Information-thermodynamics in the quantum regime

Gabriel T. Landi - University of Rochester

10/25/2023 - Colloquium - UMBC

https://www.pas.rochester.edu/~gtlandi

Prof. George Porter
YouTube

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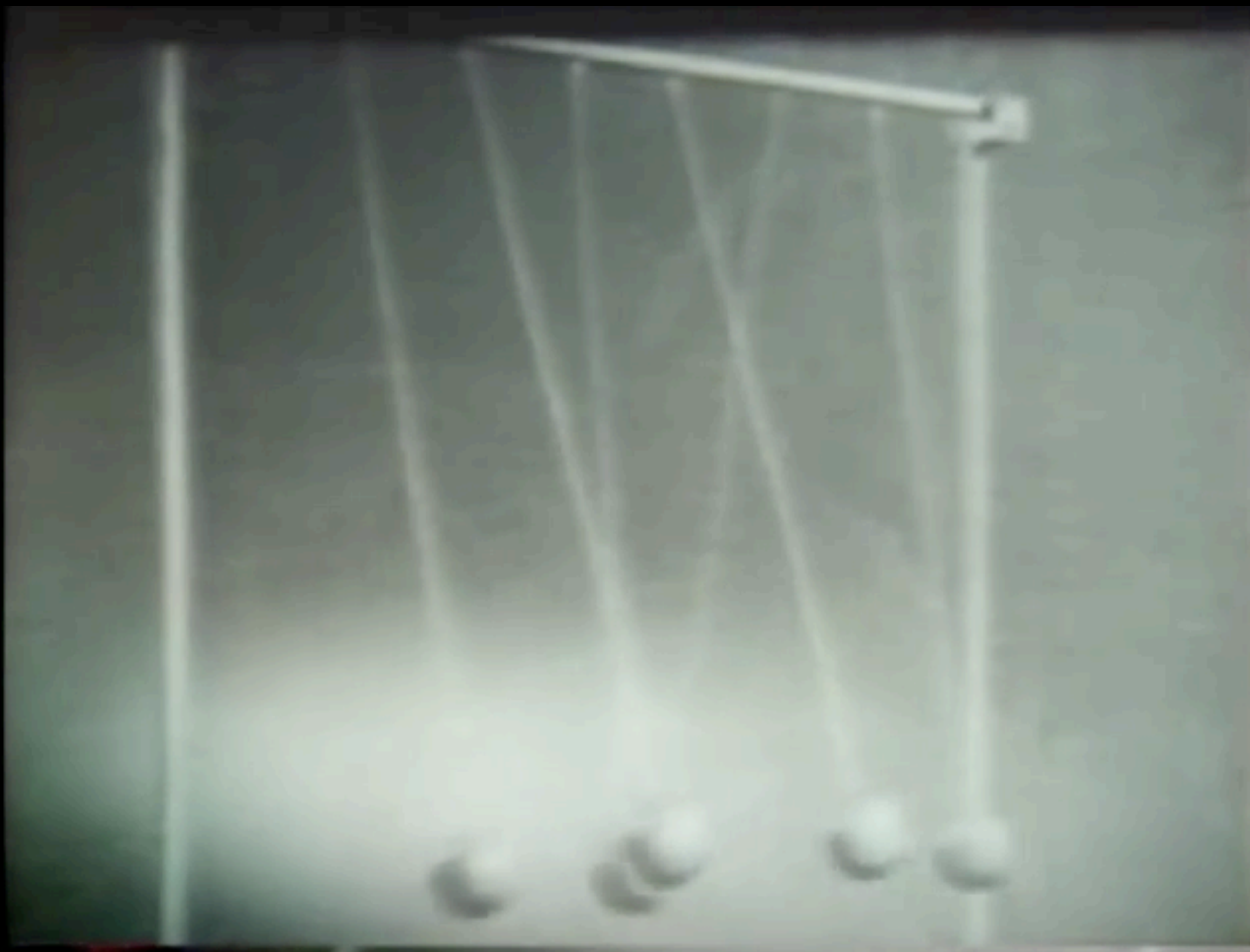

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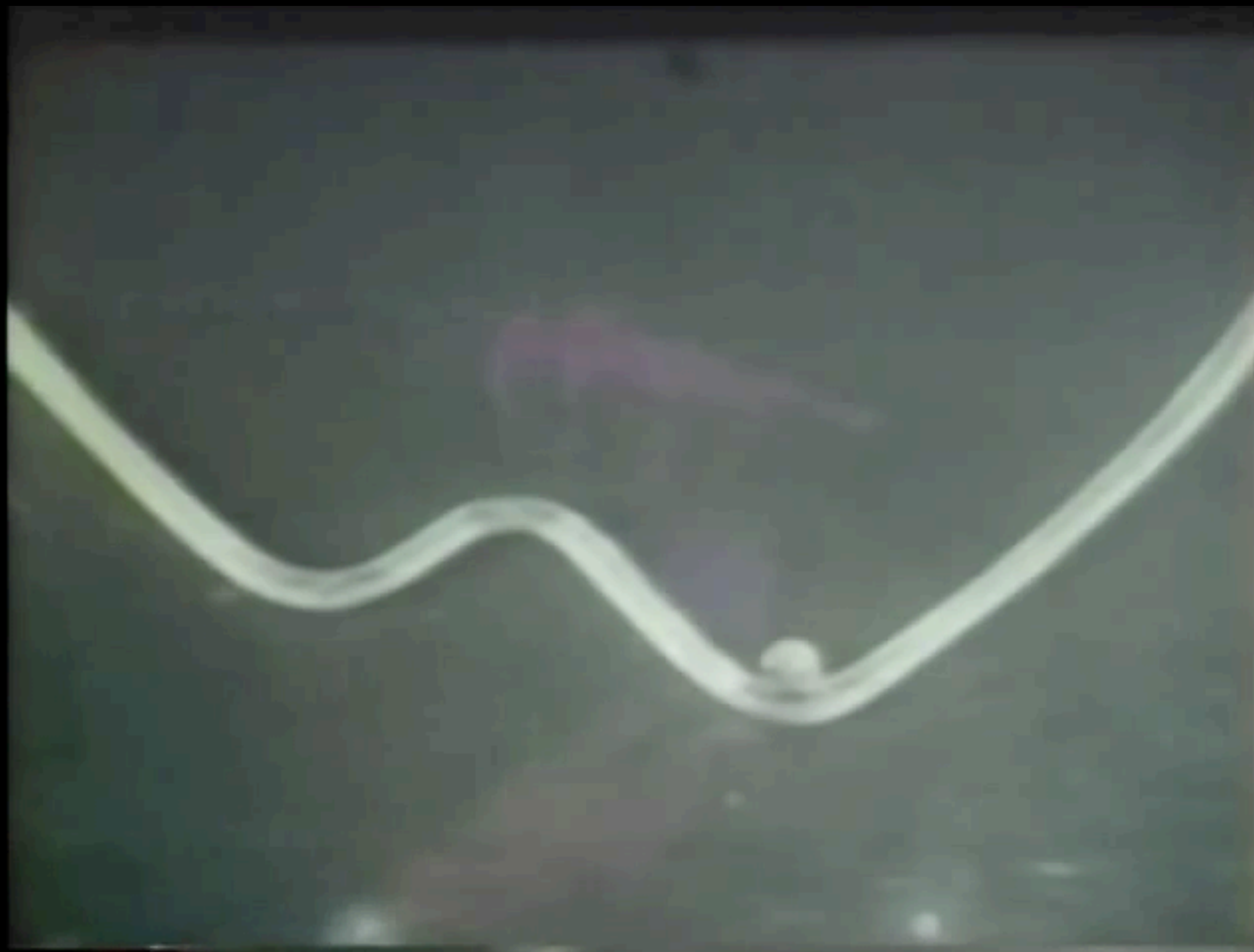
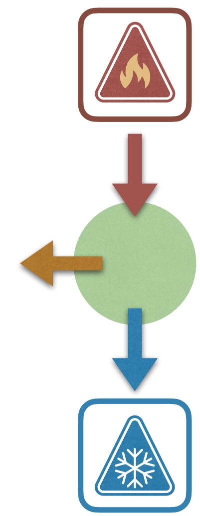


Irreversibility & Entropy production



- Clausius formulated the notion of irreversibility using entropy.
- Consider a thermodynamic process involving heat & work:

$$\Delta U = W + Q_h + Q_c \qquad \text{(1st law = balance equation)}$$

- According to Clausius, entropy does not satisfy a balance equation:

$$\Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma$$

$\sigma \geqslant 0$ is the entropy produced in the process.

