Sequential quantum measurements and the stochastic operation of thermal machines

Prof. Gabriel T. Landi
University of Rochester

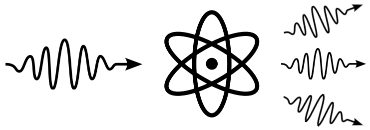
October 17
What does the future hold for AMO theory?
ITAMP - Cambridge

https://www.pas.rochester.edu/~gtlandi

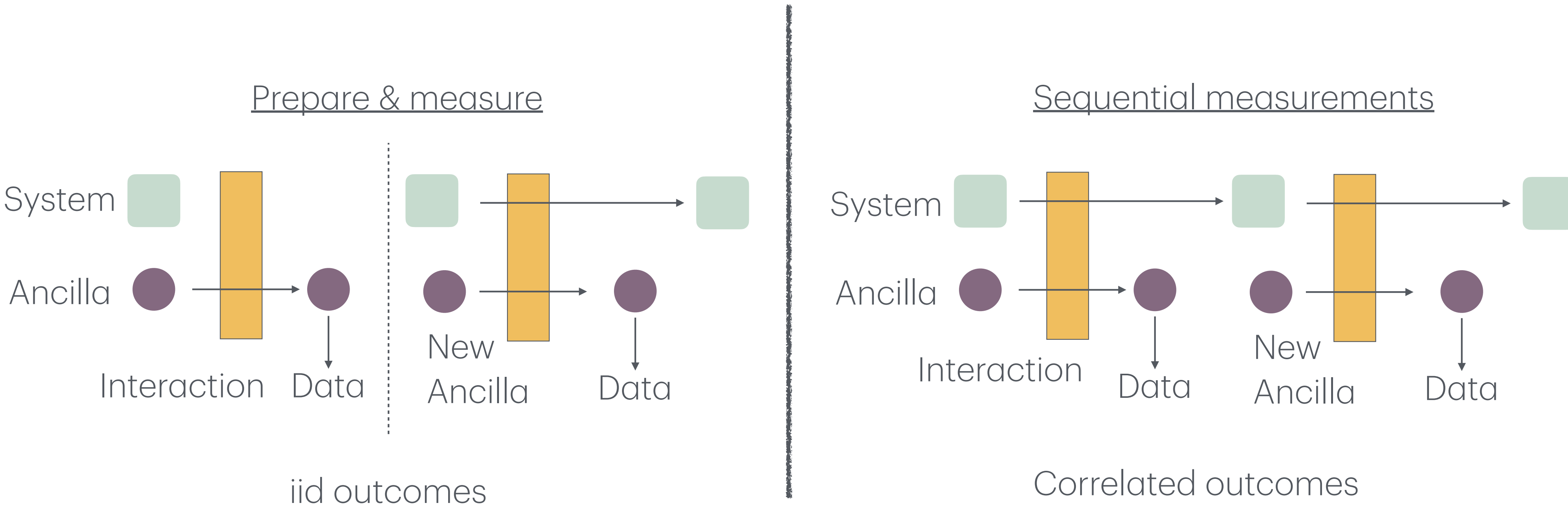
We cannot see quantum systems…

All we see is data …1110000100010011100111101100…

- To measure a system we must send in a **probe** (or **ancilla**).
    - S+A interaction encodes information about S on A.
    - Extract information by measuring A.



- **Information-back action trade-off:** the more information we want, the more we disturb the system.



A simple example

- Qubit: apply unitary U then measure in the computational basis $P_x = |x\rangle\langle x|$ where $x = 0,1$.
- Start in $|\psi_0\rangle$.

  1. Sample first outcome x_1 from $p(x_1) = |\langle x_1 | U | \psi_0 \rangle|^2$.
     Update state to $|\psi_1\rangle = |x_1\rangle$.

  2. Sample second outcome x_2 from $p(x_2 | x_1) = |\langle x_2 | U | x_1 \rangle|^2$.
     Update state to $|\psi_2\rangle = |x_2\rangle$.

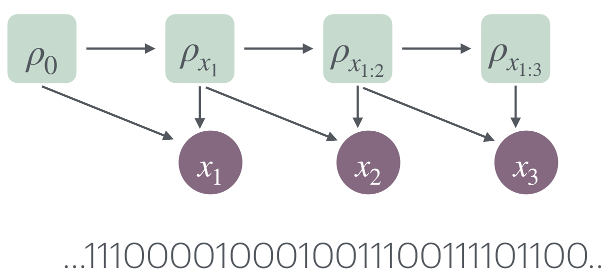
- Generates a **bitstring of emitted symbols** $x_{1:n} = (x_1, \ldots, x_n)$.
- Probability of a sequence forms a Markov chain: $P(x_1, \ldots, x_n) = p(x_n | x_{n-1}) \ldots p(x_2 | x_1) p(x_1)$.

