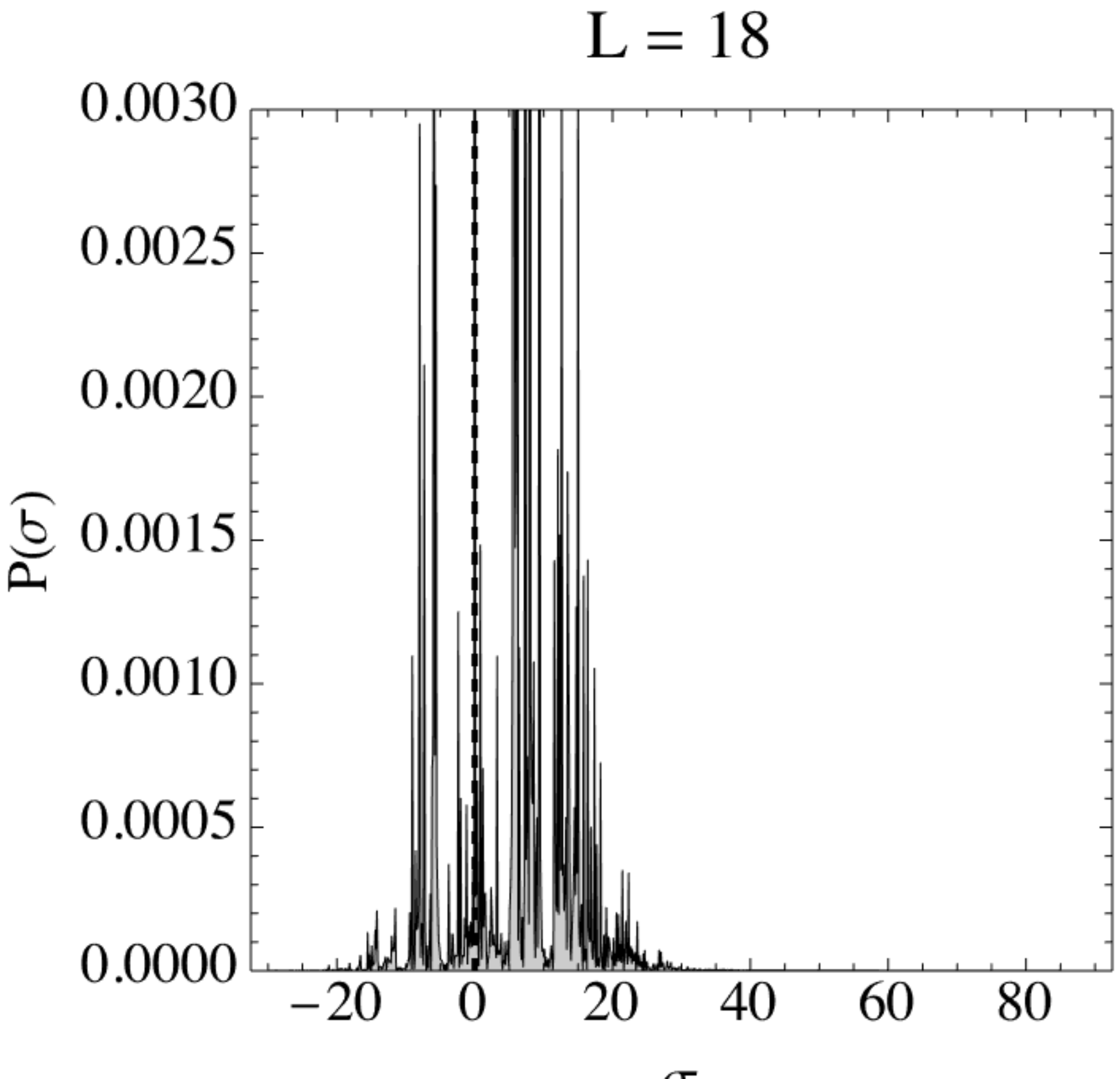
André M. Timpanaro, Jader P. Santos, and Gabriel T. Landi, *Phys. Rev. Lett.* **124**, 240601 (2020)

Fluctuations are significant in the micro-world

- Macro-world: heat flows from hot → cold.
- Micro-world: heat ***usually*** flows from hot → cold.



G. T. Landi and Dragi Karevski Phys. Rev. E **93**, 032122 (2015)



Heat Exchange Fluctuation Theorem

$$P(-\sigma) = e^{-\sigma} P(\sigma)$$

Implies 2nd law: $\langle \sigma \rangle \geqslant 0$

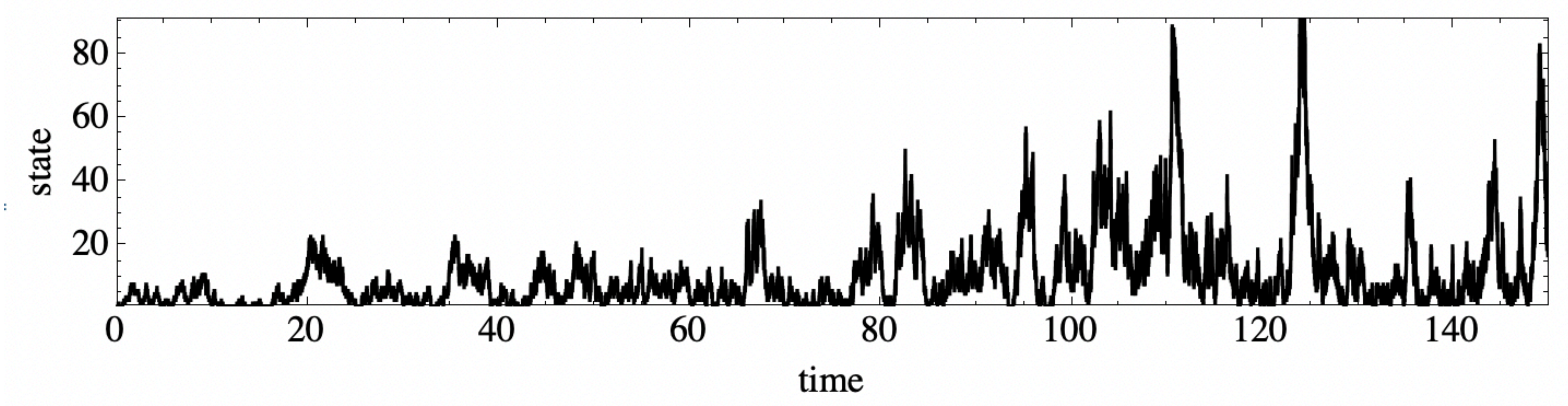
C. Jarzynski and D. Wójcik, Phys. Rev. Lett. 92, 230602 (2004)
G. E. Crooks, Journal of Statistical Physics, **90**, 1481–1487 (1998)

Entropy production and stochastic trajectories

- Fluctuations allow us to formulate the entropy production problem in terms of **trajectory probabilities**

$$\sigma[\gamma] = \ln \frac{P_F[\gamma]}{P_R[\gamma]}$$

- A process is reversible when the time-reversed process is as likely as the forward one.

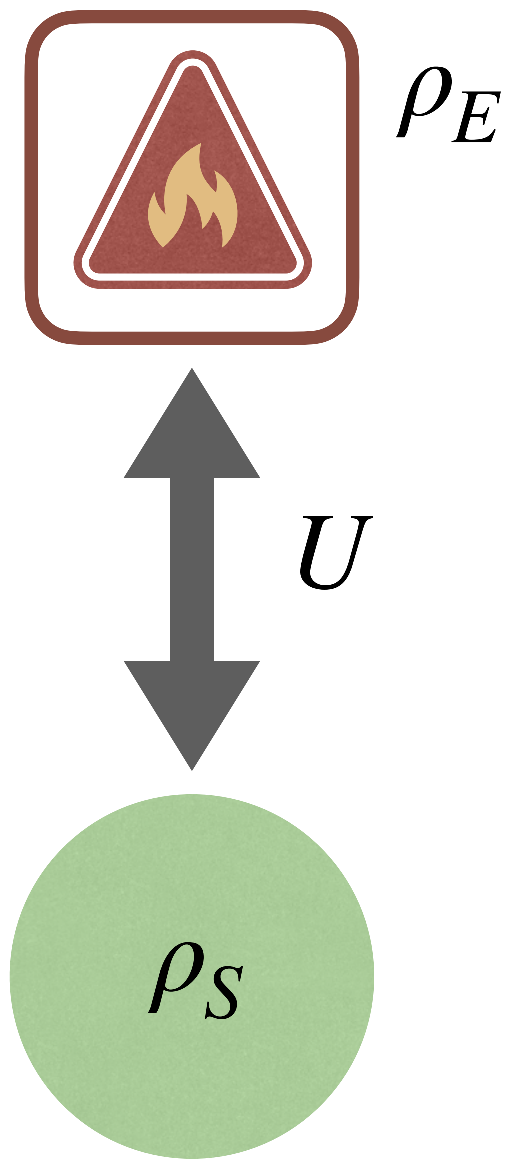


Entropy production for quantum systems

$$\rho'_{SE} = U(\rho_S \otimes \rho_E)U^\dagger$$

- Information-theoretic formulation: $\sigma = I(S:E) + D(\rho'_E || \rho_E)$
  - Operational interpretation: Characterizes irreversibility in terms of what you do not have access to:
    - System-environment correlations.
    - Changes in the environment.
- Tricky business: how to define heat currents for quantum master equations.

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

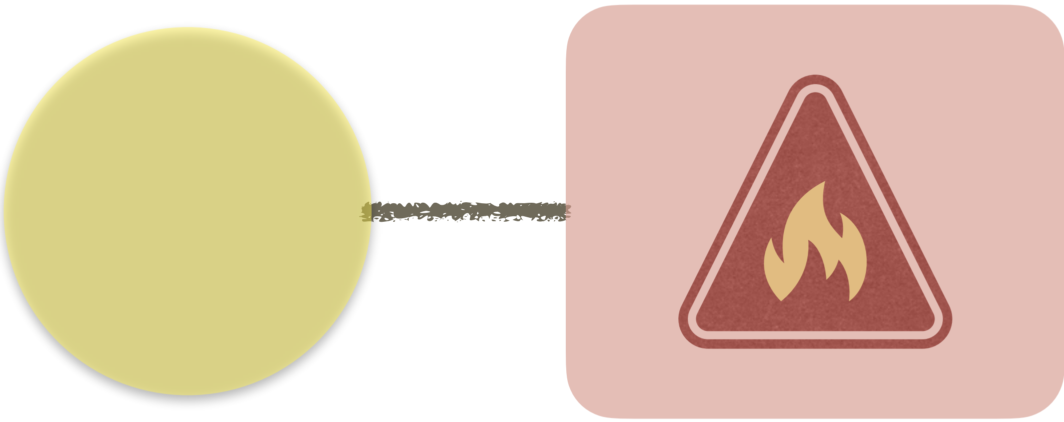


Describes an enormous variety of processes! (maybe a complicated U)

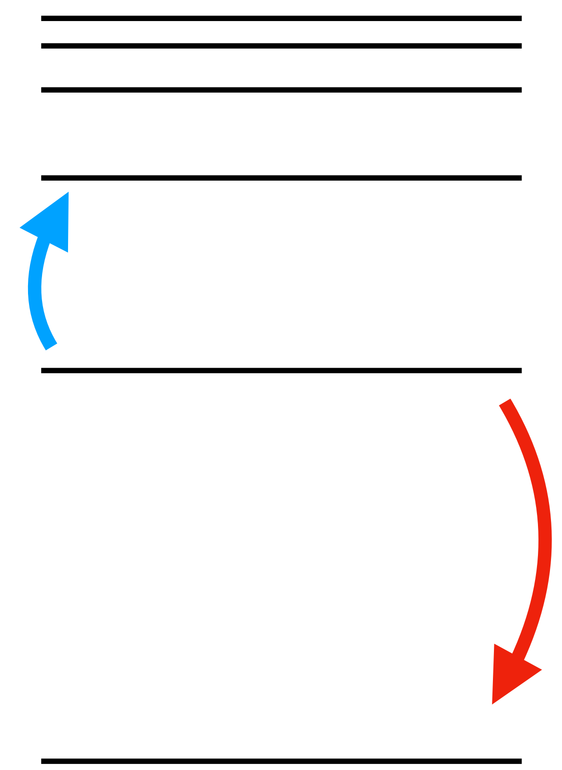
Gabriel T. Landi and Mauro Paternostro, "**Irreversible entropy production, from quantum to classical**", *Review of Modern Physics*, **93**, 035008 (2021)

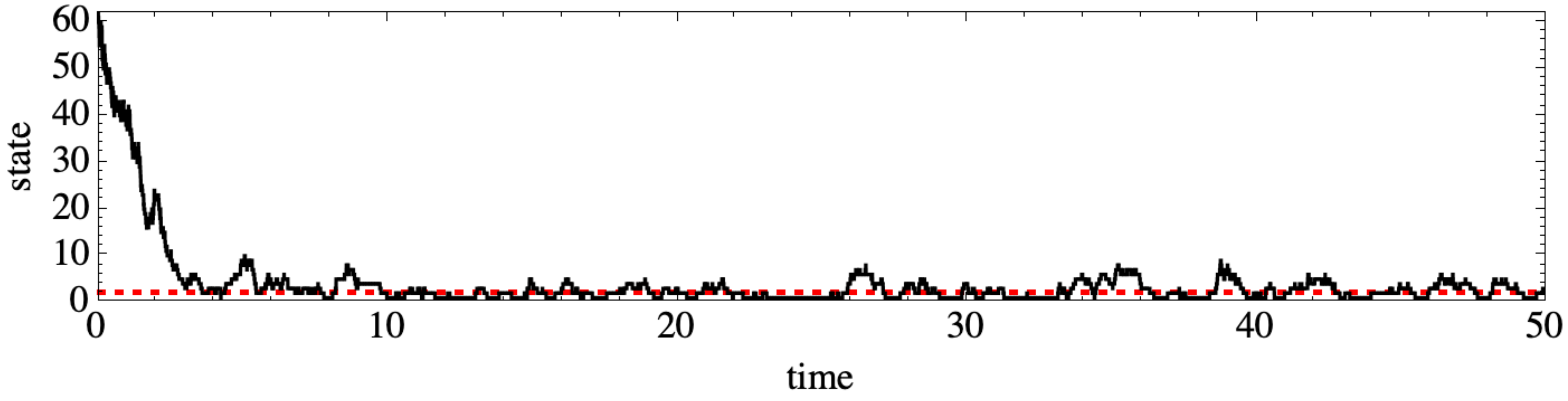
Gabriel T. Landi, Dario Poletti, Gernot Schaller, "**Nonequilibrium boundary-driven quantum systems: Models, methods, and properties.**" Reviews of Modern Physics, 94, (2022)

Relaxation towards equilibrium



- Imagine an atomic system relaxing towards equilibrium.
  - Population of energy eigenstates fluctuate until they reach thermal equilibrium.
- In addition: destroy any superpositions (**decoherence**).
- Entropy production rate can be split as $\sigma = \sigma_{\text{pop}} + \sigma_{\text{coh}}$





Additional entropy production due to coherence: Dissipation of information, without dissipation of energy.

