Information-thermodynamics in the quantum regime

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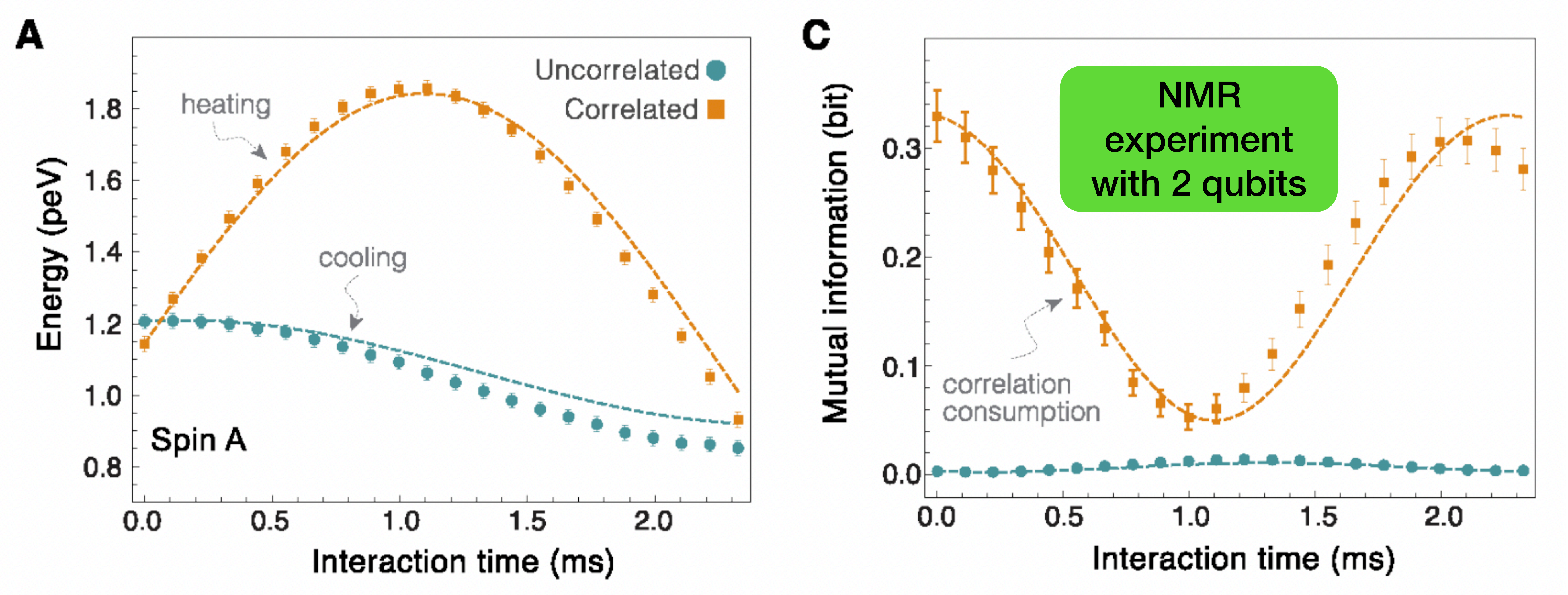
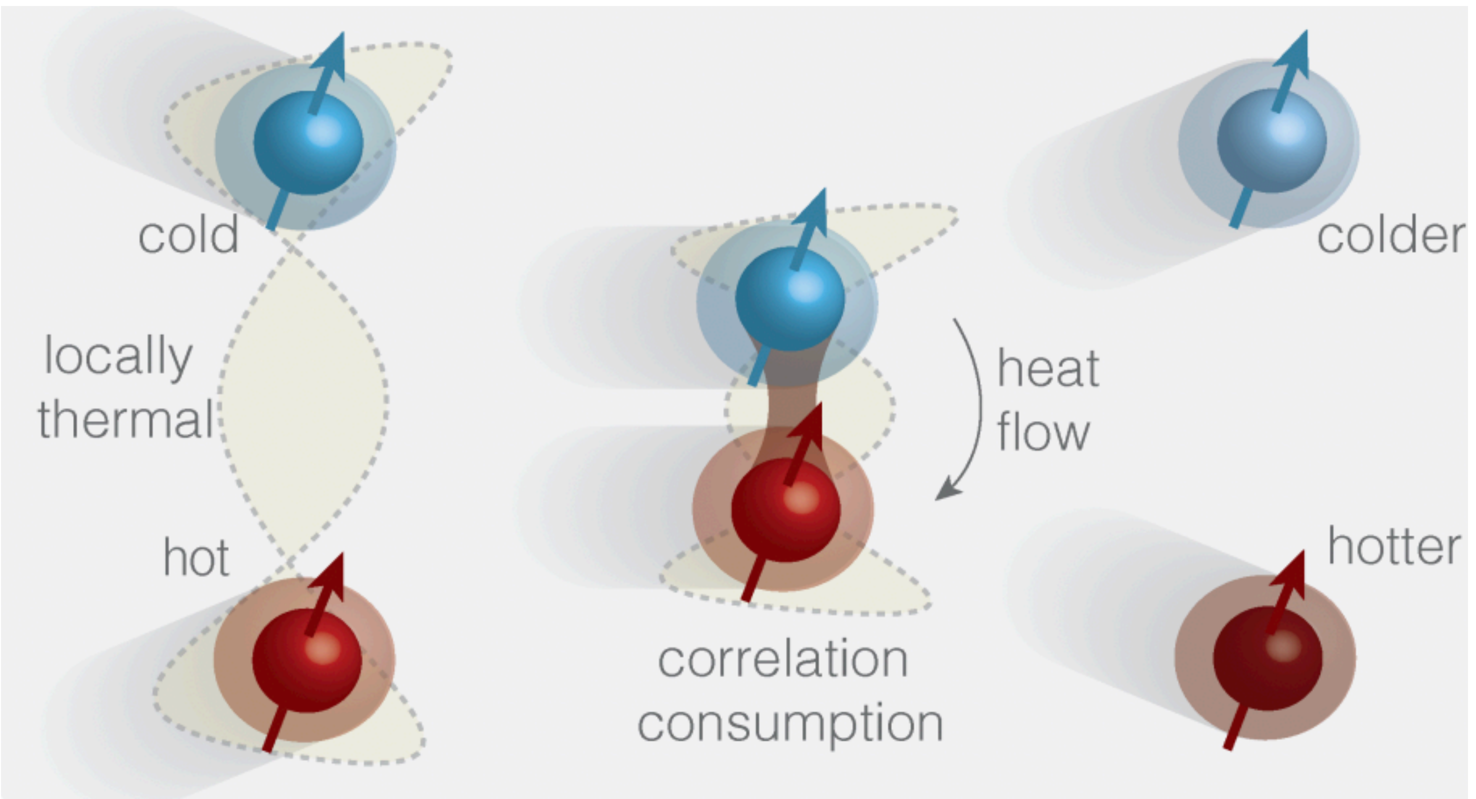
Heat flows from hot to cold

- To break that, we must pay a **price**: fridges consumes electricity (**= resource = fuel)**.
  - *“Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.”* (Clausius’ statement of the 2nd law)
- In the quantum domain, **information is also a resource.**





Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, Gabriel T. Landi, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, "**Reversing the direction of heat flow using quantum correlations**”, *Nature Communications*, **10**, 2456 (2019)

Prof. George Porter
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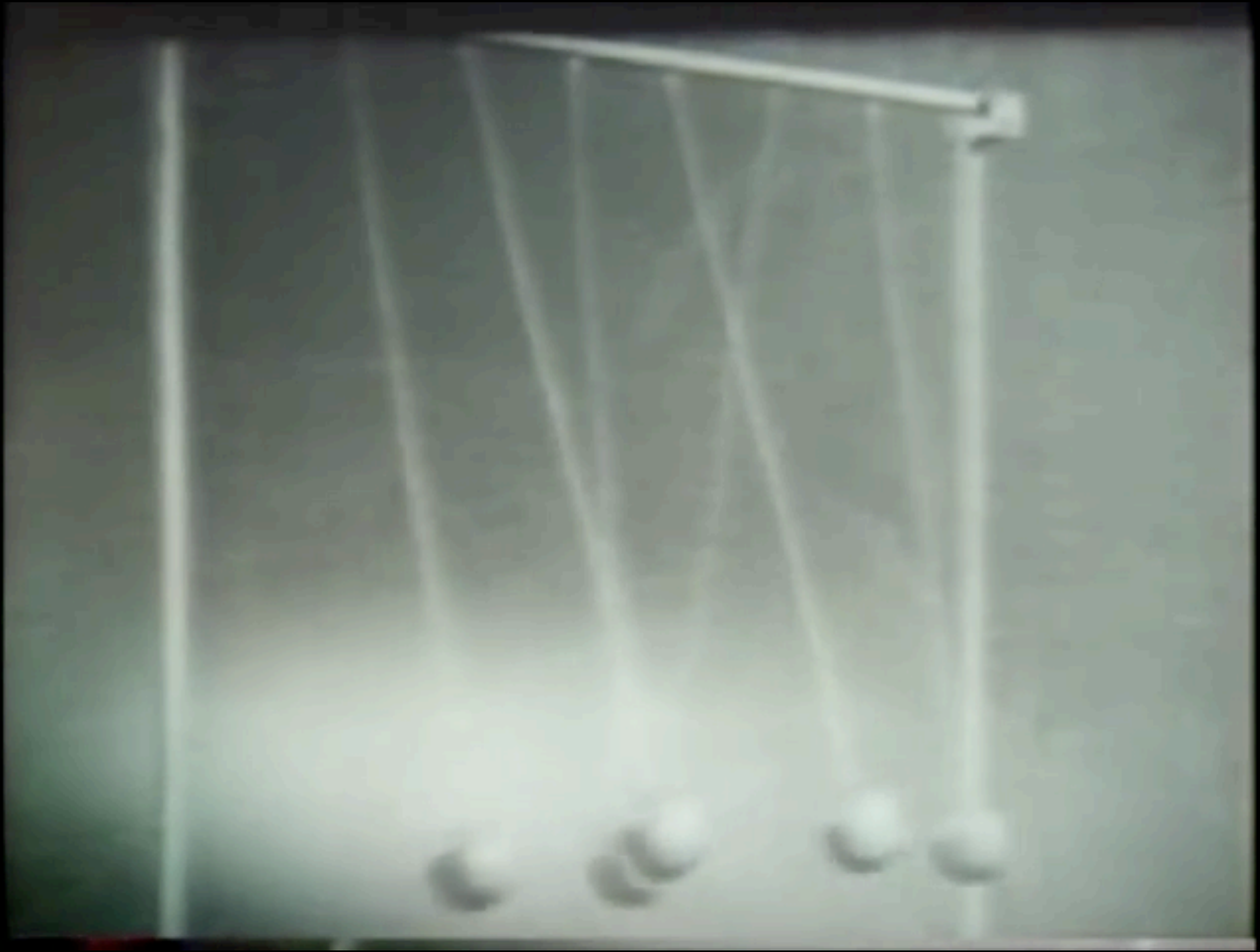

