



Information-thermodynamics in the quantum regime

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12/11/2023 - Seminar - CS Department - UofR

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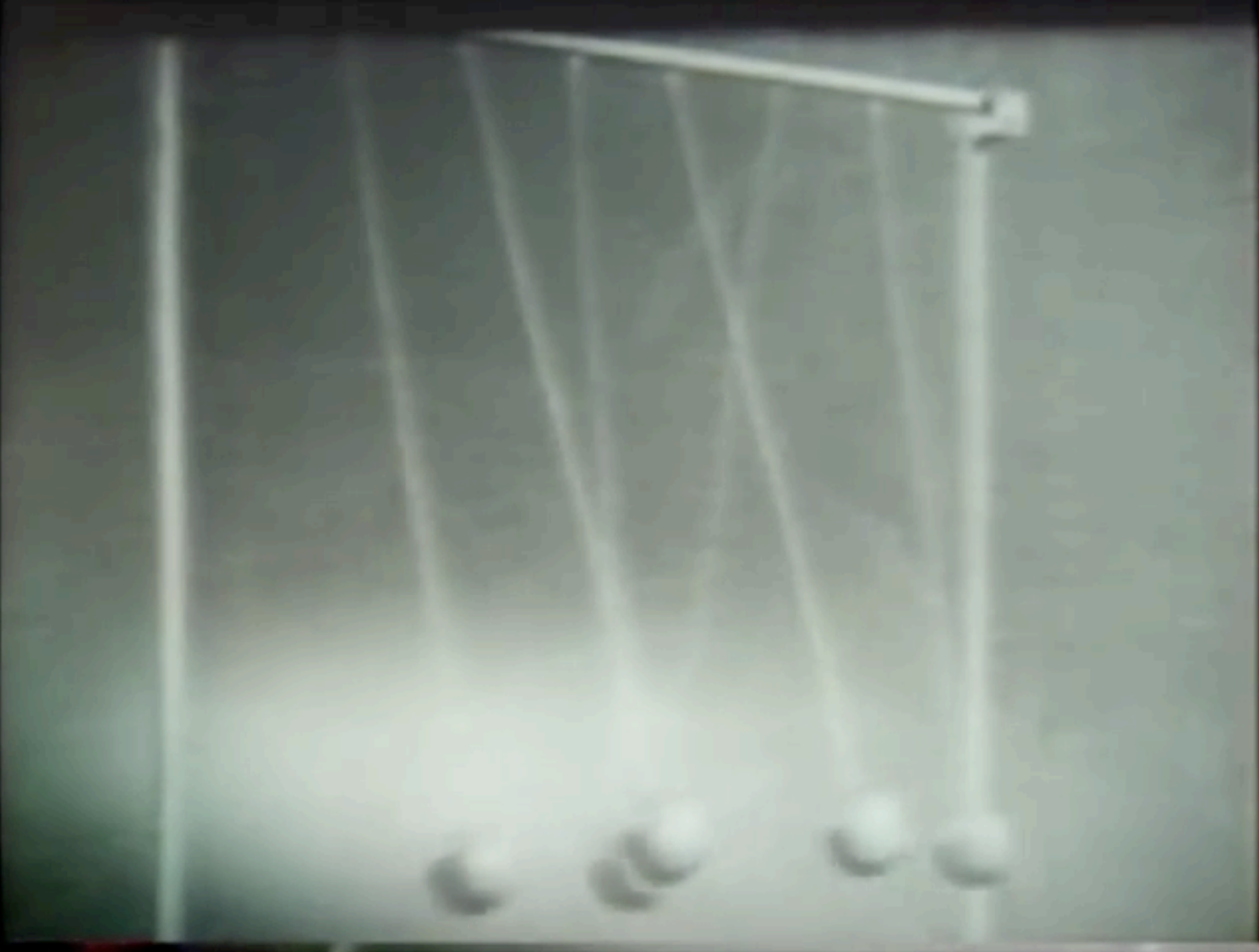
Prof. George Porter
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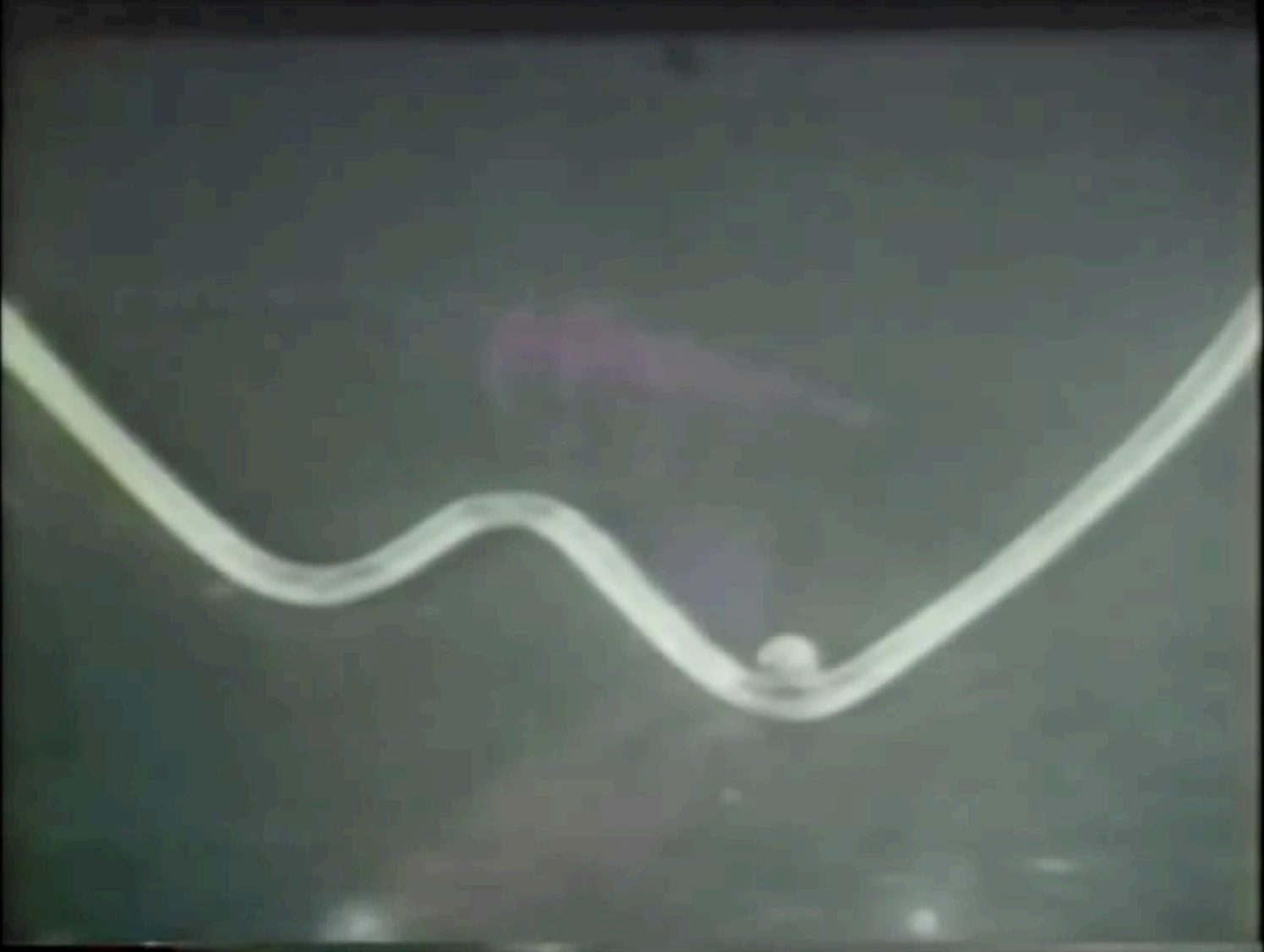
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Irreversibility & Entropy production