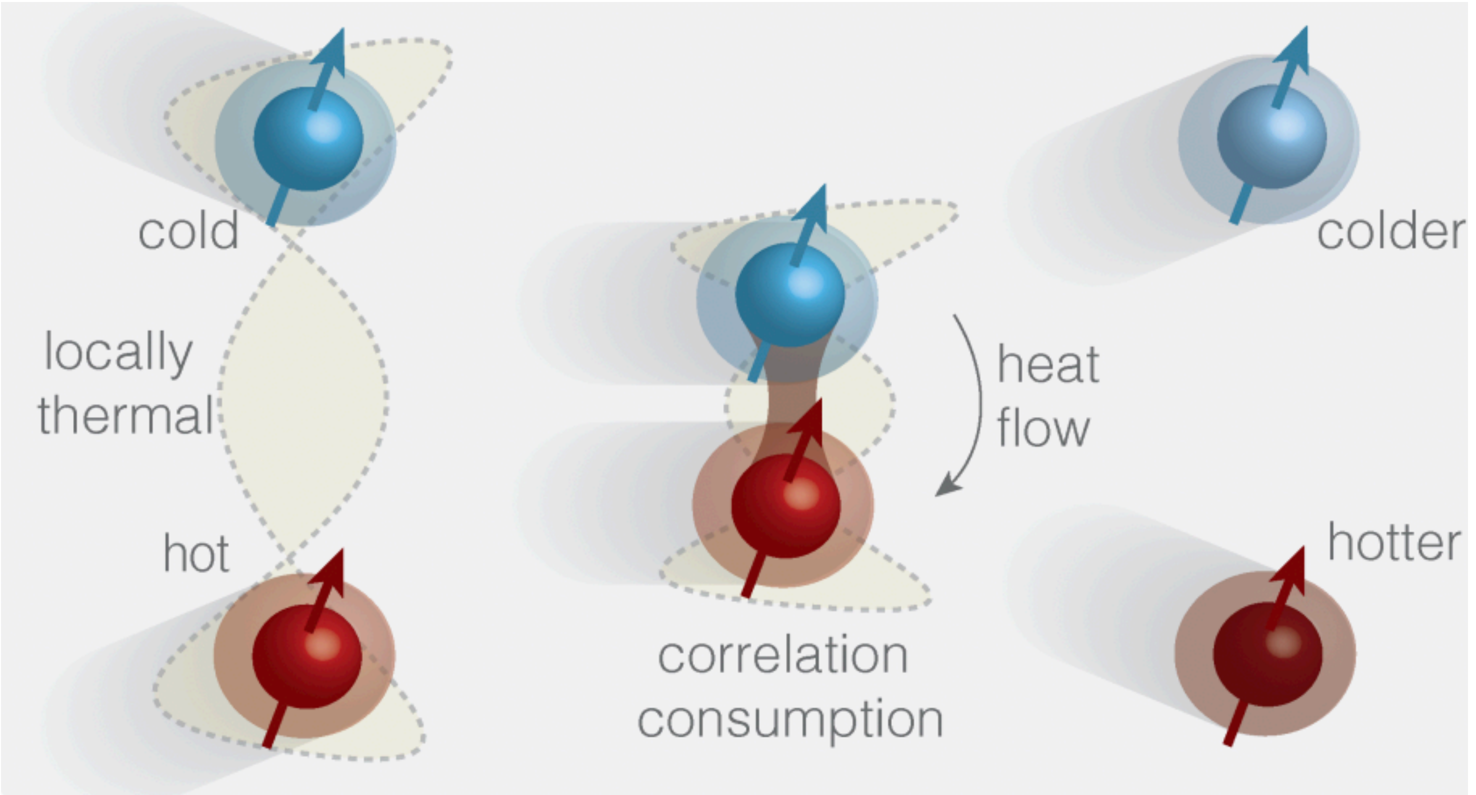
J. P. Santos, L. Céleri, GTL, M. Paternostro, npj Quantum Information **5**, 23 (2019)

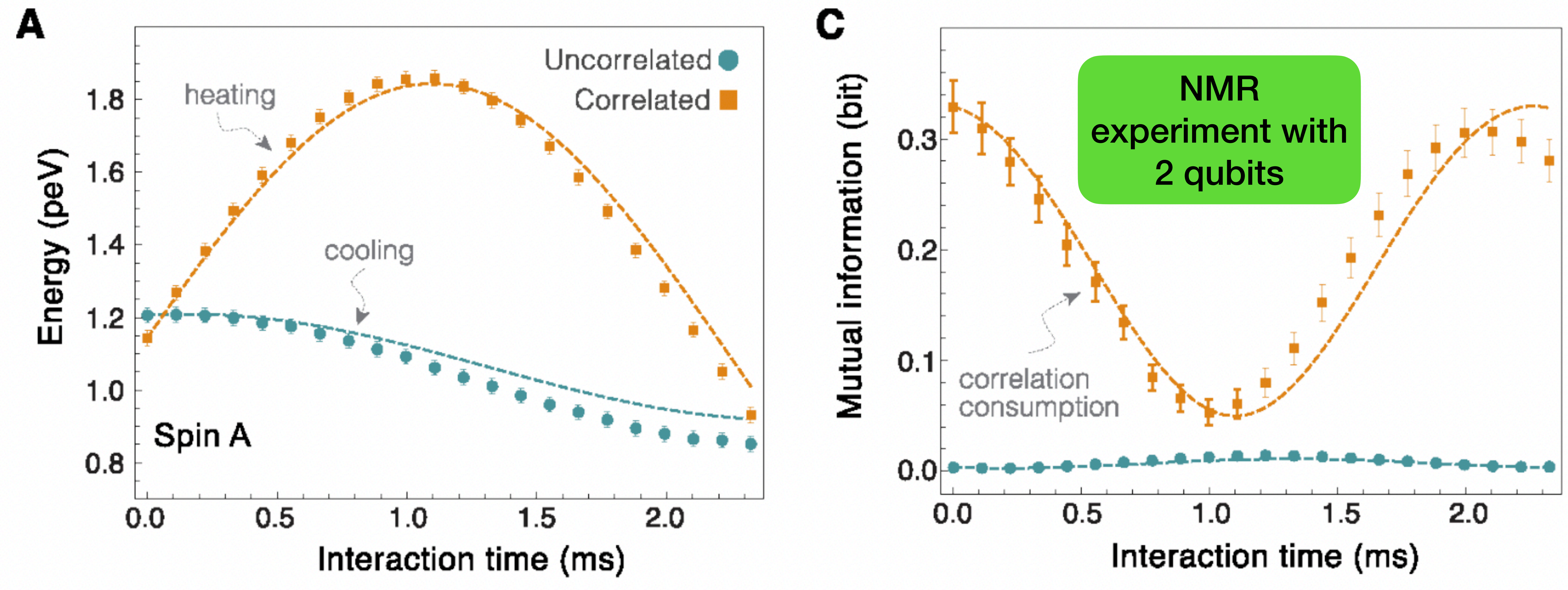
Information-thermodynamics

- In the presence of initial correlations the second law is modified to

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h \geqslant \Delta I(h:c)$$

- Heat can flow from cold to hot, provided we **consume** quantum correlations.



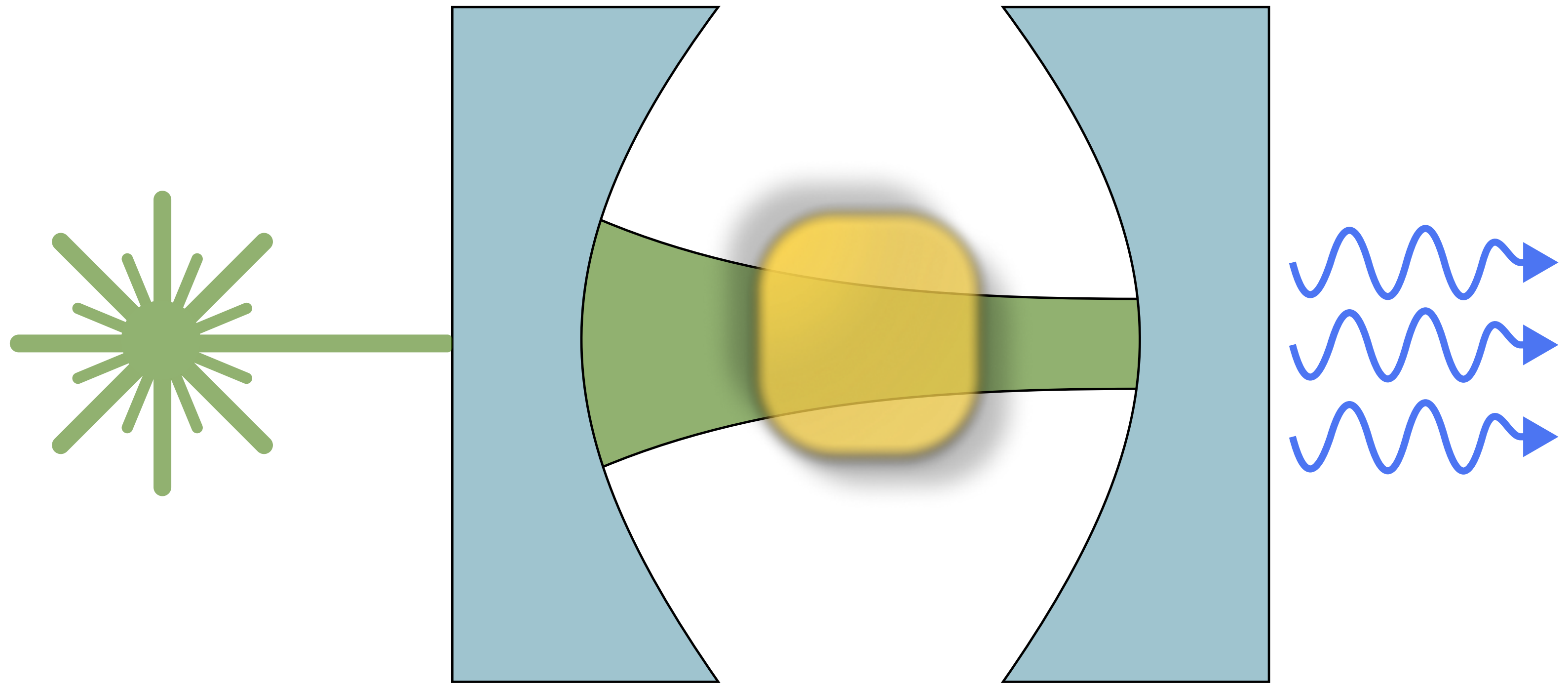


Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, "**Reversing the direction of heat flow using quantum correlations**", *Nature Communications*, **10**, 2456 (2019)

K. Micadei, G. T. Landi, E. Lutz, "Quantum fluctuation theorems beyond two-point measurements", Phys. Rev. Lett. 124, 090602 (2020)

Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Roberto M. Serra, Eric Lutz, "Experimental validation of fully quantum fluctuation theorems", Phys. Rev. Lett., 127, 180603 (2021).

Quantum phase space



- Many quantum experiments are done using optical cavities with semi-transparent mirrors.

- Photons leaking out $\simeq$ zero temperature bath.

  - Spontaneous emission: excitations can leave, but not return.

- 2nd law is buggy @ $T=0$: $\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h$.

  - Does not include vacuum fluctuations (*present in every measurement*).

- We reformulated the entropy production problem in terms of quantum phase space & the *Wigner function.*

$$\sigma = \left(\frac{1}{T_c^{\text{eff}}} - \frac{1}{T_h^{\text{eff}}}\right) Q_h$$

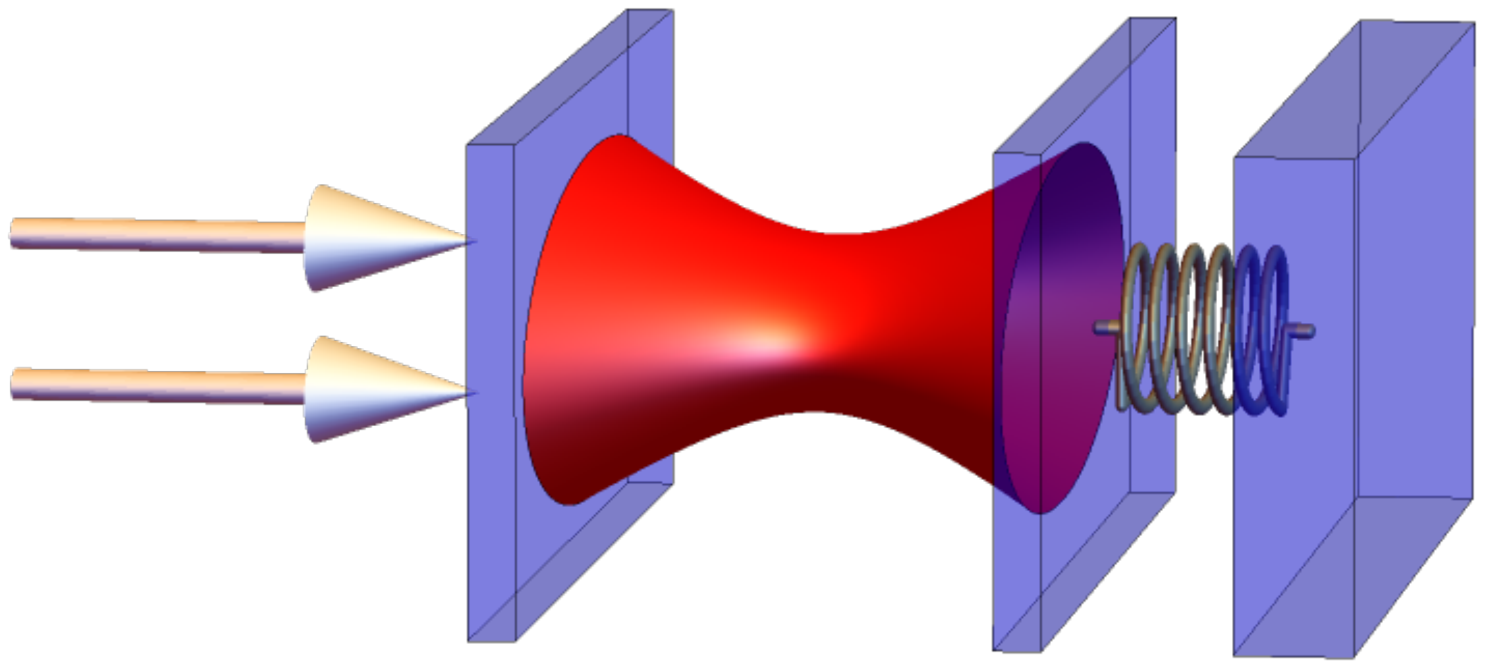
$$T^{\text{eff}} = \omega(\bar{n}+1/2), \qquad \bar{n} = \frac{1}{e^{\beta\omega}-1}$$

High temperatures: $\omega(\bar{n}+1/2) \simeq T$.

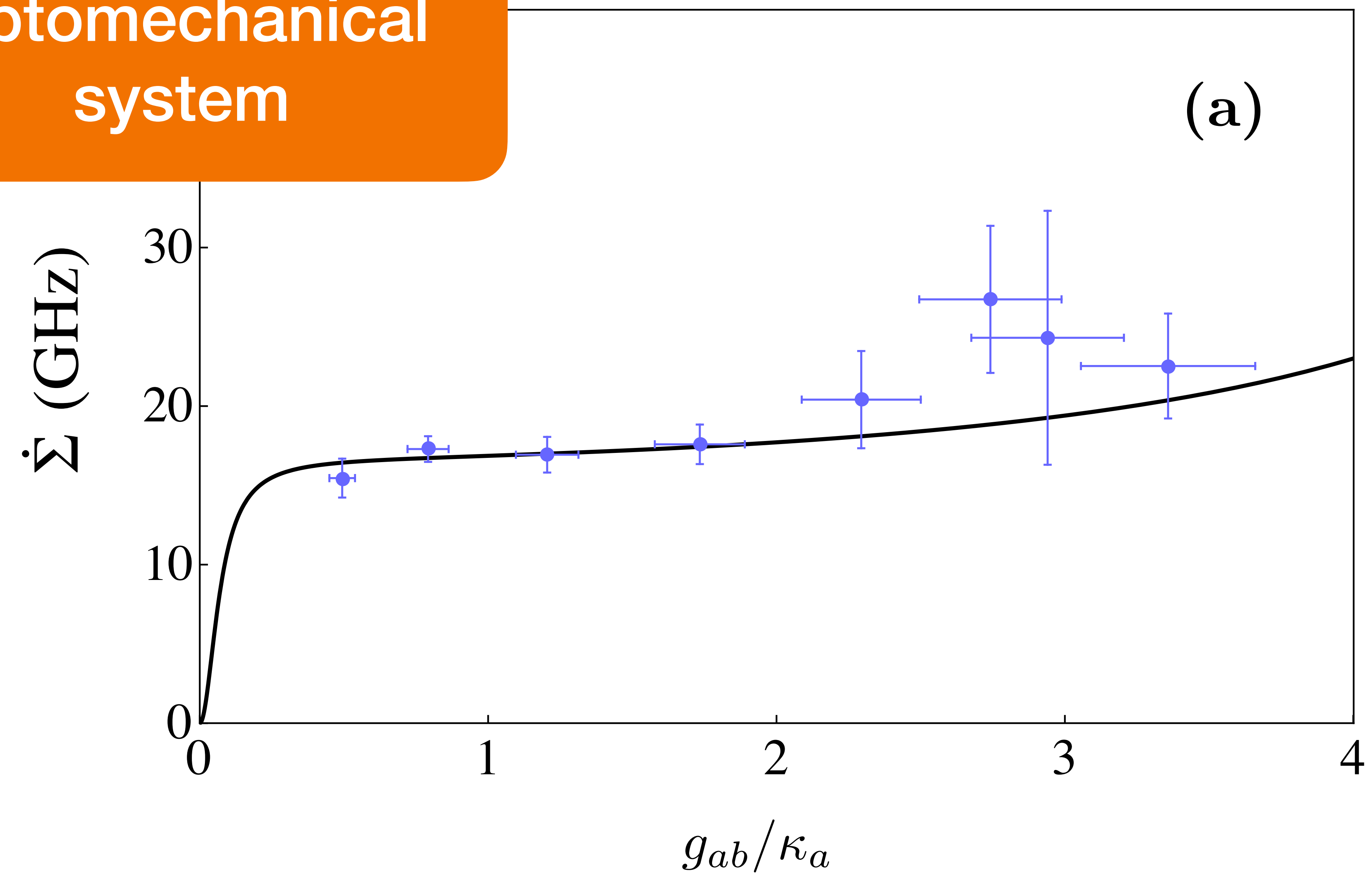
Zero temperature: $\omega(\bar{n}+1/2) = \omega/2$.