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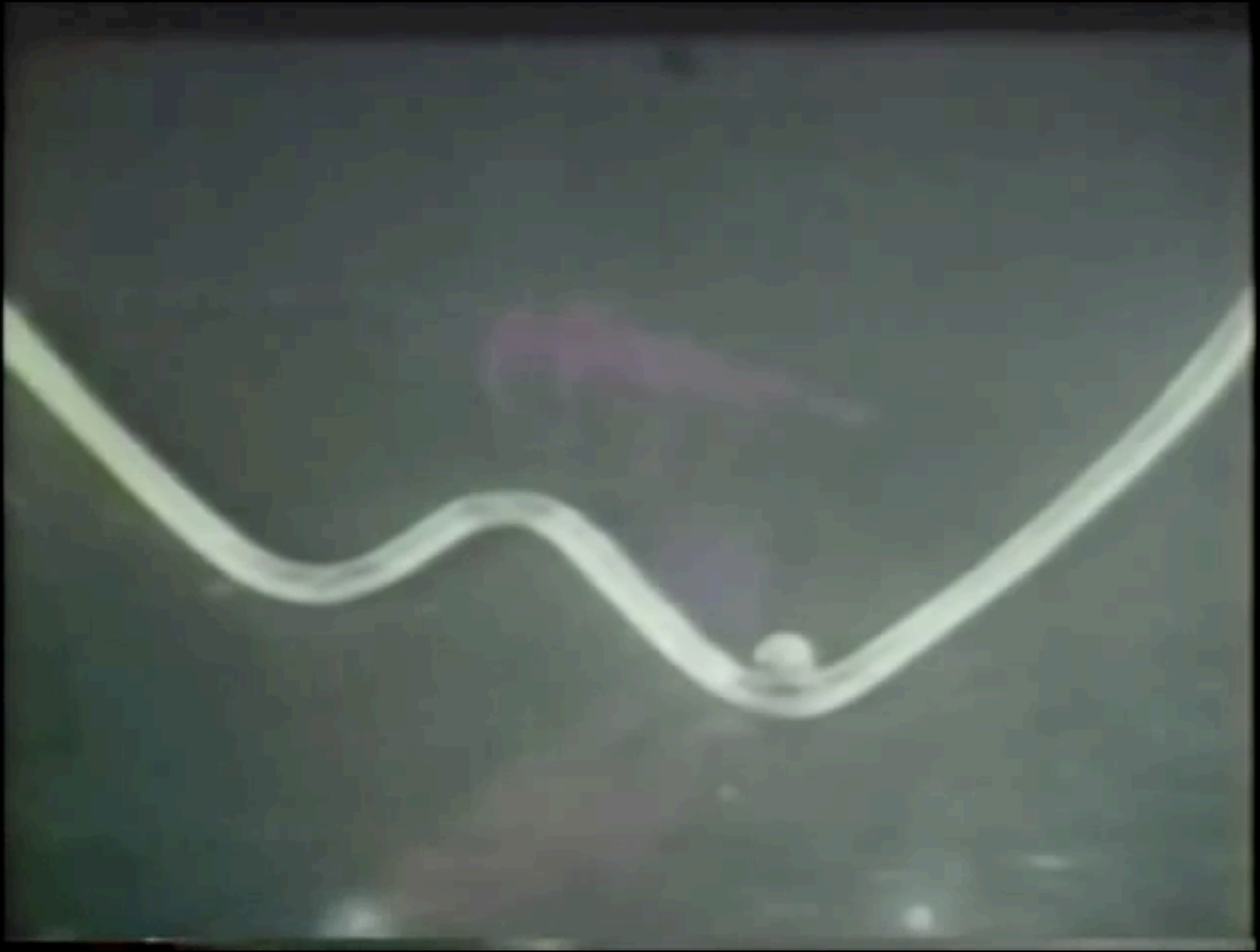


Irreversibility & Entropy production

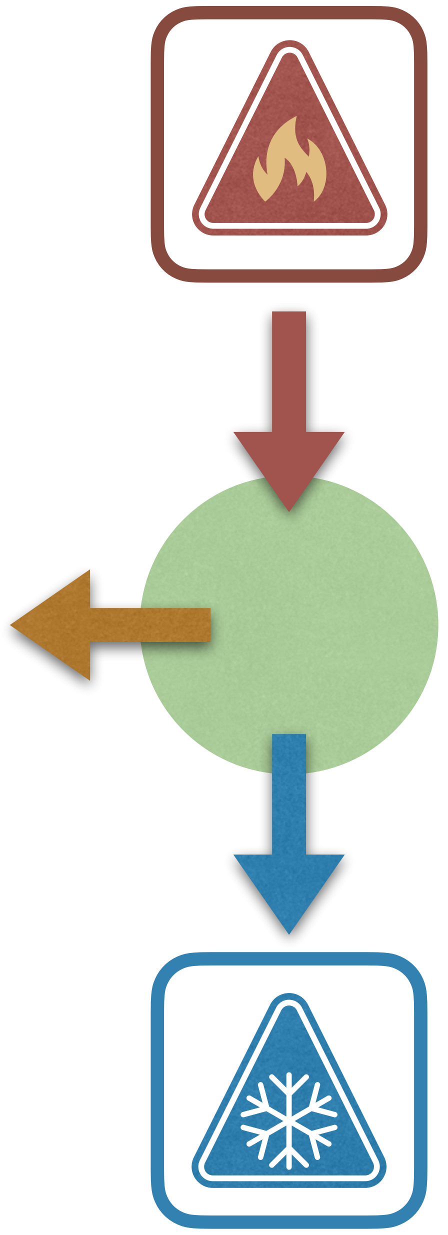
- Clausius formulated the notion of irreversibility using entropy.
- Consider a thermodynamic process involving heat & work:

$$\Delta U = W + Q_h + Q_c \qquad \text{(1st law = balance equation)}$$

- According to Clausius, entropy does not satisfy a balance equation:

$$\Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} + \sigma \qquad \sigma \geqslant 0 \text{ is the entropy produced in the process.}$$

- $\sigma \geqslant 0$ = *mathematical statement* of the 2nd law.
- σ is related to **dissipation & irreversibility:** more dissipation means the process is more irreversible.



To understand this better, let us look at cyclic processes.

$$W = -Q_h - Q_c \qquad \text{and} \qquad \sigma = -\frac{Q_h}{T_h} - \frac{Q_c}{T_c} \geqslant 0$$

To understand this better, let us look at cyclic processes.

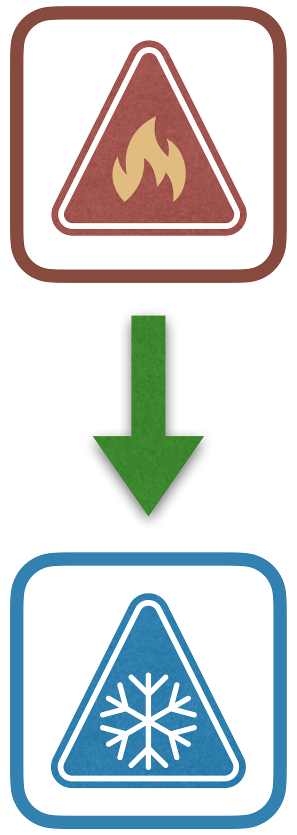
$$W = -Q_h - Q_c \quad \text{and} \quad \sigma = -\frac{Q_h}{T_h} - \frac{Q_c}{T_c} \geqslant 0$$

- **Heat flow** (no work): $Q_h = -Q_c$

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h \geqslant 0$$

Heat always flows from hot to cold

(Clausius' statement)



- **Efficiency of a heat engine:**

$$\eta = \frac{|W|}{|Q_h|} = \eta_c - \frac{T_c \sigma}{|Q_h|} < \eta_c$$

where $\eta_c = 1 - \dfrac{T_c}{T_h}$ is the **Carnot efficiency.**

The efficiency is always *lower* than Carnot's efficiency because entropy is produced.

(Carnot's statement of the 2nd law)

Other applications

- **Landauer's erasure:** Cost to erase 1 bit of information (there is no cost in writing)

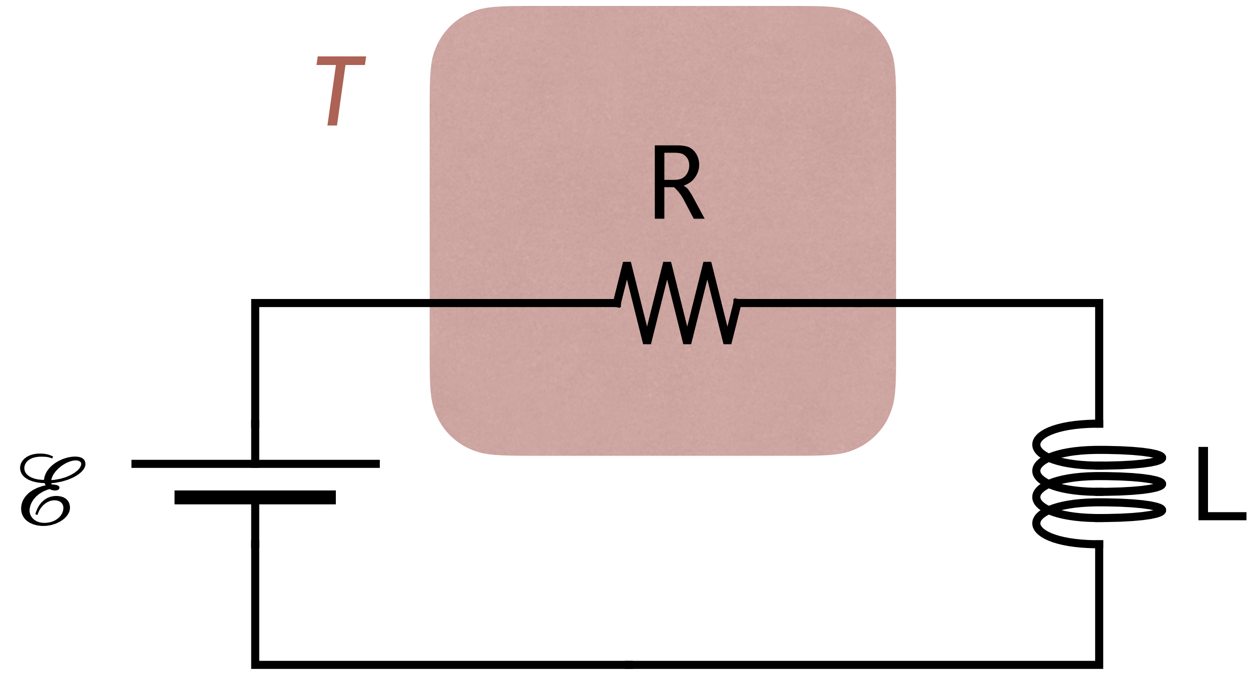
$$\Delta S = \frac{Q}{T} + \sigma \quad \rightarrow \quad \sigma = \Delta S - \frac{Q}{T} \geqslant 0$$

The entropy of a bit is at most $k_B \ln 2 \rightarrow \; Q \geqslant k_B T \ln 2$

- What about $T \simeq 0$? Very relevant for quantum computation. If eraser is a waveguide of length L:

$$Q \geqslant k_B T \ln 2 + \frac{3\hbar c}{\pi L} \ln^2(2)$$

- **Non-equilibrium steady-states:** not equilibrium.



$$\frac{dS}{dt} = \frac{\dot{Q}}{T} + \dot{\sigma} = 0 \qquad \text{so} \qquad \dot{\sigma} = -\frac{\dot{Q}}{T}$$