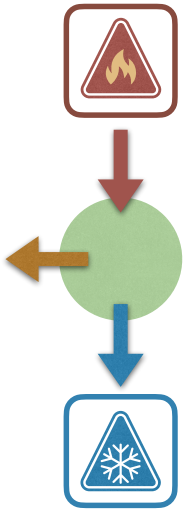
- Clausius formulated the notion of irreversibility using entropy.
- Consider a thermodynamic process involving heat & work:

$$\Delta U = W + Q_h + Q_c \quad (\text{1st law = balance equation})$$

- According to Clausius, entropy does not satisfy a balance equation:

$$\Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma \quad \sigma \geq 0 \text{ is the entropy produced in the process.}$$

- $\sigma \geq 0$ is the *mathematical statement* of the 2nd law.
- Entropy production is related to heat flow: lost to the environment and cannot be recovered.
 - Operational definition: what is accessible and what is not.
 - The irreversible videos were those that involved a lot of dissipation.



$$W = -Q_h - Q_c \quad \text{and} \quad \sigma = -\frac{Q_h}{T_h} - \frac{Q_c}{T_c} \geq 0$$

- **Operation as a heat engine:** efficiency

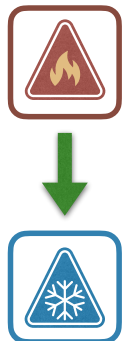
$$\eta = \frac{|W|}{|Q_h|} = \eta_c - \frac{T_c \sigma}{|Q_h|} \quad \text{where} \quad \eta_c = 1 - \frac{T_c}{T_h}$$

- The efficiency is always *lower* than Carnot's efficiency because entropy is produced (Carnot's statement of the 2nd law)

- **Heat flow** (no work): $Q_h = -Q_c$

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h} \right) Q_h \geq 0$$

Heat always flows from hot to cold
(Clausius' statement)



- **Landauer's erasure:** Minimum cost to erase information

$$\Delta Q \geq k_B T \ln 2$$

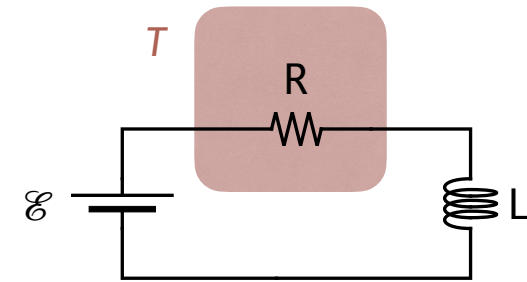
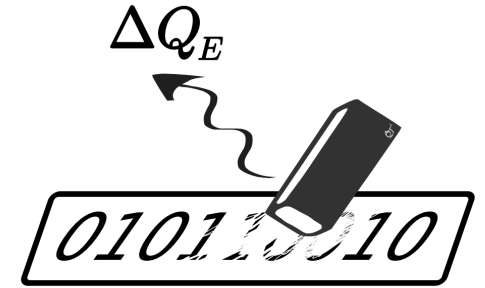
- What about $T \simeq 0$? Very relevant for quantum computation.
- If eraser is a waveguide of length L :

$$\Delta Q \geq k_B T \ln 2 + \frac{3\hbar c}{\pi L} \ln^2(2)$$

- **Non-equilibrium steady-states:** not equilibrium.

$$\frac{dS}{dt} = \frac{\dot{Q}}{T} + \dot{\sigma} = 0 \quad \text{so} \quad \dot{\sigma} = -\frac{\dot{Q}}{T}$$

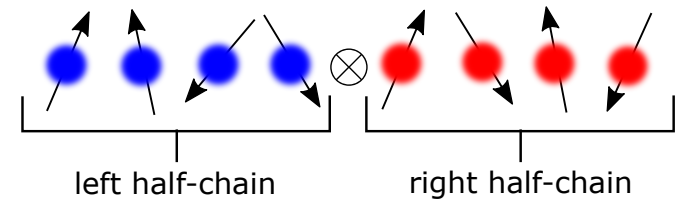
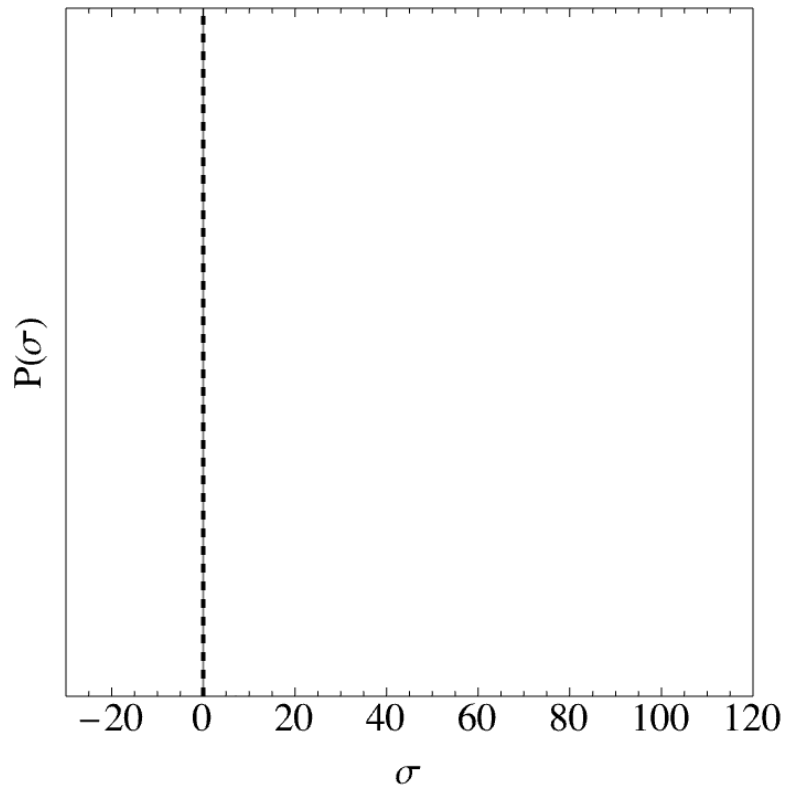
- Example: Joule heating.
Continues as long as there is juice in the battery



$$\dot{\sigma} = \frac{\mathcal{E}^2}{RT}$$

Irreversibility & the arrow of time

- Macro-world: heat flows from hot \rightarrow cold.
- Micro-world: heat *usually* flows from hot \rightarrow cold.



G. T. Landi and Dragi Karevski
Phys. Rev. E **93**, 032122 (2015)

Heat Exchange Fluctuation Theorem

$$P(-\sigma) = e^{-\sigma} P(\sigma)$$

Implies 2nd law: $\langle \sigma \rangle \geq 0$

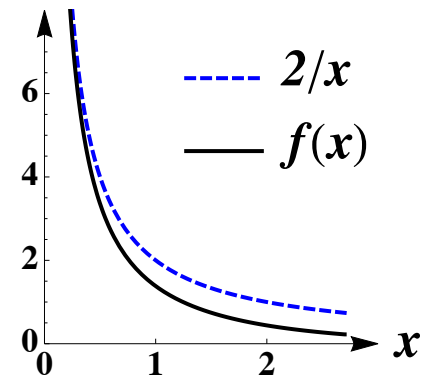
C. Jarzynski and D. Wójcik,
Phys. Rev. Lett. 92, 230602 (2004)

Thermodynamic Uncertainty Relations

- The exchange fluctuation theorem $P(-\sigma) = e^{-\sigma}P(\sigma)$ is a very special symmetry of the distribution.
- We have shown that this implies a bound on how the heat might fluctuate:

$$\frac{\text{var}(Q)}{\langle Q \rangle} \geq f(\sigma), \quad f(x) = \text{csch}(g(x/2)), \quad g(x) = \text{inverse of } x \tanh(x)$$

- This is a type of Thermodynamic Uncertainty Relation (TUR).
- TURs are a bit counter-intuitive (?):
 - To reduce fluctuations one must *increase* dissipation.