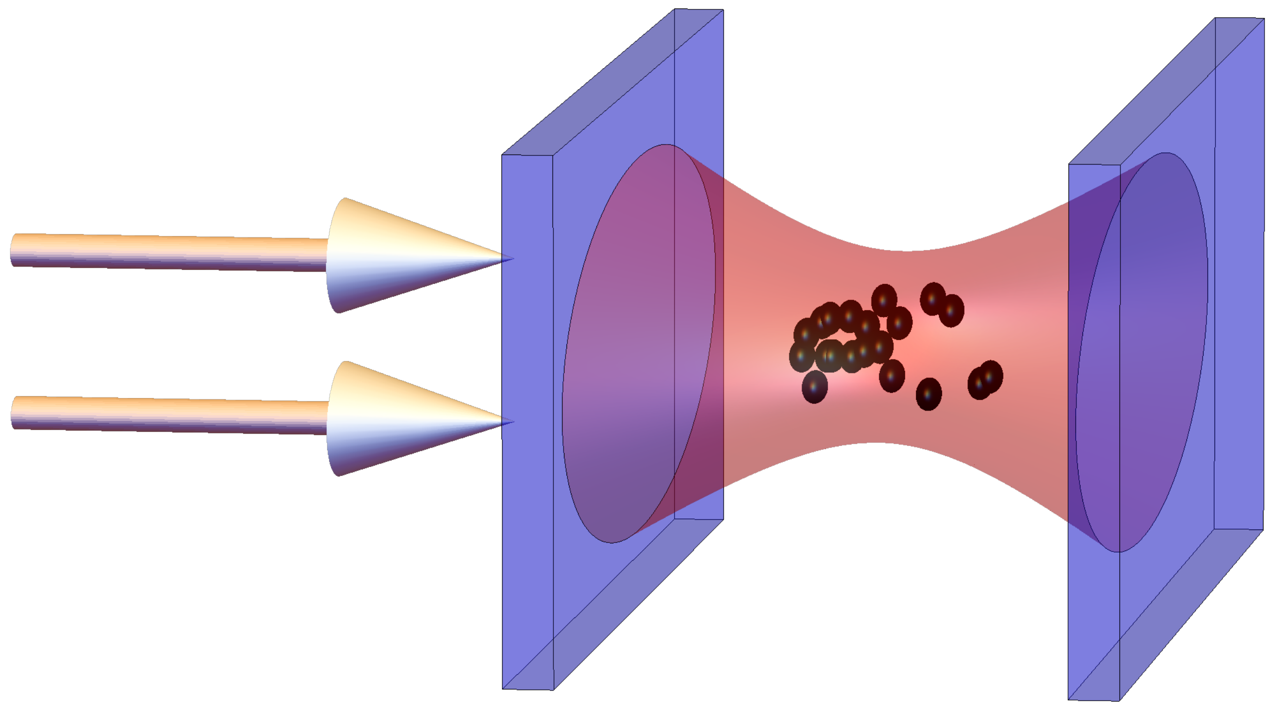
Jader P. Santos, G.T. Landi and Mauro Paternostro, Phys. Rev. Lett, **118**, 220601 (2017),

Experiments



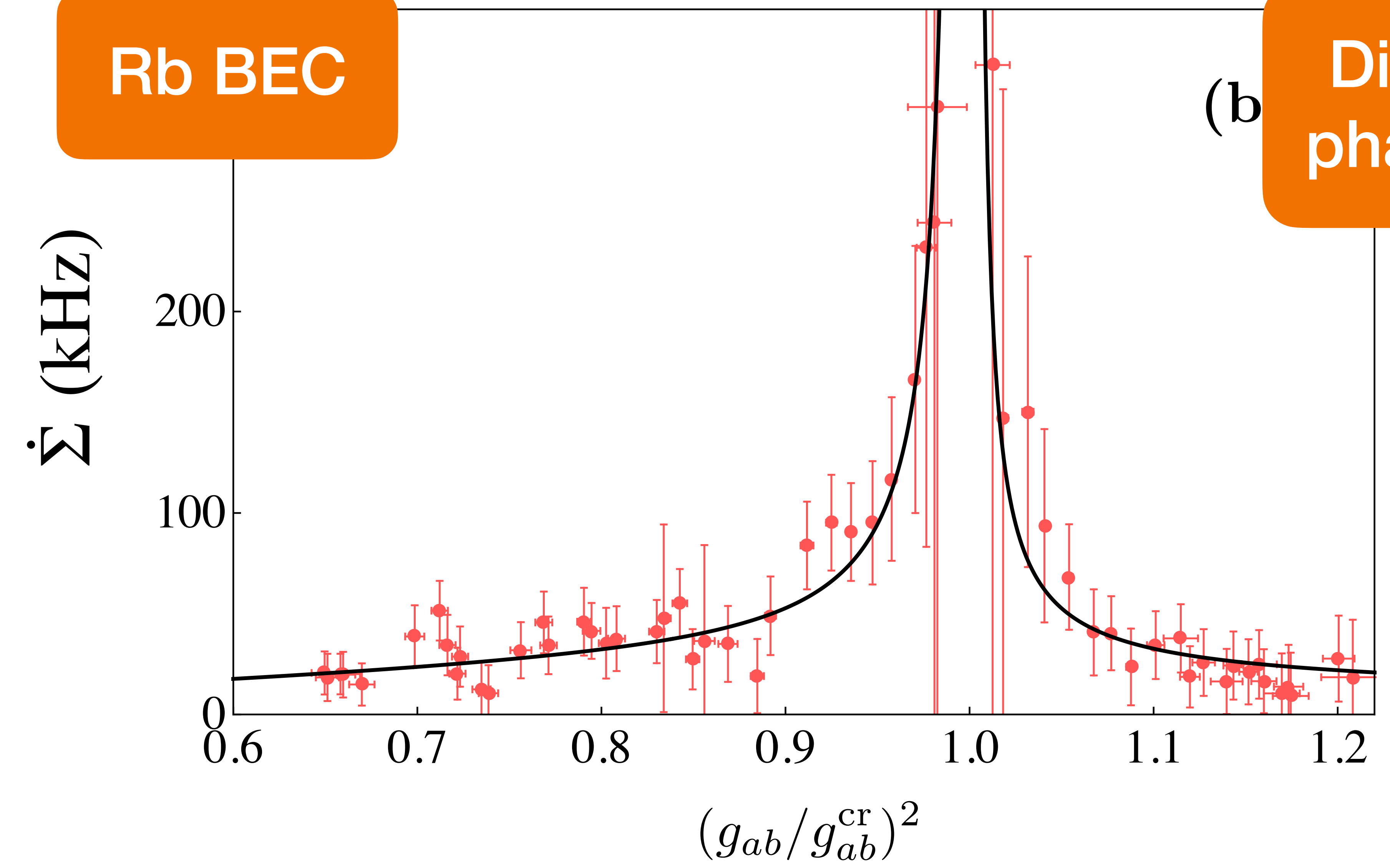
Optomechanical system





Rb BEC

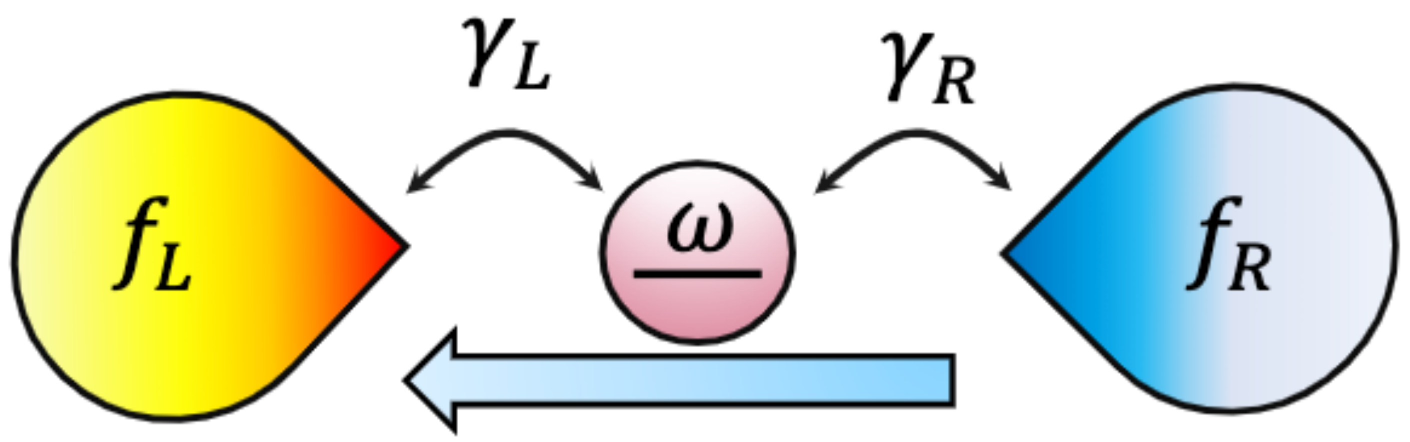
Dicke quantum phase transition.



M. Brunelli, L. Fusco, R. Landig, W. Wieczorek, J. Hoelscher-Obermaier, GTL, F Semião, A. Ferraro, N. Kiesel, T. Donner, G. De Chiara, and M. Paternostro. *Phys. Rev. Lett.*, **121**, 160604 (2018)

⚛️ Current fluctuations & thermodynamic uncertainty relations

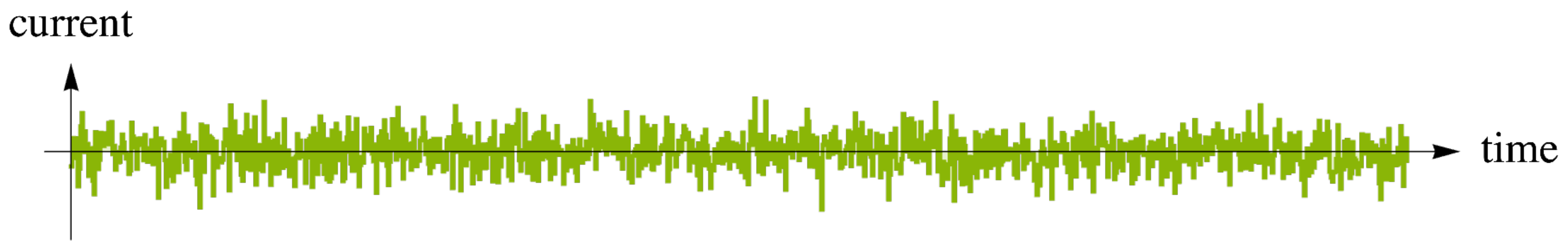
Thermodynamic uncertainty relations

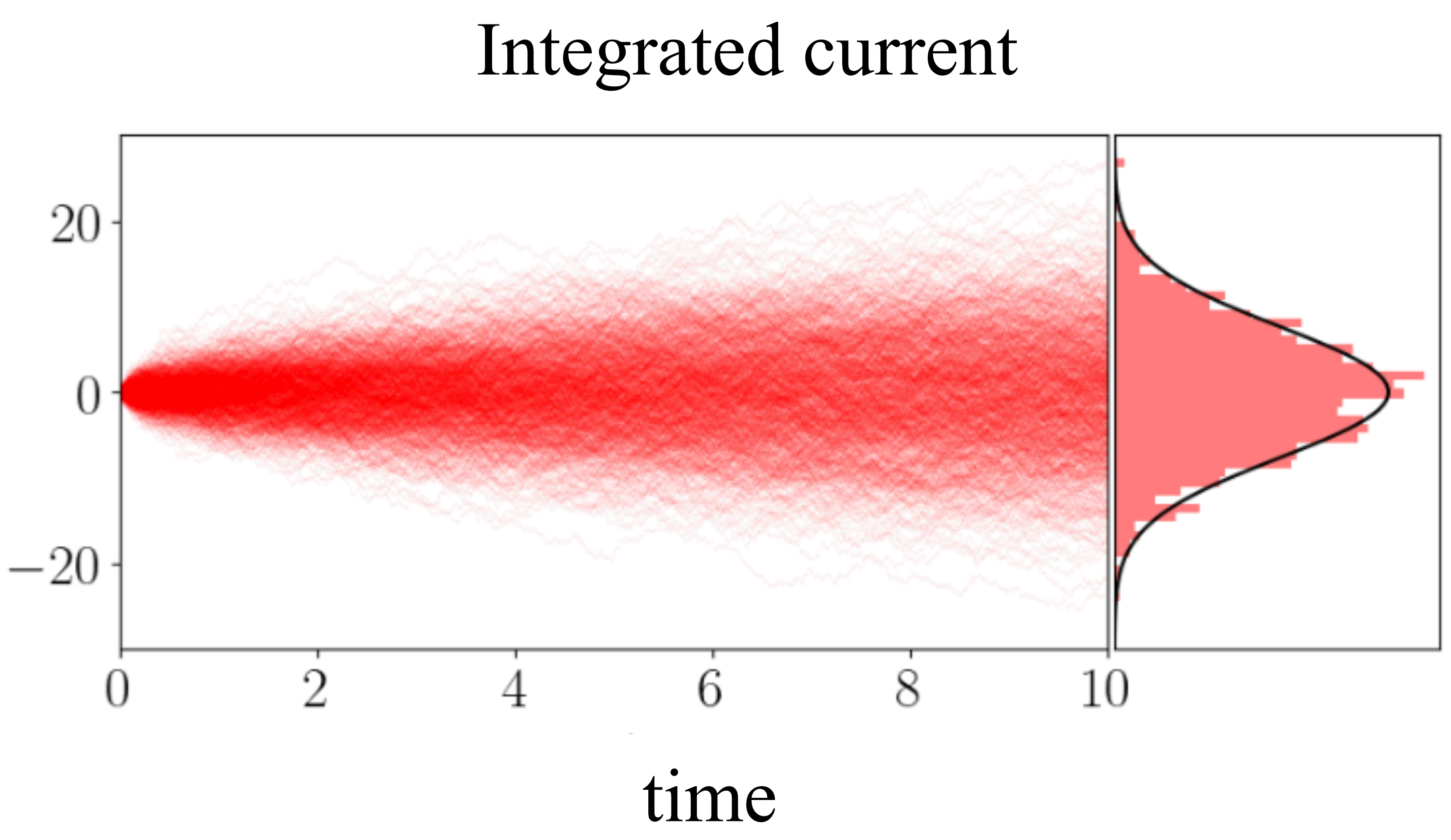


- At the μ-scale: currents fluctuate a lot:
  - Work, heat, particle current, &c.
- For classical Markov processes: TUR $\dfrac{\Delta_I^2}{I^2} \geqslant \dfrac{2}{\dot{\sigma}}$
  - Counterintuitive: must increase dissipation to curb fluctuations.
    - “Irreversibility can be good” (when fluctuations are large)

