Quantum information, thermodynamics and stochastic process in open quantum system

Gabriel T. Landi

03/31/2023 - University of Rochester

https://www.pas.rochester.edu/~gtlandi/

Entropy production

- Clausius formulated the notion of irreversibility using entropy.
- Consider a thermodynamic process involving heat & work:

$$\Delta U = W + Q_h + Q_c \qquad \text{(1st law = balance equation)}$$

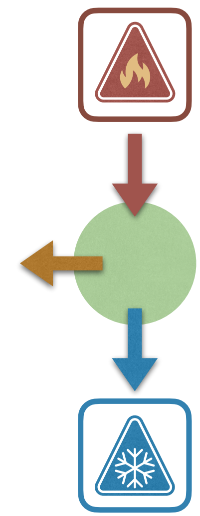
- According to Clausius, entropy does not satisfy a balance equation:

$$\Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma$$

$\sigma \geqslant 0$ is the entropy produced in the process.

- $\sigma \geqslant 0$ is the mathematical statement of the 2nd law.
- To explore the power of the 2nd law, let us consider a cyclic operation:

$$W + Q_h + Q_c = 0 \quad \text{and} \quad \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma = 0$$



$$W + Q_h + Q_c = 0 \qquad \text{and} \qquad \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma = 0$$

$$\sigma \geqslant 0$$

- **Operation as a heat engine:** efficiency

$$\eta = \frac{W}{Q_h} = \eta_c - \frac{T_c \sigma}{Q_h} \qquad \text{where} \qquad \eta_c = 1 - \frac{T_c}{T_h}$$

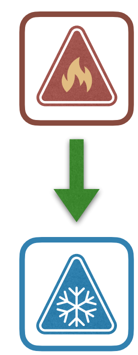
- The efficiency is always *lower* than Carnot's efficiency because entropy is produced (Carnot's statement of the 2nd law)

- **Heat flow** (no work): $Q_h = - Q_c$

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h \geqslant 0$$

Heat always flows from hot to cold

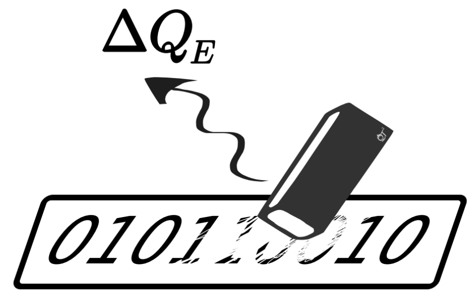
(Clausius' statement)



Otavio A. D. Molitor and Gabriel T. Landi, *Phys. Rev. A* **102**, 042217

- **Landauer's erasure:** Minimum cost to erase information

$$\Delta Q \geqslant k_B T \ln 2$$



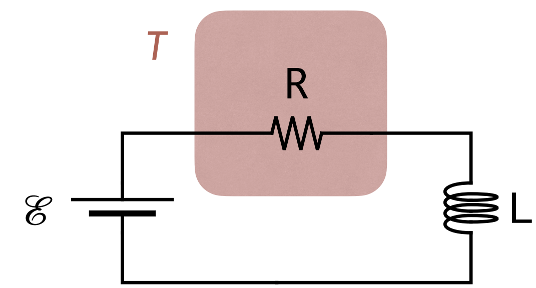
- What about $T \simeq 0$? Very relevant for quantum computation.

- If eraser is a waveguide:

$$\Delta Q \geqslant k_B T \ln 2 + \frac{3\hbar c}{\pi L} \ln^2(2)$$

- **Non-equilibrium steady-states:** not equilibrium.

$$\frac{dS}{dt} = \frac{\dot{Q}}{T} + \dot{\sigma} = 0 \qquad \text{so} \qquad \dot{\sigma} = -\frac{\dot{Q}}{T}$$



- Example: Joule heating.
  Continues as long as there is juice in the battery

$$\dot{\sigma} = \frac{\mathscr{E}^2}{RT}$$

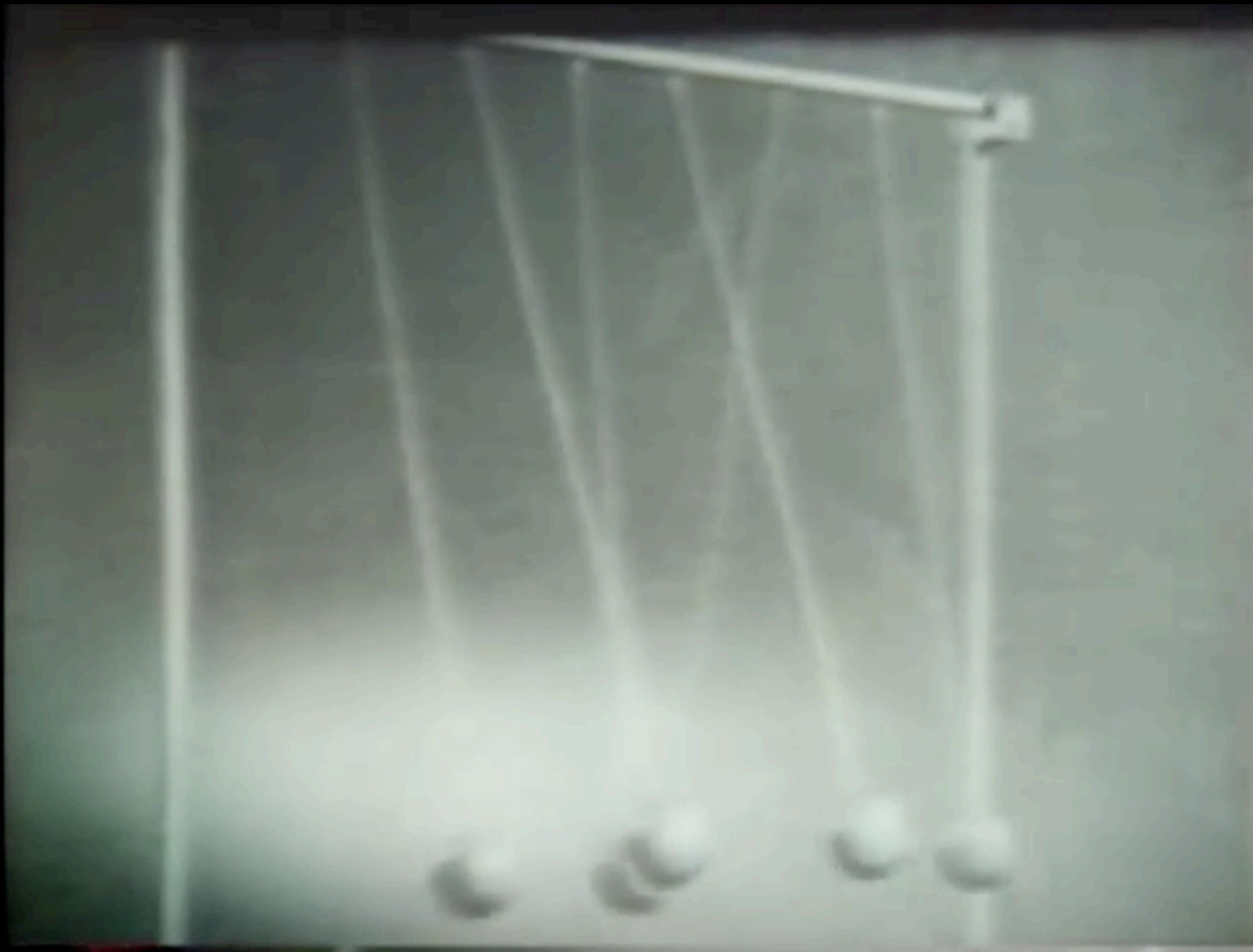
André M. Timpanaro, Jader P. Santos, and Gabriel T. Landi, *Phys. Rev. Lett.* **124**, 240601 (2020)