Non-projective measurements lead to long memory

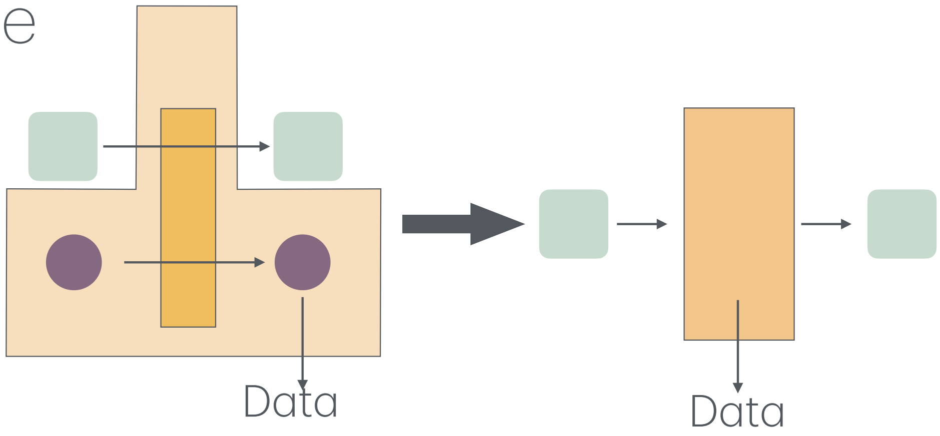
- Apply a set of Kraus operators $\sum_x F_x^\dagger F_x = 1$. Starting at ρ_0:
  1. Sample first outcome x_1 from $p(x_1) = \mathrm{tr}\{F_{x_1}\rho_0 F_{x_1}^\dagger\}$. Update state to $\rho_{x_1} = \dfrac{F_{x_1}\rho_0 F_{x_1}^\dagger}{p(x_1)}$.
  2. Sample second outcome x_2 from $p(x_2 | x_1) = \mathrm{tr}\{F_{x_2}\rho_{x_1} F_{x_2}^\dagger\}$. Update state to $\rho_{x_{1:2}} = \dfrac{F_{x_2}\rho_{x_1} F_{x_2}^\dagger}{p(x_2 | x_1)}$.

$$p(x_{n+1} | x_{1:n}) = \mathrm{tr}\left\{F_{x_{n+1}}\rho_{x_{1:n}} F_{x_{n+1}}^\dagger\right\} \quad \text{and} \quad \rho_{x_{1:n+1}} = \frac{F_{x_{n+1}}\rho_{x_{1:n}} F_{x_{n+1}}^\dagger}{p(x_{n+1} | x_{1:n})}$$

- String probability is now $P(x_{1:n}) = p(x_n | x_{1:n-1})p(x_{n-1} | x_{1:n-2})\ldots p(x_2 | x_1)p(x_1)$ which is highly non-Markovian.
  - *Evolution of the system is Markovian. But output data is not.*
- Looks like a Hidden Markov Model (HMM):
  - Quantum system is hidden.
  - Measurement outcomes (what we see) = **emitted symbols**



Instruments: simplify and generalize



- Instruments = superoperators:

$$M_x\rho = F_x\rho F_x^\dagger$$

- Update rules become:

$$p(x_{n+1}\,|\,x_{1:n}) = \mathrm{tr}\{M_{x_{n+1}}\rho_{x_{1:n}}\}$$

and

$$\rho_{x_{1:n+1}} = \frac{M_{x_{n+1}}\rho_{x_{1:n}}}{p(x_{n+1}\,|\,x_{1:n})}$$

Prob. of a string:

$$P(x_{1:n}) = \mathrm{tr}\{M_{x_N}\ldots M_{x_1}\rho_0\}$$

Conditional state

$$\rho_{x_{1:n}} = M_{x_N}\ldots M_{x_1}\rho_0/P(x_{1:n})$$

Instruments represent the most general kind of measurement possible.