For a heat engine:

$$\Delta_W^2 \geqslant 2 T_c W \frac{\eta}{\eta_C - \eta}$$





A. C. Barato, U. Seifert, *Physical Review Letters*, **114**, 158101 (2015)

Kinetic uncertainty relation

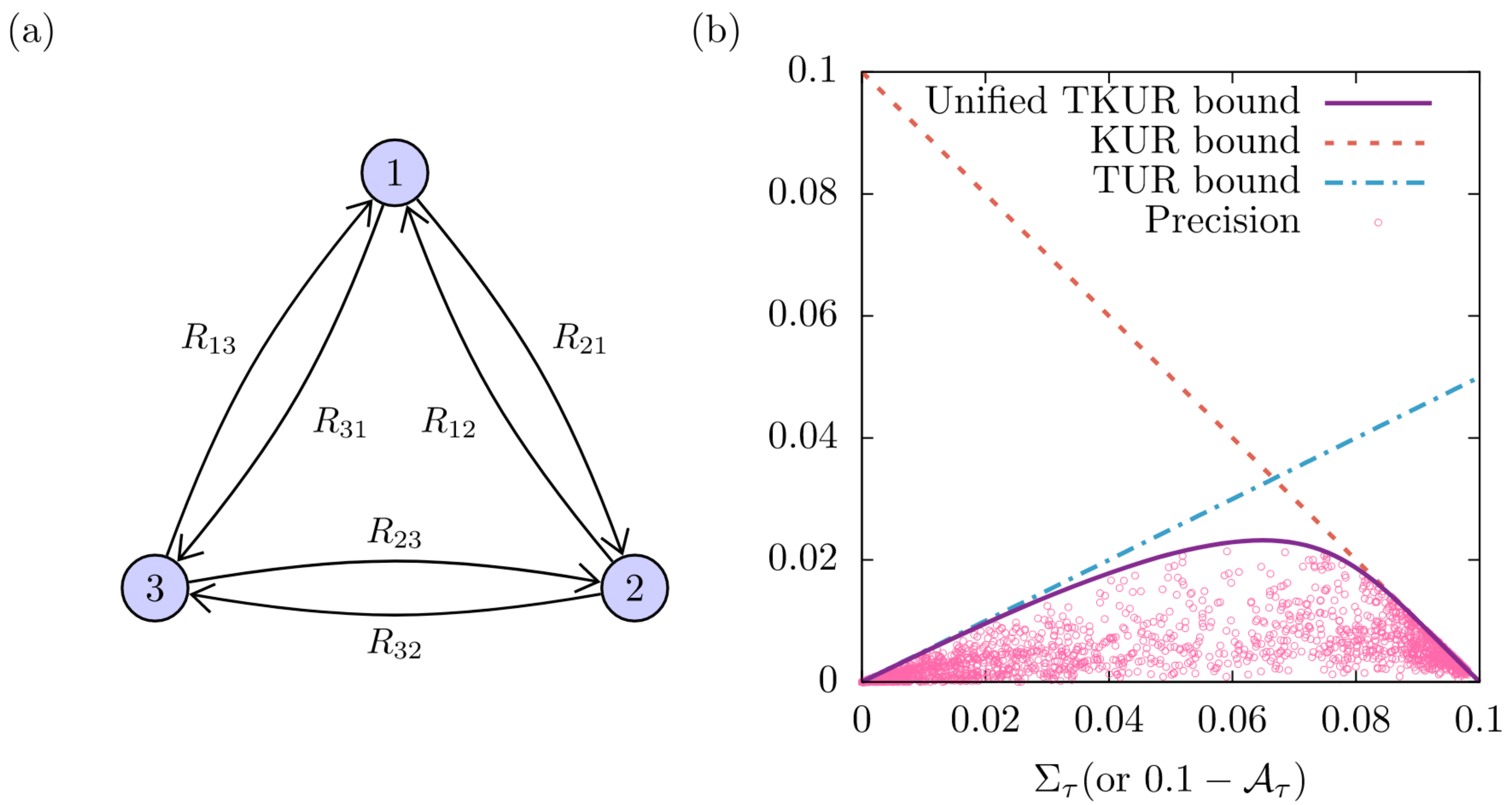
- An equivalent bound has also been recently derived, by based instead on the

  A = **Dynamical activity :=** number of transitions/unit time

$$\frac{\Delta_I^2}{I^2} \geqslant \frac{1}{A}$$

- Dissipation & activity come hand-in-hand.

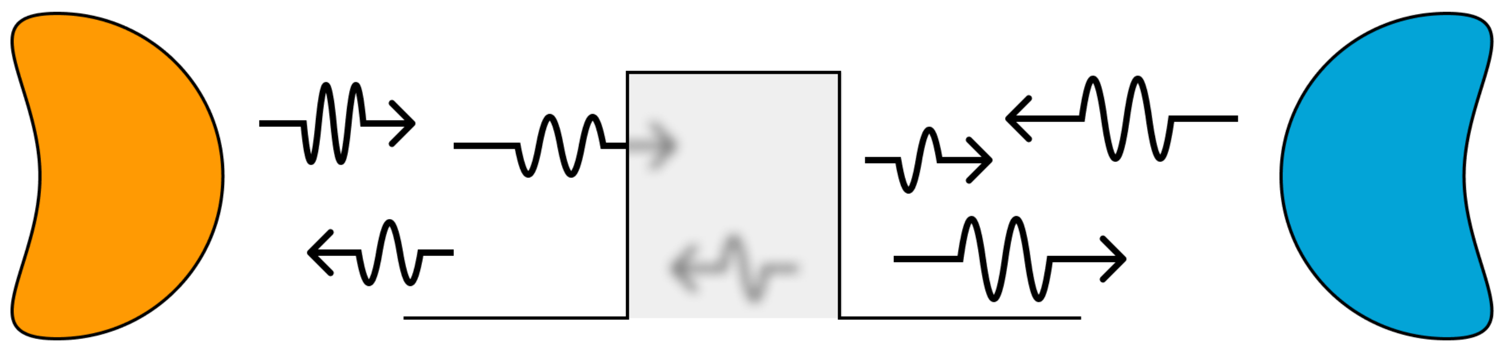
- But sometimes there can be more of one than the other.



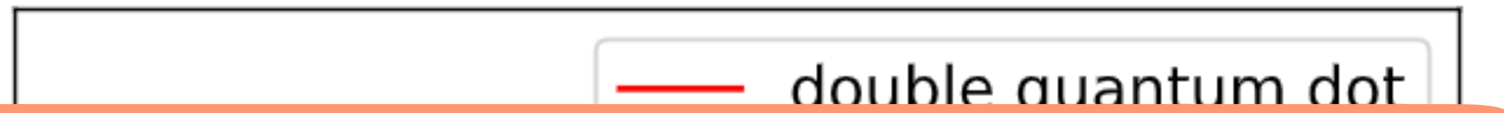
Ivan Di Terlizzi, Marco Baiesi, "**Kinetic uncertainty relation**" *J. Phys. A.*, **52**, 02LT03 (2018).

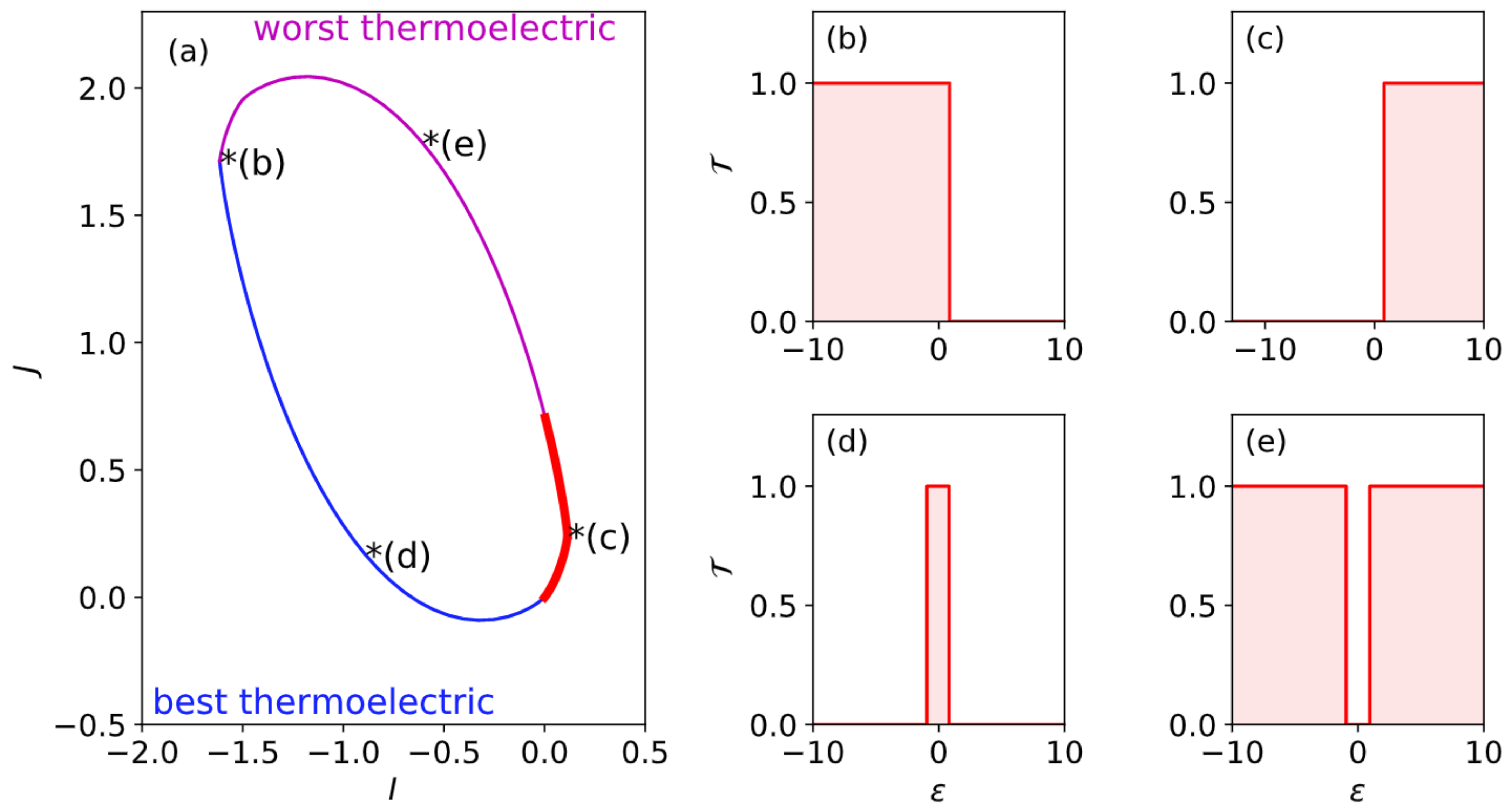
Van Tuan Vo, Tan Van Vu, Yoshihiko Hasegawa, "**Unified thermodynamic\textendashkinetic uncertainty relation**." *J. Phys. A.*, **55**, 405004 (2022)

TURs can be violated in the quantum regime



- Quantum coherent transport, e.g. through quantum dots.
- Important example: thermoelectricity.
- Practical question: what is the best thermoelectric?
  - New question: *what is the most precise thermoelectric?*





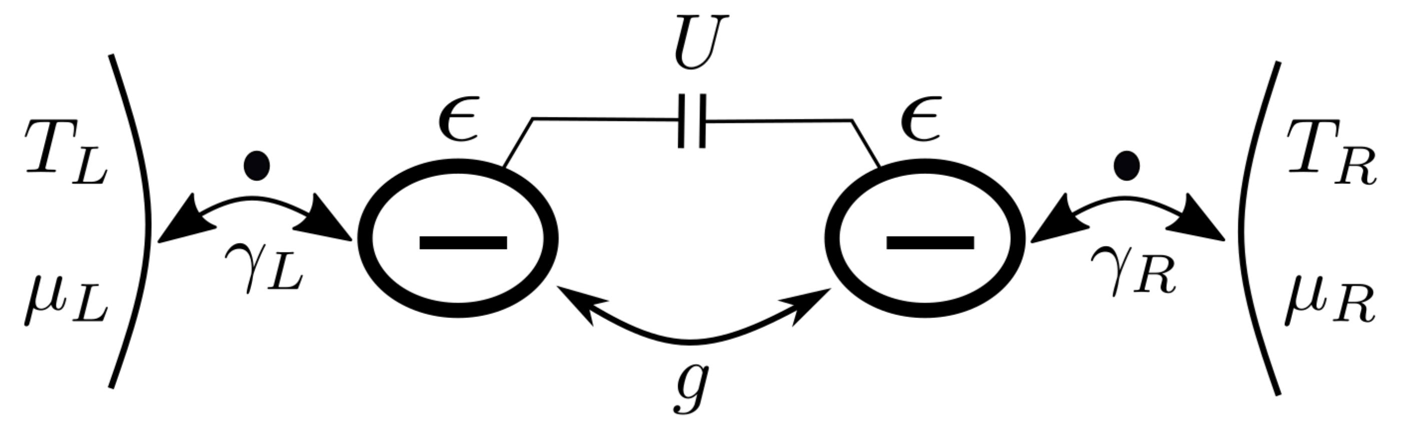
🚧 **Major open questions in the field:**

- Why quantum coherence?
- What limits the precision in the quantum world?
- Is there an actual trade-off?

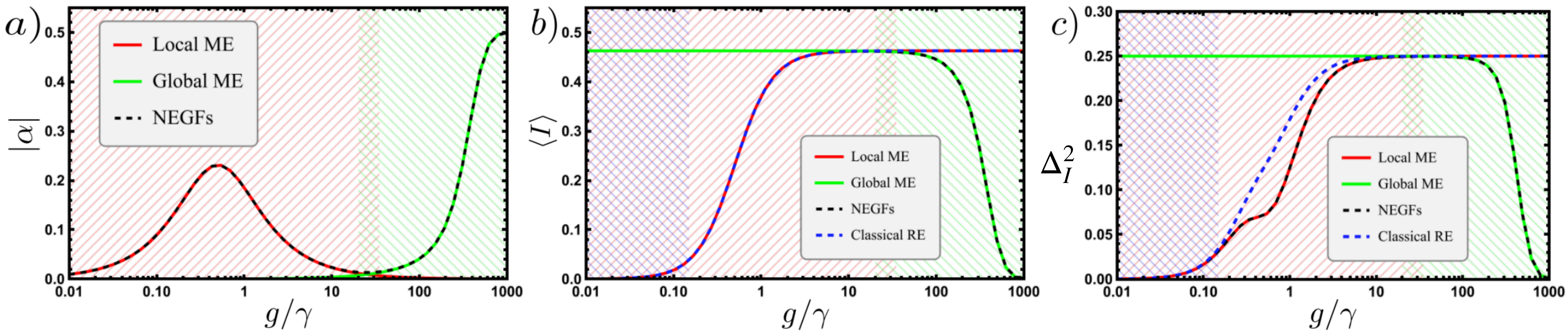
Andre M. Timpanaro, Giacomo Guarnieri, and Gabriel T. Landi, arXiv 2106.10205

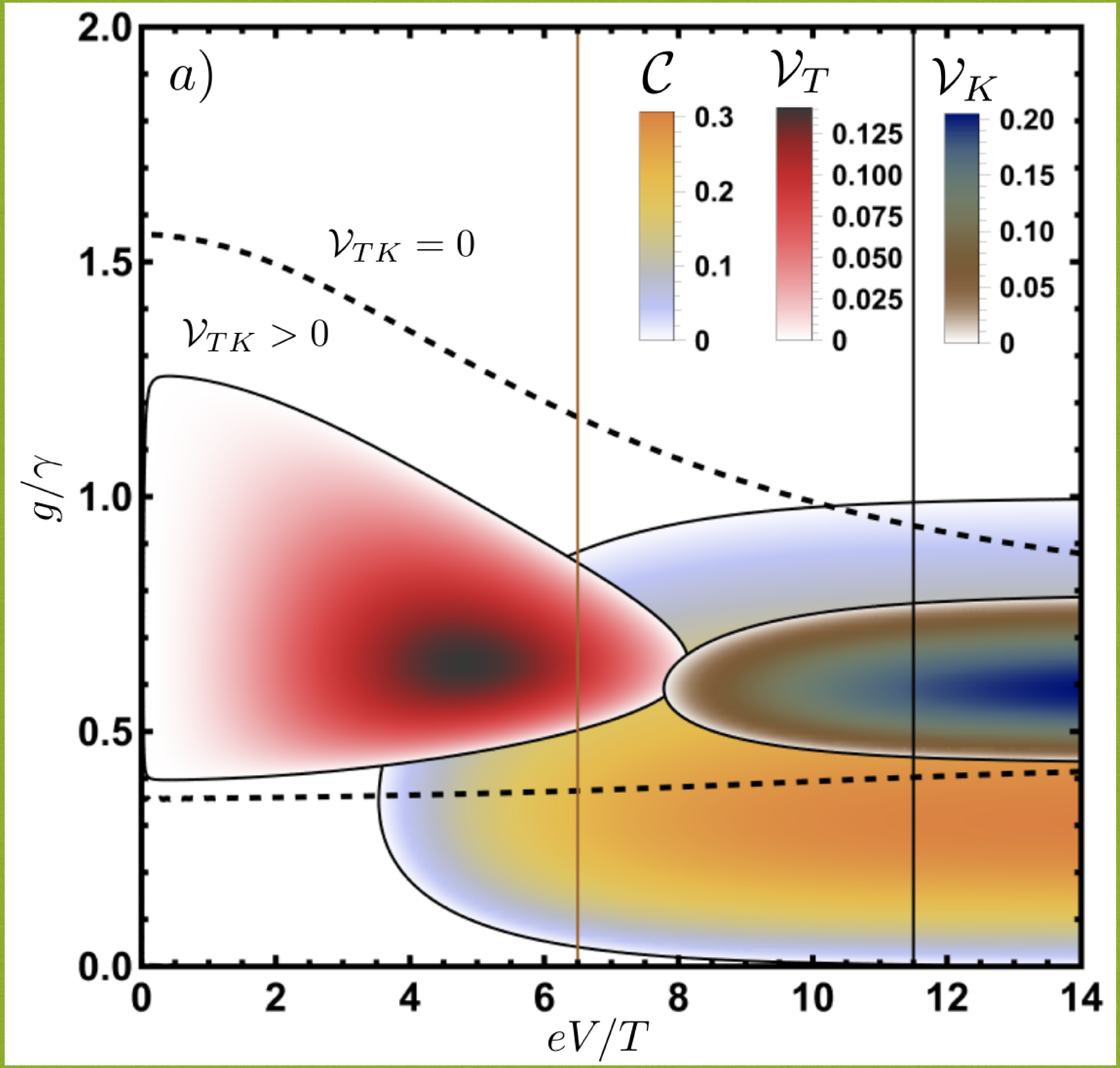
Dynamical vs. static coherence

- TUR violations must be due to quantum coherence.
    - But so is quantum entanglement.
- Is there any relation between them? *No.*
    - Entanglement is a *static* form of coherence.
    - TUR violations is dynamic.