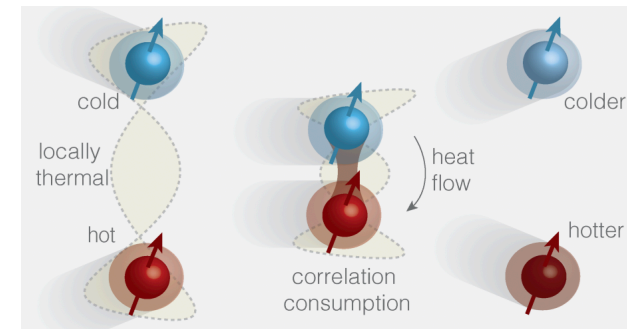
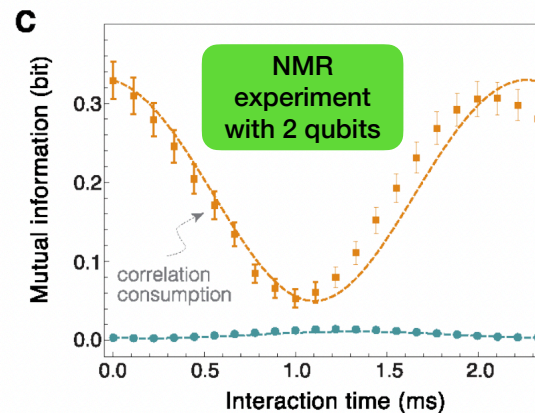
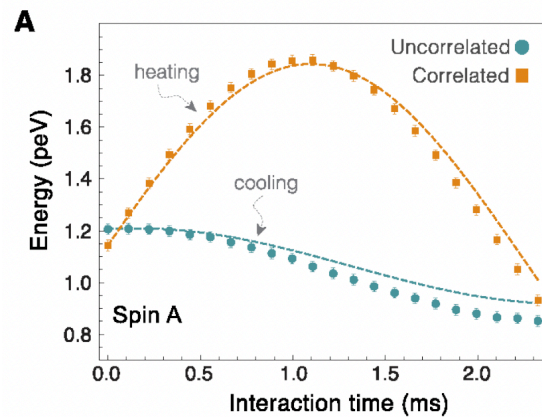
A.M. Timpanaro, G. Guarnieri, J. Goold, GTL, "Thermodynamic Uncertainty Relations from Exchange Fluctuation Theorems." Physical Review Letters, 123, (2019)

A. C. Barato and U. Seifert, Physical Review Letters 114, 158101 (2015).

Quantum Thermodynamics

Heat flows from hot to cold

- To break that, we must pay a price:
 - The fridge or the AC consume electricity.
- In the quantum domain, **information is also a resource.**



Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, GTL, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, "Reversing the direction of heat flow using quantum correlations", *Nature Communications*, **10**, 2456 (2019)

Partovi, M. H., *Phys. Rev. E*, **77**, 021110 (2008)

Jennings, D. & Rudolph, T., *Phys. Rev. E*, **81**, 061130 (2010)

Entropy production for quantum systems

- Information-theoretic formulation: $\sigma = I(S:E) + D(\rho'_E || \rho_E)$
- Operational interpretation: Characterizes irreversibility in terms of what you do not have access to.

Mutual Information:

$$I(S : E) = S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE})$$

Quantifies all correlations
(classical + quantum)

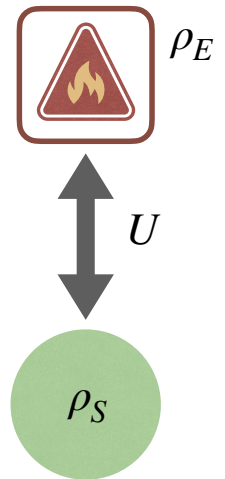
Relative entropy

$$D(\rho'_E || \rho_E) = \text{tr}(\rho'_E \ln \rho'_E - \rho'_E \ln \rho_E)$$

“Distance” between density matrices

- Here $S(\rho) = -\text{tr}(\rho \ln \rho)$ is the von Neumann entropy.

$$\rho'_{SE} = U(\rho_S \otimes \rho_E)U^\dagger$$



Describes an enormous variety of processes!
(maybe a complicated U)

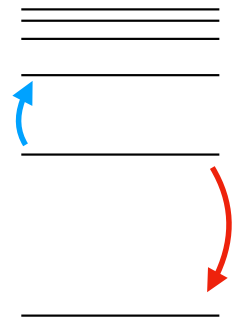
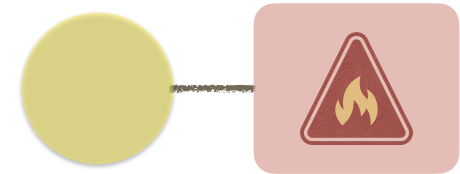
Relaxation towards equilibrium

- Imagine an atomic system relaxing towards equilibrium.
 - Population of energy eigenstates fluctuate until they reach thermal equilibrium.
- In addition: any superpositions are destroyed (**decoherence**).

• Mathematically a state $|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ is the same as the density matrix $\rho = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix}$.

- Relaxation to equilibrium then means

$$\rho = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix} \rightarrow \begin{pmatrix} p_0^{\text{th}} & 0 \\ 0 & p_1^{\text{th}} \end{pmatrix}$$



The entropy production can be split as

$$\sigma = \sigma_{\text{pop}} + \sigma_{\text{coh}}$$

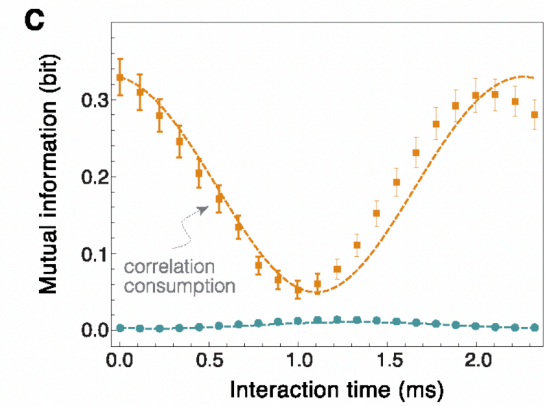
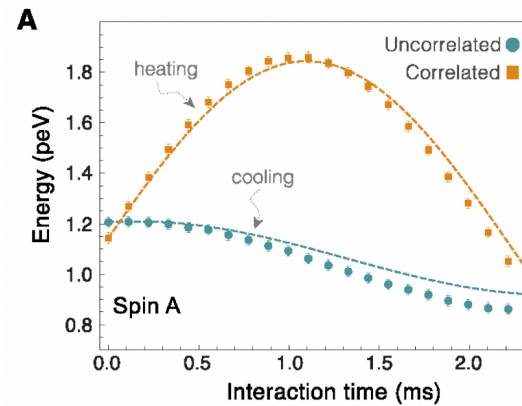
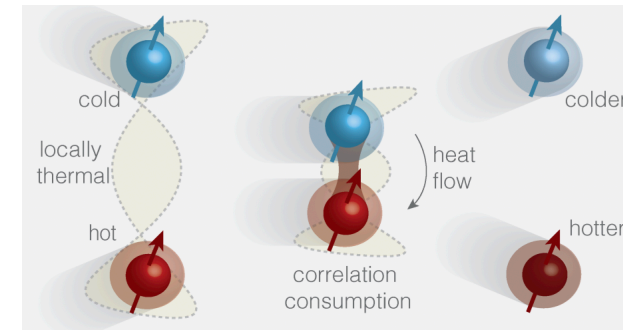
Additional entropy production due to coherence:
Dissipation of information, without dissipation of energy.

Consuming quantum correlations

- In the presence of initial correlations the second law has to be modified to

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h} \right) Q_h \geq \Delta I(h:c)$$

- Heat can flow from cold to hot, provided we **consume** quantum correlations: $\Delta I < 0$.