Irreversibility and the arrow of time

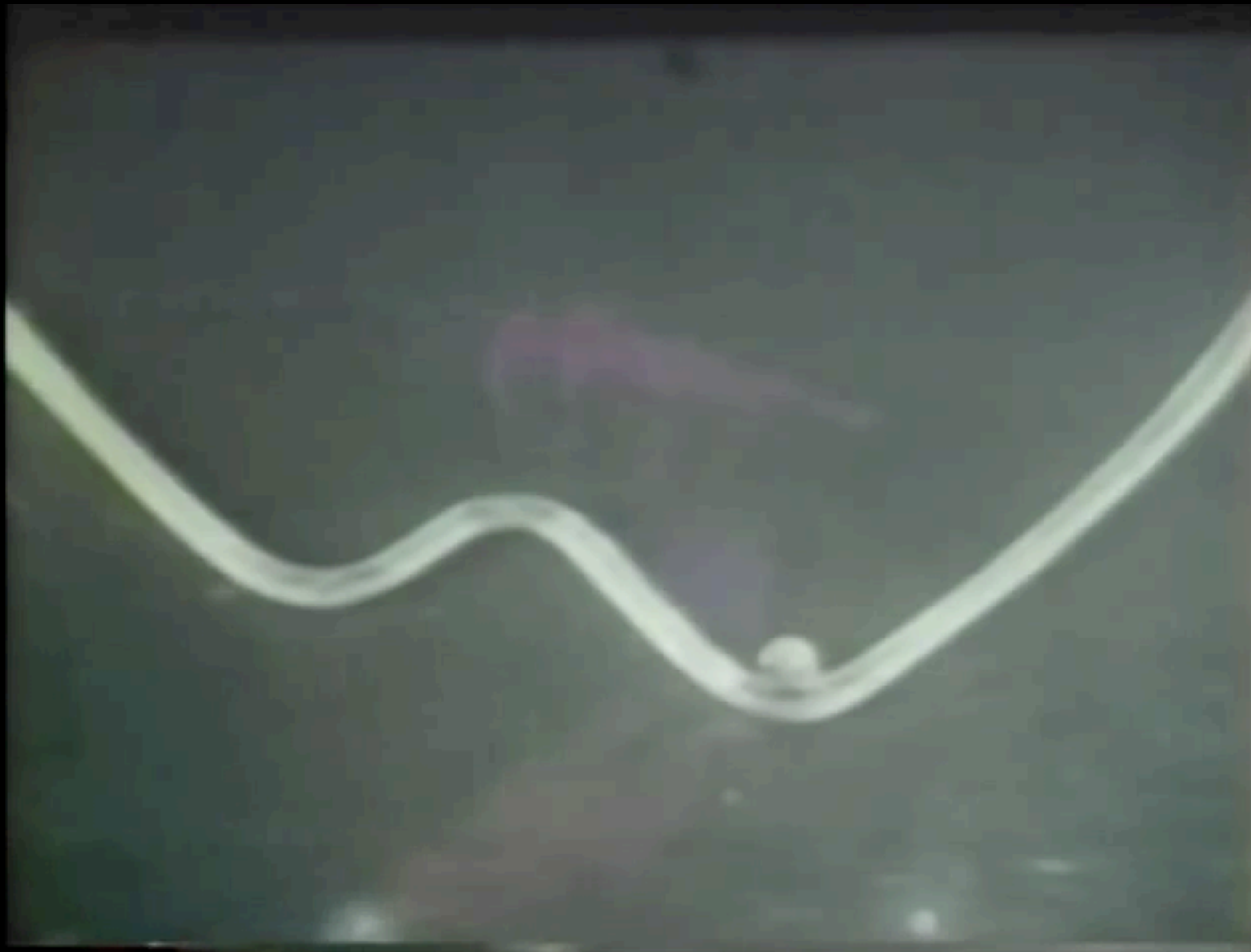
Prof. George Porter
YouTube

Prof. George Porter
YouTube

Prof. George Porter
YouTube



Prof. George Porter
YouTube



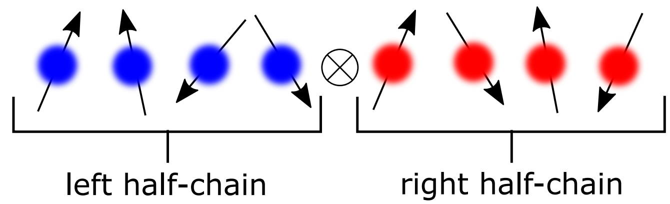
Irreversibility and the arrow of time

- Irreversibility: *how unlikely the backward process is, in comparison with the forward one.*
- But why does this happen?
    - The microscopic laws of the universe (Newton, Schrödinger, &c) are time-reversible.
- **Operational definition:** what is accessible and what is not.
    - **Dissipation:** heat lost to the environment cannot be recovered.
- Irreversible videos were those that involved a lot of dissipation.

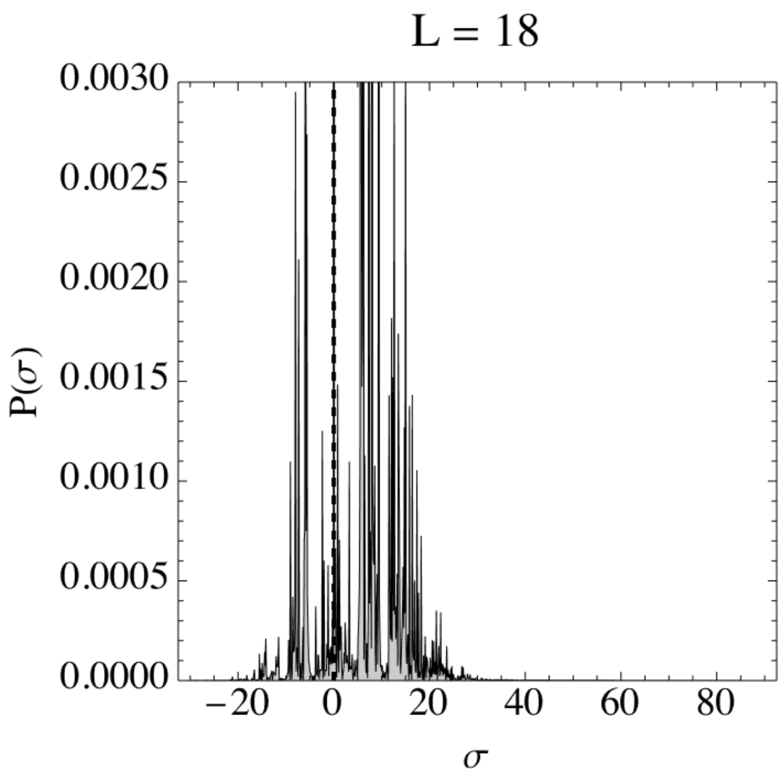
Gabriel T. Landi and Mauro Paternostro, "**Irreversible entropy production, from quantum to classical**",
Review of Modern Physics, **93**, 035008 (2021)

Fluctuations are significant in the micro-world

- Macro-world: heat flows from hot → cold.
- Micro-world: heat ***usually*** flows from hot → cold.



G. T. Landi and Dragi Karevski Phys. Rev. E **93**, 032122 (2015)



Heat Exchange Fluctuation Theorem

$$P(-\sigma) = e^{-\sigma} P(\sigma)$$

Implies 2nd law: $\langle \sigma \rangle \geqslant 0$

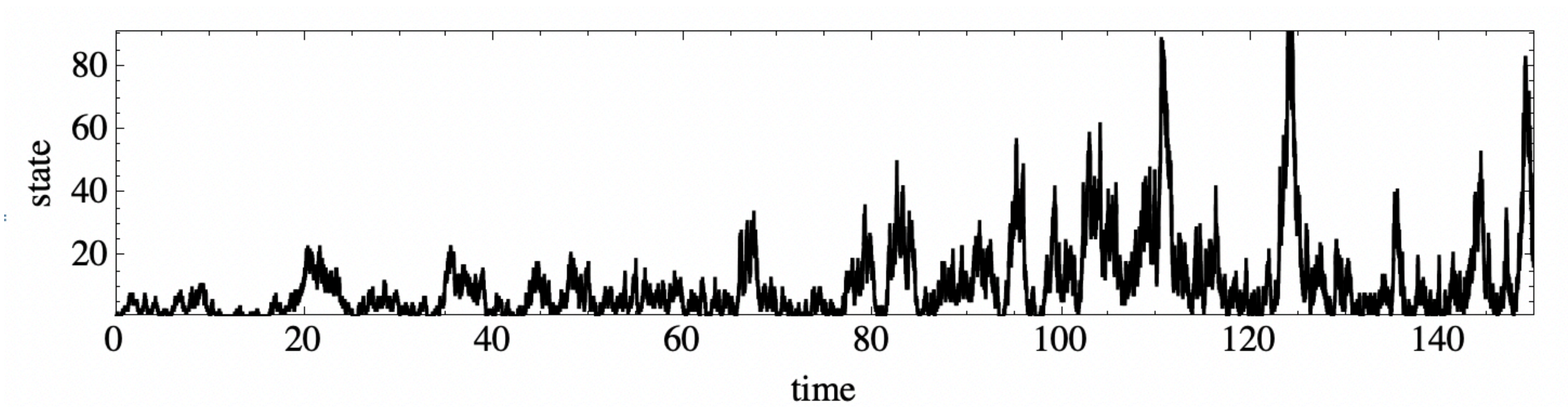
C. Jarzynski and D. Wójcik, Phys. Rev. Lett. 92, 230602 (2004)
G. E. Crooks, Journal of Statistical Physics, **90**, 1481–1487 (1998)

Entropy production and stochastic trajectories

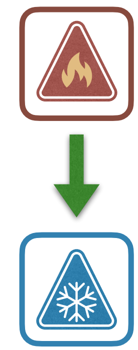
- Fluctuations allow us to formulate the entropy production problem in terms of **trajectory probabilities**

$$\sigma[\gamma] = \ln \frac{P_F[\gamma]}{P_R[\gamma]}$$

- A process is reversible when the time-reversed process is as likely as the forward one.



Connecting the two pictures



- 2 systems, A, B, in equilibrium:

$$p_n^A = \frac{e^{-\beta_A E_n^A}}{Z_A} \qquad \text{and} \qquad p_i^B = \frac{e^{-\beta_B E_i^B}}{Z_B}$$

- At $t = 0$ we put them in contact and allow them to interact with Hamiltonian H.

$$n_A, i_B \xrightarrow{e^{-iHt}} m_A, j_B \quad \text{Occurs with prob. } p(n_A, i_B \to m_A, j_B) = p_n^A p_i^B \; \langle m_A, j_B \;\; e^{-iHt} \;\; n_A, i_B \rangle \;^2$$

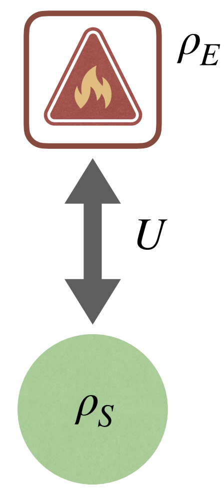
- Entropy production

$$\sigma = \ln \frac{P(n_A, i_B \to m_A, j_B)}{P(m_A, j_B \to n_A, i_B)} = \ln \frac{p_n^A p_i^B \; \langle m_A, j_B \;\; e^{-iHt} \;\; n_A, i_B \rangle \;^2}{p_m^A p_j^B \; \langle n_A, i_B \;\; e^{-iHt} \;\; m_A, j_B \rangle \;^2} = \ln \frac{p_n^A p_i^B}{p_m^A p_j^B} = \beta_A (E_m^A - E_n^A) + \beta_B (E_j^B - E_i^B)$$

C. Jarzynski and D. Wójcik, Phys. Rev. Lett. 92, 230602 (2004)

Entropy production for quantum systems

$$\rho'_{SE} = U(\rho_S \otimes \rho_E)U^\dagger$$




```
Describes an enormous
variety of processes!
(maybe a complicated U)
```

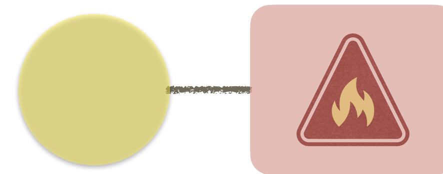
- Information-theoretic formulation: $\sigma = I(S:E) + D(\rho'_E \| \rho_E)$
  - Operational interpretation: Characterizes irreversibility in terms of what you do not have access to:
    - System-environment correlations.
    - Changes in the environment.
- Tricky business: how to define heat currents for quantum master equations.