- Example: Joule heating.
  Continues as long as there is juice in the battery

$$\dot{\sigma} = \frac{\mathscr{E}^2}{RT}$$

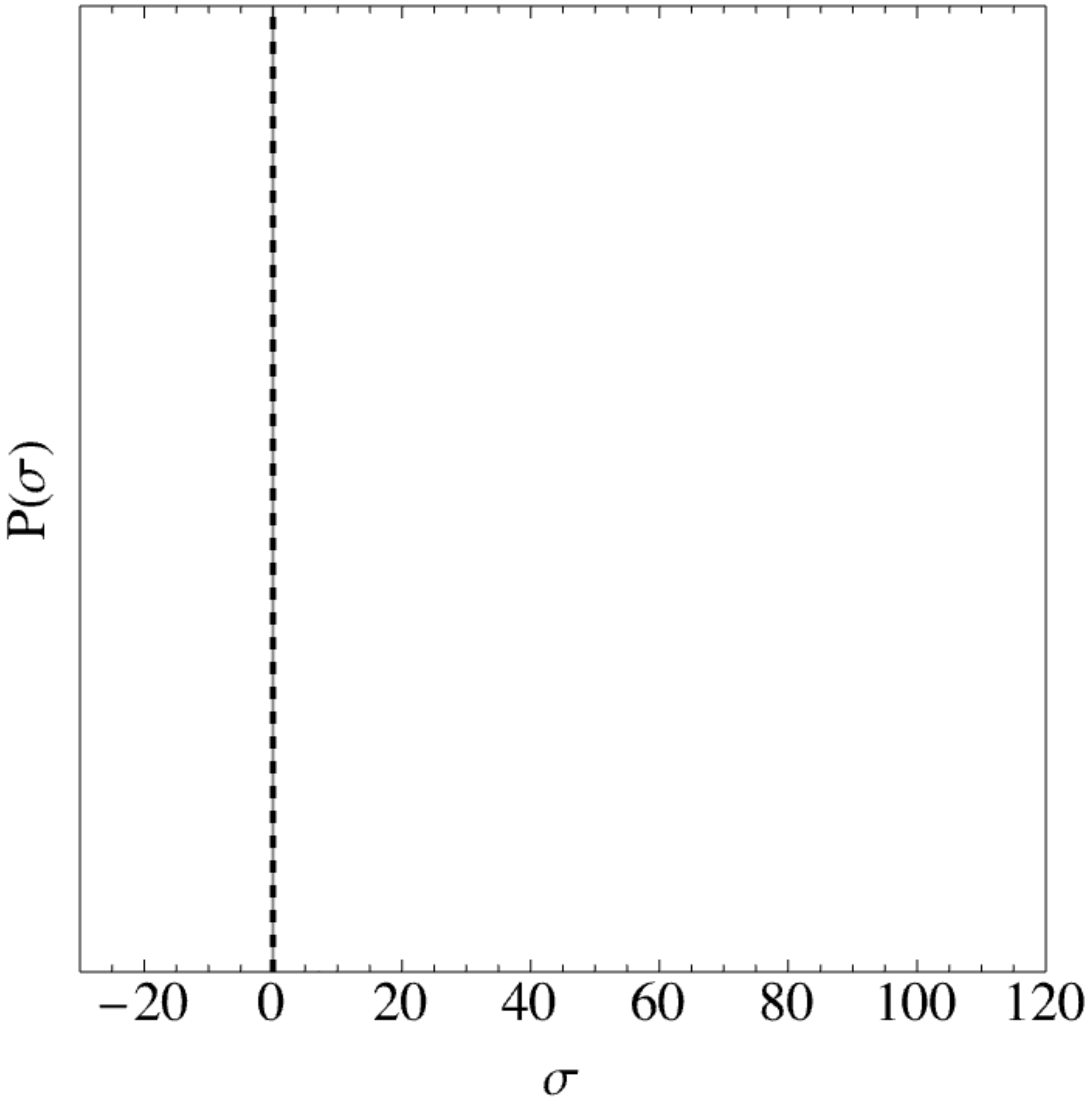
André M. Timpanaro, Jader P. Santos, and Gabriel T. Landi, *Phys. Rev. Lett.* **124**, 240601 (2020)

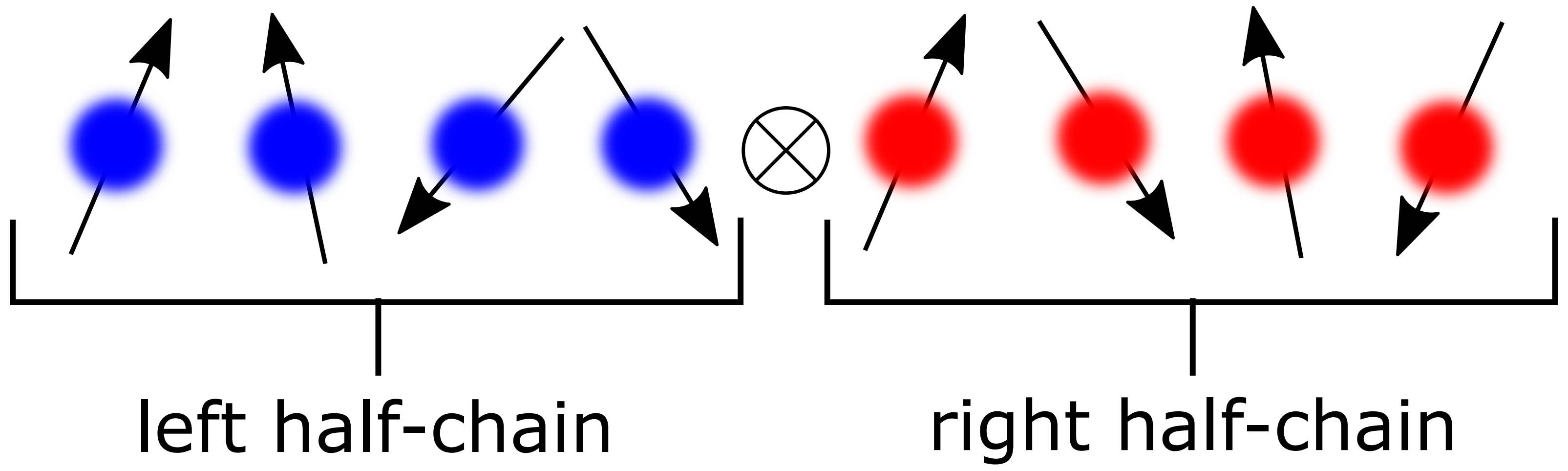
"The principle of the increase of entropy is merely an observation that in any irreversible process the entropy tends to increase."

Feynman lecture on physics.

Irreversibility & the arrow of time

- Macro-world: heat flows from hot → cold.
- Micro-world: heat ***usually*** flows from hot → cold.





G. T. Landi and Dragi Karevski
Phys. Rev. E **93**, 032122 (2015)

Heat Exchange Fluctuation Theorem

$$P(-\sigma) = e^{-\sigma} P(\sigma)$$

Implies 2nd law: $\langle \sigma \rangle \geqslant 0$

C. Jarzynski and D. Wójcik,
Phys. Rev. Lett. 92, 230602 (2004)

Thermodynamic Uncertainty Relations

- In the micro-world thermodynamic currents fluctuate. What can we say about the ***variance***?
- Thermodynamic Uncertainty Relation (TUR):

$$\frac{\Delta_Q^2}{\langle Q\rangle^2} \geqslant \frac{2}{\sigma}$$

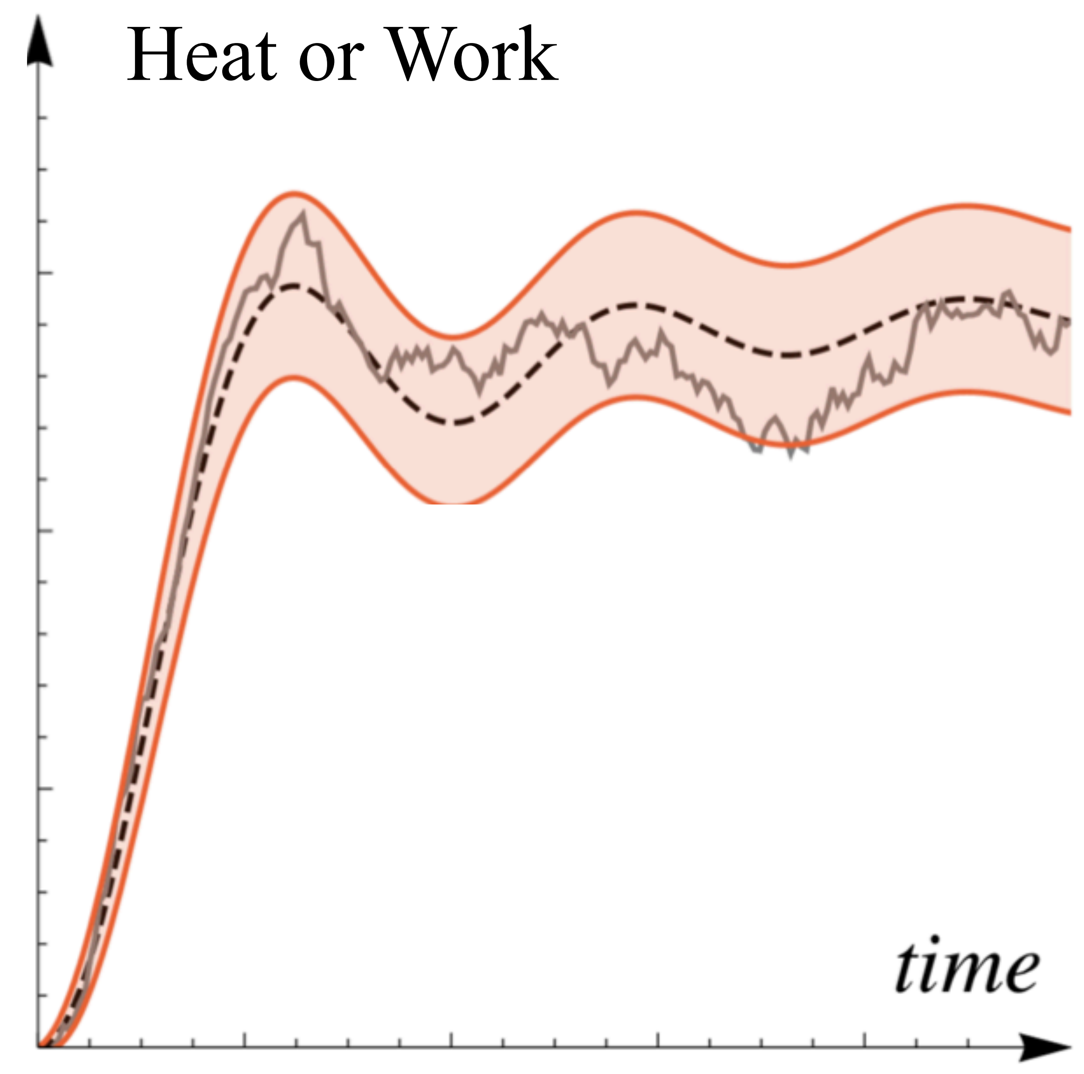
(*to reduce fluctuations one must increase dissipation*)

- Example: power extracted from a heat engine: usually high efficiency → slow engine → low power.
  In the micro-world, power fluctuations also become important:

$$\Delta_P^2 \geqslant 2T_c P \frac{\eta}{\eta_C - \eta}$$

- We showed that the xFT $P(-\sigma) = e^{-\sigma}P(\sigma)$ implies a TUR-like bound

$$\frac{\mathrm{var}(Q)}{\langle Q\rangle} \geqslant f(\sigma), \qquad f(x) = \mathrm{csch}(g(x/2)), \qquad g(x) = \text{inverse of } x\tanh(x)$$



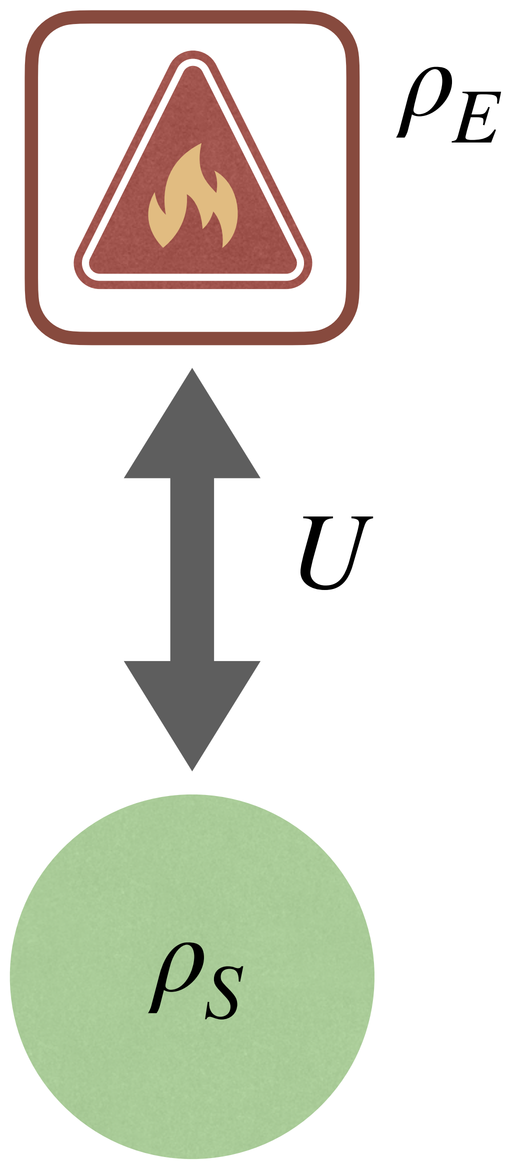
A. C. Barato and U. Seifert, *PRL* **114**, 158101 (2015).
P. Pietzonka and U. Seifert (2017), *PRL* **120**, 190602 (2018).
A. M. Timpanaro, G. Guarnieri, J. Goold, GTL, *PRL* **123**, 090604 (2019)