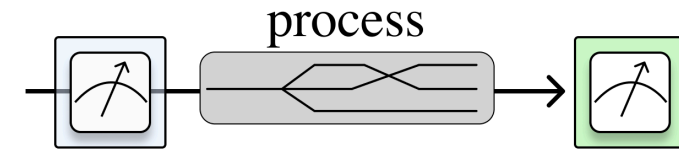
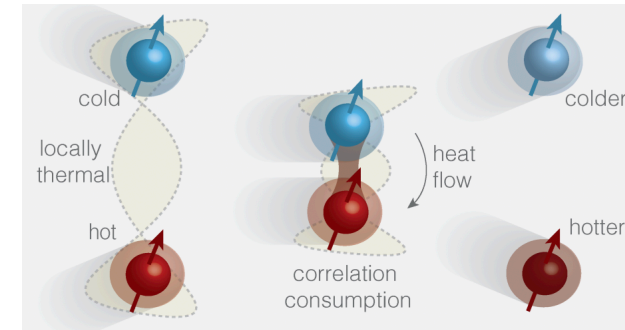
Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, GTL, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, "Reversing the direction of heat flow using quantum correlations", *Nature Communications*, **10**, 2456 (2019)

Partovi, M. H., *Phys. Rev. E*, **77**, 021110 (2008)

Jennings, D. & Rudolph, T., *Phys. Rev. E*, **81**, 061130 (2010)

How do we actually measure the heat flow?

- Heat is a property of the **process/transformation**, not a function of state.
 - To measure heat we must measure energy **before** and **after** a process (two-point measurement - TPM).
- But in quantum mechanics measurements have a back action.**
 - The first measurement will destroy the quantum correlations.
 - “Q-Thermo processes are extrinsic”
 - Can (and should) we define a Q-Thermo theory that is intrinsic? Open question.



K. Micadei, GTL, E. Lutz, “**Quantum fluctuation theorems beyond two-point measurements**”, Phys. Rev. Lett. 124, 090602 (2020)

Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, GTL, Roberto M. Serra, Eric Lutz, “**Experimental validation of fully quantum fluctuation theorems**”, Phys. Rev. Lett., 127, 180603 (2021).

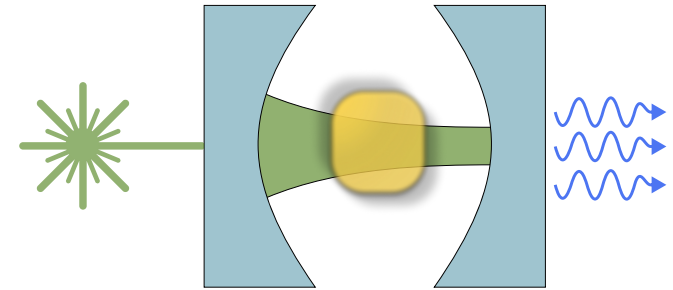
Kenji Maeda, Tharon Holdsworth, Sebastian Deffner, Akira Sone “**Detailed Fluctuation Theorem from the One-Time Measurement Scheme**,” arXiv 2306.09578

Quantum phase space

- Many quantum experiments are done using optical cavities with semi-transparent mirrors.
- Photons leaking out \simeq zero temperature bath.
 - Spontaneous emission: excitations can leave, but not return.

- 2nd law is buggy @ $T = 0$: $\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h} \right) Q_h$.

- Does not include vacuum fluctuations (*present in every measurement*).
- We reformulated the entropy production problem in terms of quantum phase space & the *Wigner function*.



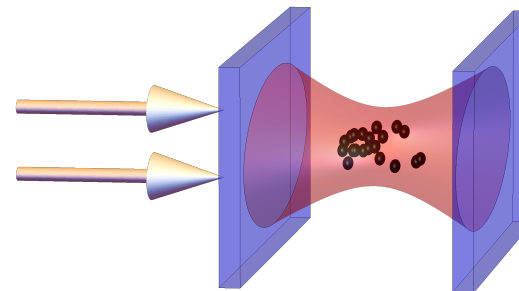
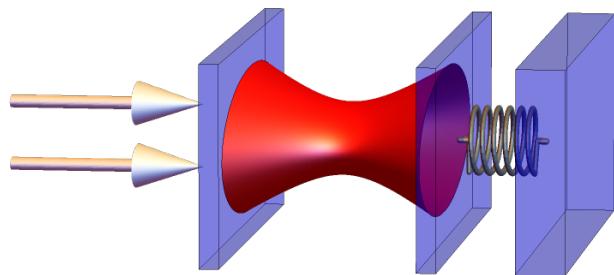
$$\sigma = \left(\frac{1}{T_c^{\text{eff}}} - \frac{1}{T_h^{\text{eff}}} \right) Q_h$$

$$T^{\text{eff}} = \omega(\bar{n} + 1/2), \quad \bar{n} = \frac{1}{e^{\beta\omega} - 1}$$

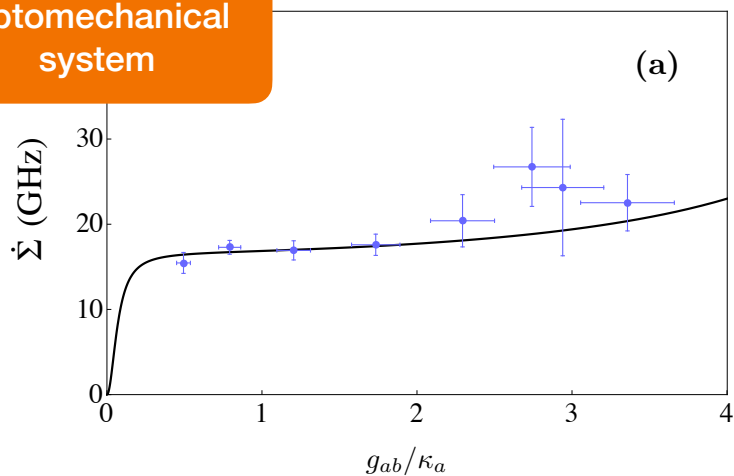
High temperatures: $\omega(\bar{n} + 1/2) \simeq T$.

Zero temperature: $\omega(\bar{n} + 1/2) = \omega/2$.

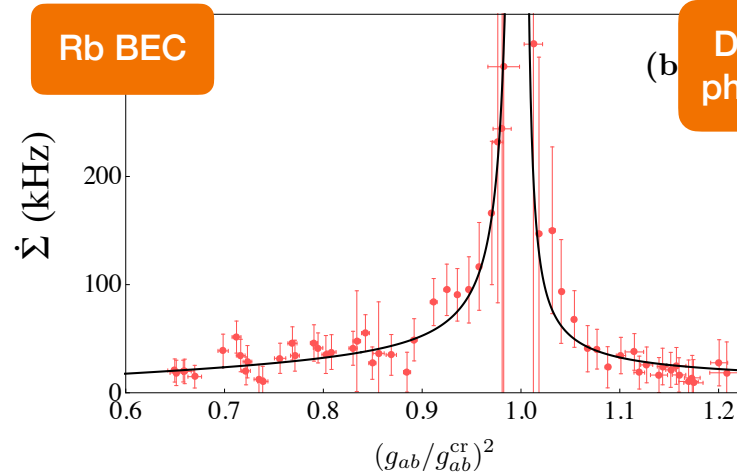
Experiments



Optomechanical system



Rb BEC



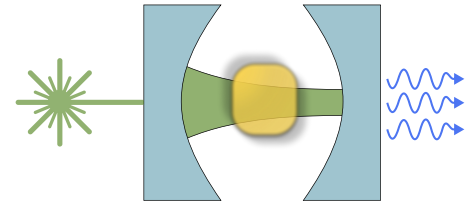
Dicke quantum phase transition.