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

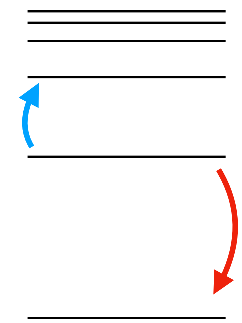
Gabriel T. Landi and Mauro Paternostro, "**Irreversible entropy production, from quantum to classical**", *Review of Modern Physics*, **93**, 035008 (2021)

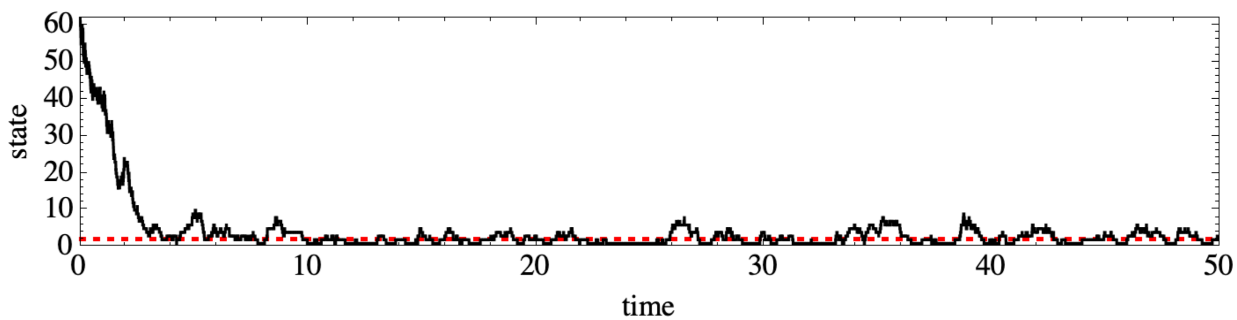
Gabriel T. Landi, Dario Poletti, Gernot Schaller, "**Nonequilibrium boundary-driven quantum systems: Models, methods, and properties.**" Reviews of Modern Physics, 94, (2022)

Relaxation towards equilibrium



- Imagine an atomic system relaxing towards equilibrium.
  - Population of energy eigenstates fluctuate until they reach thermal equilibrium.
- In addition: destroy any superpositions (**decoherence**).
- Entropy production rate can be split as $\sigma = \sigma_{\text{pop}} + \sigma_{\text{coh}}$





Additional entropy production due to coherence: Dissipation of information, without dissipation of energy.

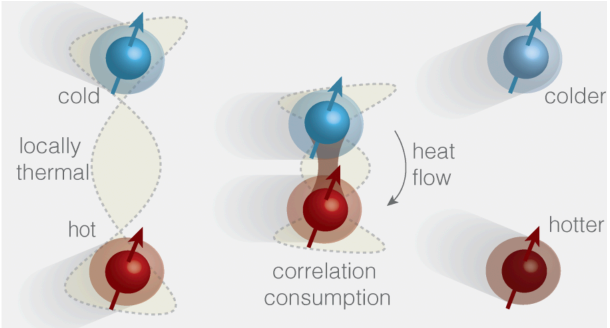
J. P. Santos, L. Céleri, GTL, M. Paternostro, npj Quantum Information **5**, 23 (2019)

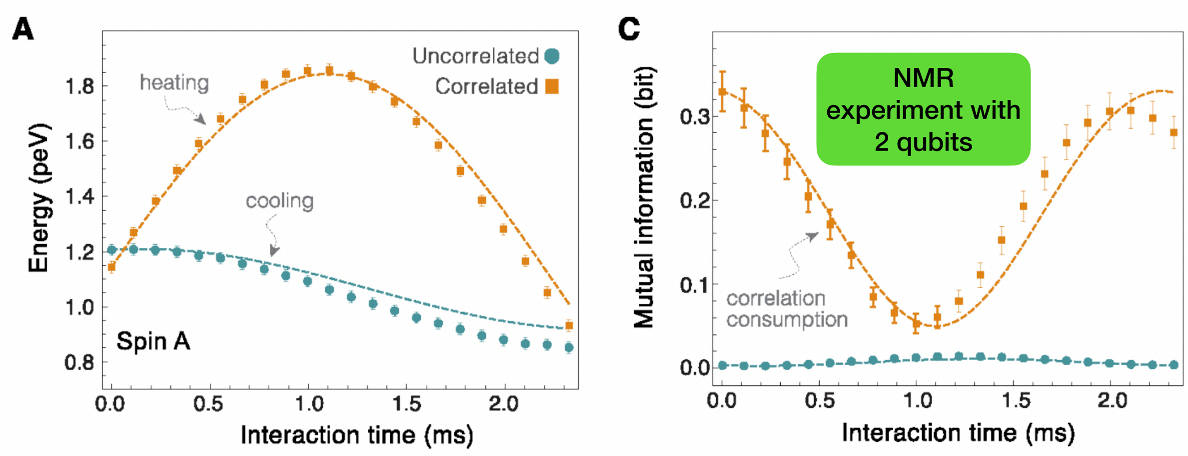
Information-thermodynamics

- In the presence of initial correlations the second law is modified to

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h \geqslant \Delta I(h:c)$$

- Heat can flow from cold to hot, provided we **consume** quantum correlations.



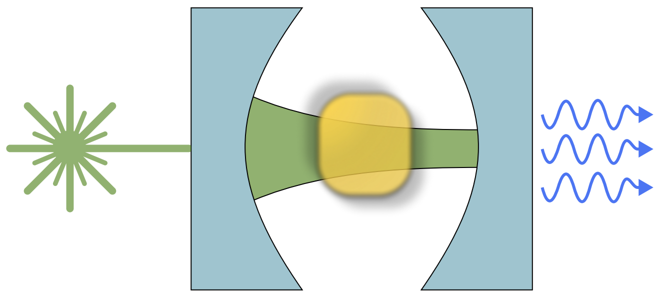


Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, "**Reversing the direction of heat flow using quantum correlations**", *Nature Communications*, **10**, 2456 (2019)

K. Micadei, G. T. Landi, E. Lutz, "Quantum fluctuation theorems beyond two-point measurements", Phys. Rev. Lett. 124, 090602 (2020)

Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Roberto M. Serra, Eric Lutz, "Experimental validation of fully quantum fluctuation theorems", Phys. Rev. Lett., 127, 180603 (2021).

Quantum phase space



- Many quantum experiments are done using optical cavities with semi-transparent mirrors.
- Photons leaking out $\simeq$ zero temperature bath.
    - Spontaneous emission: excitations can leave, but not return.
- 2nd law is buggy @ $T = 0$: $\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h.$
    - Does not include vacuum fluctuations (*present in every measurement*).
- We reformulated the entropy production problem in terms of quantum phase space & the *Wigner function.*

$$\sigma = \left(\frac{1}{T_c^{\text{eff}}} - \frac{1}{T_h^{\text{eff}}}\right) Q_h$$

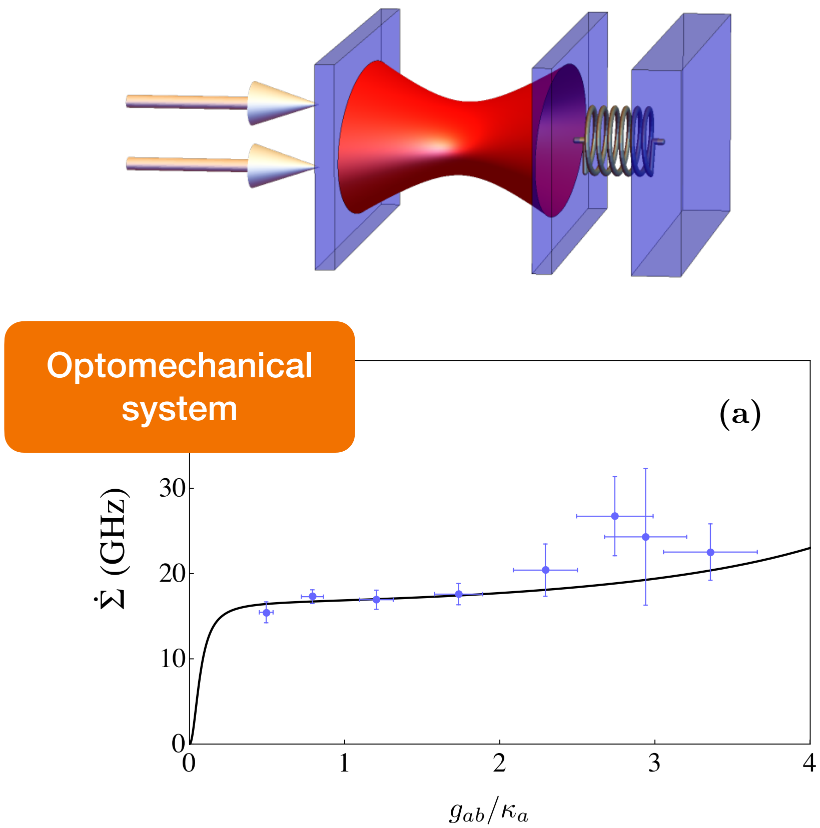
$$T^{\text{eff}} = \omega(\bar{n} + 1/2), \qquad \bar{n} = \frac{1}{e^{\beta\omega} - 1}$$

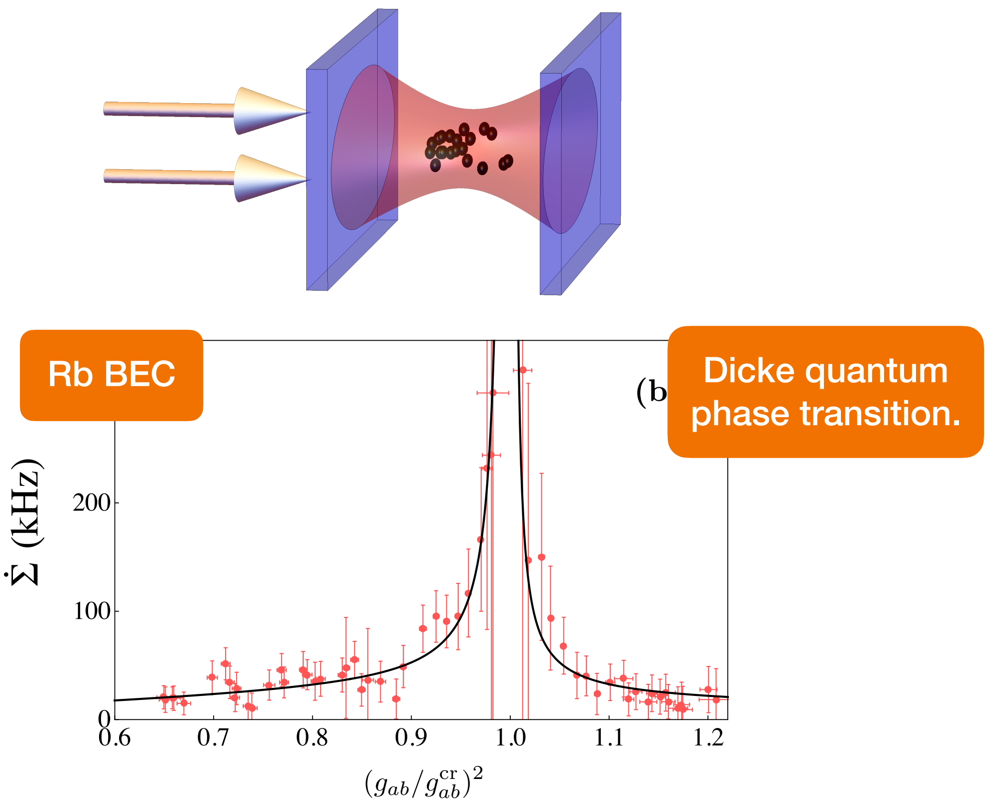
High temperatures: $\omega(\bar{n} + 1/2) \simeq T$.