Quantum Thermodynamics

Entropy production for quantum systems

- Information-theoretic formulation: $\sigma = I(S\!:\!E) + D(\rho'_E\,||\,\rho_E)$
    - Operational interpretation: Characterizes irreversibility in terms of what you do not have access to.

$$\rho'_{SE} = U(\rho_S \otimes \rho_E)U^\dagger$$



Mutual Information:

$$I'(S:E) = S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE})$$

Quantifies all correlations (classical + quantum)

Relative entropy

$$D(\rho'_E\,||\,\rho_E) = \mathrm{tr}\left(\rho'_E \ln \rho'_E - \rho'_E \ln \rho_E\right)$$

"Distance" between density matrices

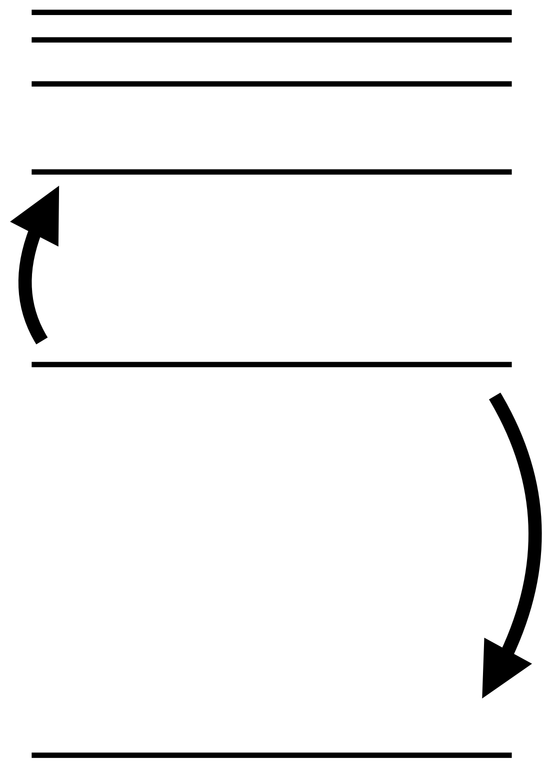
- Here $S(\rho) = -\,\mathrm{tr}(\rho \ln \rho)$ is the von Neumann entropy.

Describes an enormous variety of processes! (maybe a complicated U)

Massimiliano Esposito, Katja Lindenberg and Christian Van den Broeck, *New Journal of Physics*, 12, 013013 (2010).

Relaxation towards equilibrium

- Imagine an atomic system relaxing towards equilibrium.
    - Population of energy eigenstates fluctuate until they reach thermal equilibrium.
- In addition: any superpositions are destroyed (**decoherence**).



- Mathematically a state $|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ is the same as the density matrix $\rho = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix}$.
    - Relaxation to equilibrium then means

$$\rho = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix} \to \begin{pmatrix} p_0^{\text{th}} & 0 \\ 0 & p_1^{\text{th}} \end{pmatrix}$$

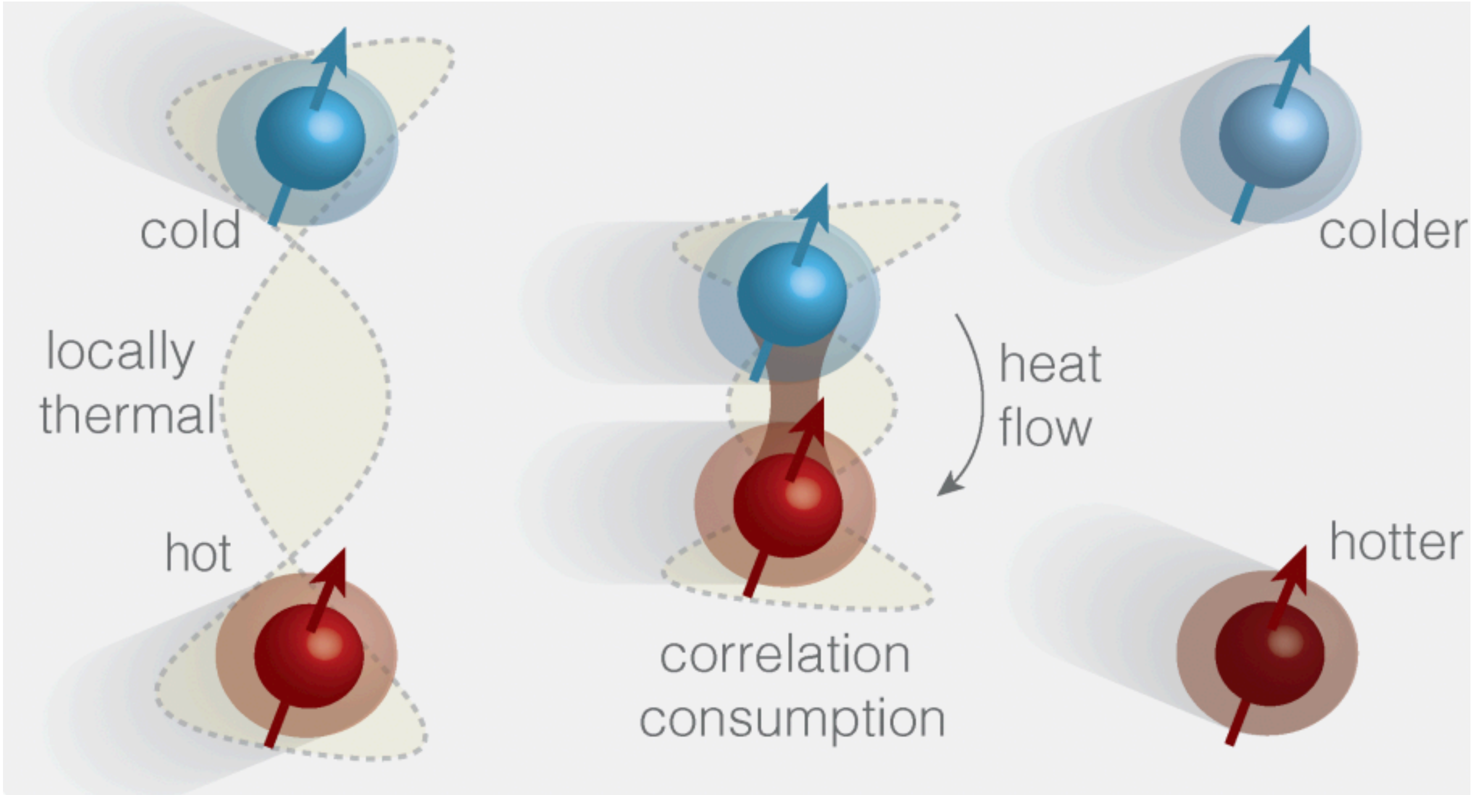
The entropy production can be split as

$$\sigma = \sigma_{\text{pop}} + \sigma_{\text{coh}}$$

Additional entropy production due to coherence:
Dissipation of information, without dissipation of energy.

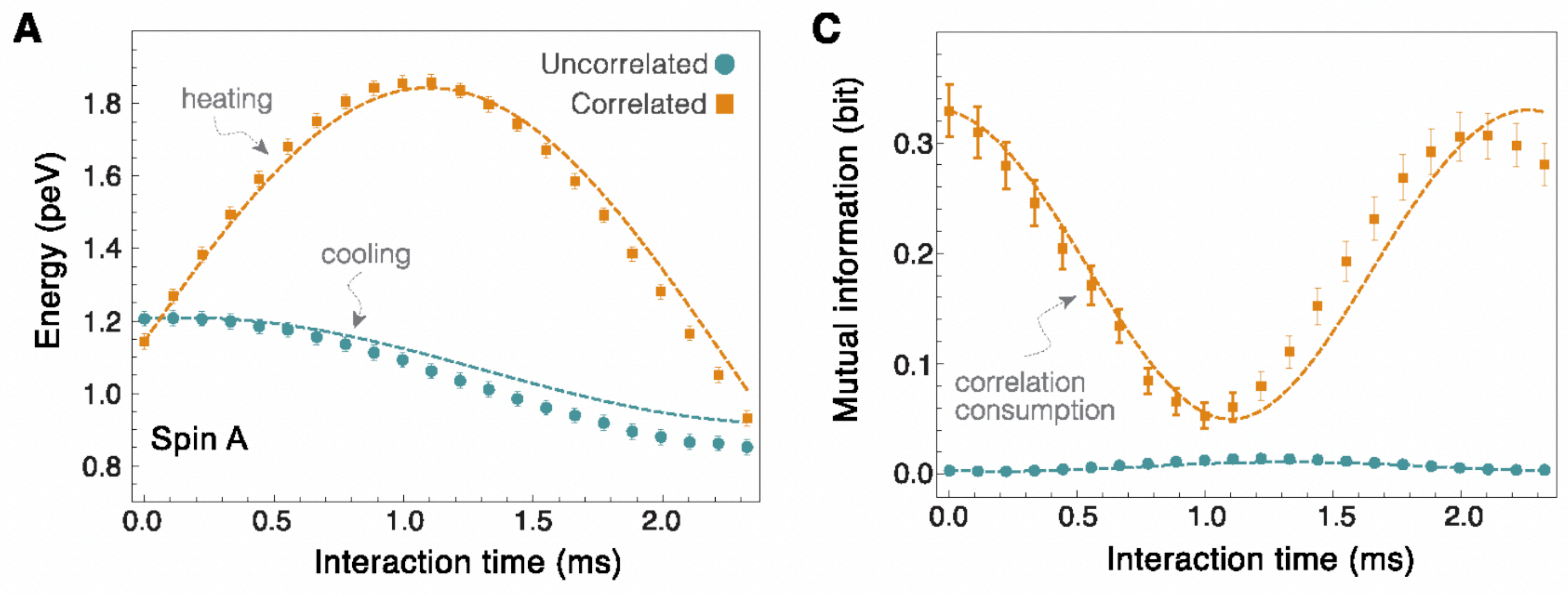
J. P. Santos, L. Céleri, GTL, M. Paternostro, npj Quantum Information **5**, 23 (2019)

Consuming quantum correlations



- In the presence of initial correlations the second law has to be modified to

$$\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h \geqslant \Delta I(h:c)$$

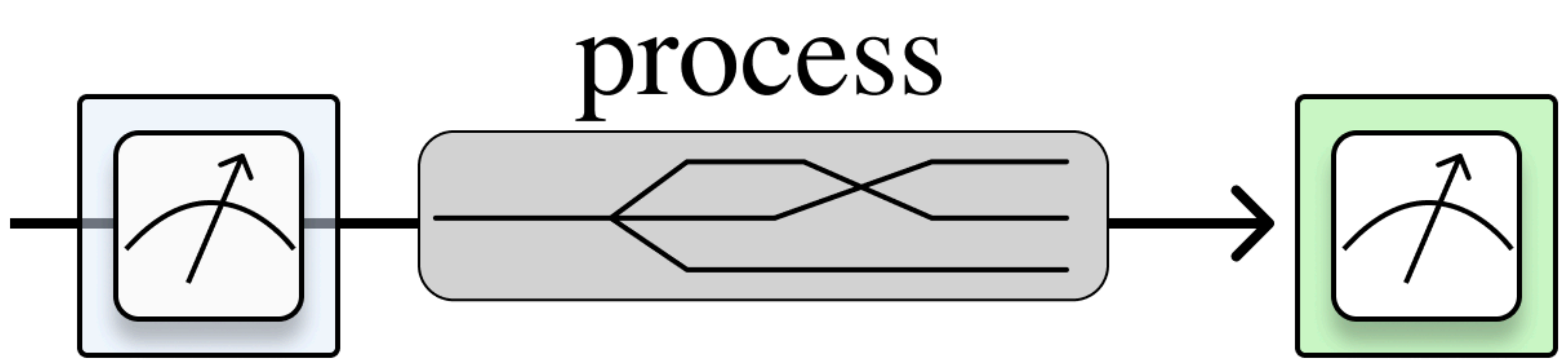


- Heat can flow from cold to hot, provided we **consume** quantum correlations: $\Delta I < 0$.

Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, GTL, Tiago B. Batalhão, Roberto M. Serra, Eric Lutz, “**Reversing the direction of heat flow using quantum correlations**”, *Nature Communications*, **10**, 2456 (2019)

Partovi, M. H., Phys. Rev. E, 77, 021110 (2008)
Jennings, D. & Rudolph, T., Phys. Rev. E, 81, 061130 (2010)

Quantum thermo is extrinsic



- Heat & work are properties of the **process/transformation**, not functions of state.
- For example, suppose process is a unitary $|\psi_f\rangle = U|\psi_i\rangle$. Change in energy is

$$\Delta H = \langle\psi_i|U^\dagger H U|\psi_i\rangle - \langle\psi_i|H|\psi_i\rangle$$

- ***But how would we actually measure this in the lab?***
  - Must measure energy **before** and **after** a process: **two-point measurement** (TPM) scheme.
  - In quantum mechanics measurements have a **back action**. What we actually get is

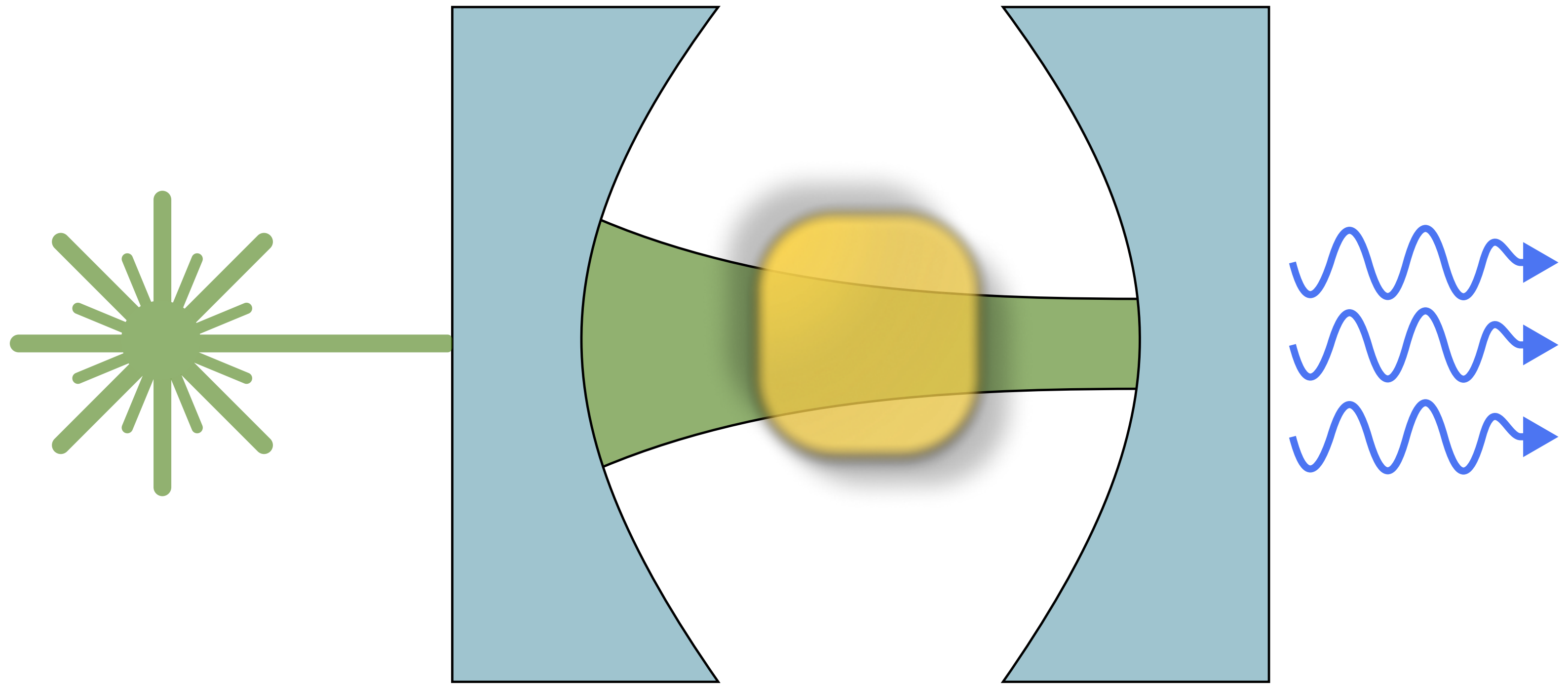
$$\Delta E = \sum_n |\langle n|\psi_i\rangle|^2 \langle n|U^\dagger H U|n\rangle - \langle\psi_i|H|\psi_i\rangle \qquad \text{(where } H|n\rangle = E_n|n\rangle\text{)}$$

  - The first measurement destroy quantum coherences!
- Can we avoid this somehow? I don't think so.

K. Micadei, GTL, E. Lutz, "**Quantum fluctuation theorems beyond two-point measurements**", Phys. Rev. Lett. 124, 090602 (2020)

Kaonan Micadei, John P. S. Peterson, Alexandre M. Souza, Roberto S. Sarthour, Ivan S. Oliveira, GTL, Roberto M. Serra, Eric Lutz, "**Experimental validation of fully quantum fluctuation theorems**", Phys. Rev. Lett., 127, 180603 (2021).

Quantum phase space



- Many quantum experiments are done using optical cavities with semi-transparent mirrors.
- Photons leaking out $\simeq$ zero temperature bath.
  - Spontaneous emission: excitations can leave, but not return.
- 2nd law is buggy @ $T = 0$: $\sigma = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) Q_h.$
  - Does not include vacuum fluctuations (*present in every measurement*).
- We reformulated the entropy production problem in terms of quantum phase space & the *Wigner function.*

$$\sigma = \left(\frac{1}{T_c^{\text{eff}}} - \frac{1}{T_h^{\text{eff}}}\right) Q_h$$

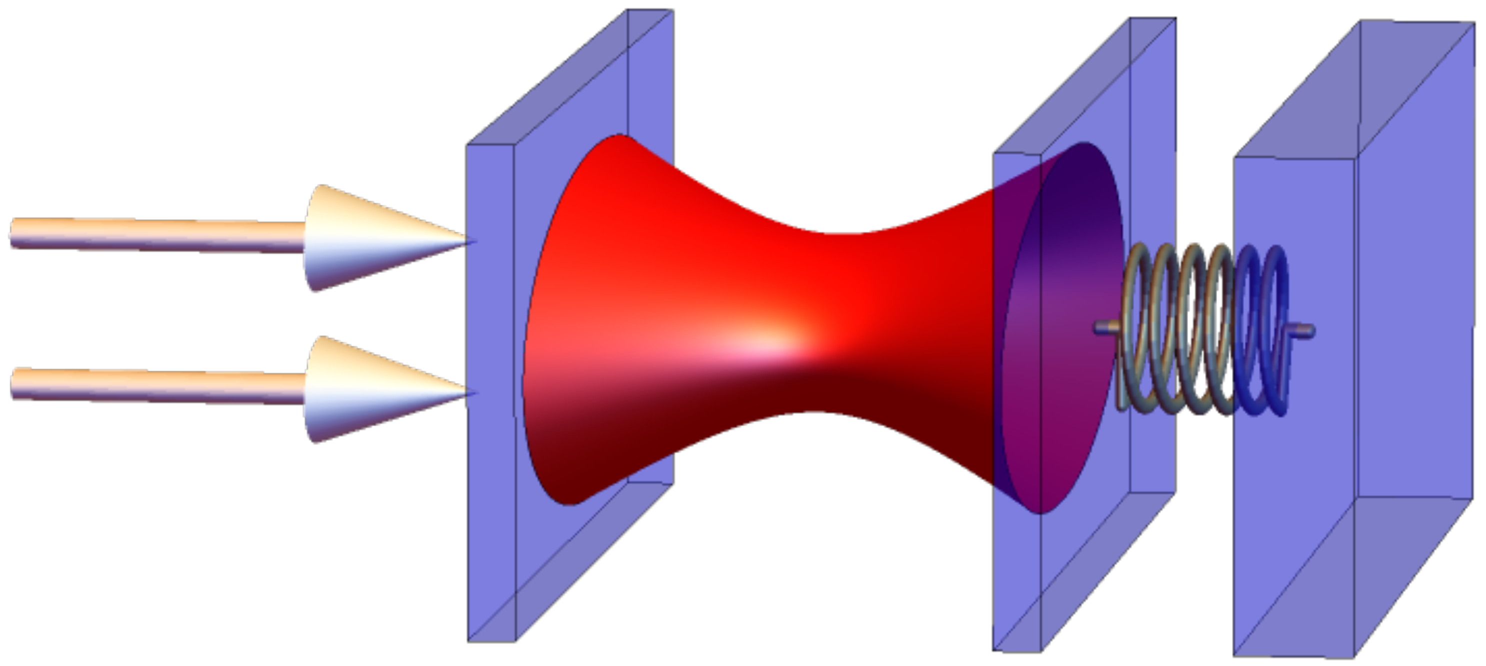
$$T^{\text{eff}} = \omega(\bar{n} + 1/2), \qquad \bar{n} = \frac{1}{e^{\beta\omega} - 1}$$

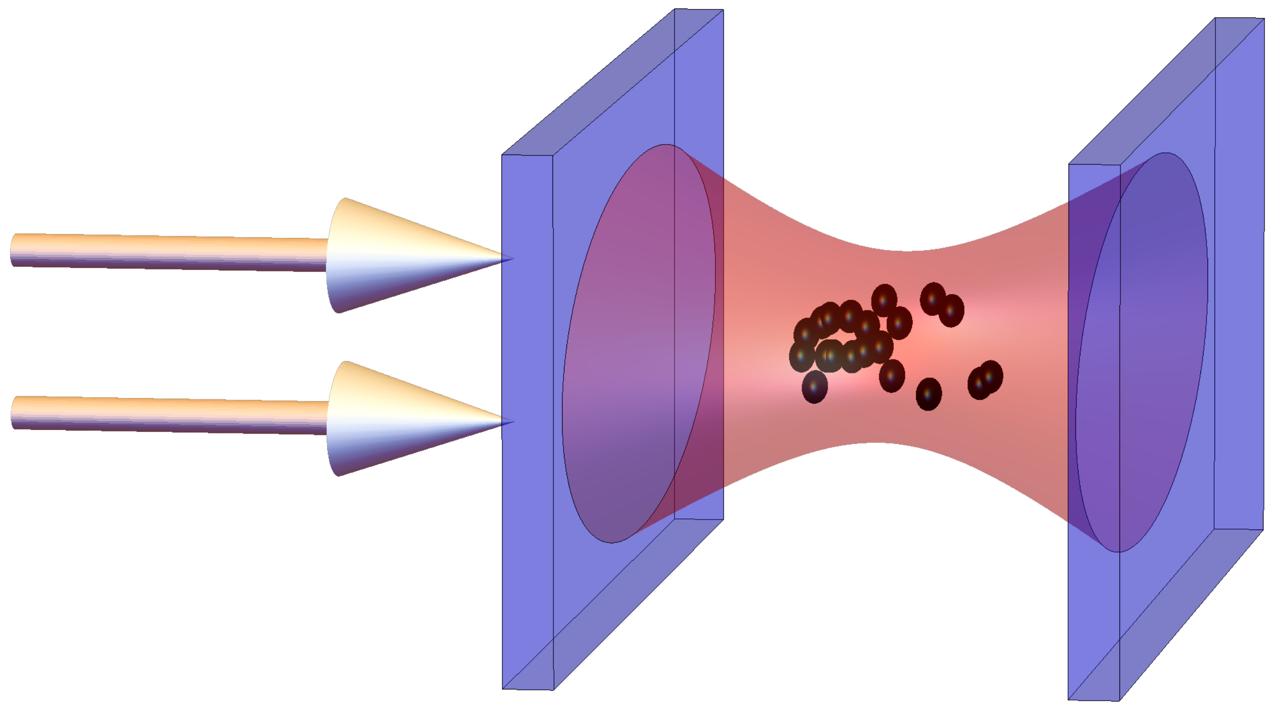
High temperatures: $\omega(\bar{n} + 1/2) \simeq T.$