M. Brunelli, L. Fusco, R. Landig, W. Wiczorek, J. Hoelscher-Obermaier, GTL, F Semião, A. Ferraro, N. Kiesel, T. Donner, G. De Chiara, and M. Paternostro. *Phys. Rev. Lett.*, **121**, 160604 (2018)

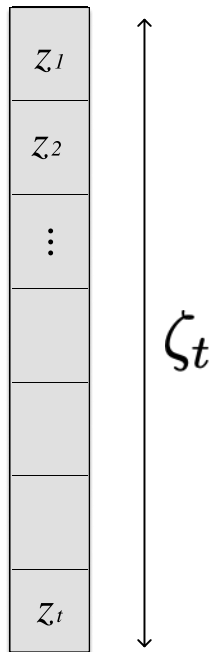
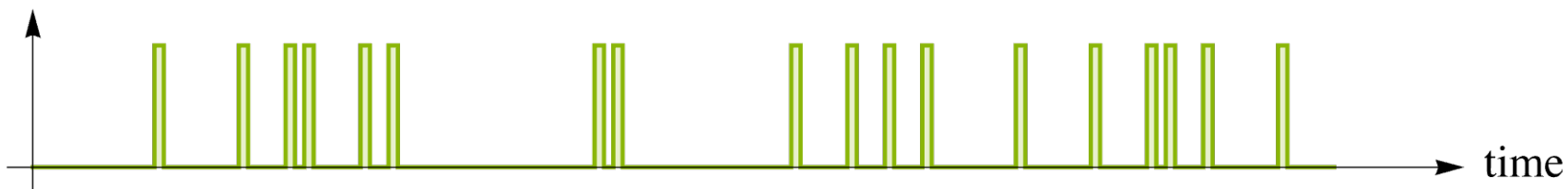
Continuously monitored quantum systems

Continuously monitored quantum systems

- Continuous monitoring of photons that leak out of the cavity.
 - Individual clicks in the detector.
- Fundamental questions: what is entropy production *given* a detection record.
 - Operational: define thermodynamics in terms of what we can actually measure.
 - Includes *information* directly in the formulation.



current



Holevo reduction to entropy production

- **Unconditional:** If we do not know the individual clicks: ρ_t
- **Conditional on the detection record:** $\rho_{t|\zeta_t}$
- **Holevo information:** accumulated information we learned from the detection.

$$I(S_t : \zeta_t) = \sum_{\zeta_t} P(\zeta_t) D(\rho_{t|\zeta_t} || \rho_t)$$

- With each new detection

$$\Delta I_t = G_t - L_t = \text{gain} - \text{loss}$$

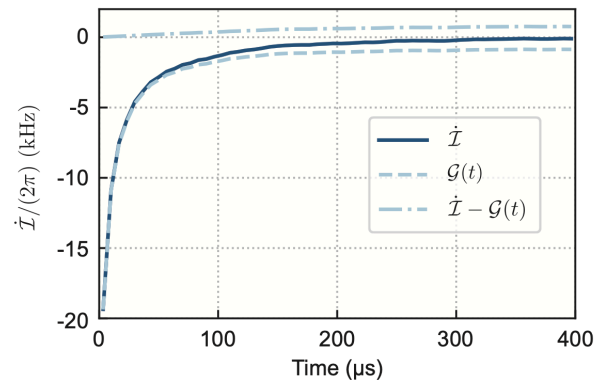
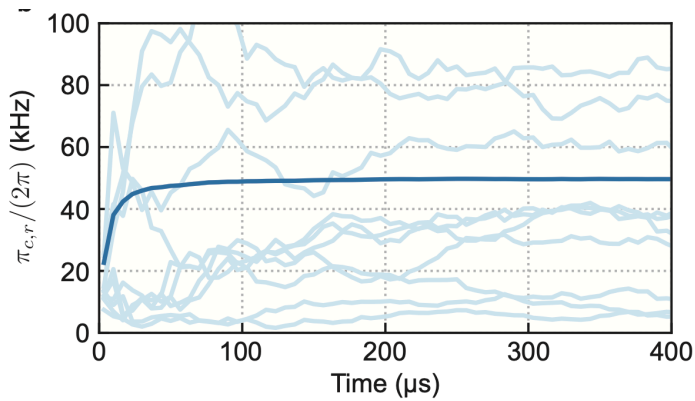
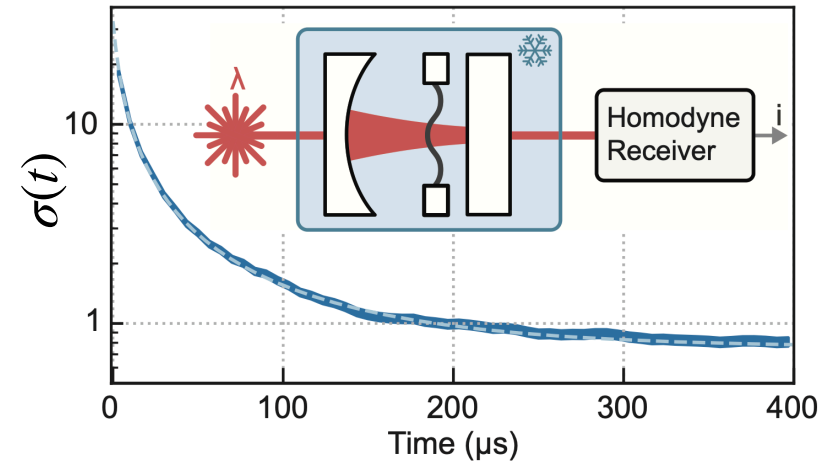
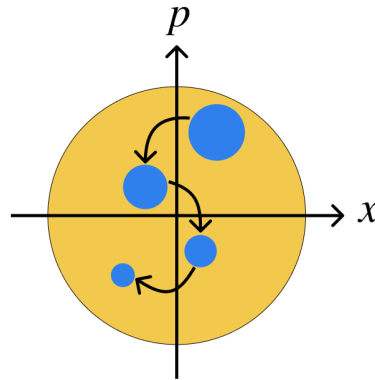
- Conditional entropy production

$$\Delta \Sigma^c = \Delta \Sigma^u - \Delta I$$

Alessio Belenchia, Luca Mancino, GTL and Mauro Paternostro, “**Entropy production in continuously measured quantum systems**”, npj Quantum Information, **6**, 97 (2020).

GTL, Mauro Paternostro and Alessio Belenchia, “**Informational steady-states and conditional entropy production in continuously monitored systems**”, PRX Quantum **3**, 010303, (2020).

Optomechanical setup



Informational steady-state:

Conditional dynamics relaxes to a colder state, which can only be maintained by continuing to monitor the system.

Massimiliano Rossi, Luca Mancino, GTL, Mauro Paternostro, Albert Schliesser, Alessio Belenchia, "**Experimental assessment of entropy production in a continuously measured mechanical resonator**", *Phys. Rev. Lett.* **125**, 080601 (2020)

Current fluctuations in the Parametric Kerr model

Michael J. Kewming, Mark T. Mitchison, GTL, "**Diverging current fluctuations in critical Kerr resonators.**" Physical Review A, 106, (2022)

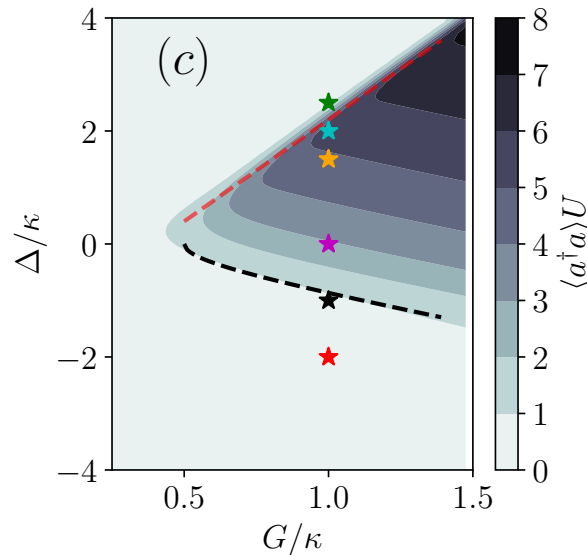
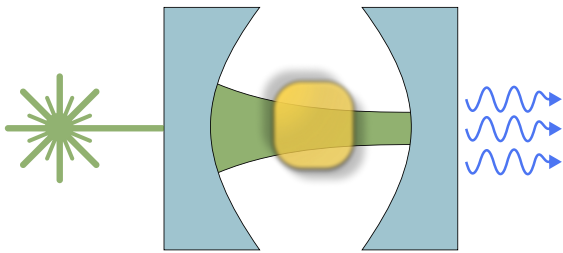
GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics,**" 2023. arxiv 303.04270