Kacper Prech, *et. al.*
"Entanglement and thermo-kinetic uncertainty relations in coherent mesoscopic transport,"
arXiv 2212.03835

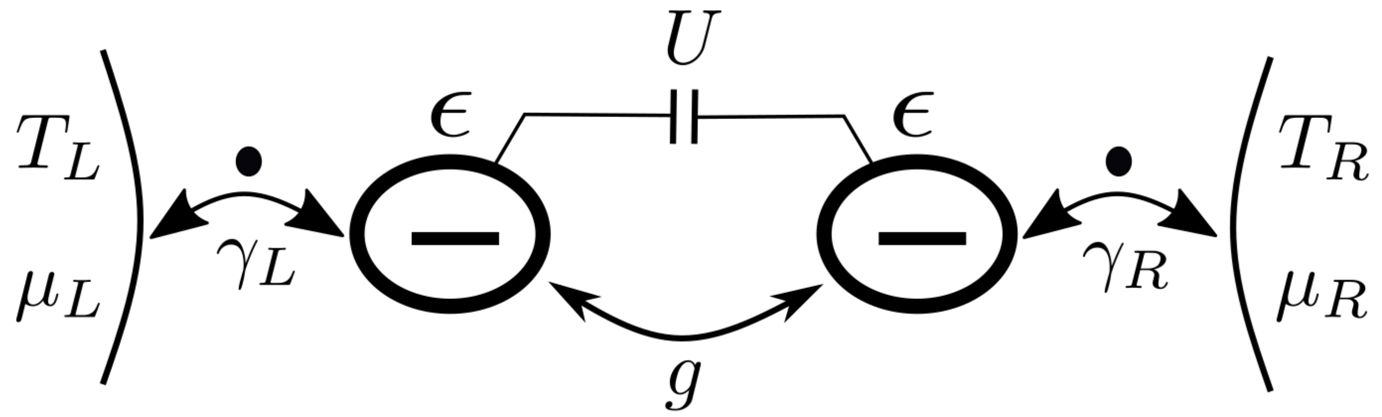


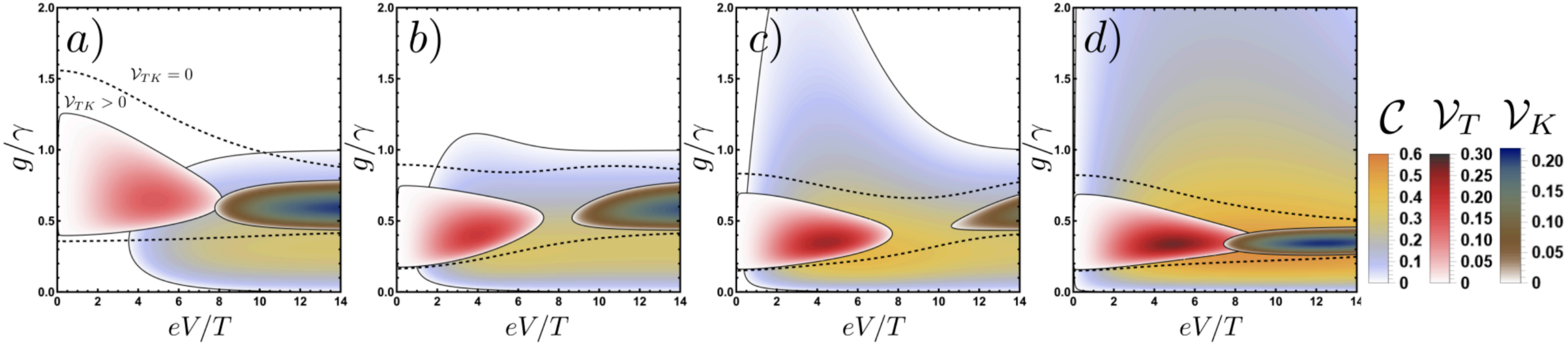


a)
$\mathcal{C}$
$\mathcal{V}_T$
$\mathcal{V}_K$
0.3
0.2
0.1
0
0.125
0.100
0.075
0.050
0.025
0
0.20
0.15
0.10
0.05
0
$\mathcal{V}_{TK} = 0$
$\mathcal{V}_{TK} > 0$
g/γ
2.0
1.5
1.0
0.5
0.0
0
2
4
6
8
10
12
14
eV/T

In the presence of Coulomb repulsion

- Coulomb repulsions tend to dramatically increase the entanglement.
- But do not substantially affect TUR and KUR violations.
- Extensive analysis:
    - Shows how entanglement and TUR/KUR depend in fundamentally different ways on the coherence.





Current fluctuations in the Parametric Kerr model

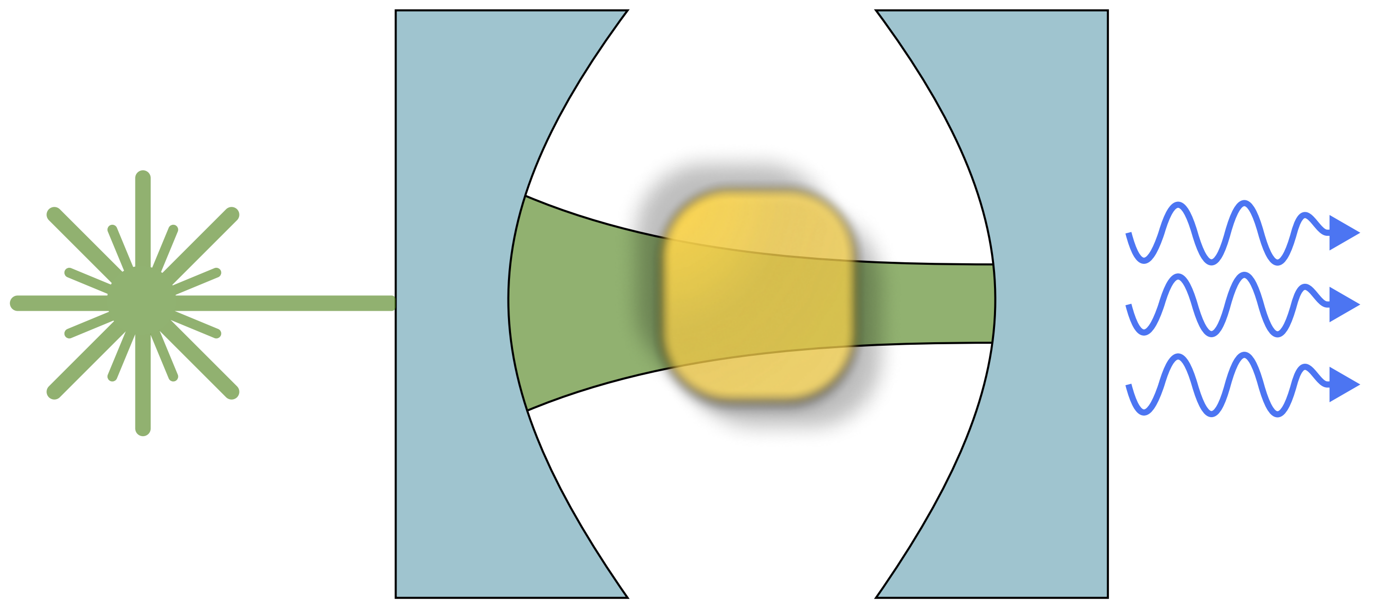
Michael J. Kewming, Mark T. Mitchison, Gabriel T. Landi, "**Diverging current fluctuations in critical Kerr resonators**." Physical Review A, 106, (2022)

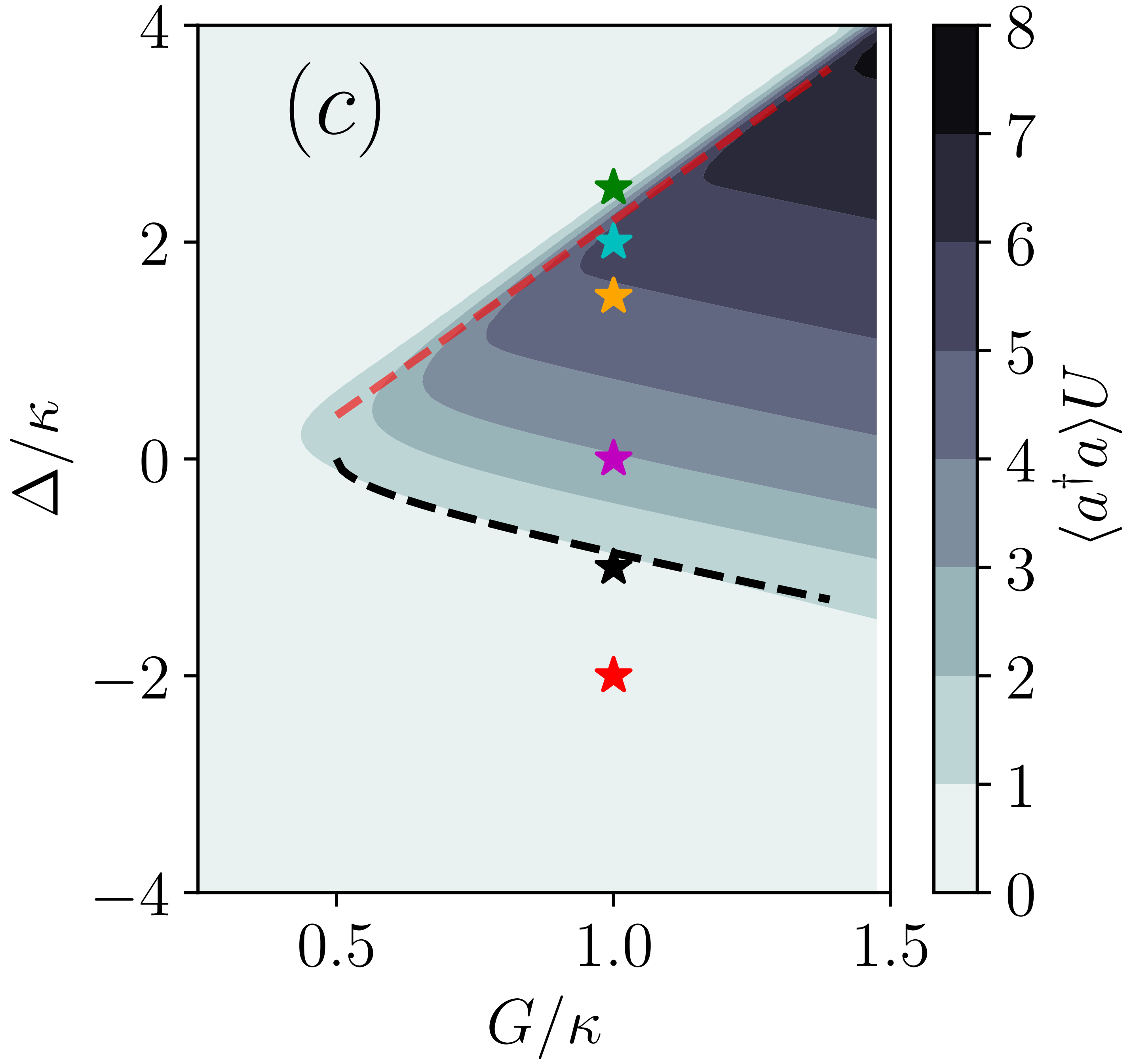
Parametric Kerr model

- Non-linear quantum harmonic oscillator:

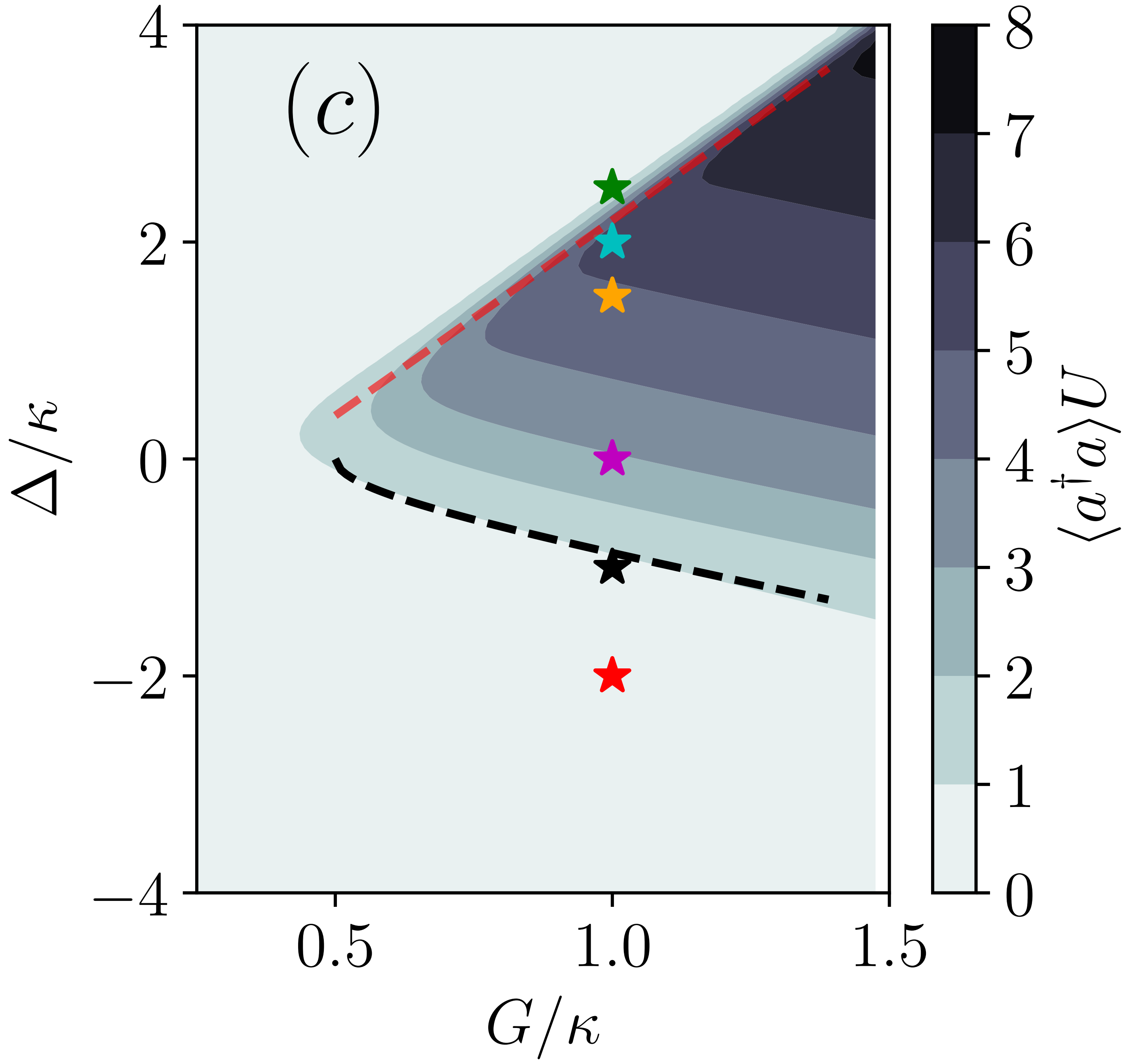
$$\frac{d\rho}{dt} = -i[H(t),\rho] + \kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a,\rho\}\right]$$

$$H = -\Delta a^\dagger a + \frac{U}{2}a^\dagger a^\dagger a a + \frac{G}{2}(a^{\dagger 2} + a^2)$$