- $\sigma \geqslant 0$ is the *mathematical statement* of the 2nd law.
- Entropy production is related to heat flow: lost to the environment and cannot be recovered.
  - Operational definition: what is accessible and what is not.
  - The irreversible videos were those that involved a lot of dissipation.

$$W = -Q_h - Q_c \qquad \text{and} \qquad \sigma = -\frac{Q_h}{T_h} - \frac{Q_c}{T_c} \geqslant 0$$

- **Operation as a heat engine:** efficiency

$$\eta = \frac{W}{Q_h} = \eta_c - \frac{T_c \sigma}{Q_h} \qquad \text{where} \qquad \eta_c = 1 - \frac{T_c}{T_h}$$

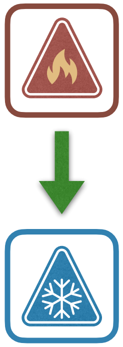
- The efficiency is always *lower* than Carnot's efficiency because entropy is produced (Carnot's statement of the 2nd law)

- **Heat flow** (no work): $Q_h = -Q_c$

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h \geqslant 0$$

Heat always flows from hot to cold

(Clausius' statement)



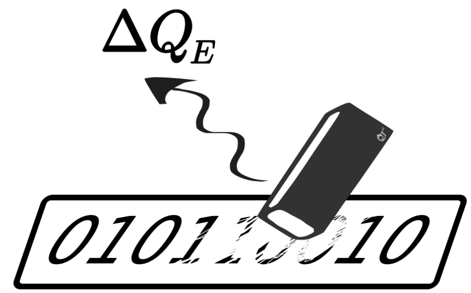
- **Landauer's erasure:** Minimum cost to erase information

$$\Delta Q \geqslant k_B T \ln 2$$

- What about $T \simeq 0$? Very relevant for quantum computation.

- If eraser is a waveguide of length L:

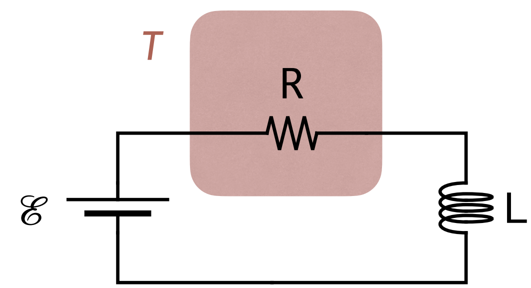
$$\Delta Q \geqslant k_B T \ln 2 + \frac{3\hbar c}{\pi L} \ln^2(2)$$



- **Non-equilibrium steady-states:** not equilibrium.

$$\frac{dS}{dt} = \frac{\dot{Q}}{T} + \dot{\sigma} = 0 \qquad \text{so} \qquad \dot{\sigma} = -\frac{\dot{Q}}{T}$$

- Example: Joule heating.
  Continues as long as there is juice in the battery



$$\dot{\sigma} = \frac{\mathscr{E}^2}{RT}$$

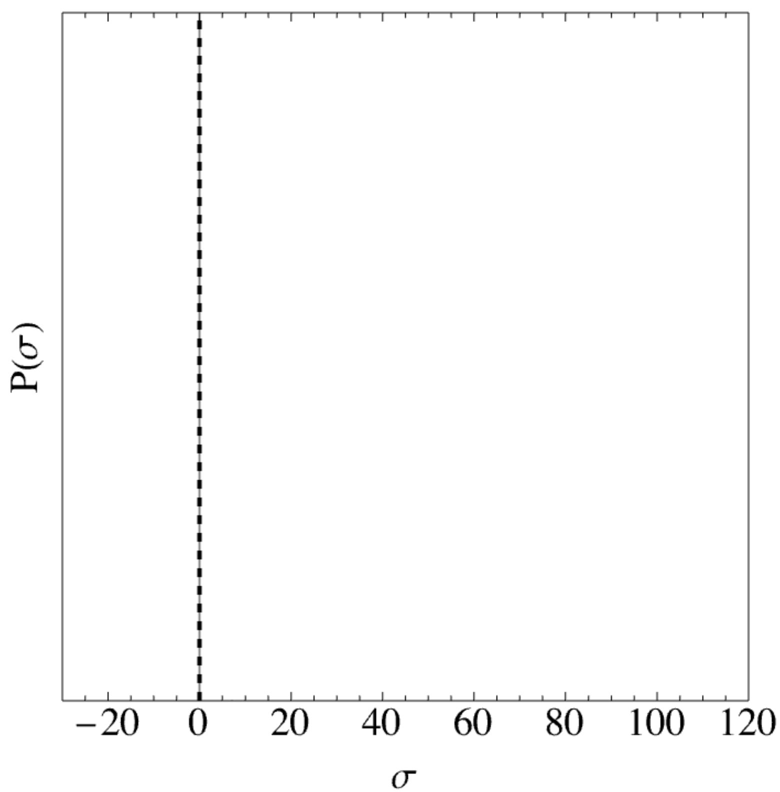
André M. Timpanaro, Jader P. Santos, and Gabriel T. Landi, *Phys. Rev. Lett.* **124**, 240601 (2020)

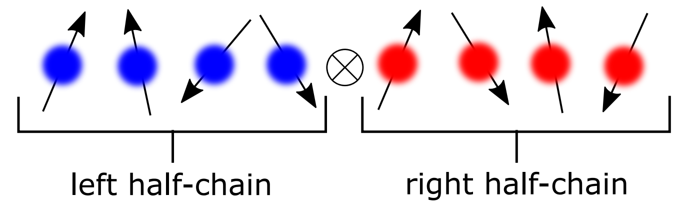
"The principle of the increase of entropy is merely an observation that in any irreversible process the entropy tends to increase."

Feynman lecture on physics.

Irreversibility & the arrow of time

- Macro-world: heat flows from hot → cold.
- Micro-world: heat ***usually*** flows from hot → cold.





G. T. Landi and Dragi Karevski
Phys. Rev. E **93**, 032122 (2015)

Heat Exchange Fluctuation Theorem

$$P(-\sigma) = e^{-\sigma} P(\sigma)$$

Implies 2nd law: $\langle \sigma \rangle \geqslant 0$

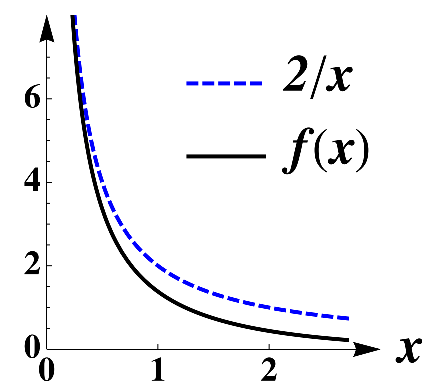
C. Jarzynski and D. Wójcik,
Phys. Rev. Lett. 92, 230602 (2004)

Thermodynamic Uncertainty Relations

- The exchange fluctuation theorem $P(-\sigma) = e^{-\sigma}P(\sigma)$ is a very special symmetry of the distribution.

- We have shown that this implies a bound on how the heat might fluctuate:

$$\frac{\mathrm{var}(Q)}{\langle Q \rangle} \geqslant f(\sigma), \qquad f(x) = \mathrm{csch}(g(x/2)), \qquad g(x) = \text{inverse of } x\tanh(x)$$



- This is a type of Thermodynamic Uncertainty Relation (TUR).