Zero temperature: $\omega(\bar{n} + 1/2) = \omega/2.$

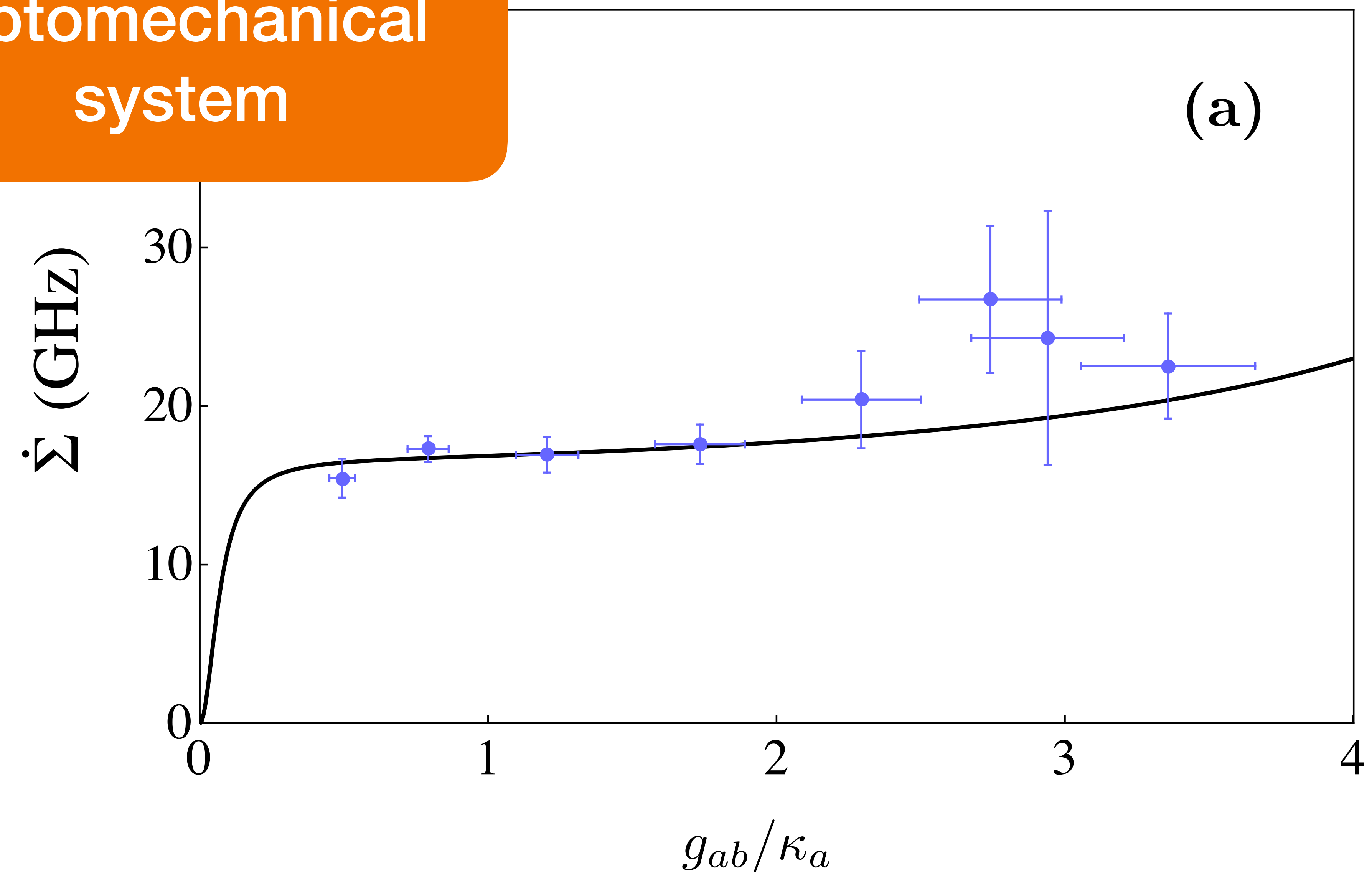
Jader P. Santos, GTL and Mauro Paternostro, Phys. Rev. Lett, **118**, 220601 (2017),

Experiments



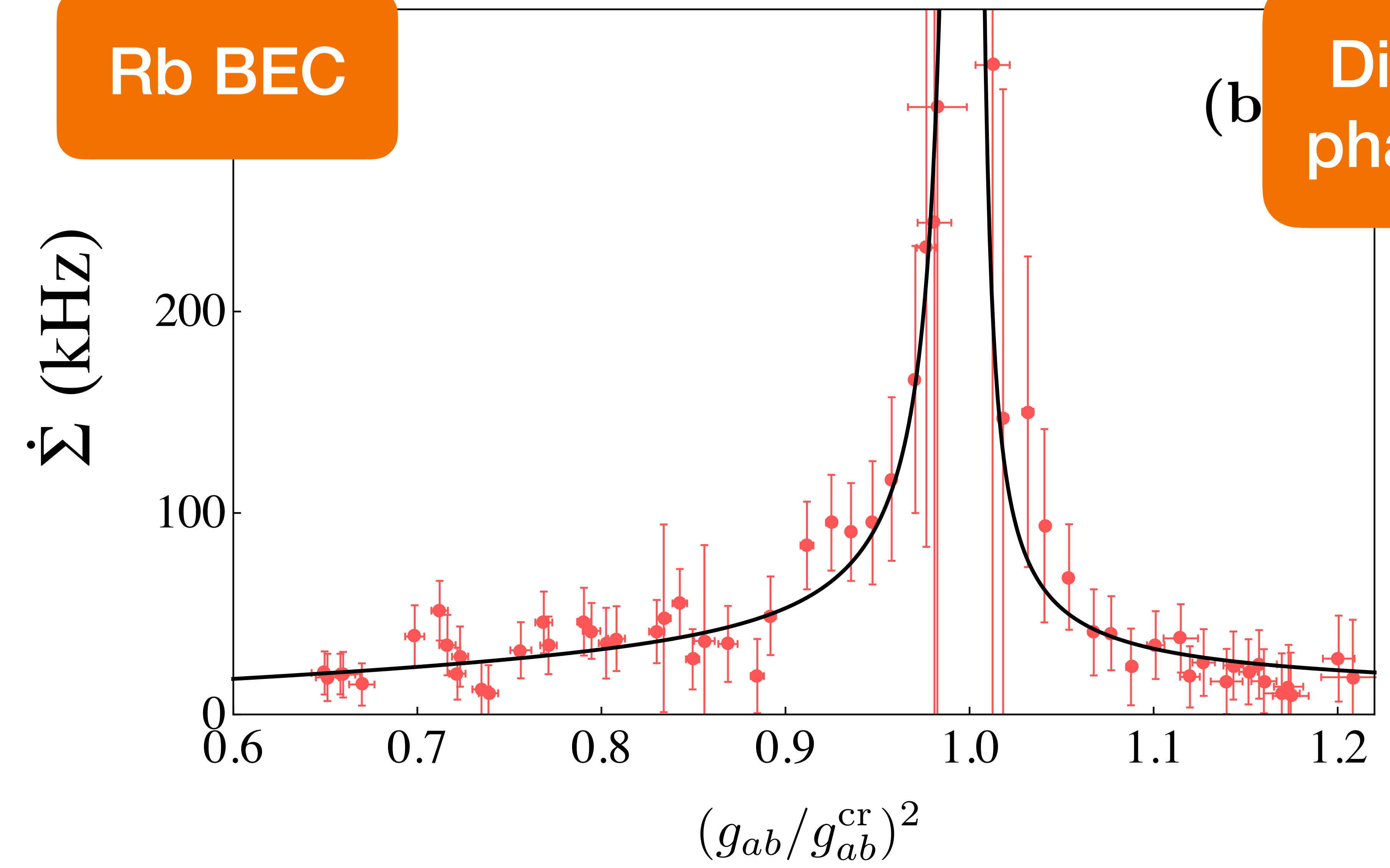


Optomechanical system



Rb BEC

Dicke quantum phase transition.

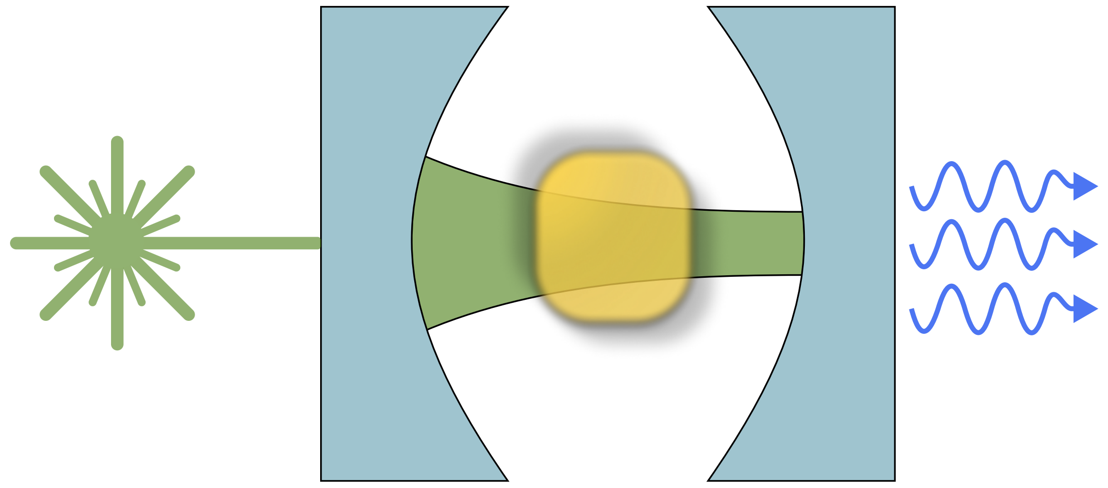


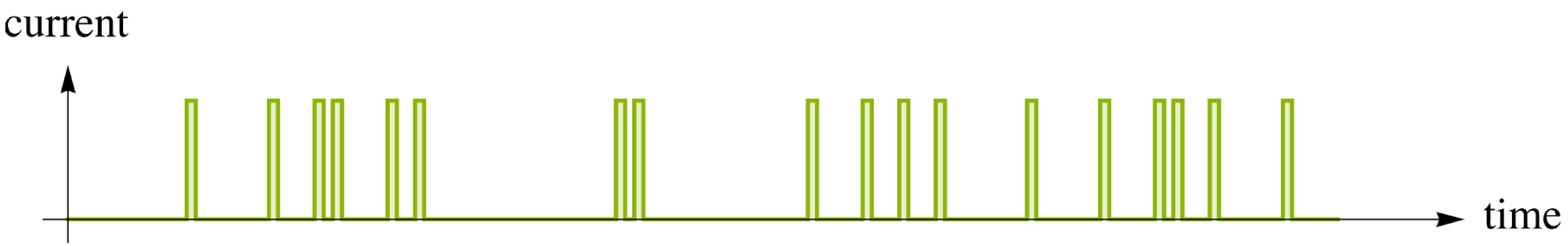
M. Brunelli, L. Fusco, R. Landig, W. Wieczorek, J. Hoelscher-Obermaier, GTL, F Semião, A. Ferraro, N. Kiesel, T. Donner, G. De Chiara, and M. Paternostro. *Phys. Rev. Lett.*, **121**, 160604 (2018)