Parametric Kerr model

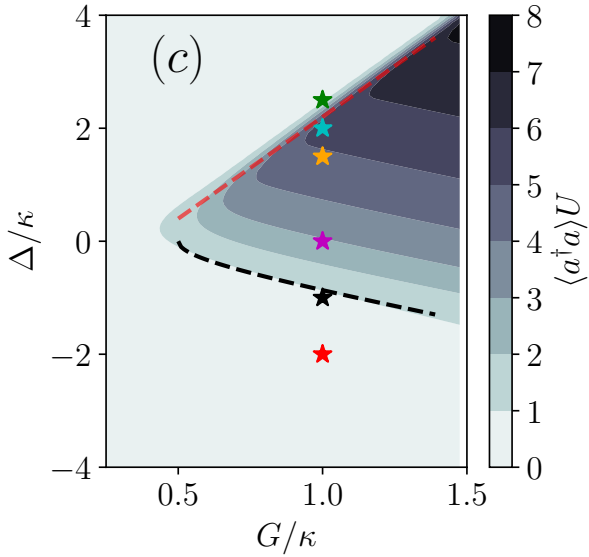
- Non-linear quantum harmonic oscillator:

$$\frac{d\rho}{dt} = -i[H(t), \rho] + \kappa \left[a\rho a^\dagger - \frac{1}{2}\{a^\dagger a, \rho\} \right]$$

$$H = -\Delta a^\dagger a + \frac{U}{2} a^\dagger a^\dagger a a + \frac{G}{2} (a^{\dagger 2} + a^2)$$

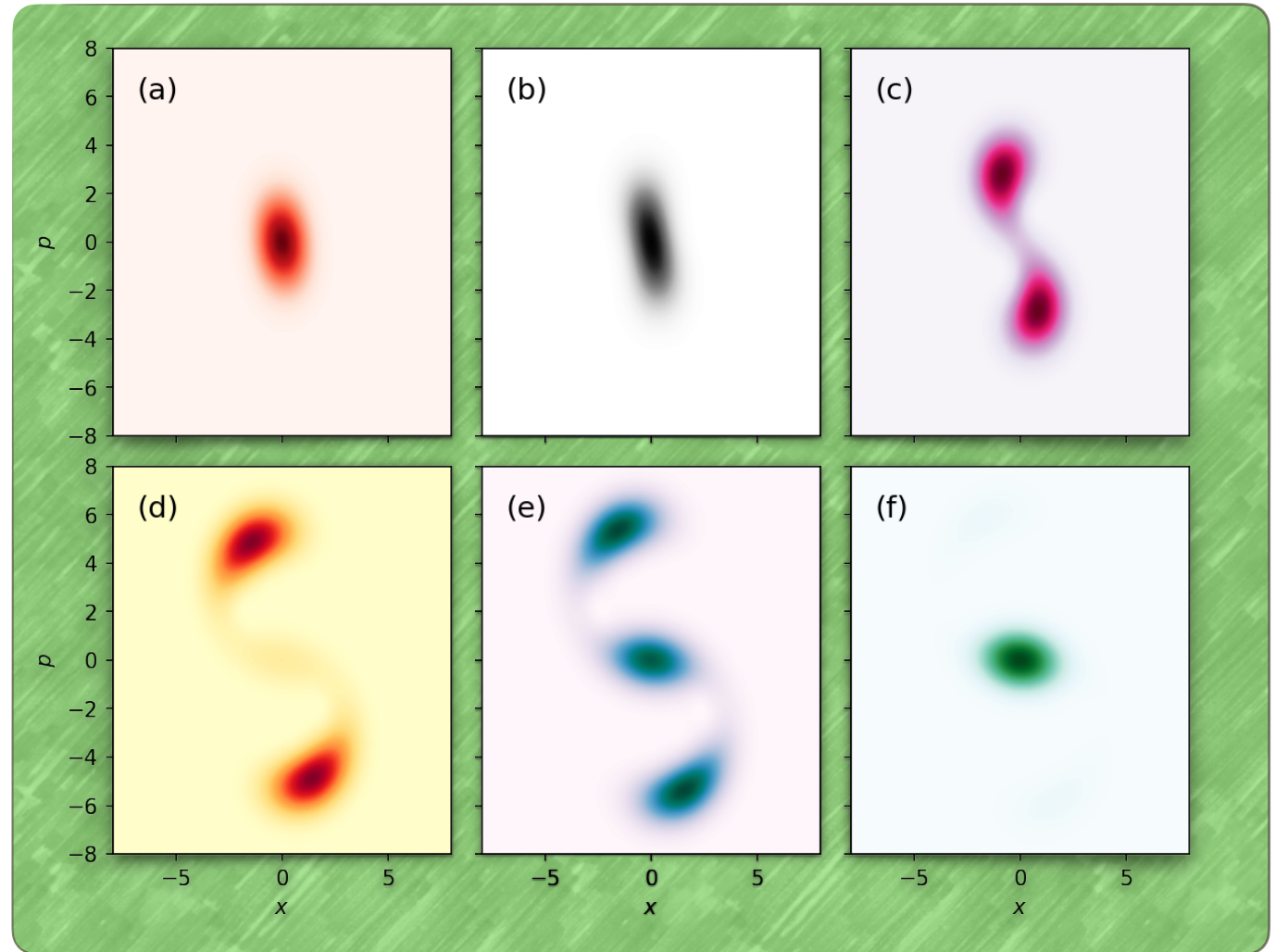


- * a = annihilation operator
photon operator for an optical cavity
- * $\Delta = \omega_p - \omega_c =$ detuning
- * U = Kerr non-linearity.
(requires a non-linear crystal inside the cavity)
- * G = 2-photon pump
(input laser produces photons in pairs)
- * κ = loss rate
rate at which photons leak out of the cavity



- 2 phase transitions, continuous and discontinuous
- Proper criticality occurs in the limit $U \rightarrow 0$ (“thermodynamic limit”)