- TURs are a bit counter-intuitive (?):

  - To reduce fluctuations one must *increase* dissipation.

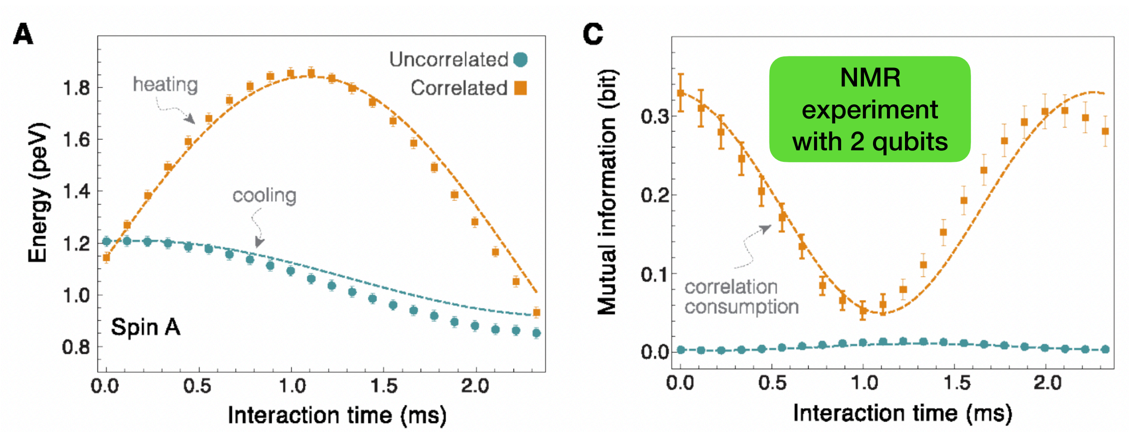
A.M. Timpanaro, G. Guarnieri, J. Goold, GTL, "**Thermodynamic Uncertainty Relations from Exchange Fluctuation Theorems.**" Physical Review Letters, 123, (2019)

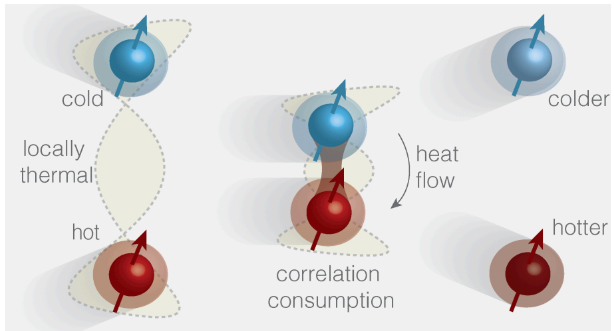
A. C. Barato and U. Seifert, Physical Review Letters 114, 158101 (2015).

Quantum Thermodynamics

Heat flows from hot to cold

- To break that, we must pay a **price**:
  - The fridge or the AC consume electricity.
- In the quantum domain, **information is also a resource.**





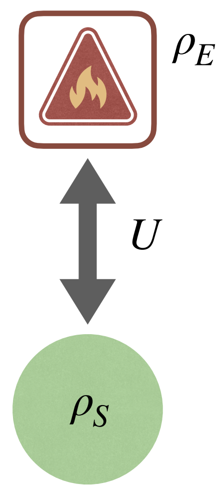
Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, GTL, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, "**Reversing the direction of heat flow using quantum correlations**", *Nature Communications*, **10**, 2456 (2019)

Partovi, M. H., Phys. Rev. E, 77, 021110 (2008)
Jennings, D. & Rudolph, T., Phys. Rev. E, 81, 061130 (2010)

Entropy production for quantum systems

- Information-theoretic formulation: $\sigma = I(S:E) + D(\rho'_E \| \rho_E)$
  - Operational interpretation: Characterizes irreversibility in terms of what you do not have access to.

$$\rho'_{SE} = U(\rho_S \otimes \rho_E)U^\dagger$$



Describes an enormous variety of processes! (maybe a complicated U)

Mutual Information:

$$I'(S:E) = S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE})$$

Quantifies all correlations (classical + quantum)

Relative entropy

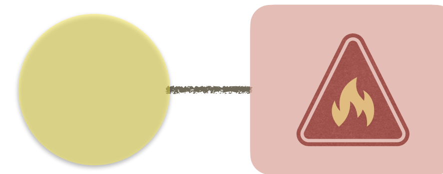
$$D(\rho'_E \| \rho_E) = \mathrm{tr}\left(\rho'_E \ln \rho'_E - \rho'_E \ln \rho_E\right)$$

"Distance" between density matrices

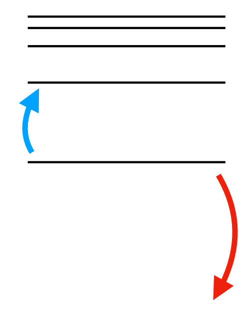
- Here $S(\rho) = -\,\mathrm{tr}(\rho \ln \rho)$ is the von Neumann entropy.

Massimiliano Esposito, Katja Lindenberg and Christian Van den Broeck, *New Journal of Physics*, 12, 013013 (2010).

Relaxation towards equilibrium



- Imagine an atomic system relaxing towards equilibrium.
    - Population of energy eigenstates fluctuate until they reach thermal equilibrium.
- In addition: any superpositions are destroyed (**decoherence**).



- Mathematically a state $\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ is the same as the density matrix $\rho = \begin{pmatrix} a^{2} & ab^* \\ a^*b & b^{2} \end{pmatrix}$.
    - Relaxation to equilibrium then means

$$\rho = \begin{pmatrix} a^{2} & ab^* \\ a^*b & b^{2} \end{pmatrix} \to \begin{pmatrix} p_0^{\text{th}} & 0 \\ 0 & p_1^{\text{th}} \end{pmatrix}$$

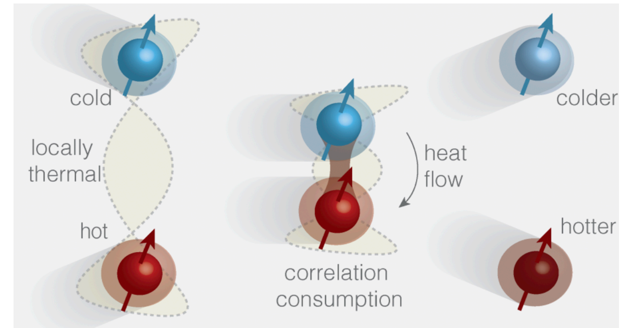
The entropy production can be split as

$$\sigma = \sigma_{\text{pop}} + \sigma_{\text{coh}}$$

Additional entropy production due to coherence: Dissipation of information, without dissipation of energy.