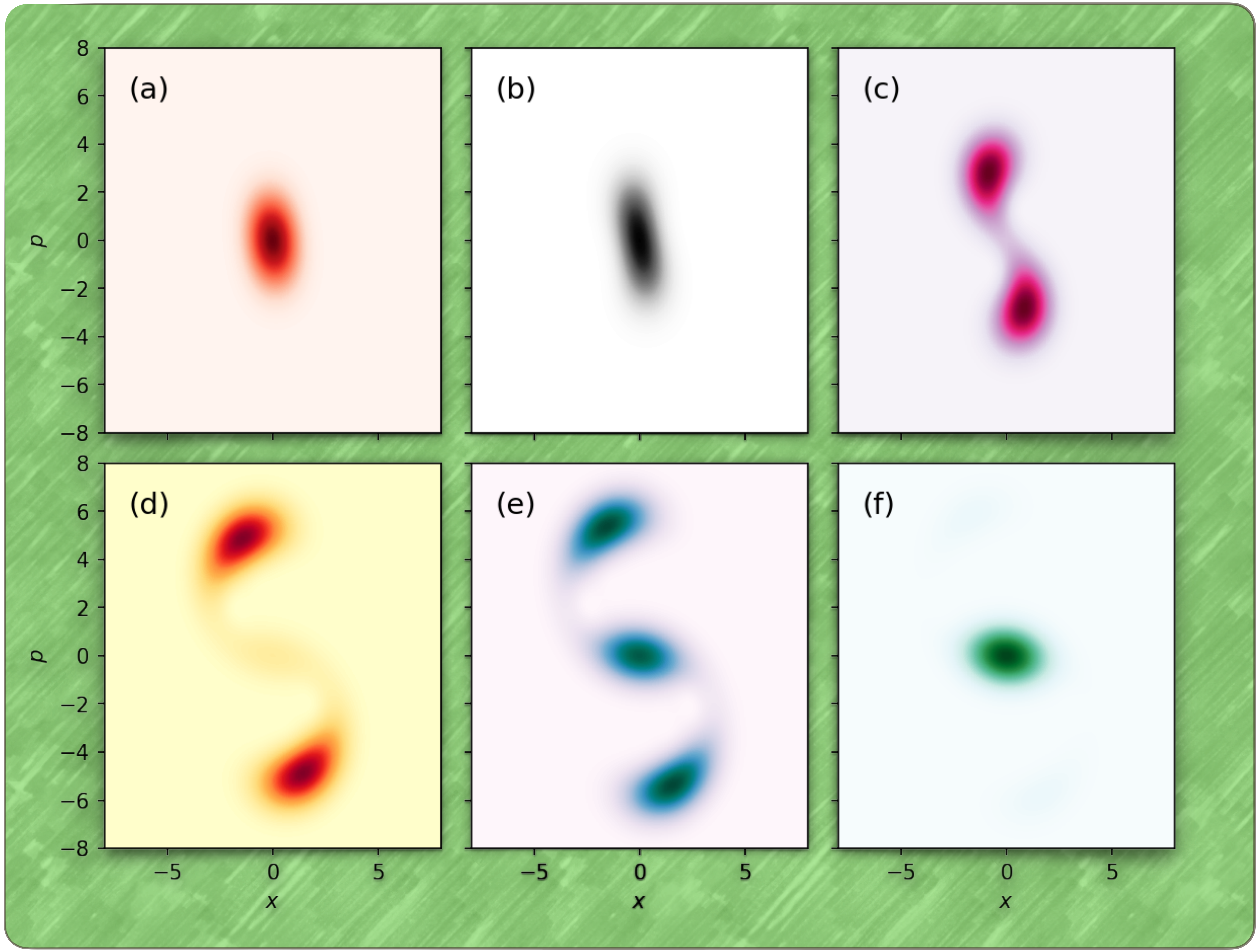


- a = annihilation operator photon operator for an optical cavity
- $\Delta = \omega_p - \omega_c$ = detuning
- U = Kerr non-linearity. (requires a non-linear crystal inside the cavity)
- G = 2-photon pump (input laser produces photons in pairs)
- κ = loss rate rate at which photons leak out of the cavity



- 2 phase transitions, continuous and discontinuous
- Proper criticality occurs in the limit $U \to 0$ ("thermodynamic limit")

Wigner function

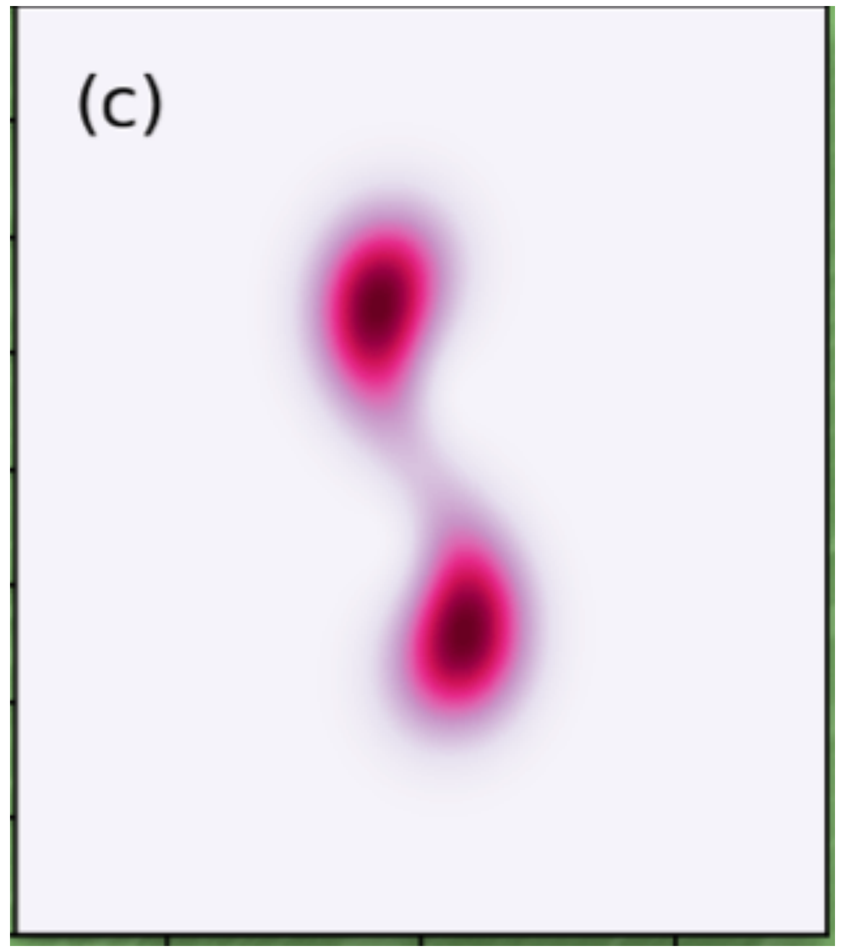


Cat qubits

- Steady-state is a mixture of two Schrödinger cat states

$$|S\rangle = |\alpha\rangle + |-\alpha\rangle$$

$$|A\rangle = |\alpha\rangle - |-\alpha\rangle$$

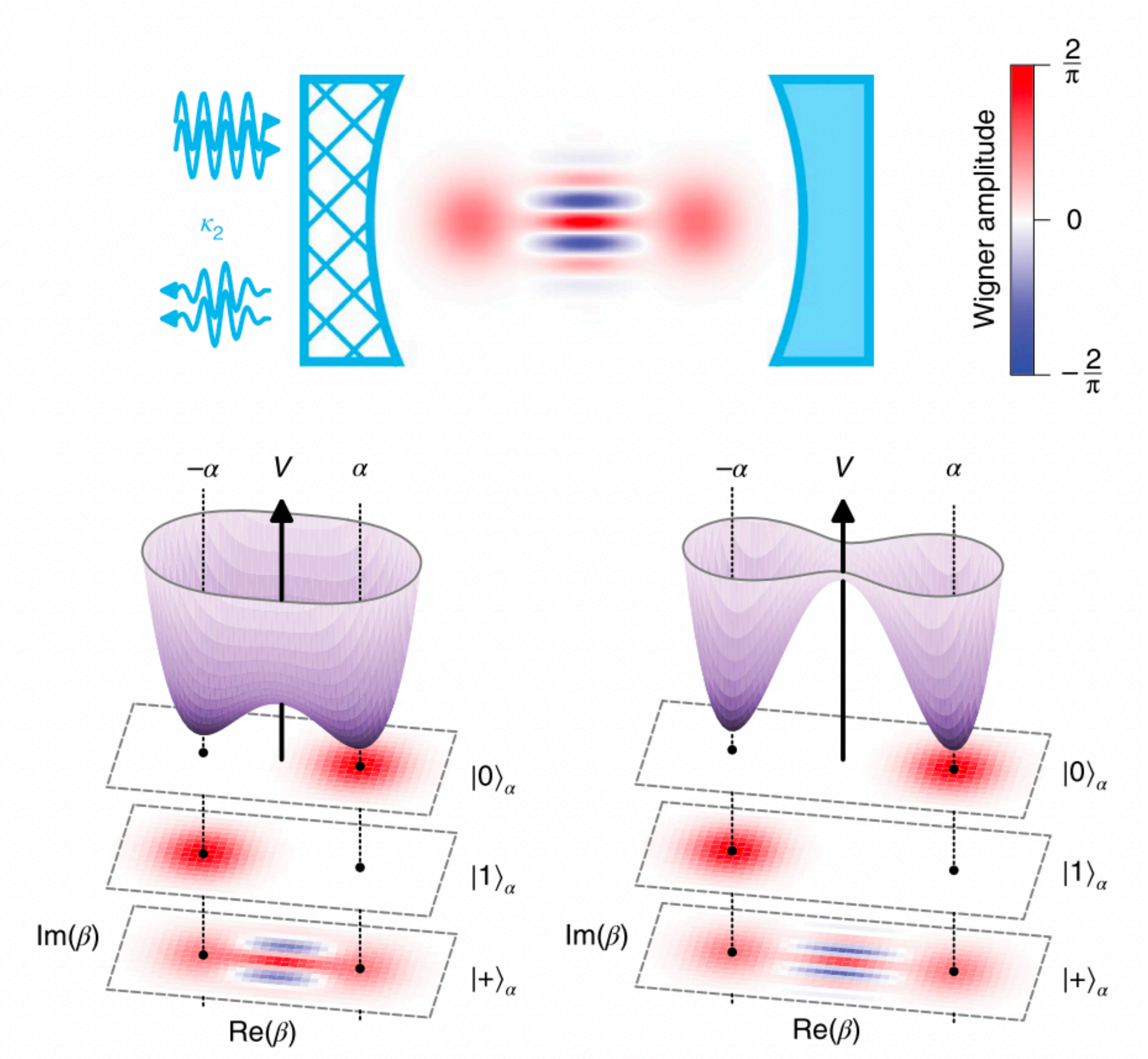


- Use this to define cat qubits:

$$|0\rangle = |\alpha\rangle$$

$$|1\rangle = |-\alpha\rangle$$

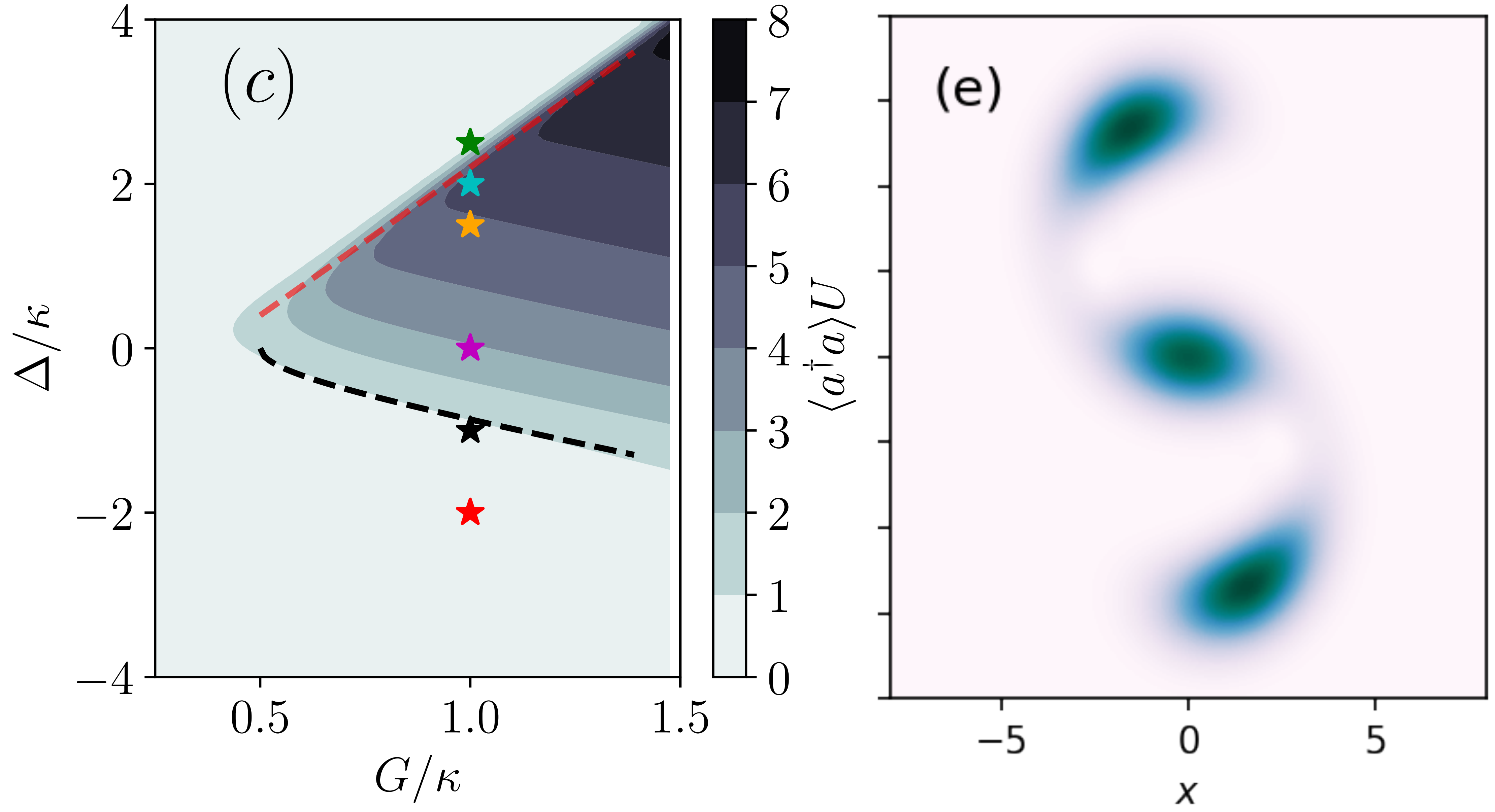
- Cat qubits are more robust against errors.
  - Quantum computing with Kerr cats.

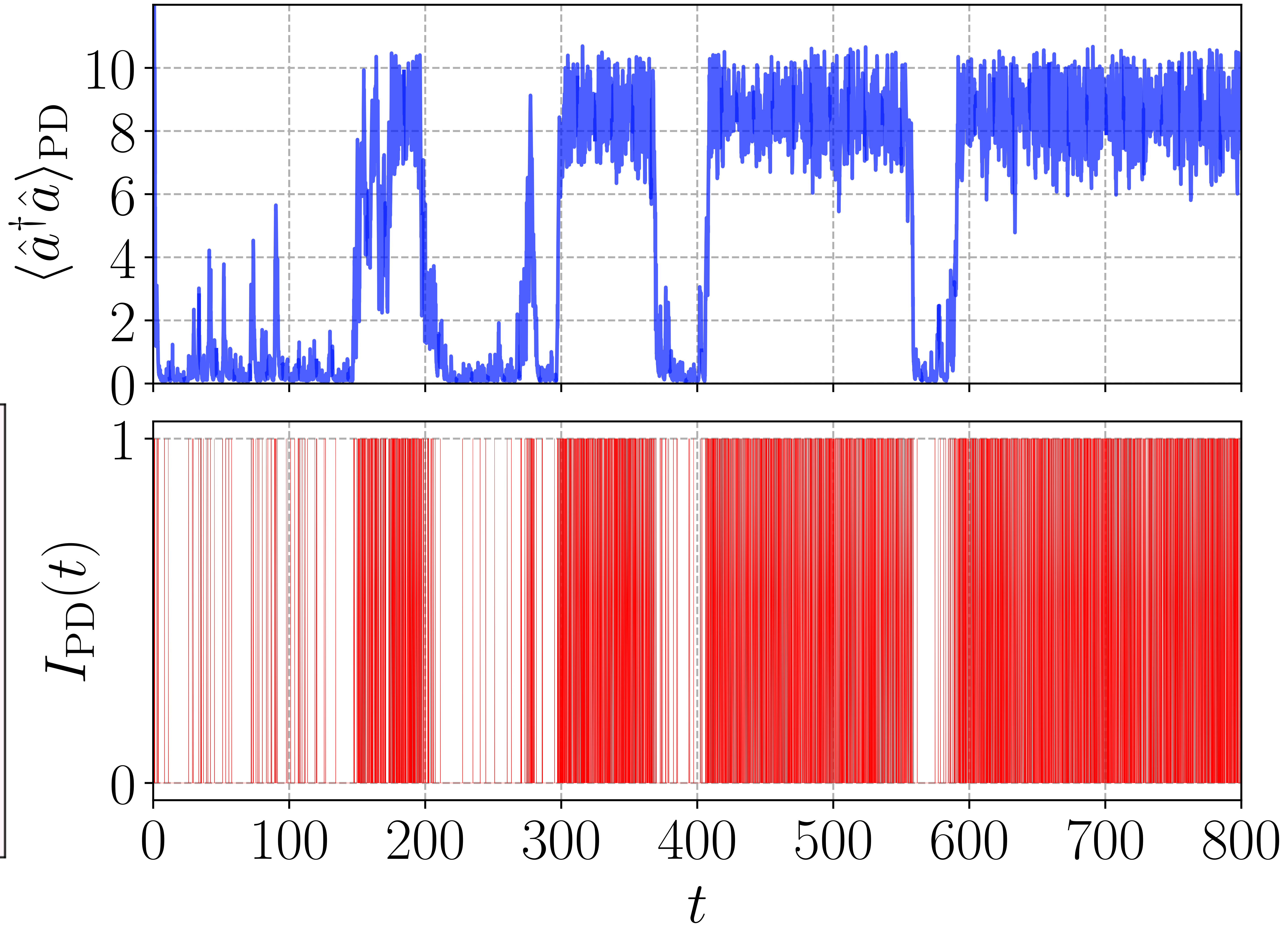


Lescanne, *et. al.*, Nature, **16**, 509-513 (2020)

Photo-detection current

- @ discontinuous transition: on/off (telegraph) behavior of the current.
- Photo-detection cannot resolve upper vs. lower blobs.