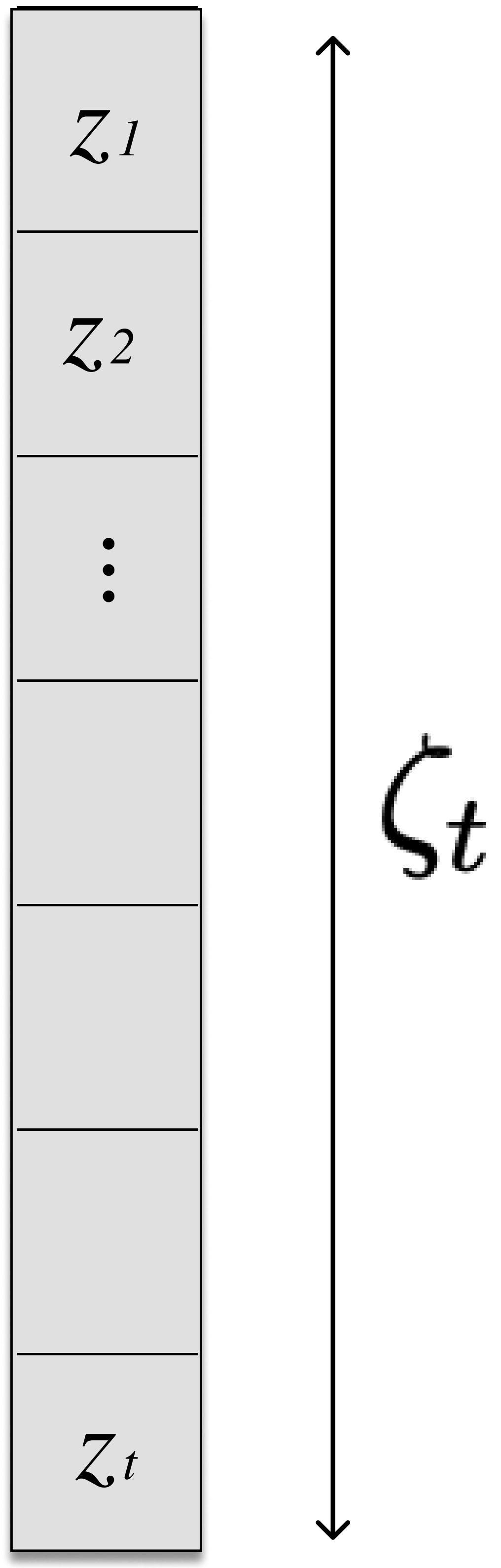
Continuously monitored quantum systems

Continuously monitored quantum systems

- Continuous monitoring of photons that leak out of the cavity.
  - Individual clicks in the detector.
- Fundamental questions: what is entropy production *given* a detection record.
  - Operational: define thermodynamics in terms of what we can actually measure.
  - Includes *information* directly in the formulation.



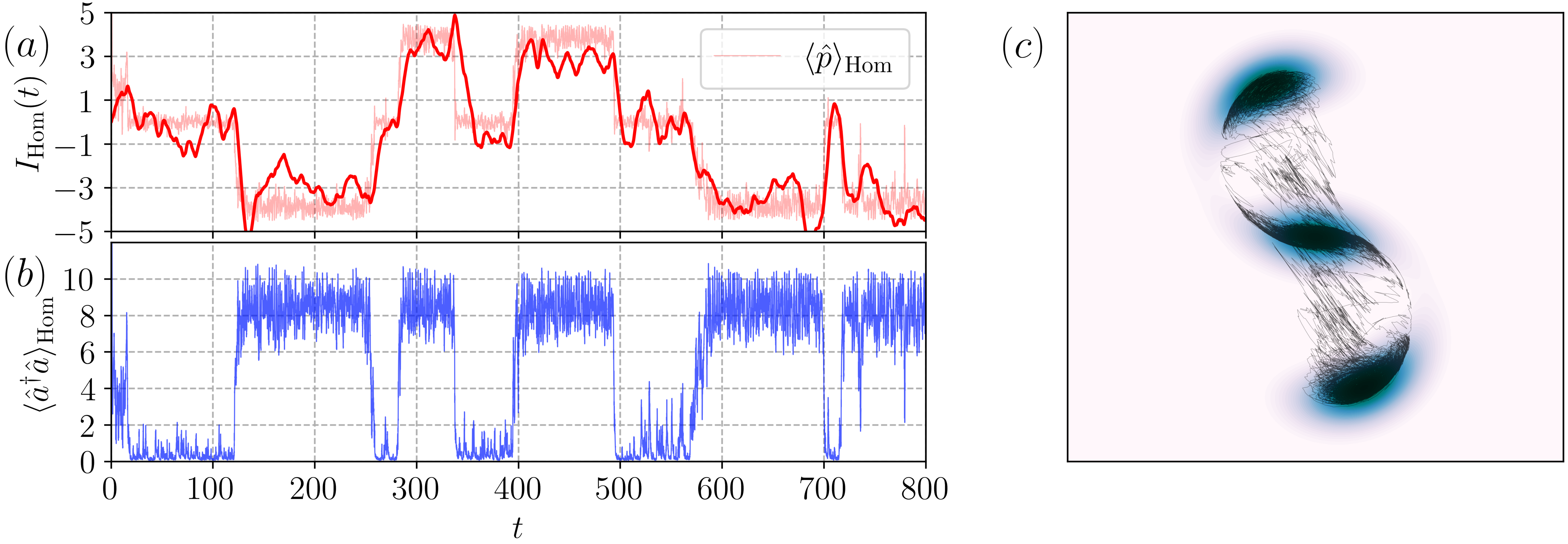




Example: parametric Kerr model

- $H = -\Delta a^\dagger a + \frac{U}{2} a^\dagger a^\dagger a a + \frac{G}{2}(a^{\dagger 2} + a^2)$

- We can reconstruct our best guess for the system's state: ***quantum trajectories***



GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics**," PRX Quantum 5, 020201 (2024)

Holevo reduction to entropy production

- **Unconditional:** If we do not know the individual clicks: ρ_t
- **Conditional on the detection record:** $\rho_{t|\zeta_t}$
- **Holevo information:** accumulated information we learned from the detection.

$$I(S_t : \zeta_t) = \sum_{\zeta_t} P(\zeta_t) D\left(\rho_{t|\zeta_t} || \rho_t\right)$$

- With each new detection

$$\Delta I_t = G_t - L_t = \text{gain} - \text{loss}$$

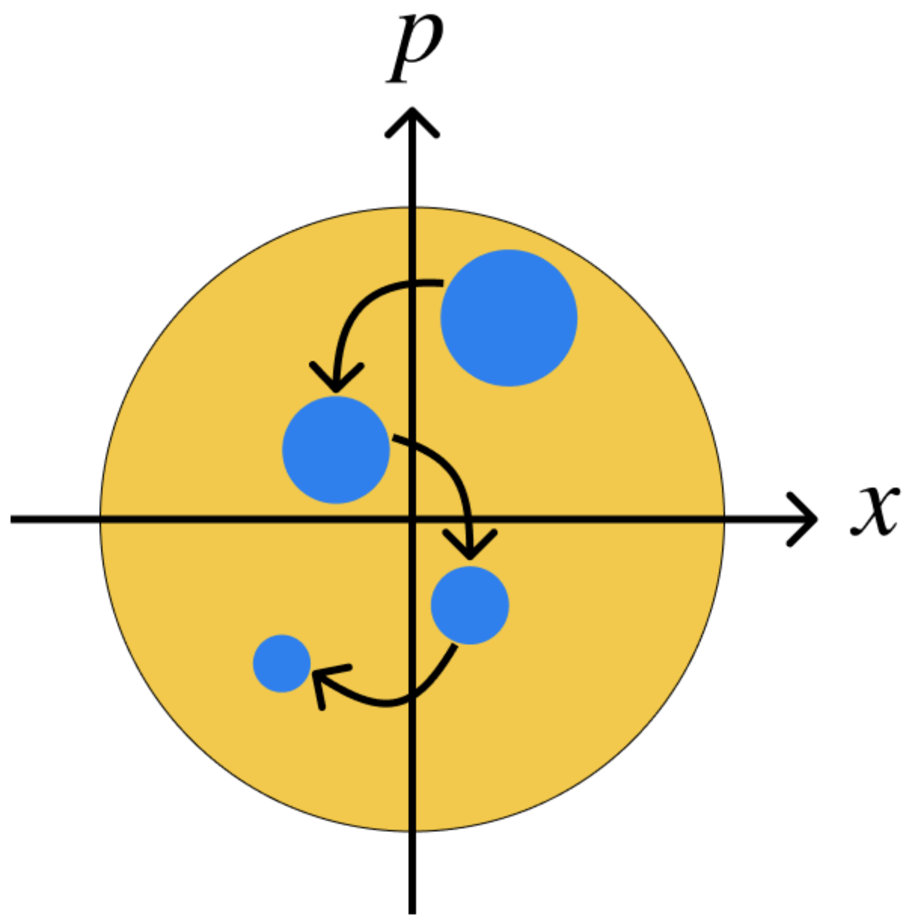
- Conditional entropy production

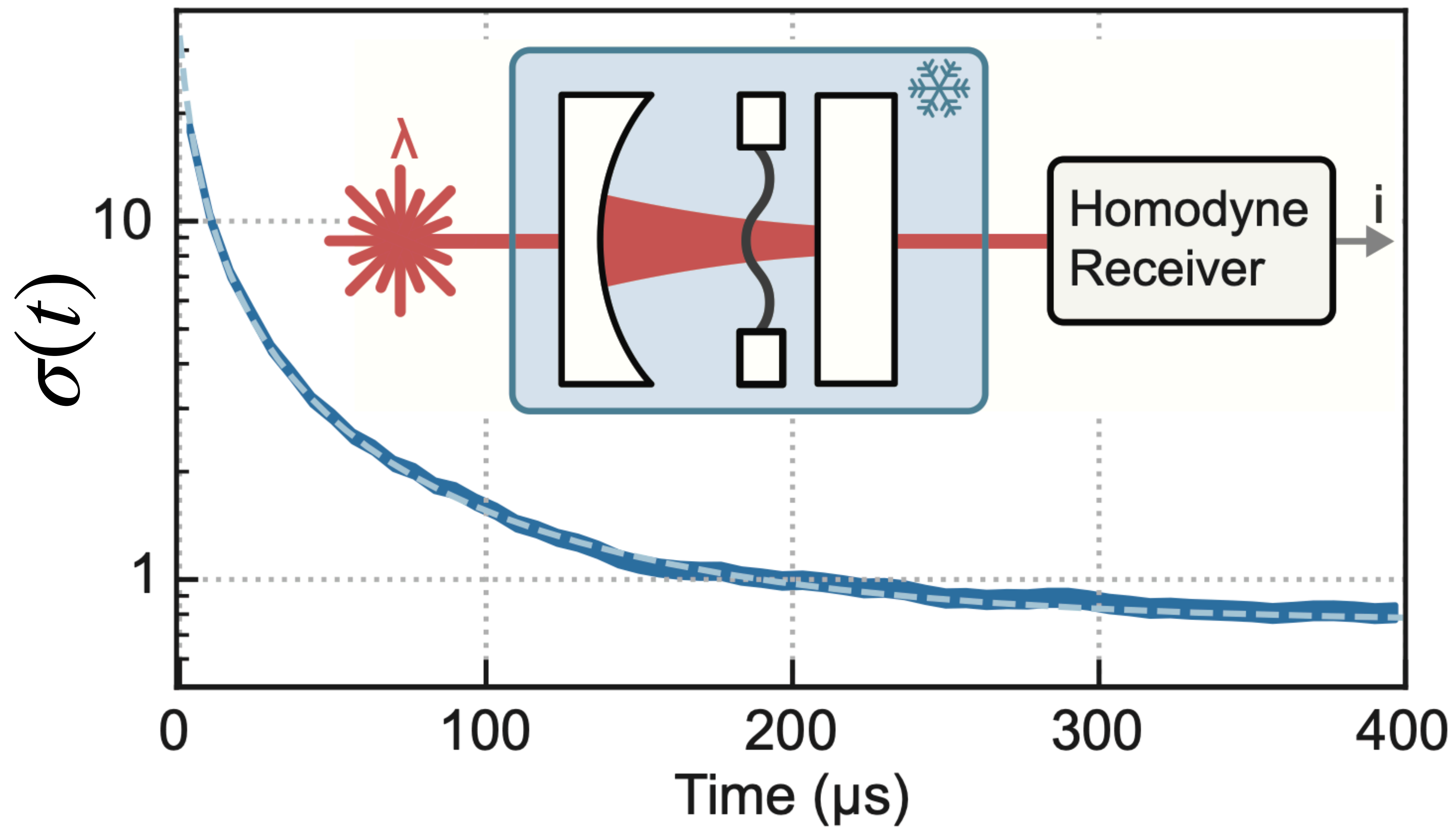
$$\Delta \Sigma^c = \Delta \Sigma^u - \Delta I$$