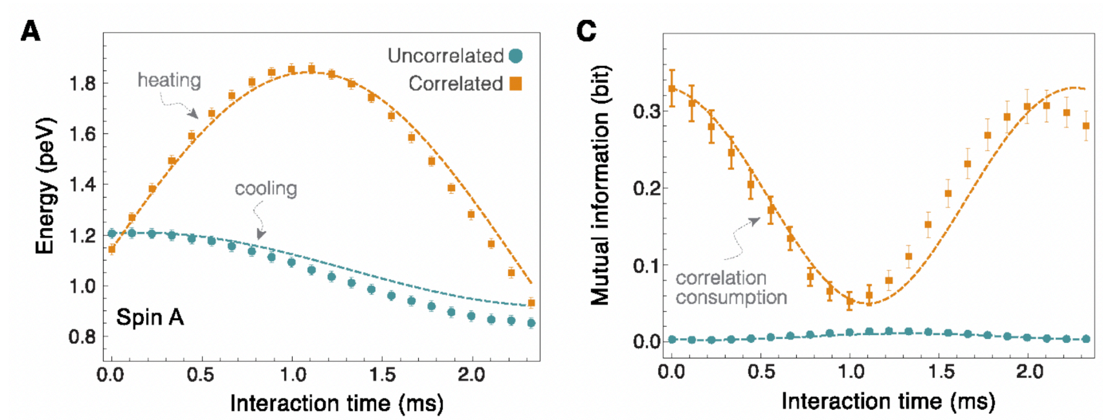
J. P. Santos, L. Céleri, GTL, M. Paternostro, npj Quantum Information **5**, 23 (2019)

Consuming quantum correlations



- In the presence of initial correlations the second law has to be modified to

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h \geqslant \Delta I(h:c)$$



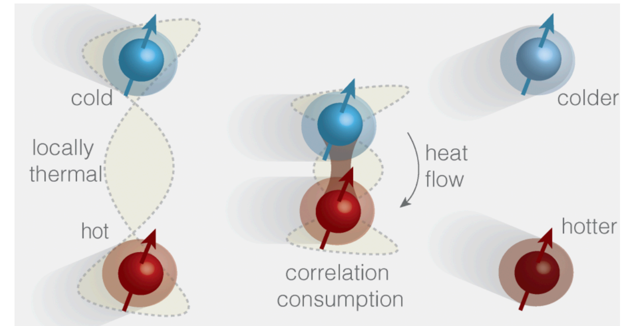
- Heat can flow from cold to hot, provided we **consume** quantum correlations: $\Delta I < 0$.

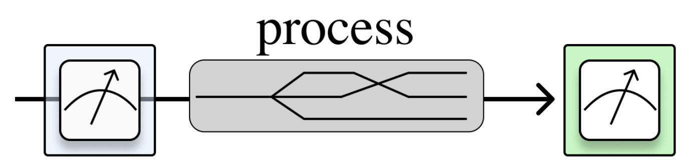
Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, GTL, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, "**Reversing the direction of heat flow using quantum correlations**", *Nature Communications*, **10**, 2456 (2019)

Partovi, M. H., Phys. Rev. E, 77, 021110 (2008)
Jennings, D. & Rudolph, T., Phys. Rev. E, 81, 061130 (2010)

How do we actually measure the heat flow?

- Heat is a property of the **process/transformation**, not a function of state.
    - To measure heat we must measure energy **before** and **after** a process (two-point measurement - TPM).
- **But in quantum mechanics measurements have a back action.**





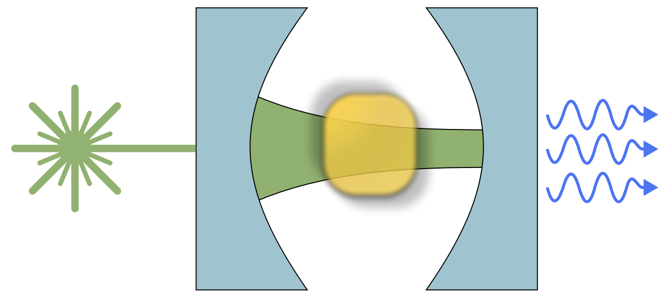
- The first measurement will destroy the quantum correlations.
    - “Q-Thermo processes are extrinsic”
- Can (and should) we define a Q-Thermo theory that is intrinsic? Open question.

K. Micadei, GTL, E. Lutz, “**Quantum fluctuation theorems beyond two-point measurements**”, Phys. Rev. Lett. 124, 090602 (2020)

Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, GTL, Roberto M. Serra, Eric Lutz, “**Experimental validation of fully quantum fluctuation theorems**”, Phys. Rev. Lett., 127, 180603 (2021).

Kenji Maeda, Tharon Holdsworth, Sebastian Deffner, Akira Sone "**Detailed Fluctuation Theorem from the One-Time Measurement Scheme**," arXiv 2306.09578

Quantum phase space



- Many quantum experiments are done using optical cavities with semi-transparent mirrors.
- Photons leaking out $\simeq$ zero temperature bath.
  - Spontaneous emission: excitations can leave, but not return.
- 2nd law is buggy @ $T = 0$: $\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h$.
  - Does not include vacuum fluctuations (*present in every measurement*).
- We reformulated the entropy production problem in terms of quantum phase space & the *Wigner function.*

$$\sigma = \left(\frac{1}{T_c^{\text{eff}}} - \frac{1}{T_h^{\text{eff}}}\right) Q_h$$

$$T^{\text{eff}} = \omega(\bar{n} + 1/2), \qquad \bar{n} = \frac{1}{e^{\beta\omega} - 1}$$

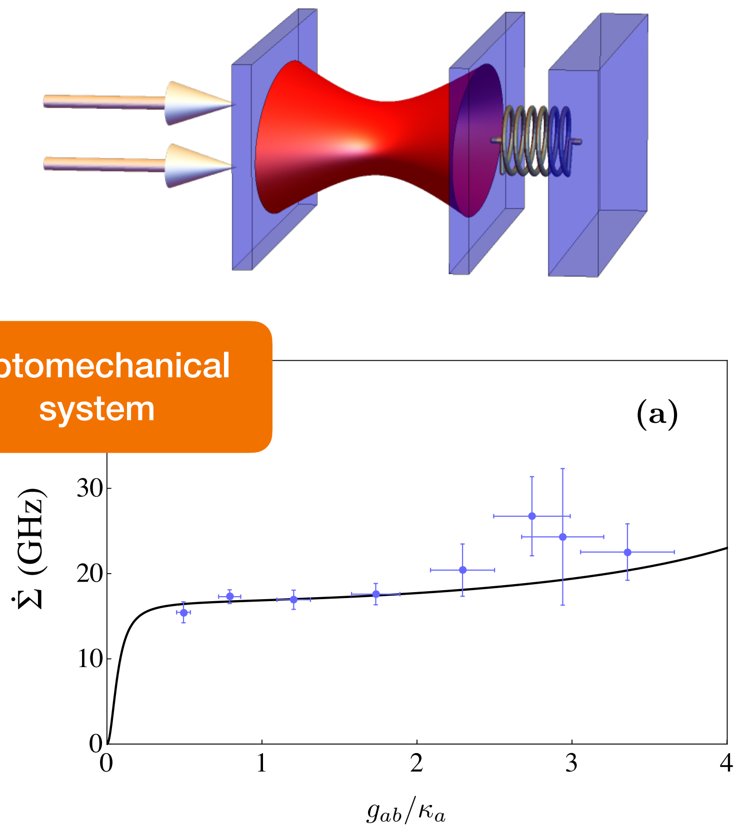
High temperatures: $\omega(\bar{n} + 1/2) \simeq T$.

Zero temperature: $\omega(\bar{n} + 1/2) = \omega/2$.

Jader P. Santos, GTL and Mauro Paternostro, Phys. Rev. Lett, **118**, 220601 (2017),