Zero temperature: $\omega(\bar{n} + 1/2) = \omega/2$.

Jader P. Santos, G.T. Landi and Mauro Paternostro, Phys. Rev. Lett, **118**, 220601 (2017),

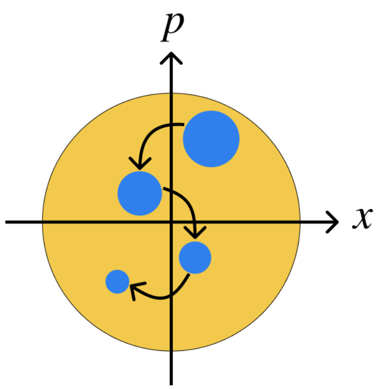
Experiments

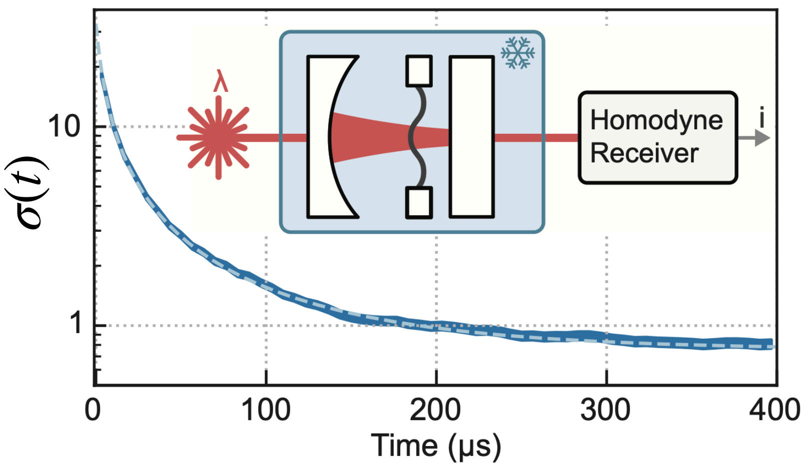




M. Brunelli, L. Fusco, R. Landig, W. Wieczorek, J. Hoelscher-Obermaier, GTL, F Semião, A. Ferraro, N. Kiesel, T. Donner, G. De Chiara, and M. Paternostro. *Phys. Rev. Lett.*, **121**, 160604 (2018)

🇩🇰 Continuously measured vibrating membrane



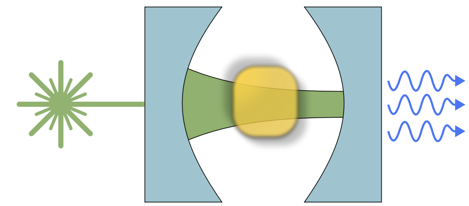


Informational steady-state:

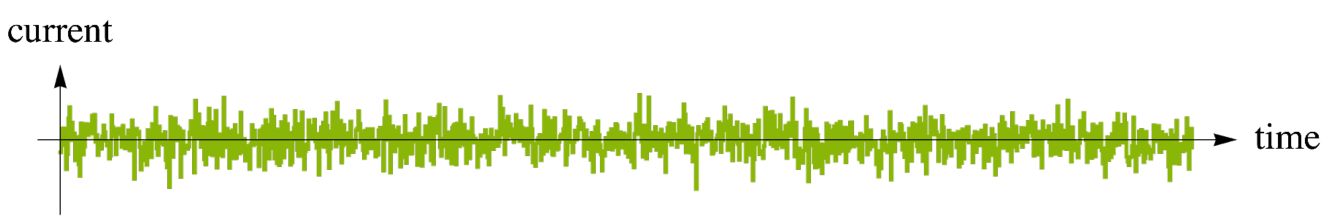
Conditional dynamics relaxes to a colder state, which can only be maintained by continuously monitoring S.

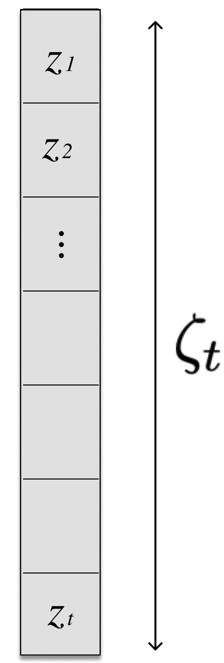
Massimiliano Rossi, Luca Mancino, Gabriel T. Landi, Mauro Paternostro, Albert Schliesser, Alessio Belenchia, "**Experimental assessment of entropy production in a continuously measured mechanical resonator**", Phys. Rev. Lett. **125**, 080601 (2020)

Continuously monitored quantum systems



- Continuous monitoring of photons that leak out of the cavity.
  - Individual clicks in the detector.
- Fundamental questions: what is entropy production *given* a detection record.
  - Operation: define thermodynamics in terms of what we can actually measure.
  - Includes *information* directly in the formulation.





Holevo information

- **Unconditional:** If we do not know the individual clicks: ρ_t
- **Conditional on the detection record:** $\rho_{t\ \zeta_t}$
- **Holevo information:** accumulated information we learned from the detection.

$$I(S_t : \zeta_t) = \sum_{\zeta_t} P(\zeta_t) D\big(\rho_{t\ \zeta_t} \quad \rho_t\big)$$

- With each new detection

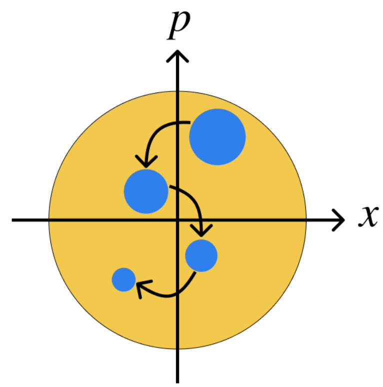
$$\Delta I_t = G_t - L_t = \text{gain} - \text{loss}$$

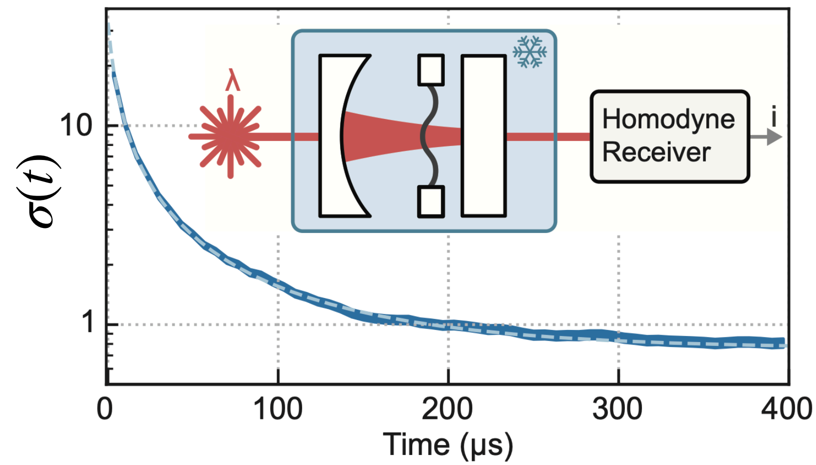
- Conditional entropy production

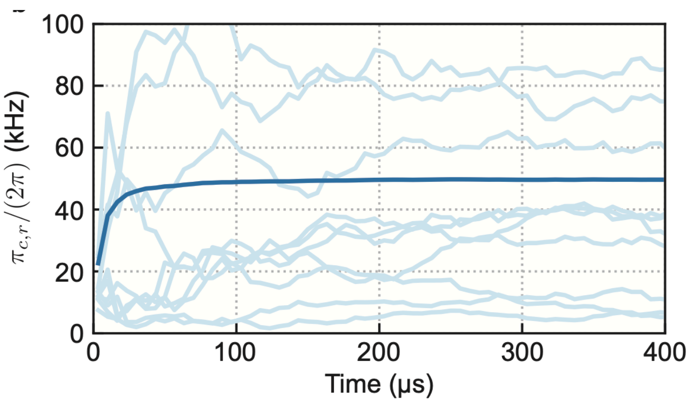
$$\Delta\Sigma^c = \Delta\Sigma^u - \Delta I$$

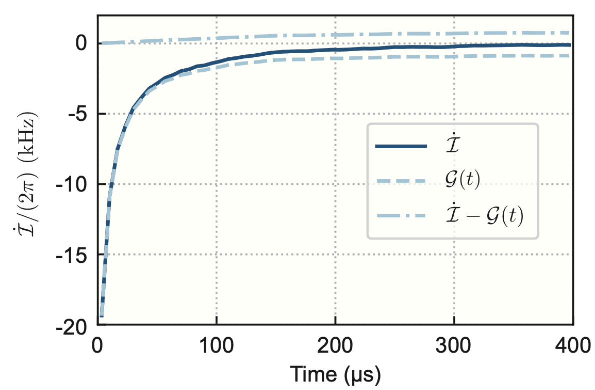
Gabriel T. Landi, Mauro Paternostro, Alessio Belenchia, PRX Quantum, 3, (2022)
Alessio Belenchia, Luca Mancino, Gabriel T. Landi, Mauro Paternostro, npj Quantum Information, 6, (2020)

🇩🇰 Mechanical setup









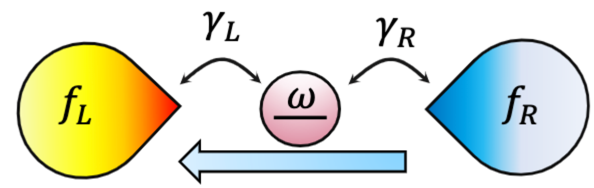
Informational steady-state:

Conditional dynamics relaxes to a colder state, which can only be maintained by continuously monitoring S.