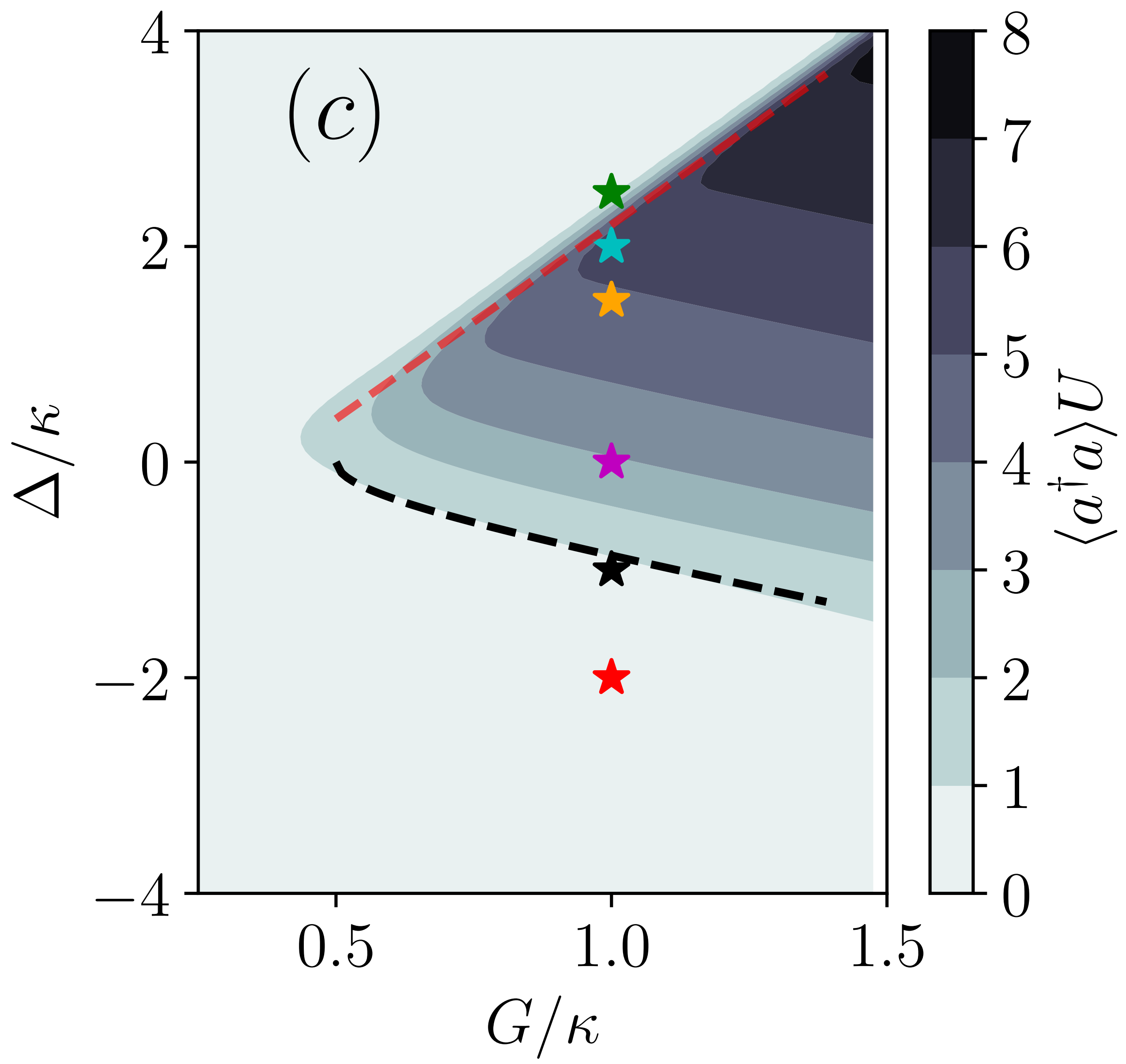


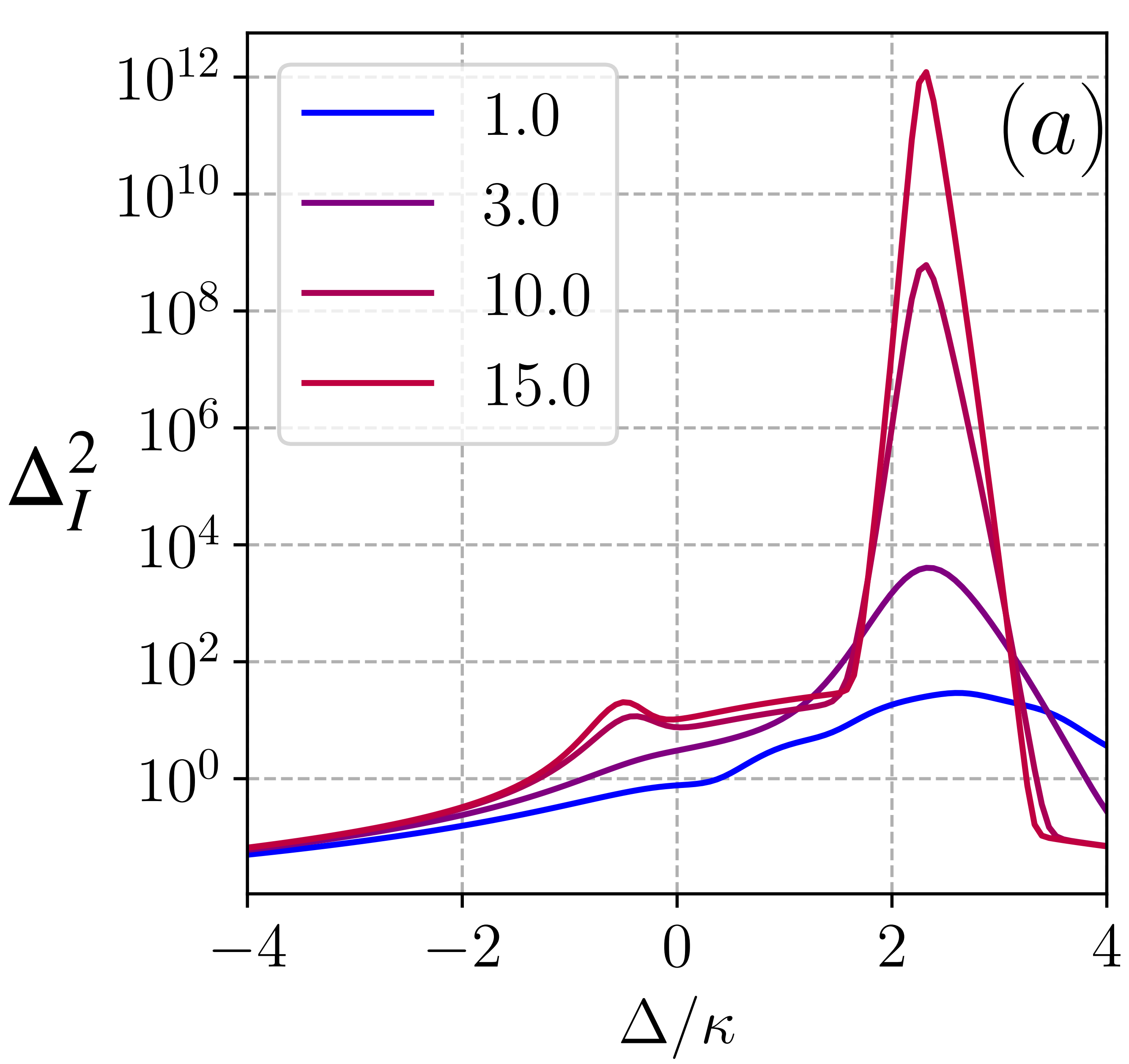


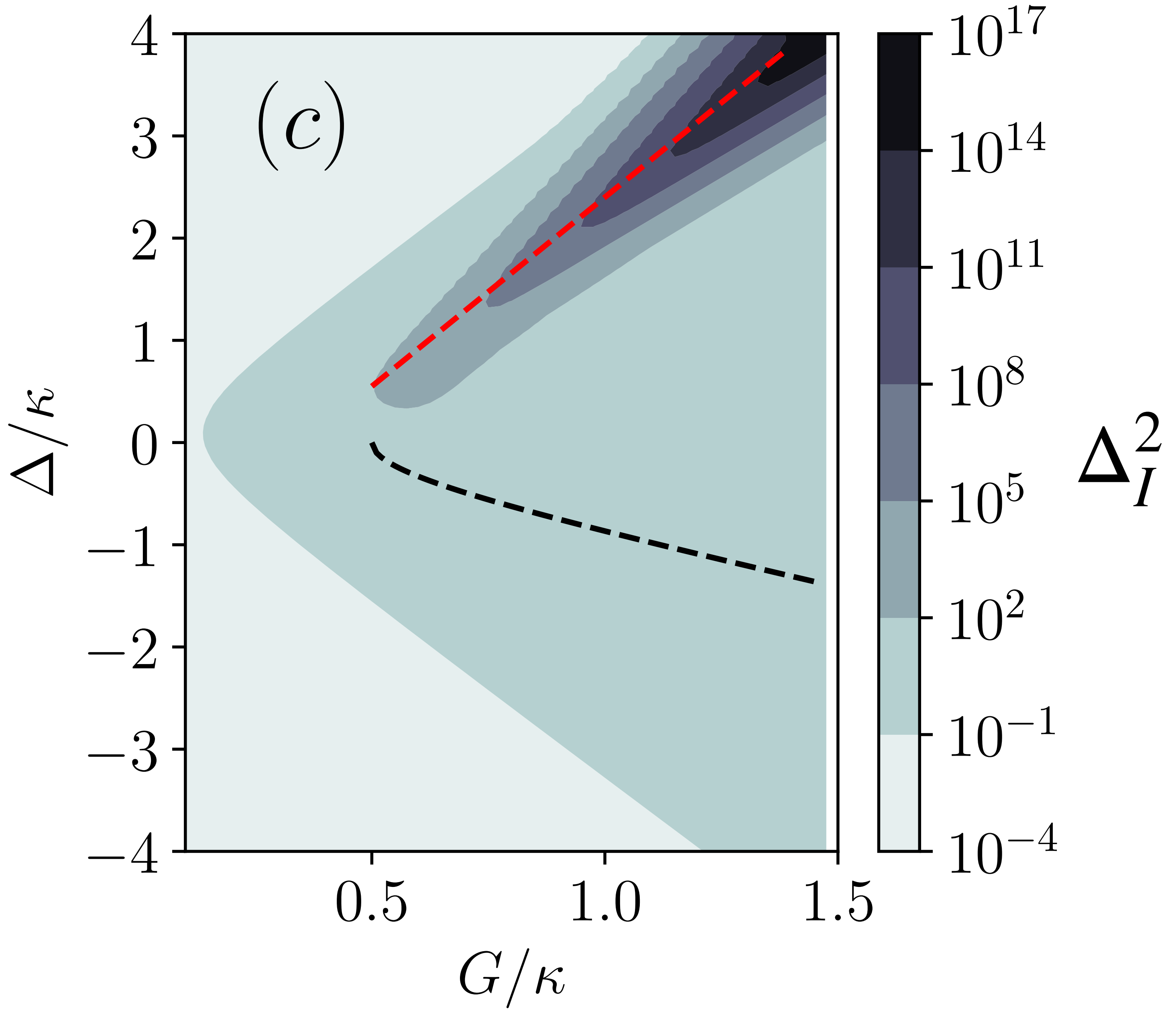
Divergence of the diffusion coefficient

- "Thermodynamic limit:" $U \to 0$
- In the discontinuous transition ($\Delta > 0$)