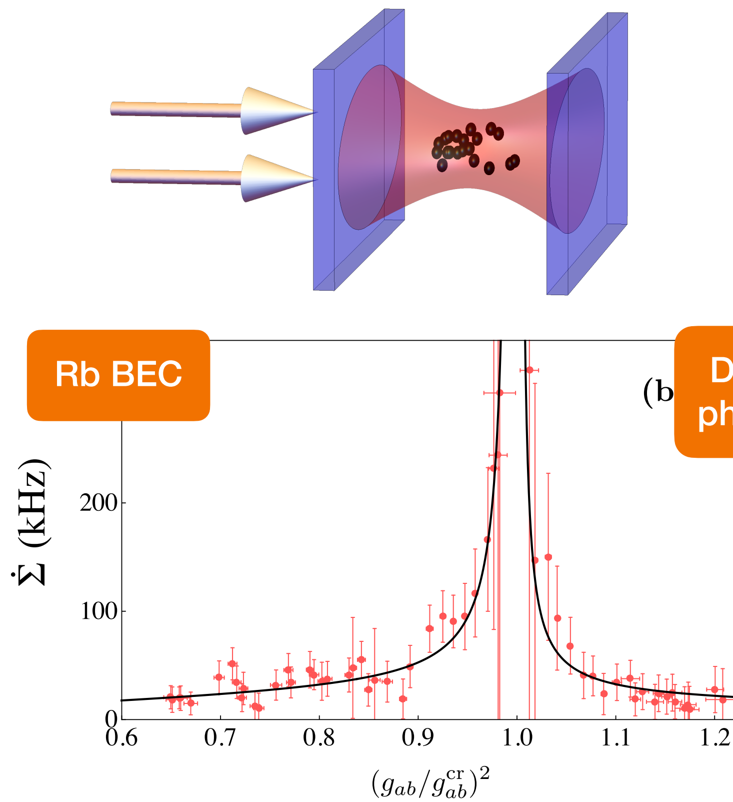
Experiments

Optomechanical system



Rb BEC

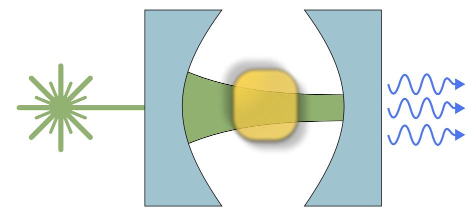
Dicke quantum phase transition.



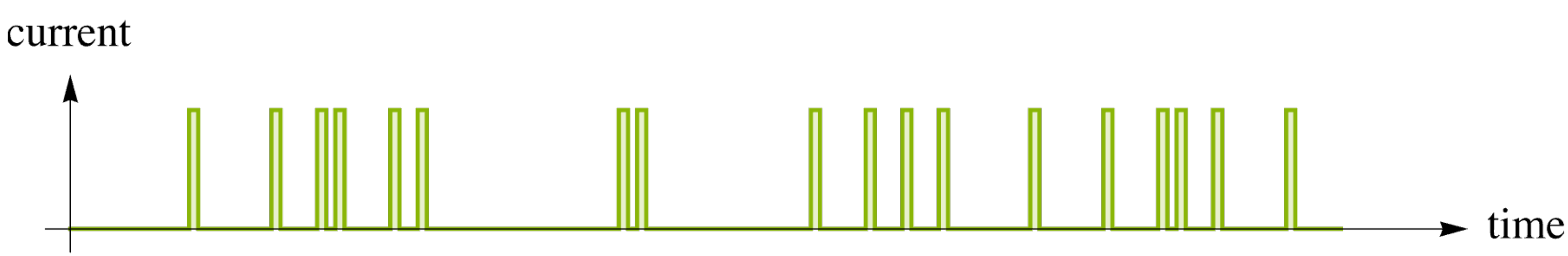
M. Brunelli, L. Fusco, R. Landig, W. Wieczorek, J. Hoelscher-Obermaier, GTL, F Semião, A. Ferraro, N. Kiesel, T. Donner, G. De Chiara, and M. Paternostro. *Phys. Rev. Lett.*, **121**, 160604 (2018)

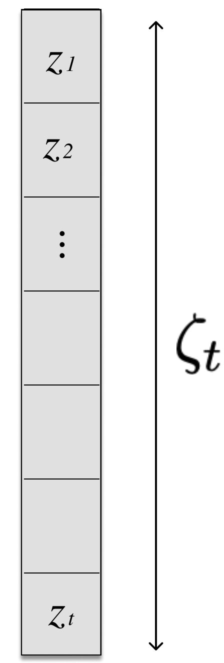
Continuously monitored quantum systems

Continuously monitored quantum systems



- Continuous monitoring of photons that leak out of the cavity.
    - Individual clicks in the detector.
- Fundamental questions: what is entropy production *given* a detection record.
    - Operational: define thermodynamics in terms of what we can actually measure.
    - Includes *information* directly in the formulation.





Holevo reduction to entropy production

- **Unconditional:** If we do not know the individual clicks: ρ_t
- **Conditional on the detection record:** $\rho_{t\ \zeta_t}$
- **Holevo information:** accumulated information we learned from the detection.

$$I(S_t : \zeta_t) = \sum_{\zeta_t} P(\zeta_t) D\big(\rho_{t\ \zeta_t} \quad \rho_t\big)$$

- With each new detection

$$\Delta I_t = G_t - L_t = \text{gain} - \text{loss}$$

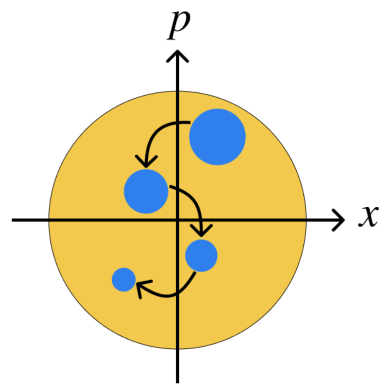
- Conditional entropy production

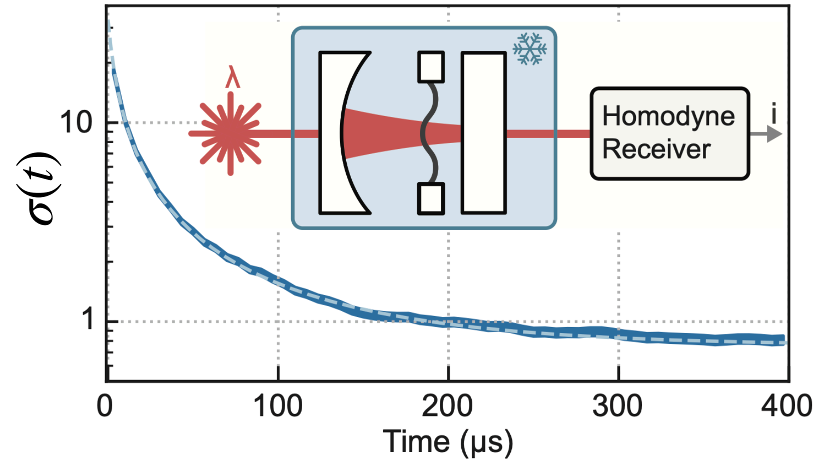
$$\Delta \Sigma^c = \Delta \Sigma^u - \Delta I$$

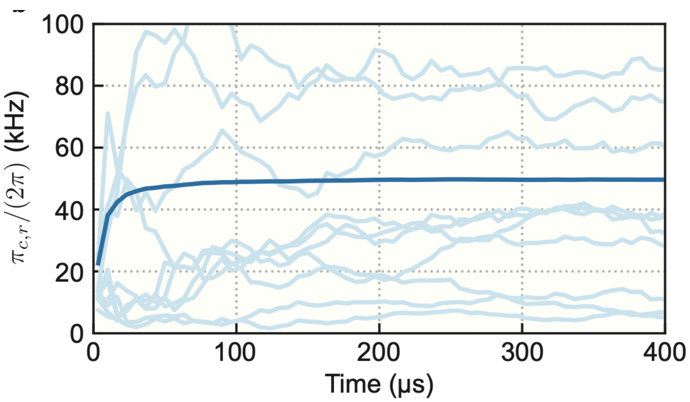
Alessio Belenchia, Luca Mancino, GTL and Mauro Paternostro, "**Entropy production in continuously measured quantum systems**", npj Quantum Information, **6**, 97 (2020).

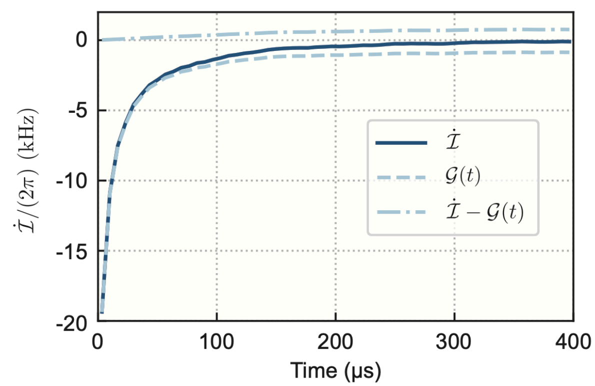
GTL, Mauro Paternostro and Alessio Belenchia, "**Informational steady-states and conditional entropy production in continuously monitored systems**", PRX Quantum **3**, 010303, (2020).