Wigner function



Cat qubits

- Steady-state is a mixture of two Schrödinger cat states

$$|S\rangle = |\alpha\rangle + |-\alpha\rangle$$

$$|A\rangle = |\alpha\rangle - |-\alpha\rangle$$

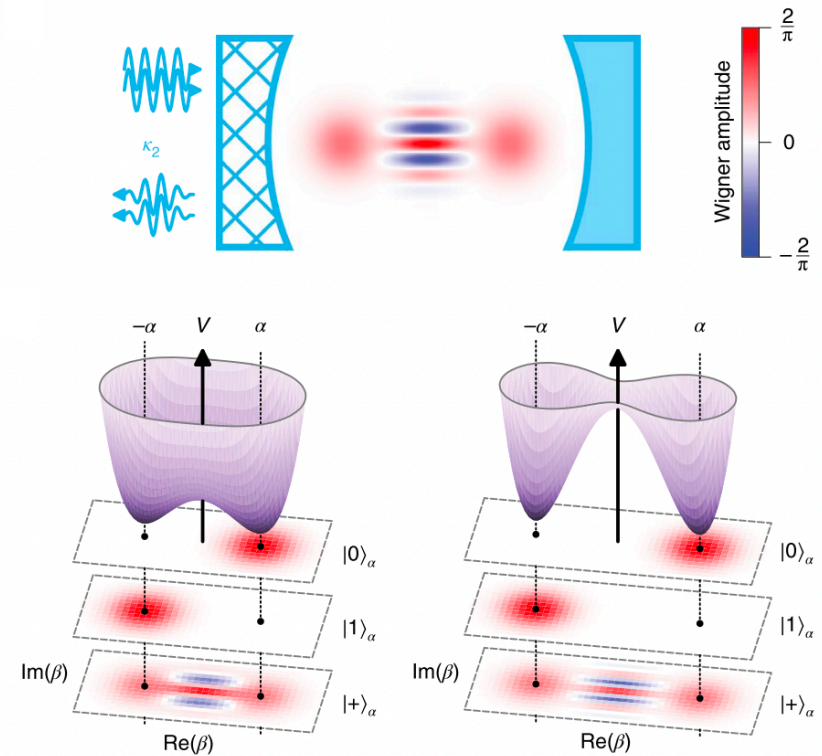
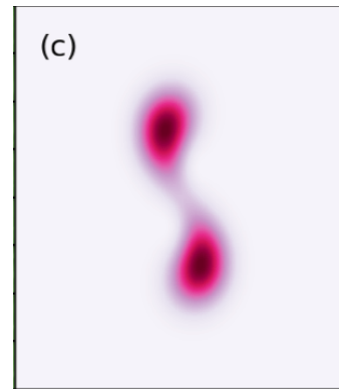
- Use this to define cat qubits:

$$|0\rangle = |\alpha\rangle$$

$$|1\rangle = |-\alpha\rangle$$

- Cat qubits are more robust against errors.

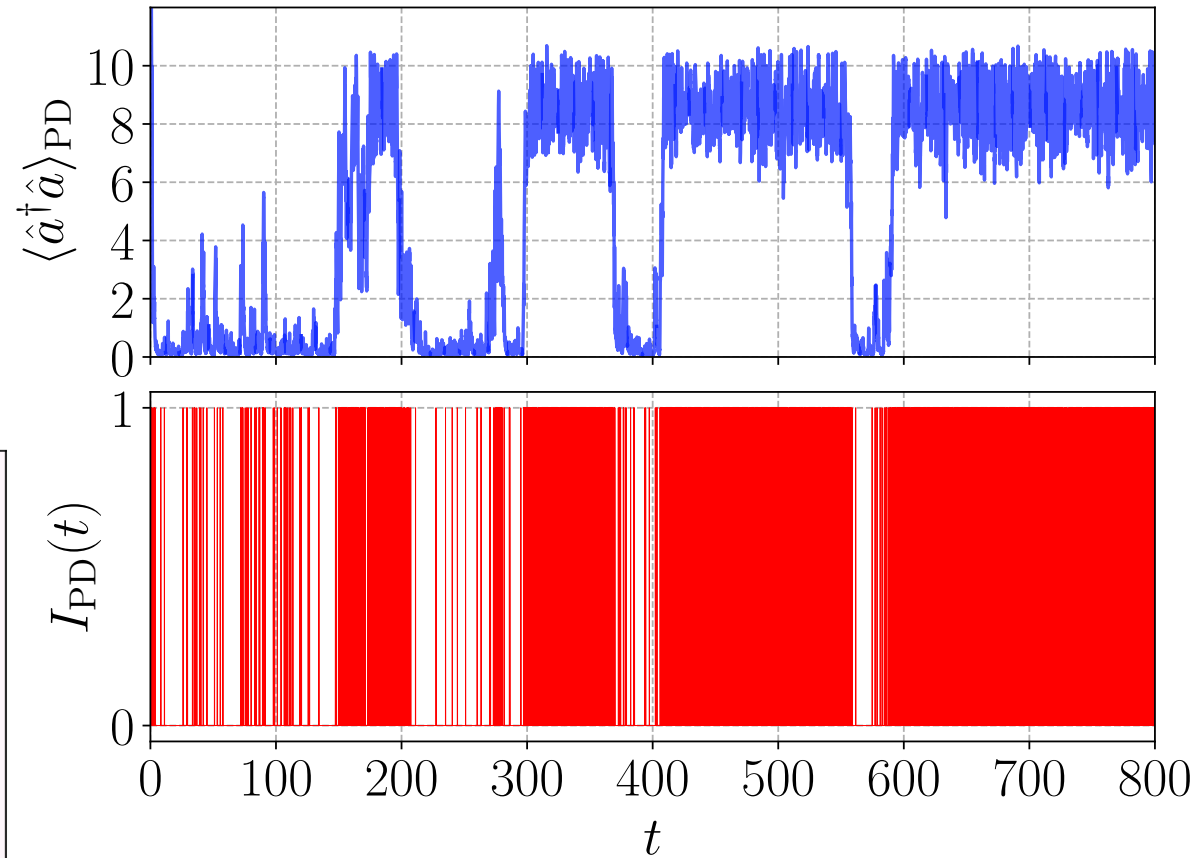
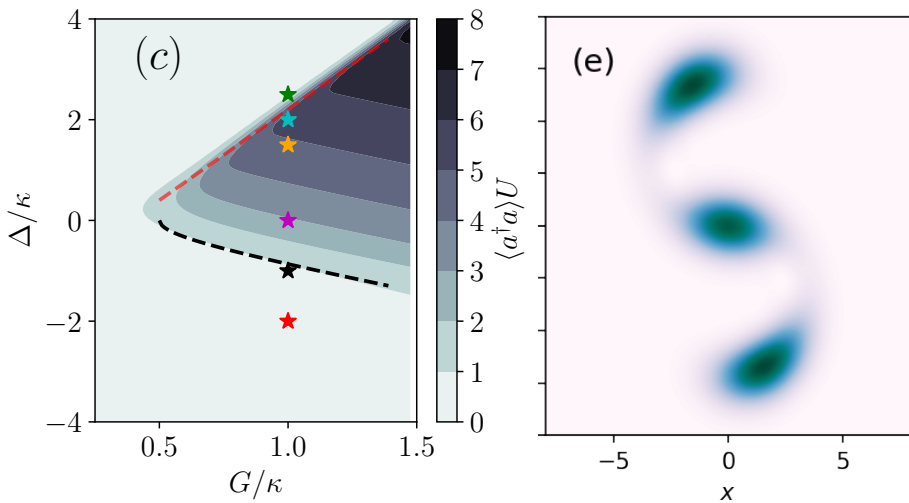
- Quantum computing with Kerr cats.



Lescanne, *et. al.*, Nature, **16**, 509-513 (2020)