Massimiliano Rossi, Luca Mancino, Gabriel T. Landi, Mauro Paternostro, Albert Schliesser, Alessio Belenchia, "**Experimental assessment of entropy production in a continuously measured mechanical resonator**", Phys. Rev. Lett. **125**, 080601 (2020)

Thermodynamic uncertainty relations

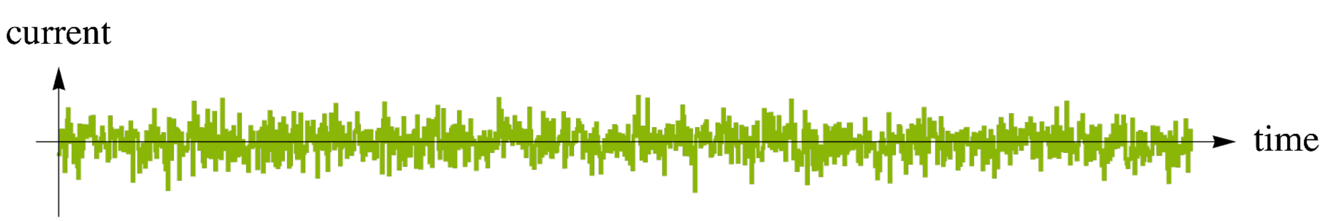
Thermodynamic uncertainty relations

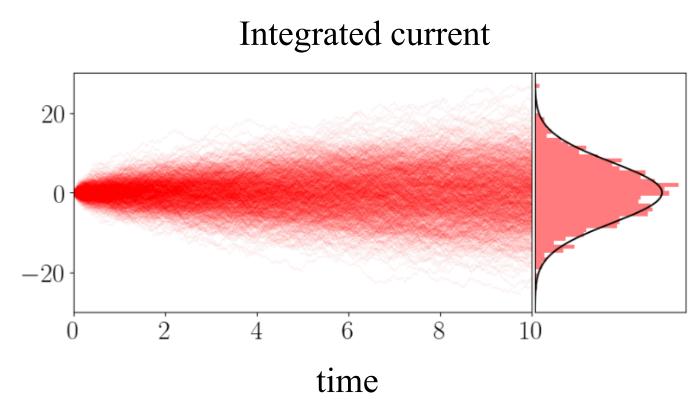


- At the μ-scale: currents fluctuate a lot:
  - Work, heat, particle current, &c.
- For classical Markov processes: TUR $\dfrac{\Delta_I^2}{I^2} \geqslant \dfrac{2}{\dot{\sigma}}$
  - Counterintuitive: must increase dissipation to curb fluctuations.
    - "Irreversibility can be good" (when fluctuations are large)

For a heat engine:

$$\Delta_W^2 \geqslant 2T_c W \frac{\eta}{\eta_C - \eta}$$

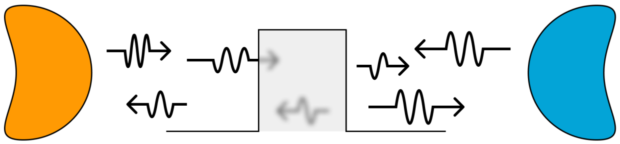


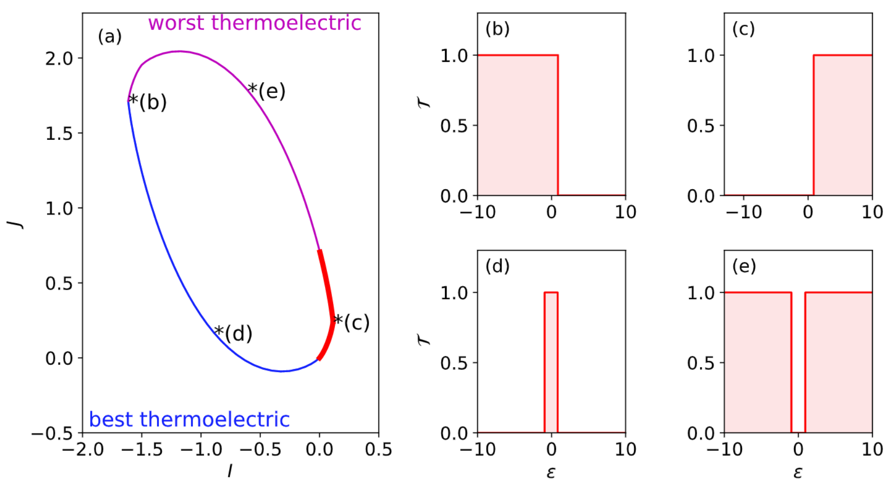


A. C. Barato, U. Seifert, *Physical Review Letters*, **114**, 158101 (2015)

TURs can be violated in the quantum regime

- Quantum coherent transport, e.g. through quantum dots.
- Important example: thermoelectricity.
- Practical question: what is the best thermoelectric?
  - New question: *what is the most precise thermoelectric?*





🚧 **Major open questions in the field:**

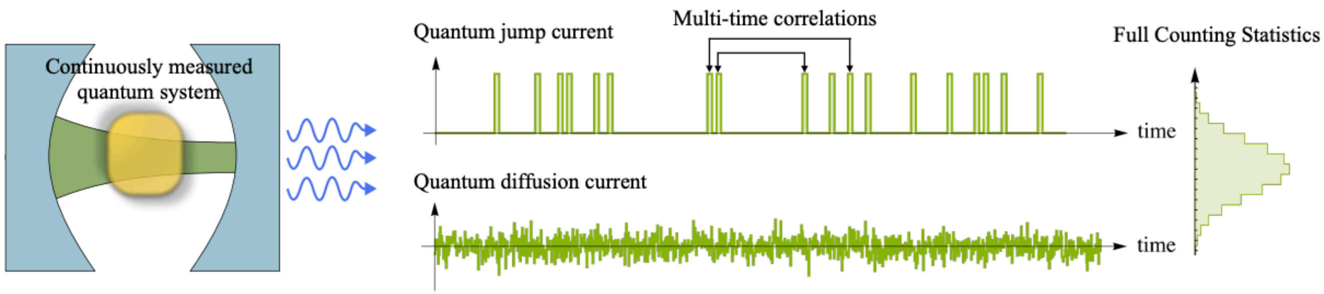
- Why quantum coherence?

- What limits the precision in the quantum world?

- Is there an actual trade-off?

Andre M. Timpanaro, Giacomo Guarnieri, and Gabriel T. Landi, arXiv 2106.10205

Understanding current fluctuations



- New tutorial on how to compute current fluctuations in open quantum systems.

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_k L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho\}$$

- Pedagogical. Meant for PhD students.

- Connects tools using in Quantum Optics with tools used in Cond. Mat. and Statistical Physics.

Gabriel T. Landi, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics.**" arXiv 2303.04270

Quantum jumps (Sec. II.C)

Superoperators:

$$\mathcal{J}\rho = \sum_k \nu_k L_k \rho L_k^\dagger, \quad K = \sum_k \nu_k^2 \langle L_k^\dagger L_k \rangle$$

Stochastic current:

$dN(t) = \nu_k$ when jump k occurs.

Averages to $J = \sum_k \nu_k \langle L_k^\dagger L_k \rangle$

Quantum diffusion ((Sec. II.D)

Superoperators:

$$\mathcal{H}\rho = \sum_k \nu_k (L_k \rho e^{-i\phi_k} + \rho L_k^\dagger e^{i\phi_k}), \quad K_{\text{diff}} = \sum_k \nu_k^2$$

Stochastic current:

$$I_{\text{diff}} = \sum_k \nu_k \left(\langle L_k e^{-i\phi_k} + L_k^\dagger e^{i\phi_k} \rangle + \frac{dW_k}{dt} \right)$$

Averages to $J_{\text{diff}} = \sum_k \nu_k \langle L_k e^{-i\phi_k} + L_k^\dagger e^{i\phi_k} \rangle$

Tilted Liouvillians (Sec. IV)

Quantum Jumps: $\mathcal{L}_\chi \rho = -i[H,\rho] + \sum_k e^{i\nu_k \chi} L_k \rho L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k\}$

Quantum diffusion:

$$\mathcal{L}_\chi \rho = -i[H,\rho] + i\chi \sum_k \nu_k (L_k \rho e^{-i\phi_k} + \rho L_k^\dagger e^{i\phi_k}) - \frac{\chi^2}{2} \rho \sum_k \nu_k^2$$

Generalized superoperators from $\mathscr{L}_\chi$**:**

$$\mathcal{J} = -i\partial_\chi \mathcal{L}_\chi\big|_{\chi=0} \qquad K = -\partial_\chi^2 \mathcal{L}_\chi\big|_{\chi=0}$$

Fluctuations of the output current (Sec. III)

Two-point function: $F(\tau) = E\big(I(t)I(t+\tau)\big) - J^2 = K\delta(\tau) + \text{tr}\{\mathcal{J} e^{\mathcal{L}\tau} \mathcal{J}\rho\} - J^2$

Power spectrum: $S(\omega) = \int_{-\infty}^{\infty} d\tau \; e^{-i\omega\tau} F(\tau)$

Noise: $D = \frac{d}{dt}\text{Var}\big(N(t)\big) = S(0) = 2\int_0^\infty d\tau \; F(\tau)$

For diffusion change

$\mathcal{J} \to \mathcal{H}$

$K \to K_{\text{diff}}$

Full counting statistics (Sec. IV)