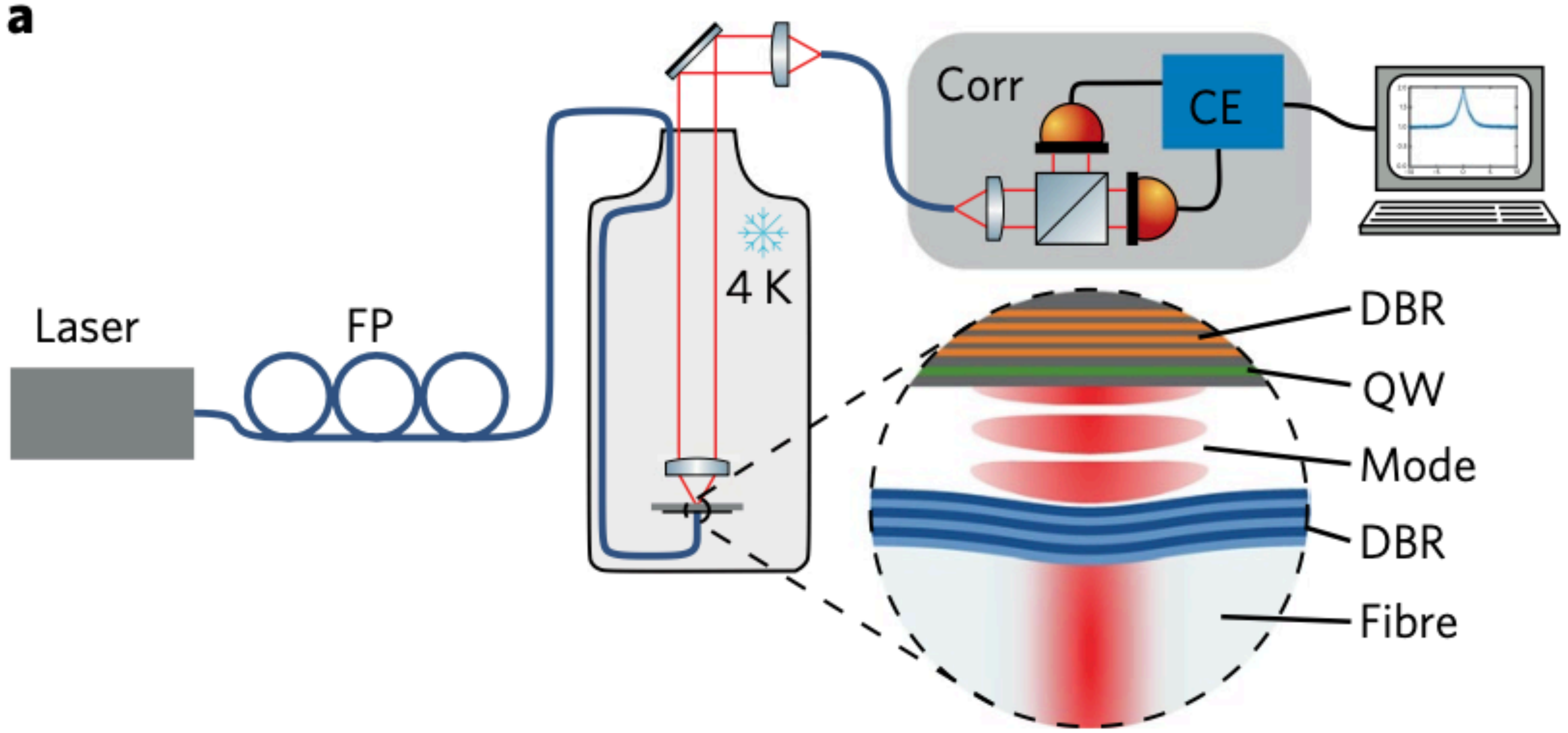
$$\Delta_I^2 \sim e^{1/U}$$

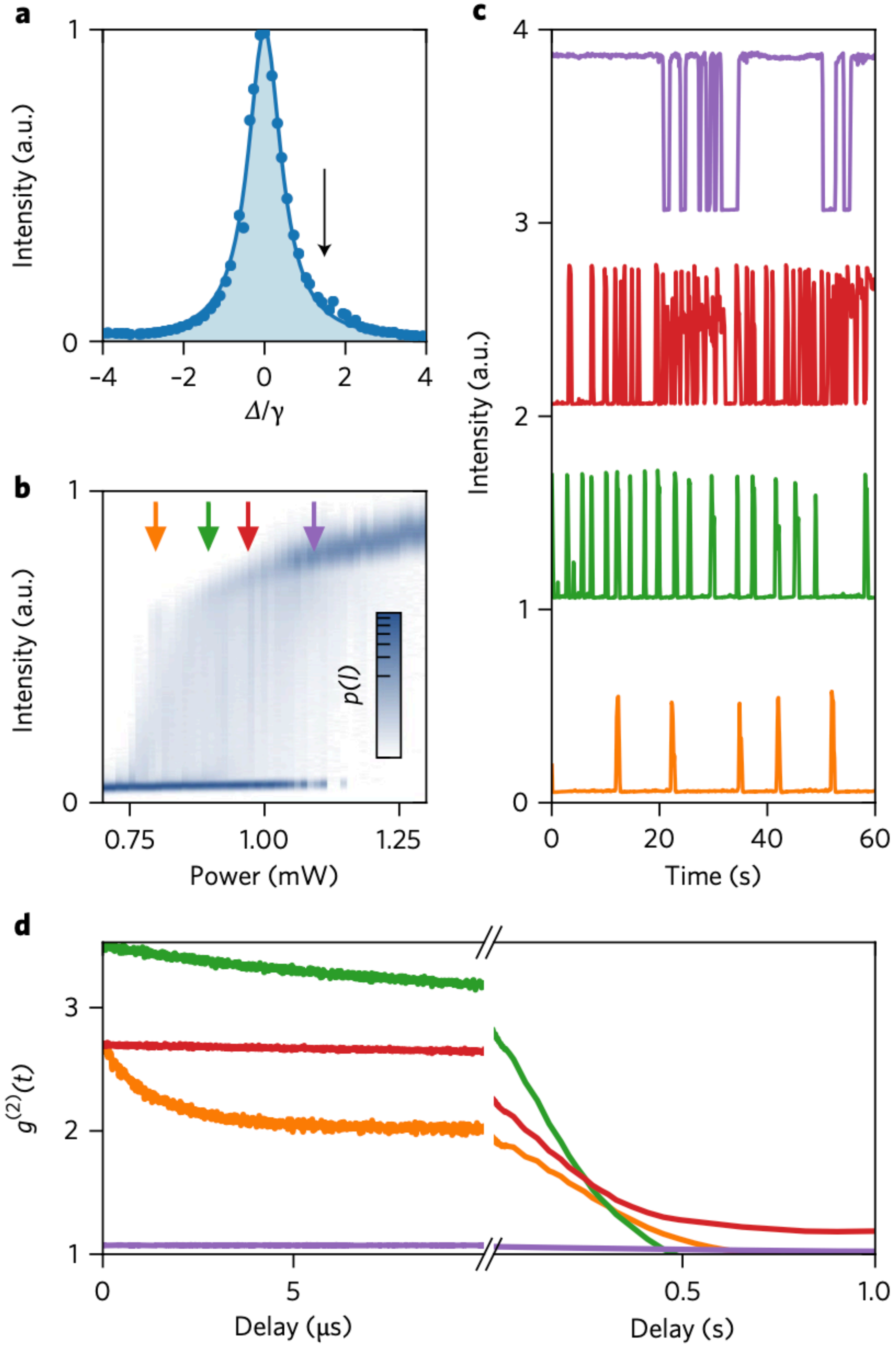






GaAs cavity polaritons.



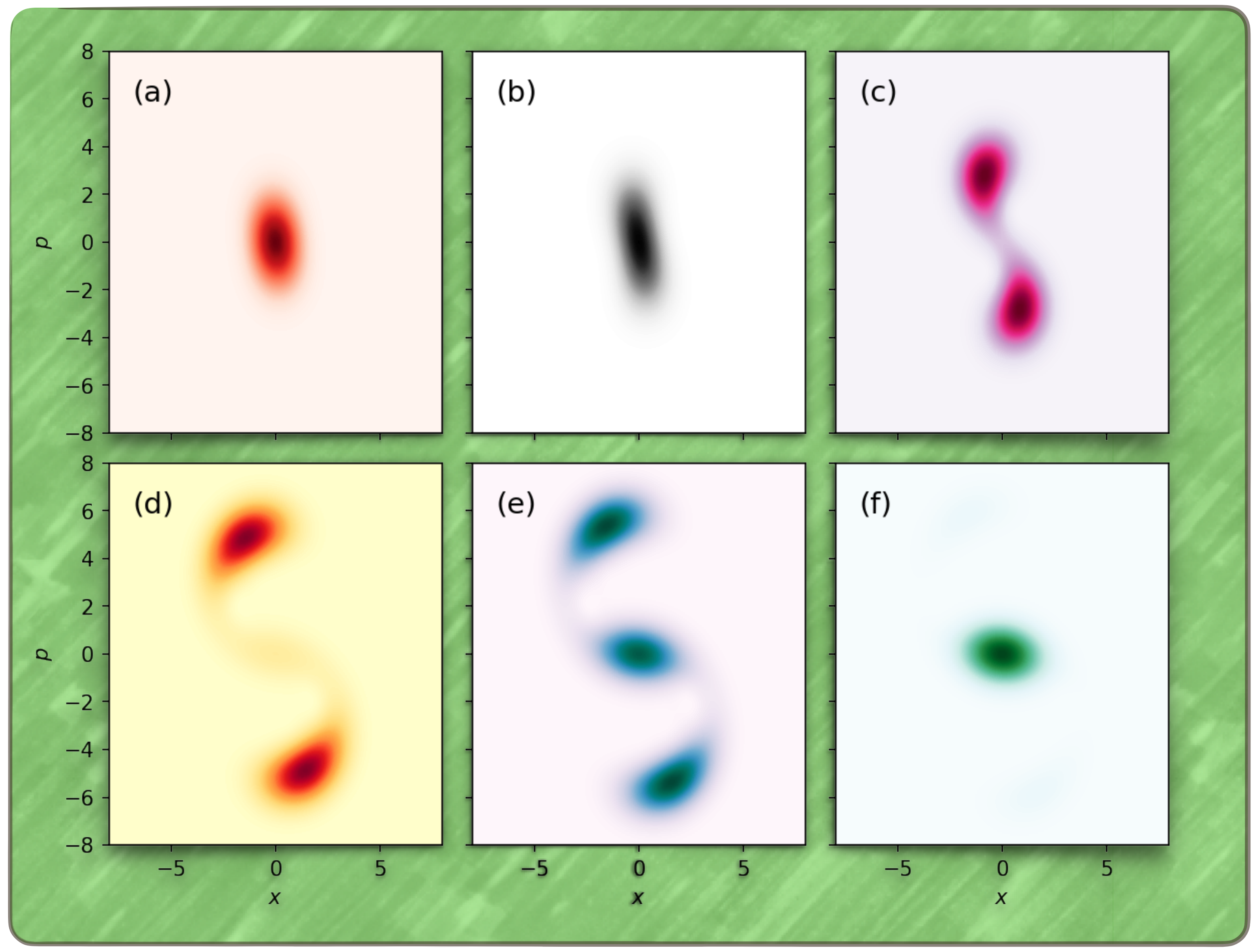


T. Fink, *et. al.*, *Nature Physics,* **14**, 365 (2018)

Homodyne current (in $p = i(a^\dagger - a)$)

- The homodyne current switches between 3 values (+,0,-).
- Captures the tunneling between the 3 blobs.

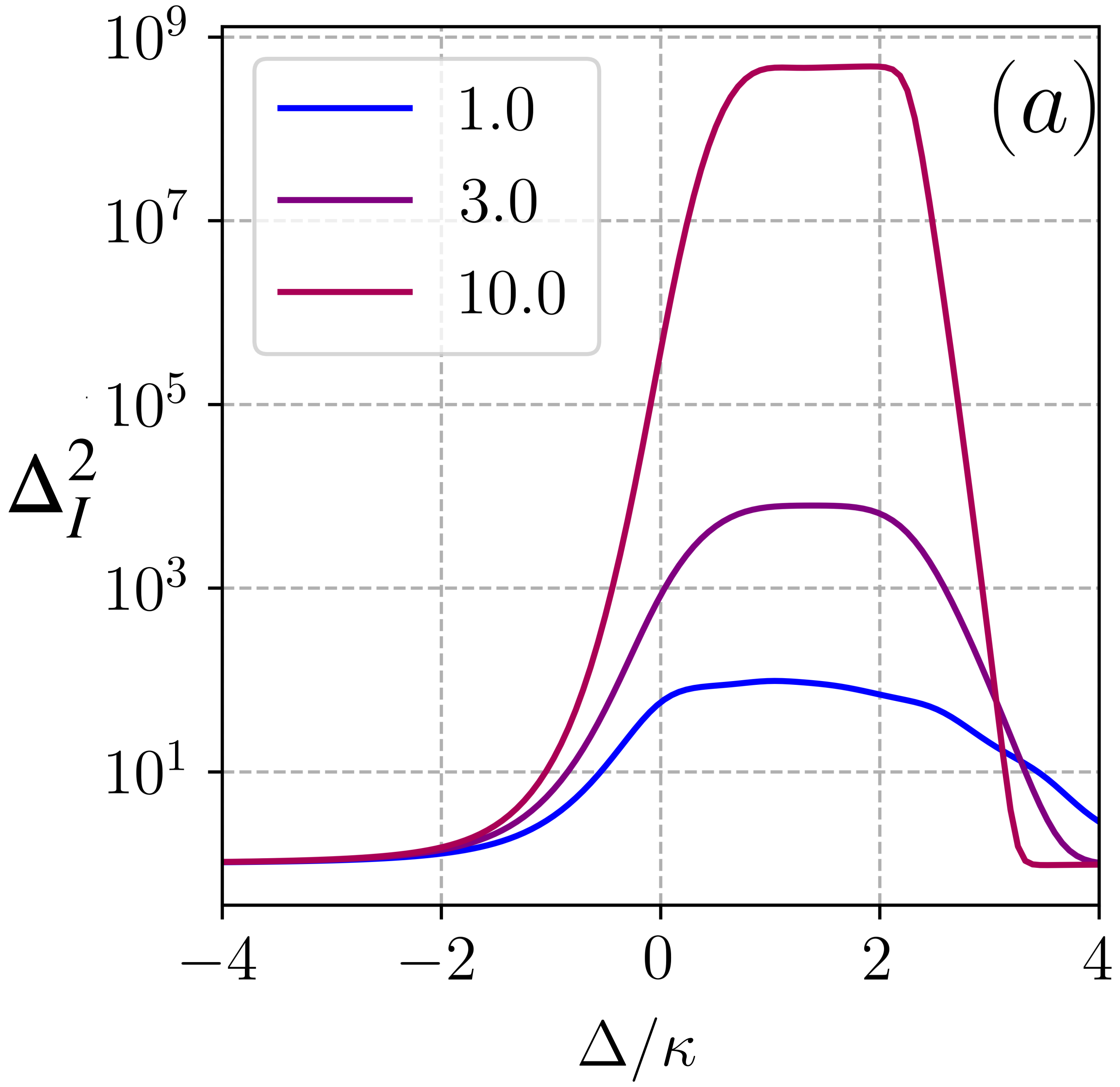
Divergence of the diffusion coefficient

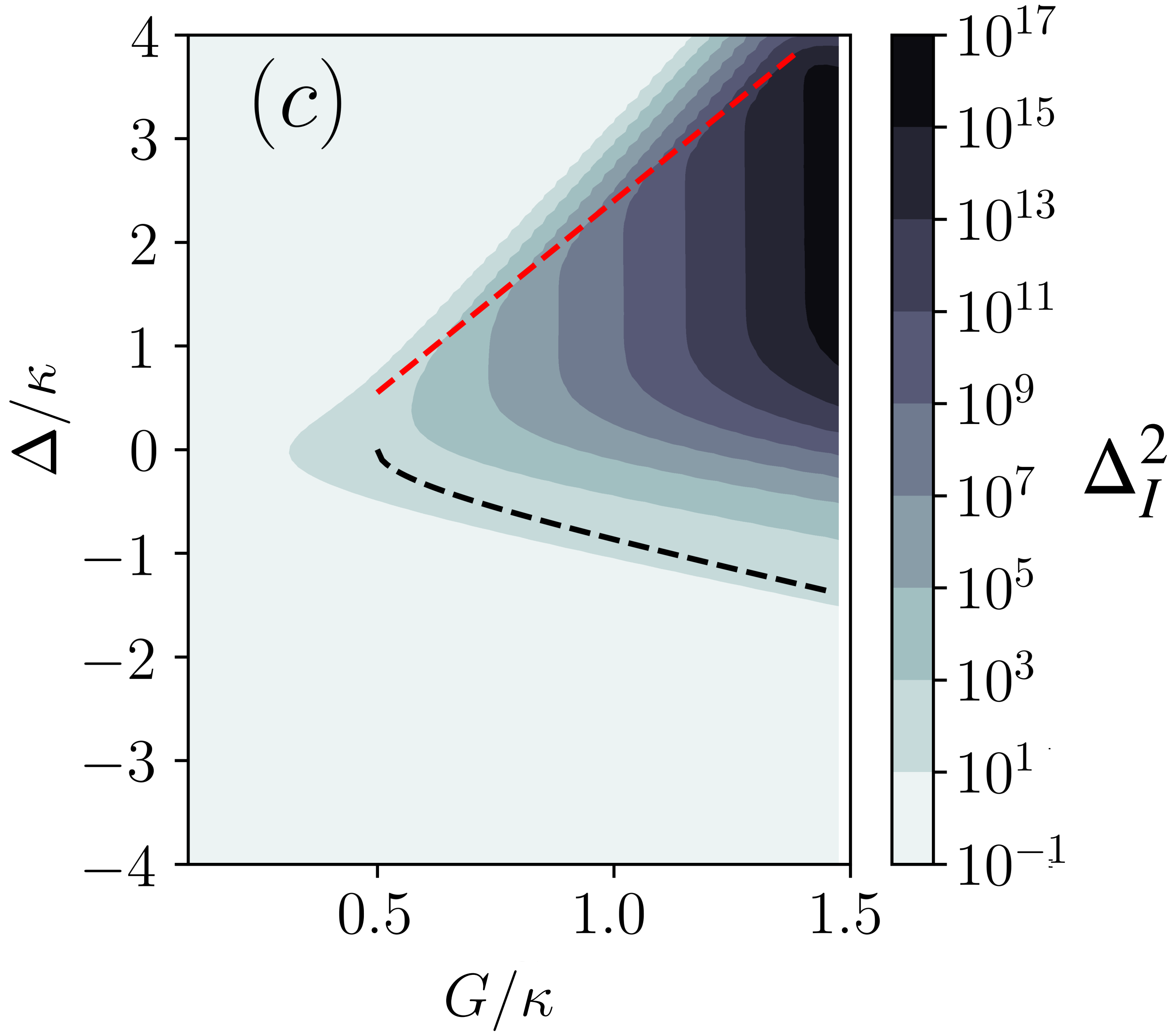


- Homodyne current noise diverges exponentially in a much broader region.

$$\Delta_I^2 \sim e^{1/U}$$

- Reflects sensitivity to all 3 blobs.





Conclusions

Thank you!

- **Entropy production:** quantifies dissipation/irreversibility.
- In the quantum realm:
  - How to define it.
  - What does it imply?
    - Quantum coherences and correlations.
- **Thermodynamic Uncertainty Relations:**
  - To curb fluctuations, we must increase dissipation.
  - No TUR in the quantum regime.
    - What are the reasons/implications?

G.T. Landi, M.J. Kewming, M.T. Mitchison and P. Potts,

"Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics"

To appear soon as a tutorial in PRX Quantum.

https://www.pas.rochester.edu/~gtlandi/