🇩🇰 Optomechanical setup









Informational steady-state:

Conditional dynamics relaxes to a colder state, which can only be maintained by continuing to monitor the system.

Massimiliano Rossi, Luca Mancino, GTL, Mauro Paternostro, Albert Schliesser, Alessio Belenchia, "**Experimental assessment of entropy production in a continuously measured mechanical resonator**", Phys. Rev. Lett. **125**, 080601 (2020)

Current fluctuations in the Parametric Kerr model

Michael J. Kewming, Mark T. Mitchison, GTL, "**Diverging current fluctuations in critical Kerr resonators**." Physical Review A, 106, (2022)

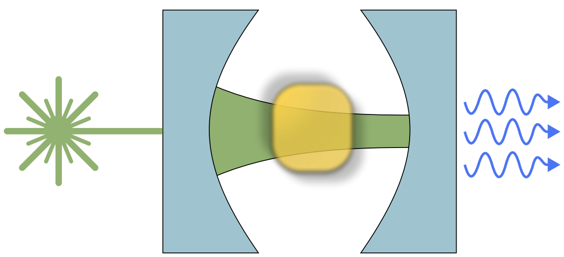
GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics**," 2023. arxiv 303.04270

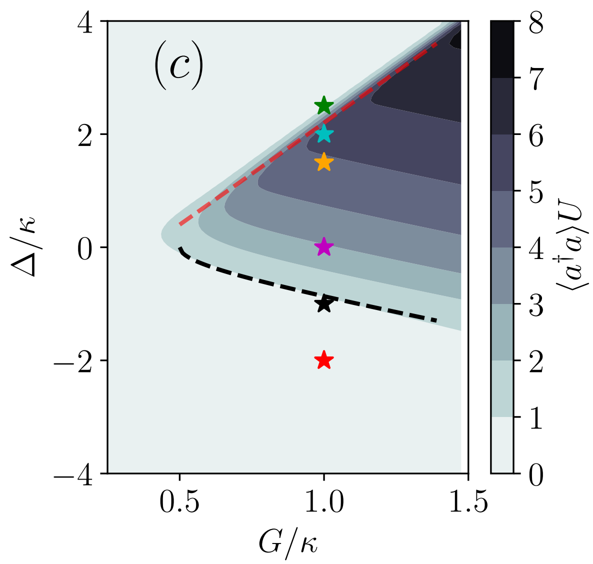
Parametric Kerr model

- Non-linear quantum harmonic oscillator:

$$\frac{d\rho}{dt} = -i[H(t),\rho] + \kappa\left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a,\rho\}\right]$$

$$H = -\Delta a^\dagger a + \frac{U}{2}a^\dagger a^\dagger a a + \frac{G}{2}(a^{\dagger 2} + a^2)$$





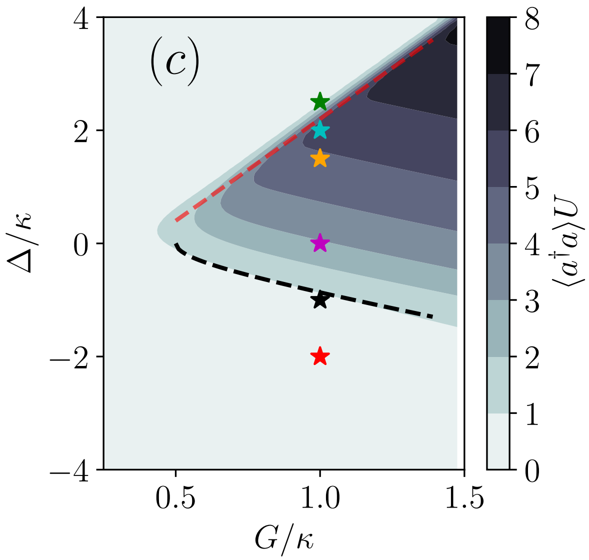
✱ a = annihilation operator
photon operator for an optical cavity

✱ $\Delta = \omega_p - \omega_c$ = detuning

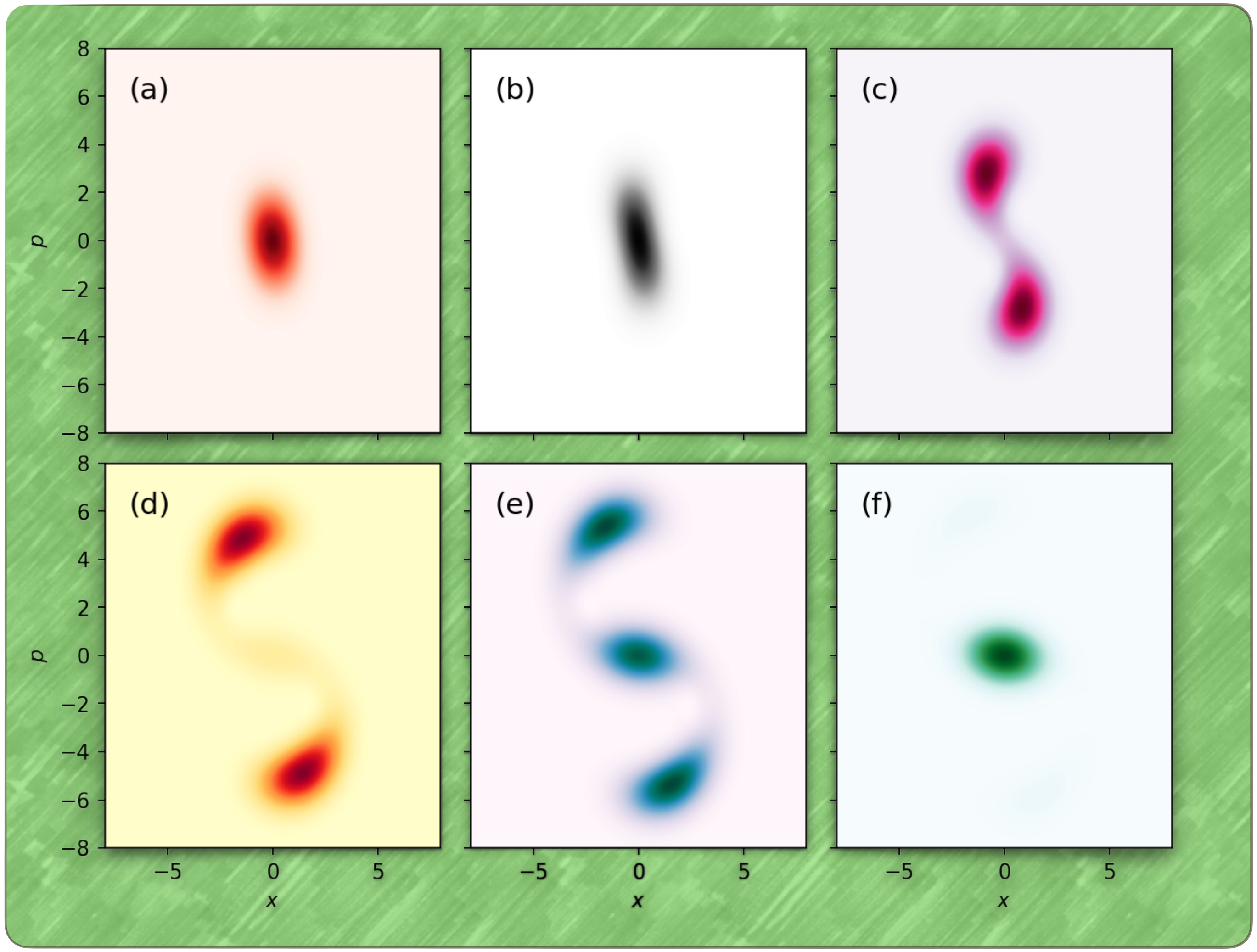
✱ U = Kerr non-linearity.
(requires a non-linear crystal inside the cavity)