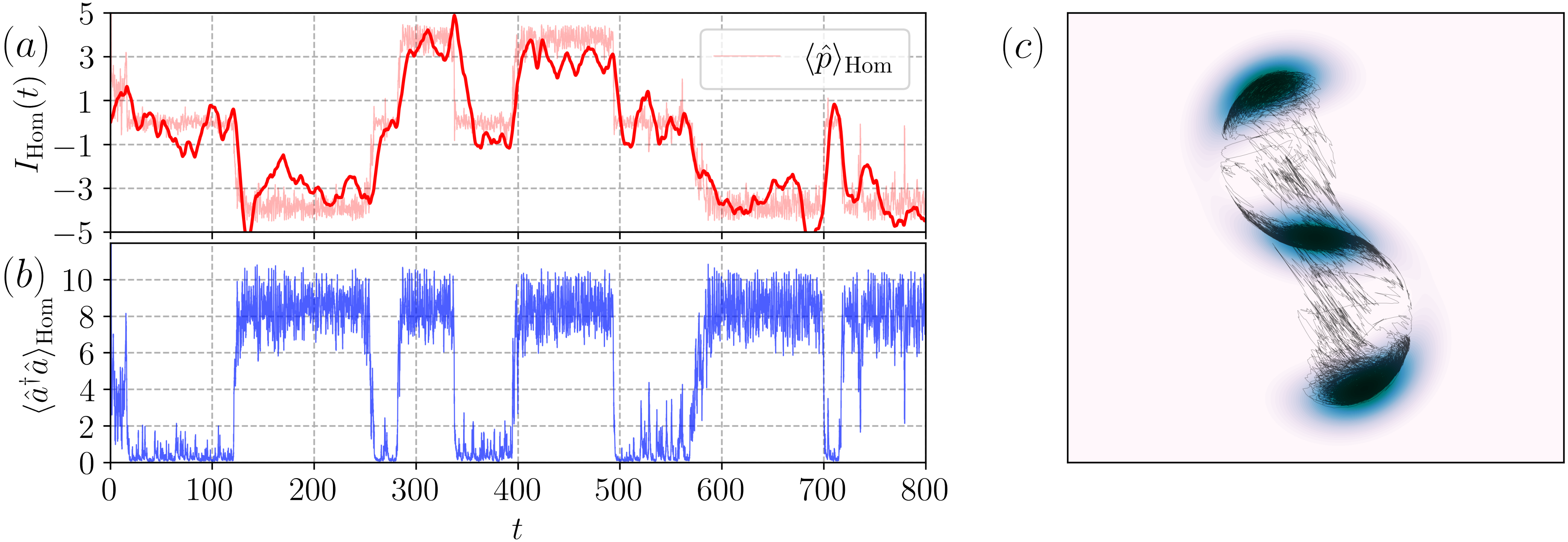
Generalized QME: $\frac{d\rho_\chi}{dt} = \mathcal{L}_\chi \rho_\chi \quad \to \quad \rho_\chi(t) = e^{\mathcal{L}_\chi t}\rho(0)$

Prob. dist. of $N(t)$**:** $P(n,t) = \int \frac{d\chi}{2\pi} e^{-in\chi} \text{tr}\rho_\chi(t)$

Scaled CGF: $C(\chi) = \lim_{t\to\infty} \partial_t \ln \text{tr}\rho_\chi(t) = \lambda_0(\chi) = \text{max eig of } \mathcal{L}_\chi$

Quantum trajectories and stochastic currents

- Parametric Kerr model: $H = -\Delta a^\dagger a + \frac{U}{2} a^\dagger a^\dagger a a + \frac{G}{2}(a^{\dagger 2} + a^2)$
- Quantum dynamics → stochastic current (which is what an experimentalist measures!)



Conclusions

Thank you!

- **Entropy production:** quantifies dissipation/irreversibility.
- In the quantum realm:
  - How to define it.
  - What does it imply?
    - Quantum coherences and correlations.
- **Current fluctuations:**
  - TURs: To curb fluctuations, we must increase dissipation.
  - How to describe them.
  - Connection between the quantum realm and the classical world.

G. T. Landi, M. J. Kewming, M. T. Mitchison and P. Potts,

"**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics**"

Tutorial in PRX Quantum.
arXiv 2303.04270

`https://www.pas.rochester.edu/~gtlandi/`

Current fluctuations in the Parametric Kerr model

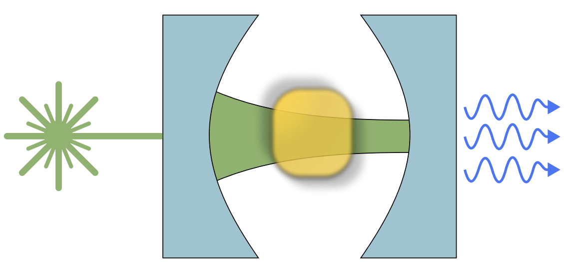
Michael J. Kewming, Mark T. Mitchison, Gabriel T. Landi, "**Diverging current fluctuations in critical Kerr resonators**." Physical Review A, 106, (2022)

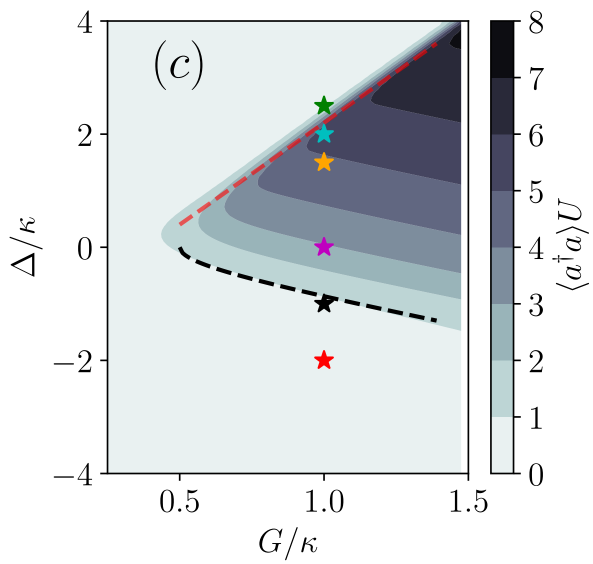
Parametric Kerr model

- Non-linear quantum harmonic oscillator:

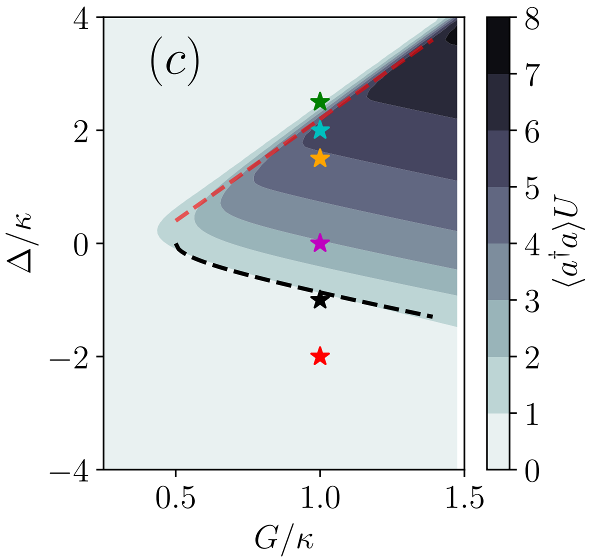
$$\frac{d\rho}{dt} = -i[H(t), \rho] + \kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\}\right]$$

$$H = -\Delta a^\dagger a + \frac{U}{2} a^\dagger a^\dagger a a + \frac{G}{2}(a^{\dagger 2} + a^2)$$

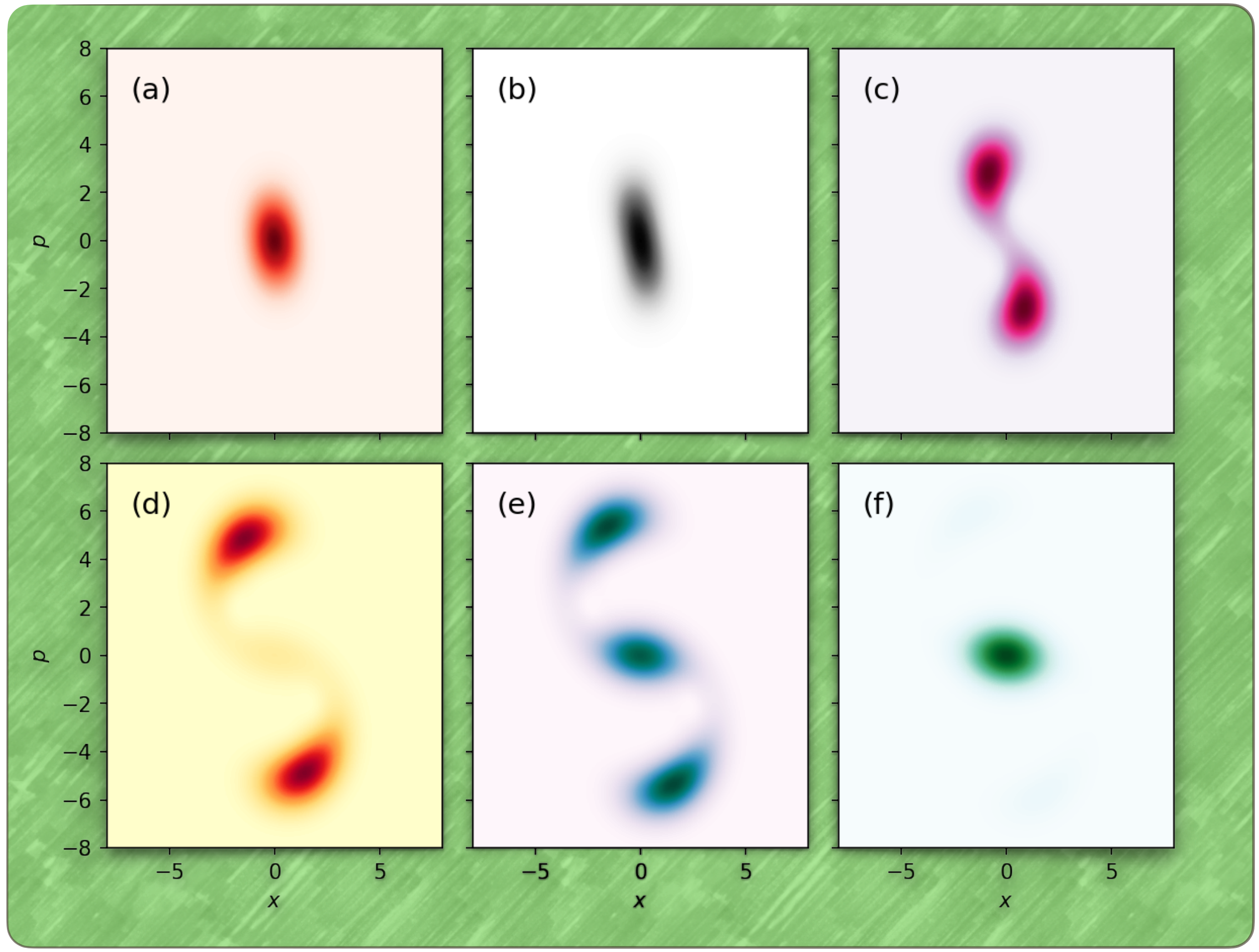




- a = annihilation operator
  photon operator for an optical cavity
- $\Delta = \omega_p - \omega_c$ = detuning
- U = Kerr non-linearity.
  (requires a non-linear crystal inside the cavity)
- G = 2-photon pump
  (input laser produces photons in pairs)
- κ = loss rate
  rate at which photons leak out of the cavity