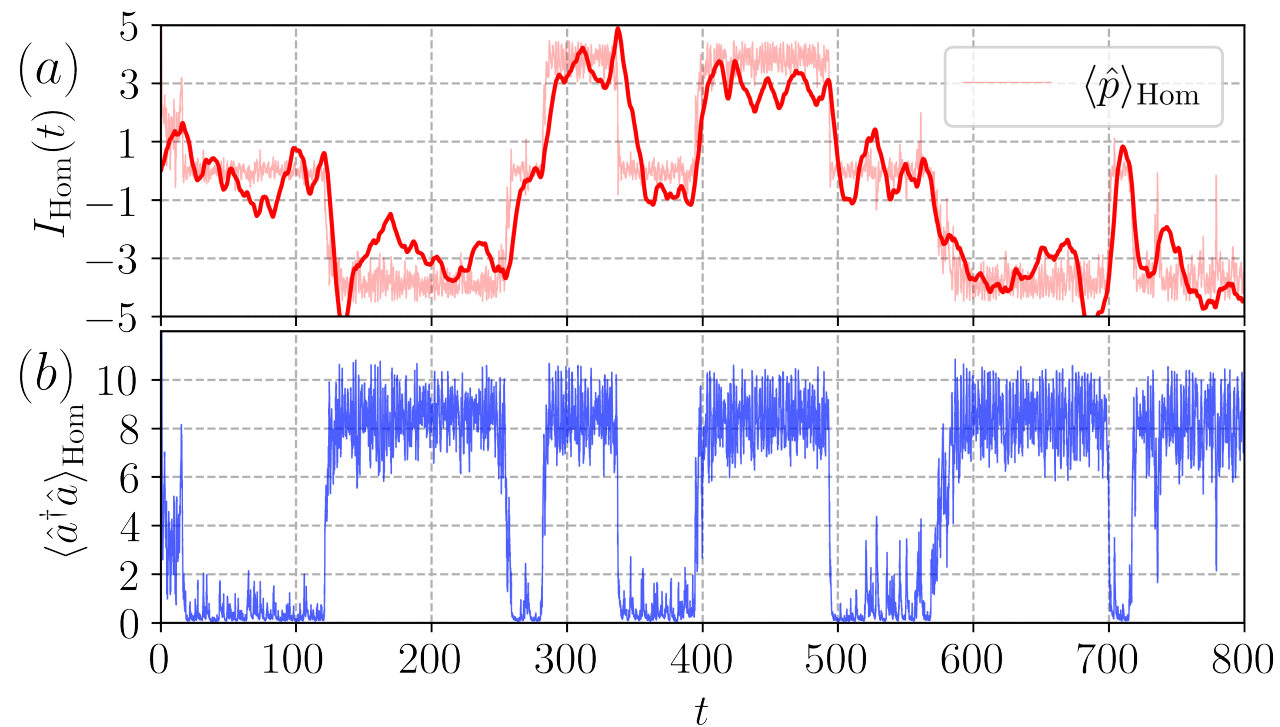
Photo-detection current

- @ discontinuous transition: on/off (telegraph) behavior of the current.
- Photo-detection cannot resolve upper vs. lower blobs.

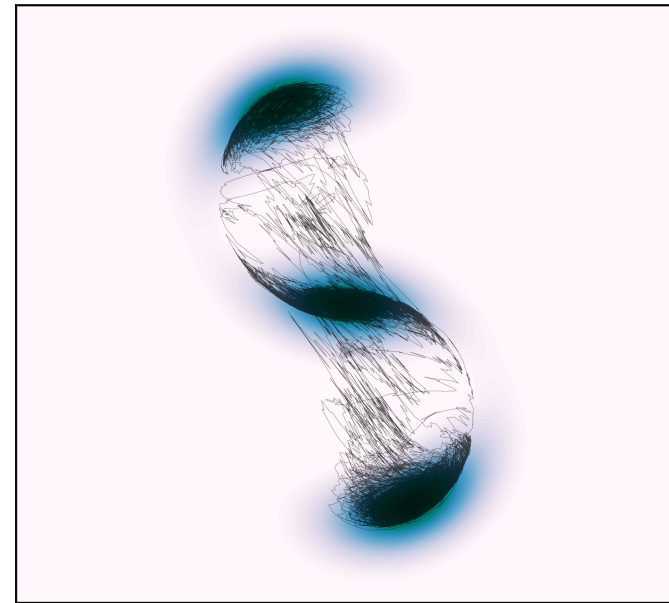


Homodyne current

- Observable is now $p = i(a^\dagger - a)$.
- The homodyne current switches between 3 values (+,0,-).
- Captures the tunneling between the 3 blobs.



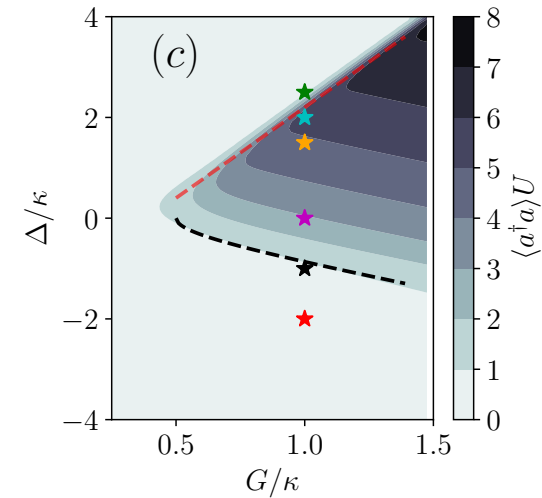
(c)



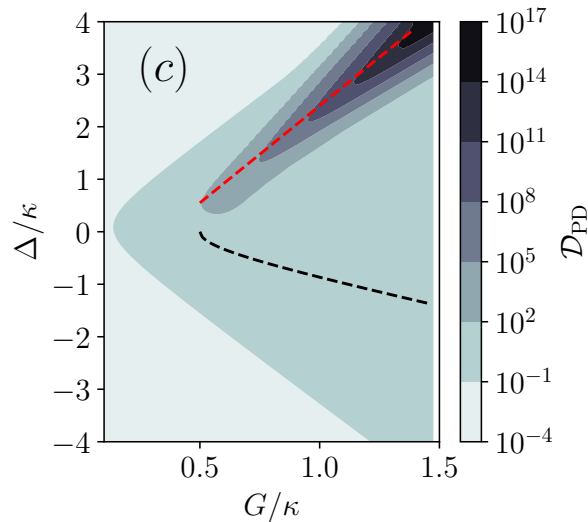
Exponential divergence of the noise

- “Thermodynamic limit:” $U \rightarrow 0$
- In the discontinuous transition ($\Delta > 0$)

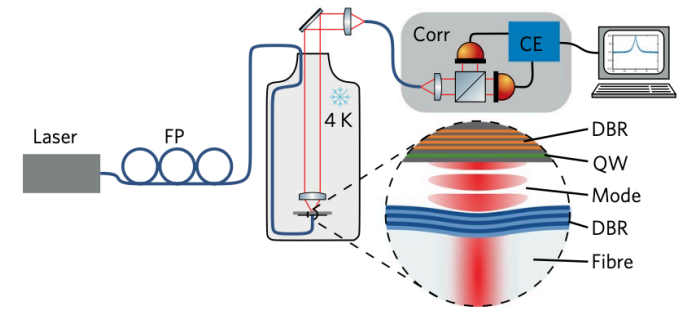
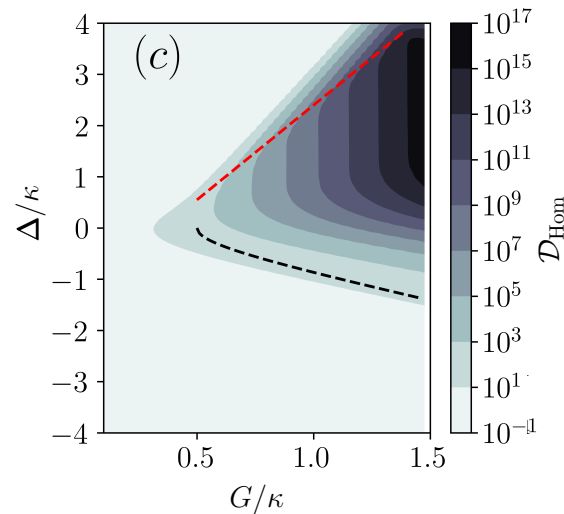
$$D \sim e^{1/U}$$



Direct photo-detection



Homodyne



T. Fink, *et. al.*, *Nature Physics*, **14**, 365 (2018)

Conclusions

- **Entropy production** quantifies the irreversibility of a process.
- In the quantum realm it characterizes:
 - Decoherence & loss of information.
 - Quantum correlations.
 - Zero-temperature fluctuations.
 - Measurement back action.

Thank you!



<https://www.pas.rochester.edu/~gtlandi>

GTL and Mauro Paternostro, "**Irreversible entropy production, from quantum to classical**", *Review of Modern Physics*, **93**, 035008 (2021)

GTL, Dario Poletti, Gernot Schaller, "**Nonequilibrium boundary-driven quantum systems: Models, methods, and properties.**" *Reviews of Modern Physics*, 94, (2022)

GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics,**" 2023. arxiv 303.04270