✱ G = 2-photon pump
(input laser produces photons in pairs)

✱ κ = loss rate
rate at which photons leak out of the cavity



Wigner function



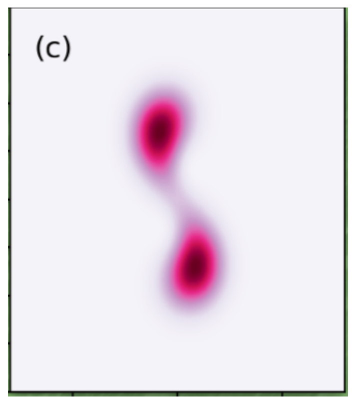
- 2 phase transitions, continuous and discontinuous
- Proper criticality occurs in the limit $U \to 0$ ("thermodynamic limit")

Cat qubits

- Steady-state is a mixture of two Schrödinger cat states

$$S\rangle = \alpha\rangle + -\alpha\rangle$$

$$A\rangle = \alpha\rangle - -\alpha\rangle$$

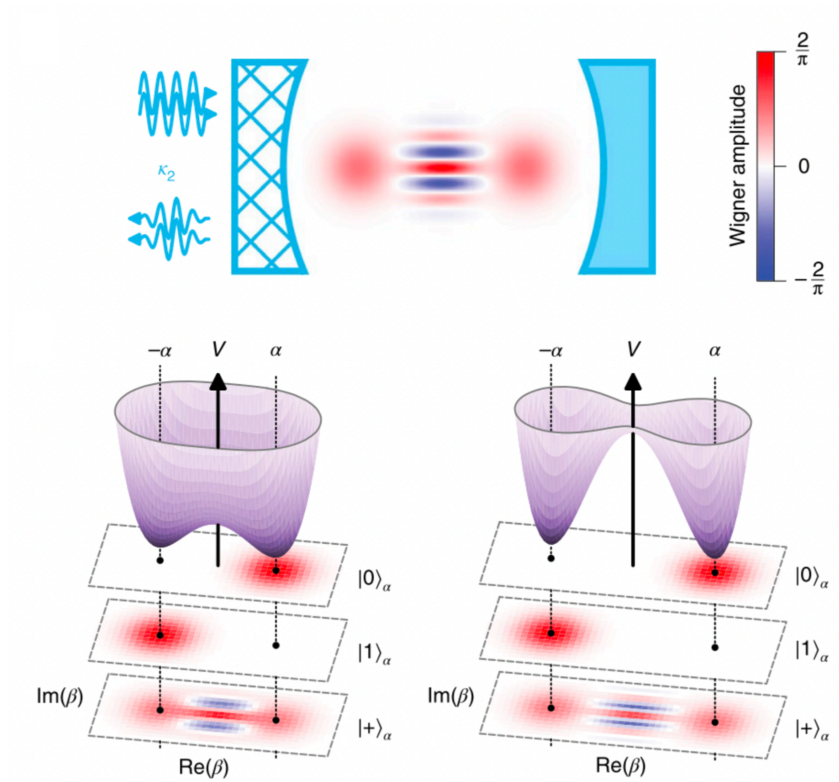


- Use this to define cat qubits:

$$0\rangle = \alpha\rangle$$

$$1\rangle = -\alpha\rangle$$

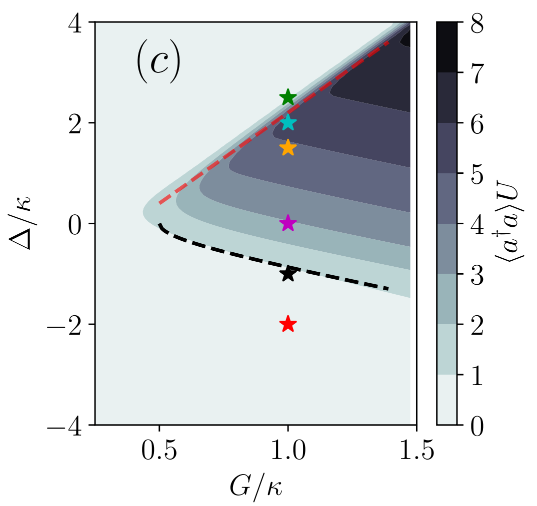
- Cat qubits are more robust against errors.
    - Quantum computing with Kerr cats.

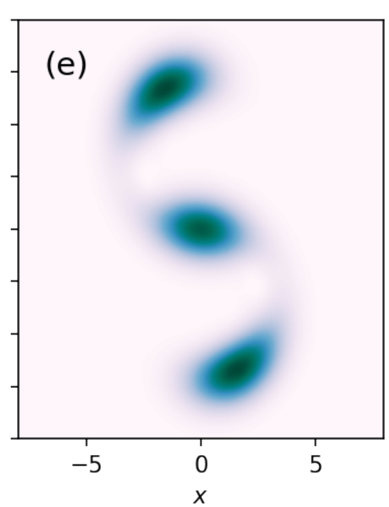


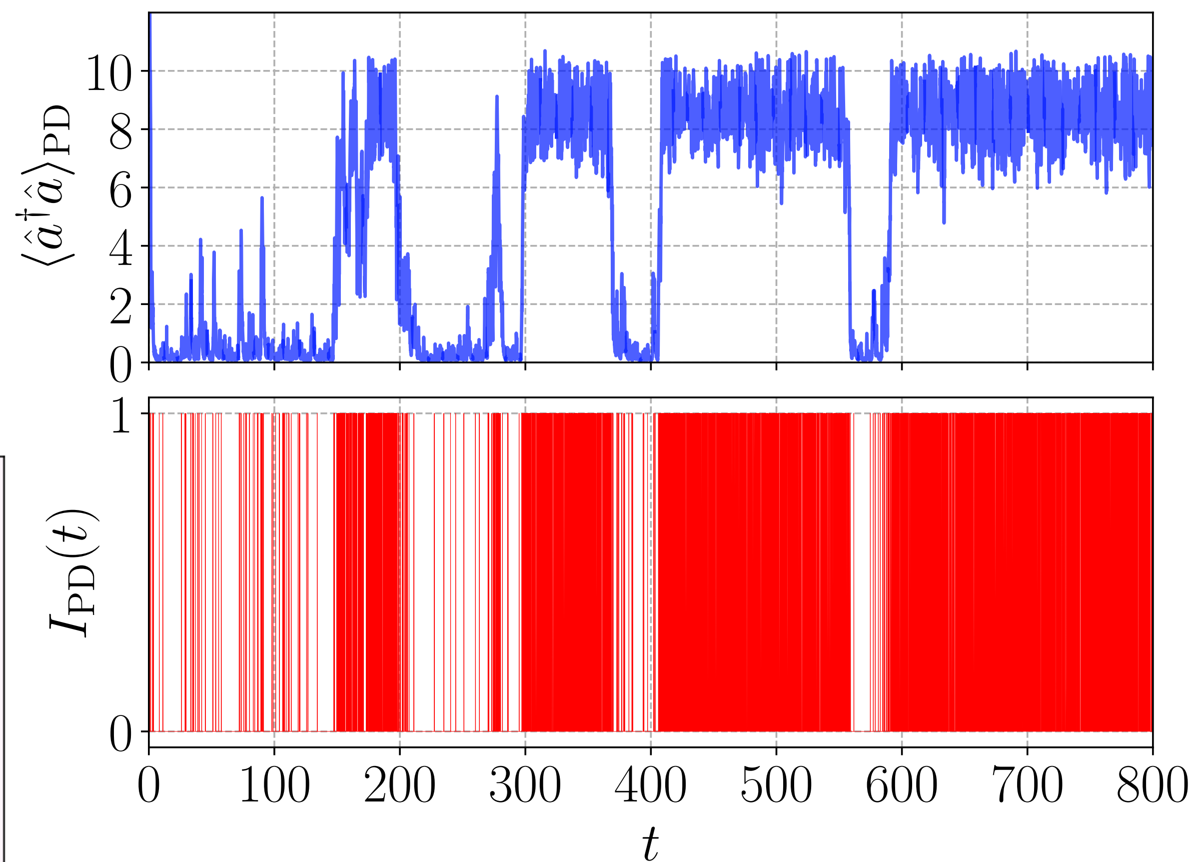
Lescanne, *et. al.*, Nature, **16**, 509-513 (2020)

Photo-detection current

- @ discontinuous transition: on/off (telegraph) behavior of the current.
- Photo-detection cannot resolve upper vs. lower blobs.