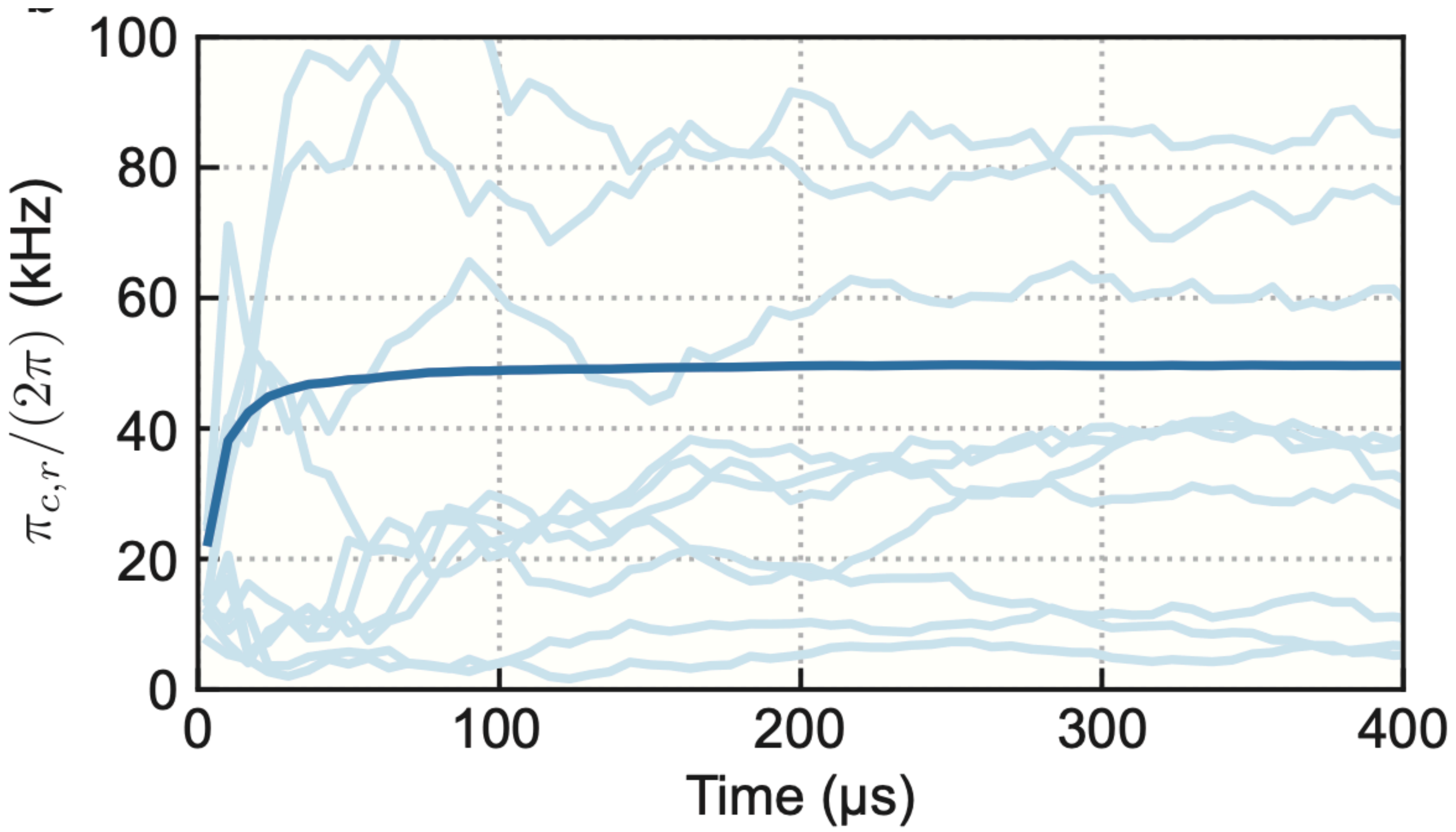
Alessio Belenchia, Luca Mancino, GTL and Mauro Paternostro, "**Entropy production in continuously measured quantum systems**", npj Quantum Information, **6**, 97 (2020).

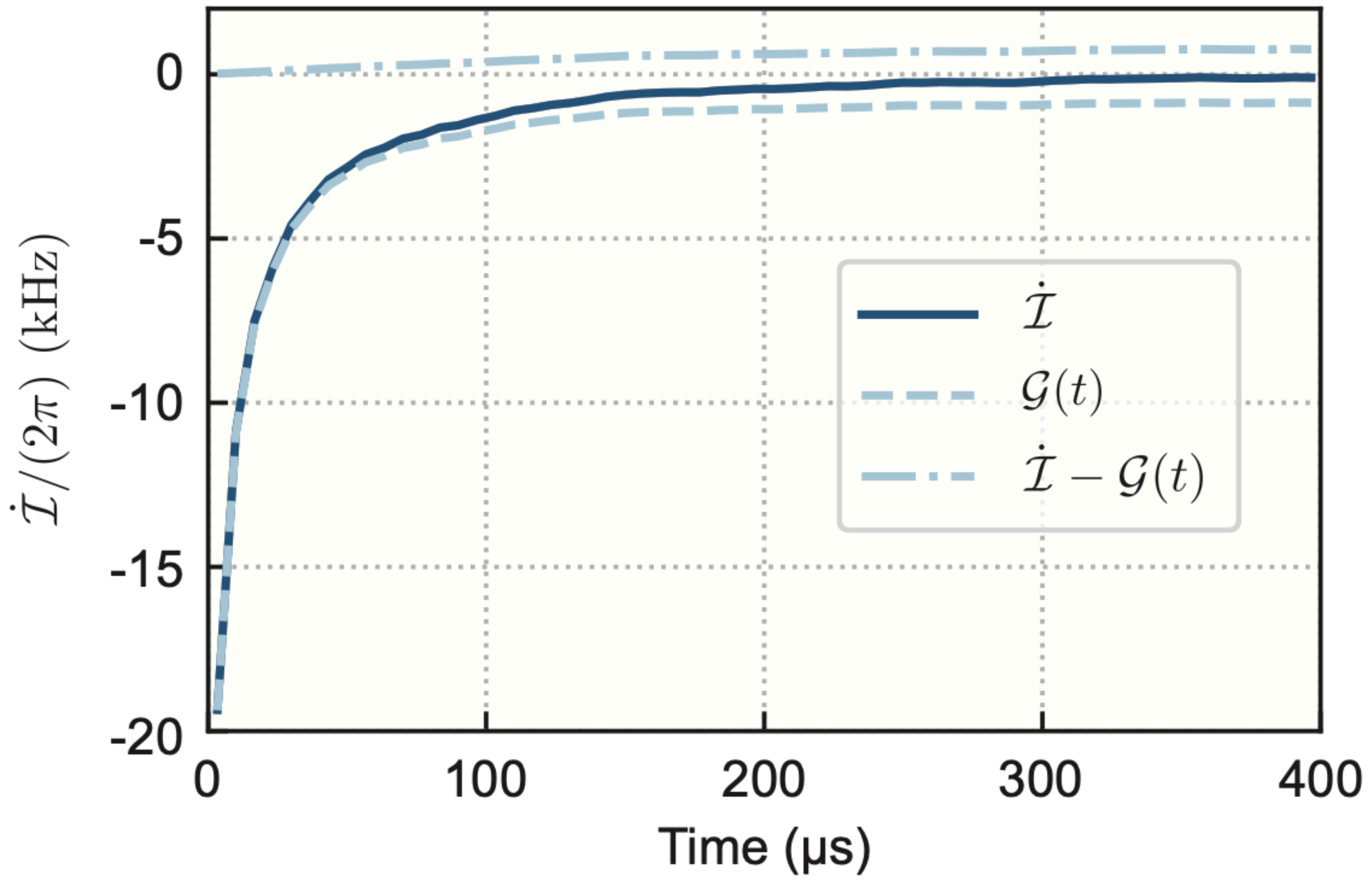
GTL, Mauro Paternostro and Alessio Belenchia, "**Informational steady-states and conditional entropy production in continuously monitored systems**", PRX Quantum **3**, 010303, (2020).

🇩🇰 Optomechanical setup









Informational steady-state:

Conditional dynamics relaxes to a colder state, which can only be maintained by continuing to monitor the system.

Massimiliano Rossi, Luca Mancino, GTL, Mauro Paternostro, Albert Schliesser, Alessio Belenchia, "**Experimental assessment of entropy production in a continuously measured mechanical resonator**", Phys. Rev. Lett. **125**, 080601 (2020)

Conclusions

Thank you!

- **Entropy production** quantifies the irreversibility of a process.
- In the quantum realm it characterizes:
  - Decoherence & loss of information.
  - Quantum correlations.
  - Zero-temperature fluctuations.
  - Measurement back action.

https://www.pas.rochester.edu/~gtlandi

GTL and Mauro Paternostro, "**Irreversible entropy production, from quantum to classical**", *Review of Modern Physics*, **93**, 035008 (2021)

GTL, Dario Poletti, Gernot Schaller, "**Nonequilibrium boundary-driven quantum systems: Models, methods, and properties.**" Reviews of Modern Physics, 94, (2022)

GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics**," PRX Quantum 5, 020201 (2024)

- Suppose a system evolves according to some unitary $|\psi_f\rangle = U|\psi_i\rangle$.
- Say we want to measure the energy change:

$$\Delta H = \langle\psi_i|U^\dagger H U|\psi_i\rangle - \langle\psi_i|H|\psi_i\rangle$$

- ***How would we actually measure this in the lab?***
- Measure the energy before and after the process. ***Two-point measurement (TPM) scheme.***
- Basis $H|n\rangle = E_n|n\rangle$.
  - Before: measure $|\psi_i\rangle$ obtain energy E_n with probability $p_n = |\langle n|\psi_i\rangle|^2$.
    - System collapses to $|n\rangle$.
  - Evolve: $|n\rangle \to U|n\rangle$
  - Measure again. Obtain E_m with probability $p_{m|n} = |\langle m|U|n\rangle|^2$.
  - Change in energy was

$$\Delta E = \sum_{n,m} (E_m - E_n)|\langle m|U|n\rangle|^2|\langle n|\psi_i\rangle|^2$$