Wigner function



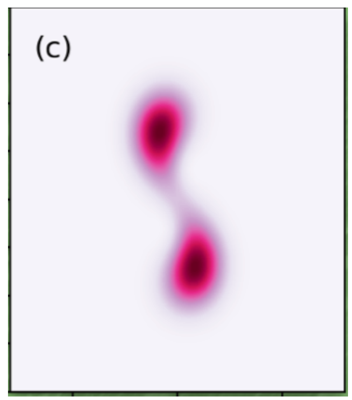
- 2 phase transitions, continuous and discontinuous
- Proper criticality occurs in the limit $U \to 0$ ("thermodynamic limit")

Cat qubits

- Steady-state is a mixture of two Schrödinger cat states

$$S\rangle = \alpha\rangle + -\alpha\rangle$$

$$A\rangle = \alpha\rangle - -\alpha\rangle$$

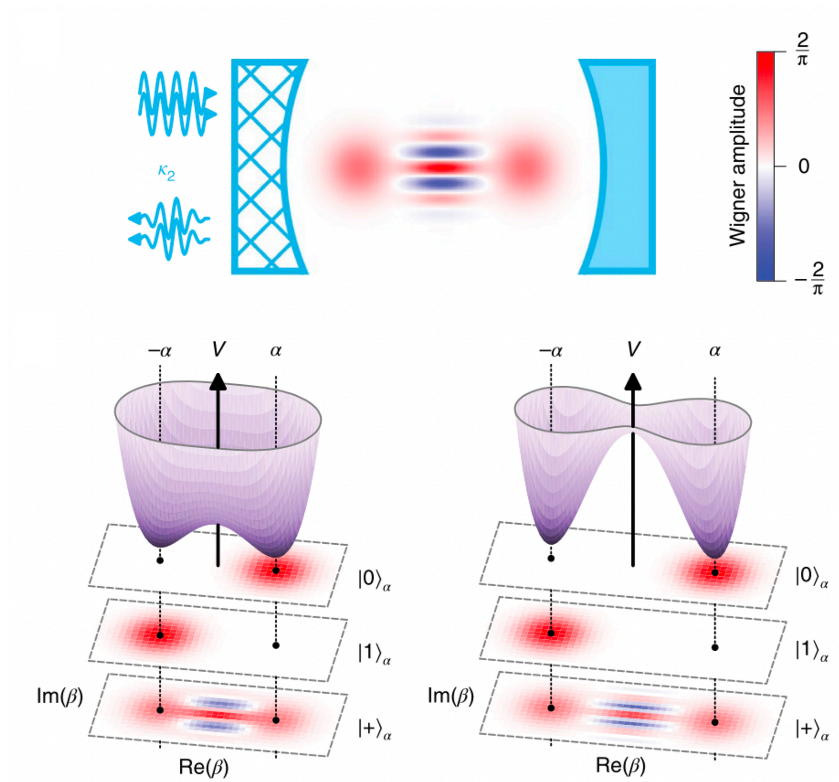


- Use this to define cat qubits:

$$0\rangle = \alpha\rangle$$

$$1\rangle = -\alpha\rangle$$

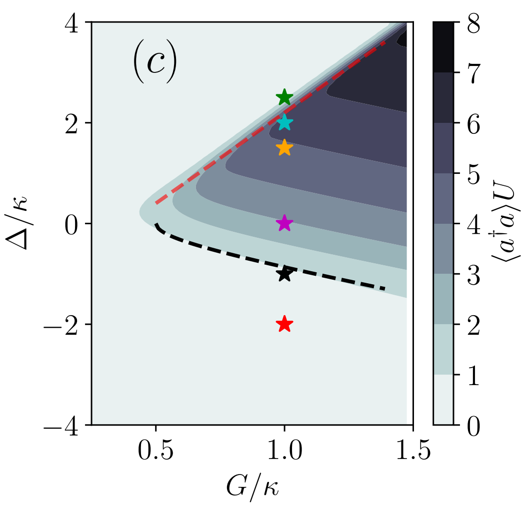
- Cat qubits are more robust against errors.
    - Quantum computing with Kerr cats.

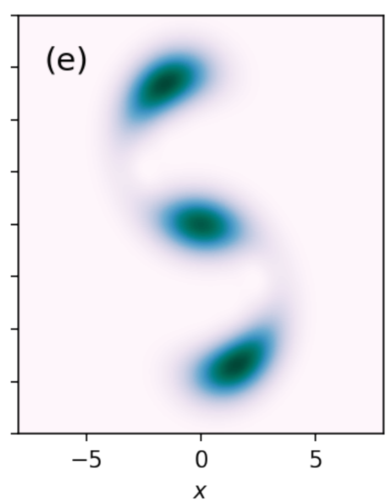


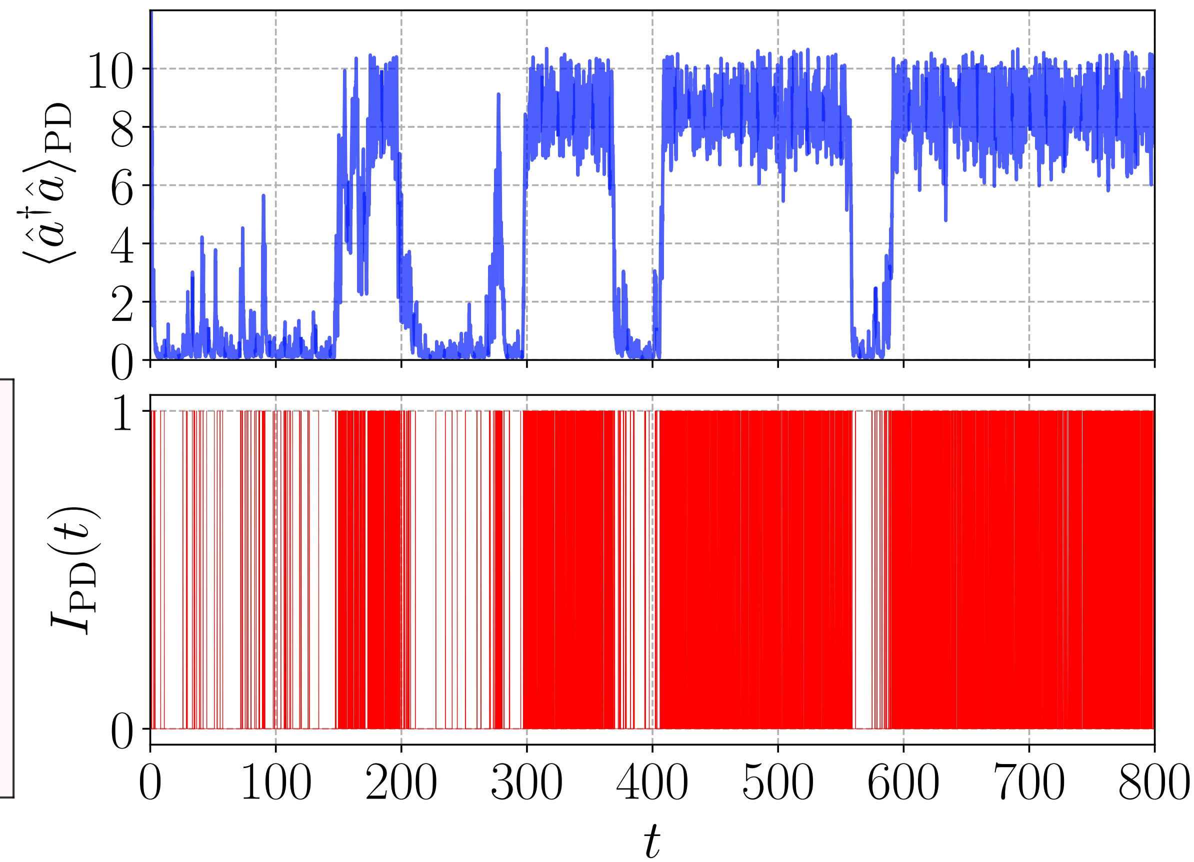
Lescanne, *et. al.*, Nature, **16**, 509-513 (2020)

Photo-detection current

- @ discontinuous transition: on/off (telegraph) behavior of the current.
- Photo-detection cannot resolve upper vs. lower blobs.