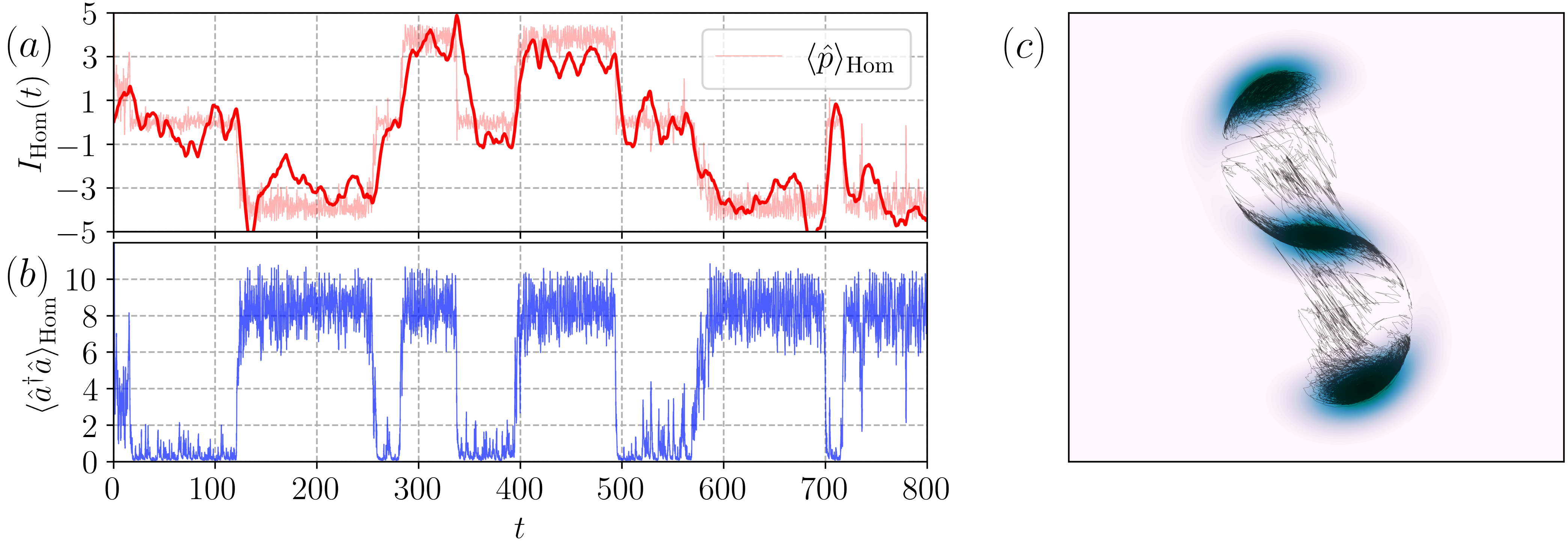






Homodyne current

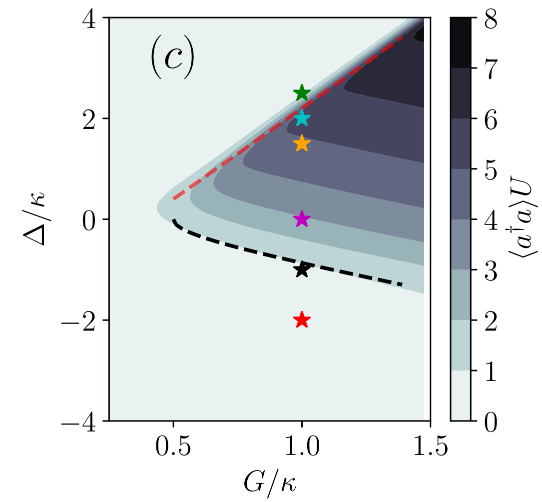
- Observable is now $p = i(a^\dagger - a)$.
- The homodyne current switches between 3 values (+,0,-).
- Captures the tunneling between the 3 blobs.



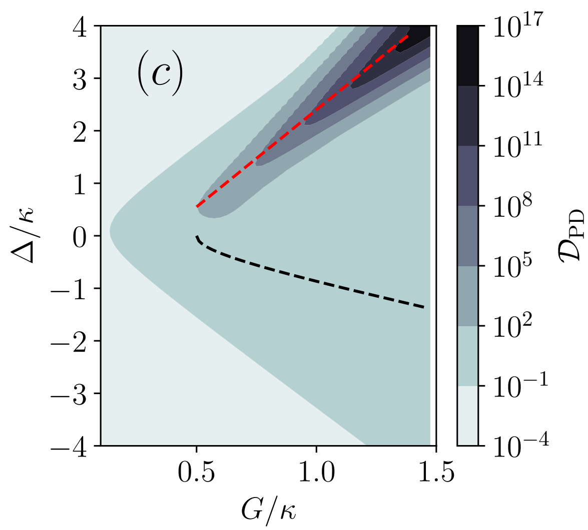
Exponential divergence of the noise

- "Thermodynamic limit:" $U \to 0$
- In the discontinuous transition ($\Delta > 0$)

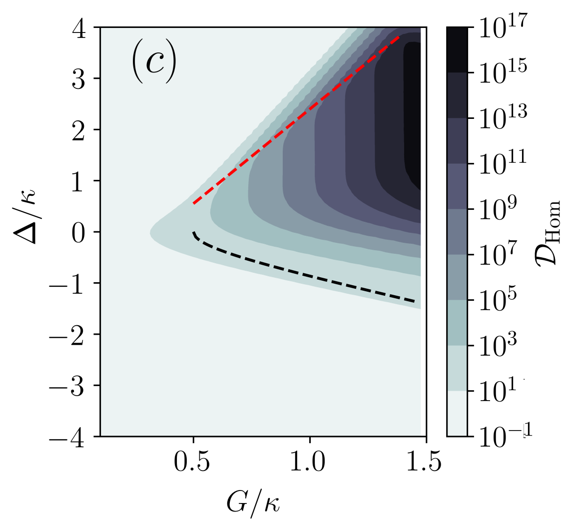
$$D \sim e^{1/U}$$

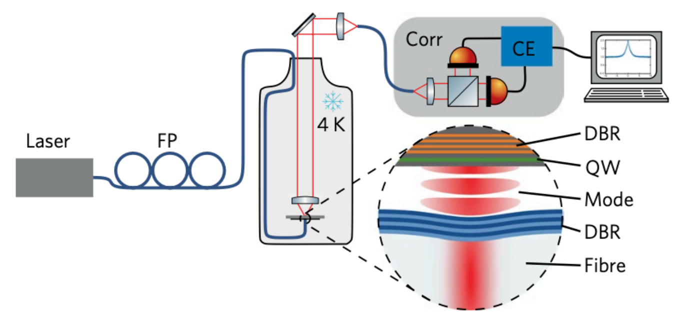


Direct photo-detection



Homodyne





T. Fink, *et. al.*, *Nature Physics,* **14**, 365 (2018)

Conclusions

Thank you!

- **Entropy production** quantifies the irreversibility of a process.
- In the quantum realm it characterizes:
  - Decoherence & loss of information.
  - Quantum correlations.
  - Zero-temperature fluctuations.
  - Measurement back action.

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