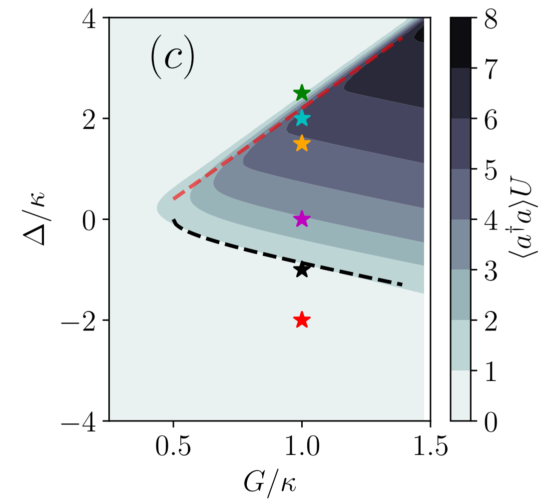


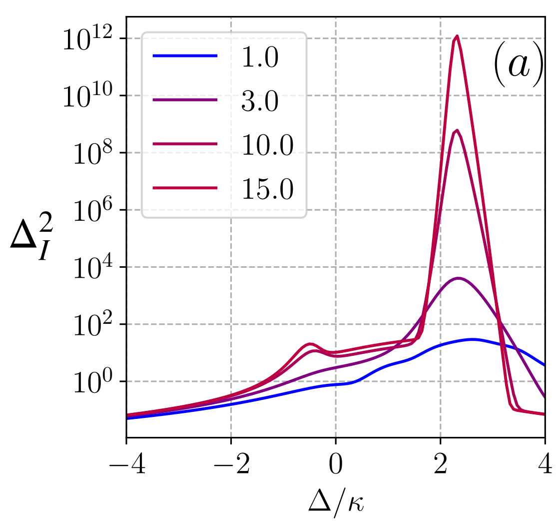


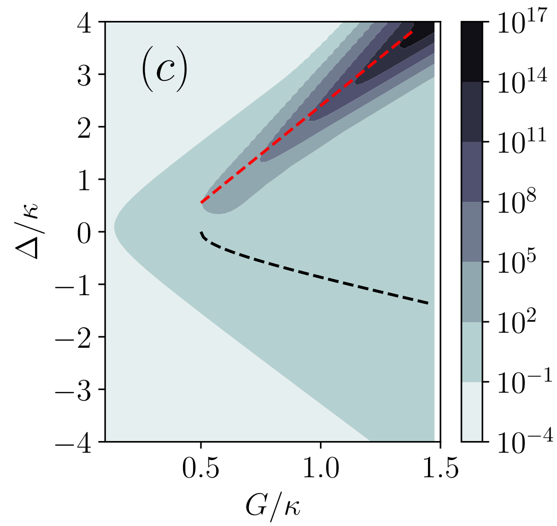
Divergence of the diffusion coefficient

- “Thermodynamic limit:” $U \to 0$
- In the discontinuous transition ($\Delta > 0$)

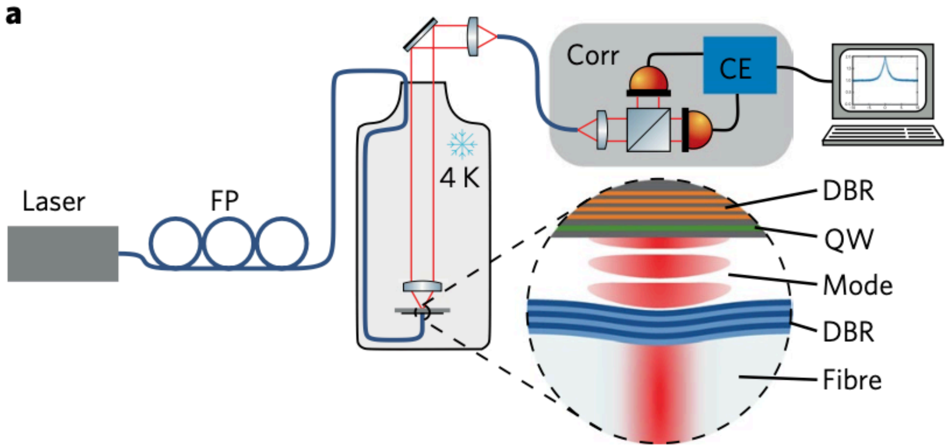
$$\Delta_I^2 \sim e^{1/U}$$

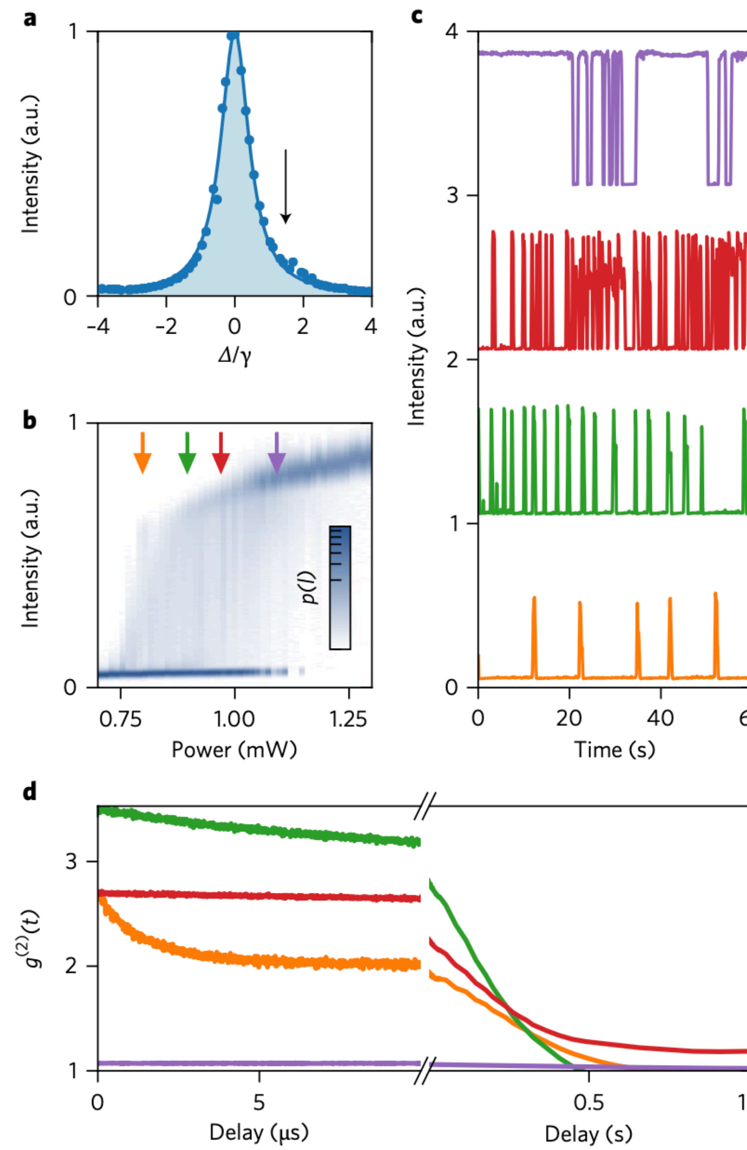






GaAs cavity polaritons.

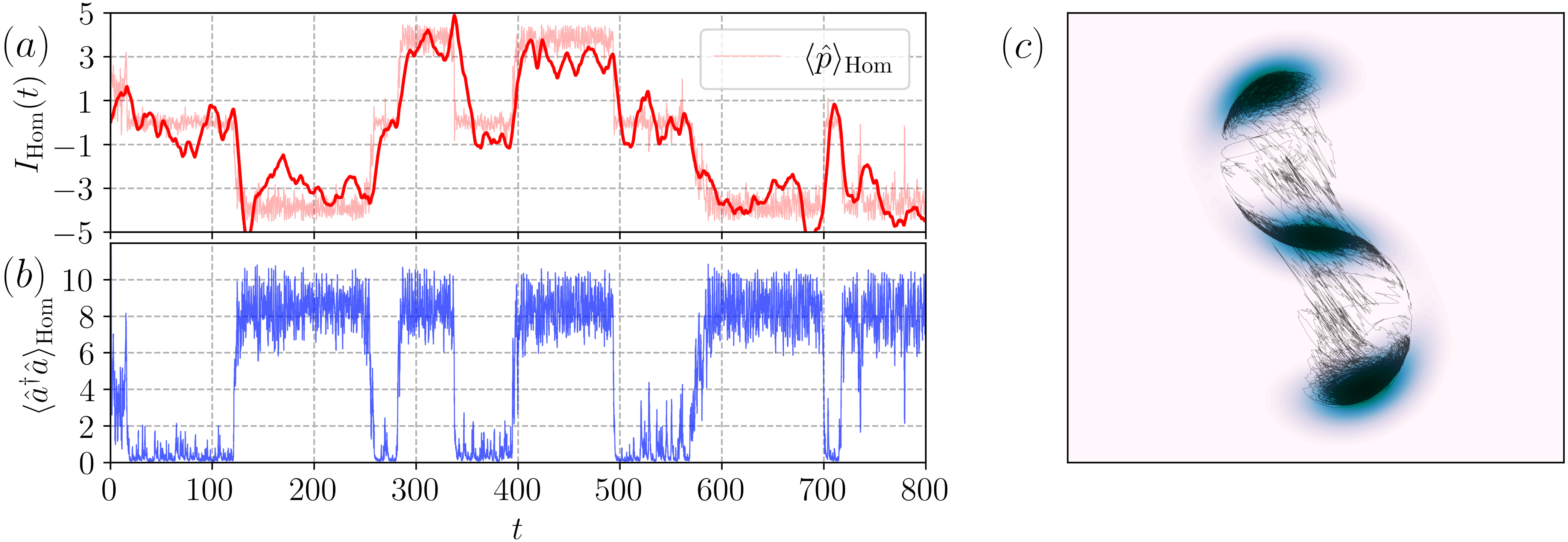




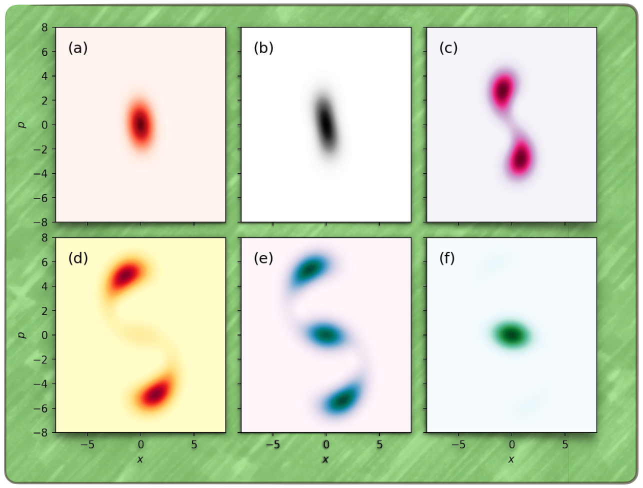
T. Fink, *et. al.*, *Nature Physics,* **14**, 365 (2018)

Homodyne current (in $p = i(a^\dagger - a)$)

- The homodyne current switches between 3 values (+,0,-).
- Captures the tunneling between the 3 blobs.



Divergence of the diffusion coefficient



- Homodyne current noise diverges exponentially in a much broader region.

$$\Delta_I^2 \sim e^{1/U}$$

- Reflects sensitivity to all 3 blobs.