Also encompass inefficient measurements

$$M_x\rho = \sum_{k\in x} F_k\rho F_k^\dagger$$

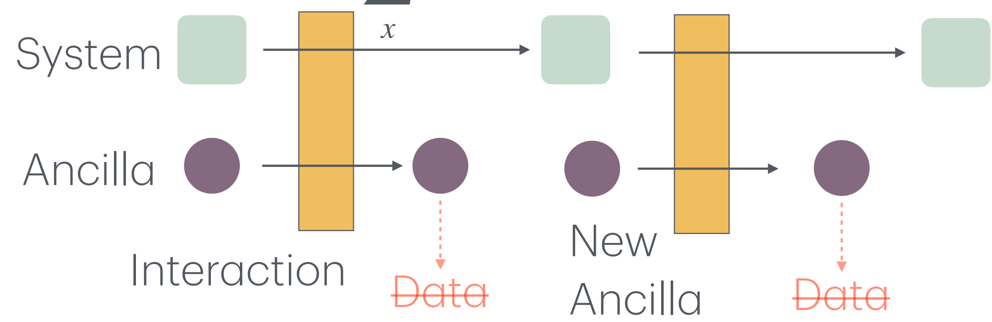
Wiseman, H. M. & Milburn, G. J. **Quantum Measurement and Control.** (Cambridge University Press, New York, 2009)

Unconditional dynamics

- If we measure but don't record the outcome the state of the system still changes (measurement back action)

- Ex: collision model or master equation.

$$\rho' = \sum_x p_x \rho'_x = \sum_x M_x \rho = \mathscr{M}\rho \qquad \text{where} \qquad \mathscr{M} = \sum_x M_x$$

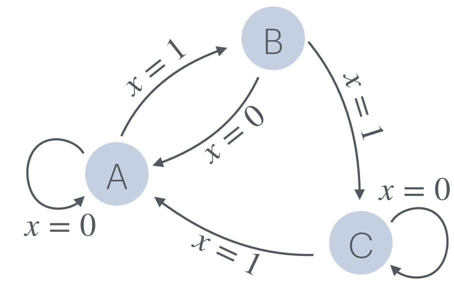


- $\mathscr{M}$ is a quantum channel.

- After n steps: $\rho_n = \mathscr{M}^n \rho_0$.

- Describes the average impact that the interaction with the ancilla causes in the system.

Connection to Hidden Markov Models



- $P(x,\sigma\,|\,\sigma')$ = prob. that system goes from $\sigma' \to \sigma$ while emitting a symbol x.
  - If HMM state is $\pi(\sigma')$ the prob. that we observe symbol x is

$$p(x) = \sum_{\sigma,\sigma'} P(x,\sigma\,|\,\sigma')\pi(\sigma')$$

- If outcome was x, bayesian update the state of the hidden layer:

$$\pi(\sigma\,|\,x) = \frac{P(x,\sigma)}{p(x)} = \frac{\sum_{\sigma'} P(x,\sigma\,|\,\sigma')\pi(\sigma')}{p(x)}$$

- Define substochastic matrices: $(M_x)_{\sigma,\sigma'} = P(x,\sigma\,|\,\sigma')$ and $\langle 1\,| = (1,\ldots,1)$. Then

$$p(x) = \langle 1\,|M_x|\,\pi\rangle \qquad \text{and} \qquad |\,\pi_x\rangle = \frac{M_x\,|\,\pi\rangle}{p(x)}$$

Compare with

$$p(x) = \mathrm{tr}\{M_x\rho\}$$

and

$$\rho_x = \frac{M_x\rho}{p(x)}$$

Milz, S. & Modi, K. "**Quantum Stochastic Processes and Quantum non-Markovian Phenomena**".
PRX Quantum 2, 030201 (2021)

Prediction

- **Unifilar models:** if we know $\rho_{x_{1:n}}$ and we observe x_{n+1} we know with certainty that the system evolved to $\rho_{x_{1:n+1}}$.

- Usefulness: data compression

$$p(x_{n+1} \,|\, x_{1:n}) = p(x_{n+1} \,|\, \rho_{x_{1:n}})$$

If we can know the internal state, we can make statistical predictions of future outcomes.

- Example: figuring out the internal state of a large language model.

F. Binder, J. Thompson, M. Gu, "**Practical unitary simulator for non-Markovian complex processes**," *Phys. Rev. Lett.* **120** 240502 (2018).

Quantum jumps

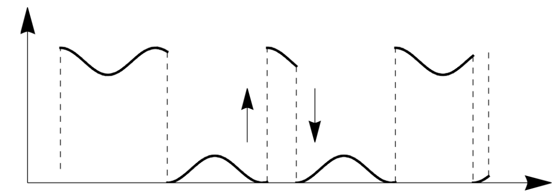
Mark Mitchison

Michael Kewming

Patrick Potts

GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts **"Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics,"** PRX Quantum 5, 020201 (2024)

GTL "**Patterns in the jump-channel statistics of open quantum systems**," arXiv 2305.07957



- Consider a quantum master equation

$$\frac{d\rho}{dt} = \mathscr{L}\rho = -i[H,\rho] + \sum_{x=1}^{r} L_x\rho L_x^\dagger - \frac{1}{2}\{L_x^\dagger L_x, \rho\}$$

- The infinitesimal evolution can be written as a set of instruments:

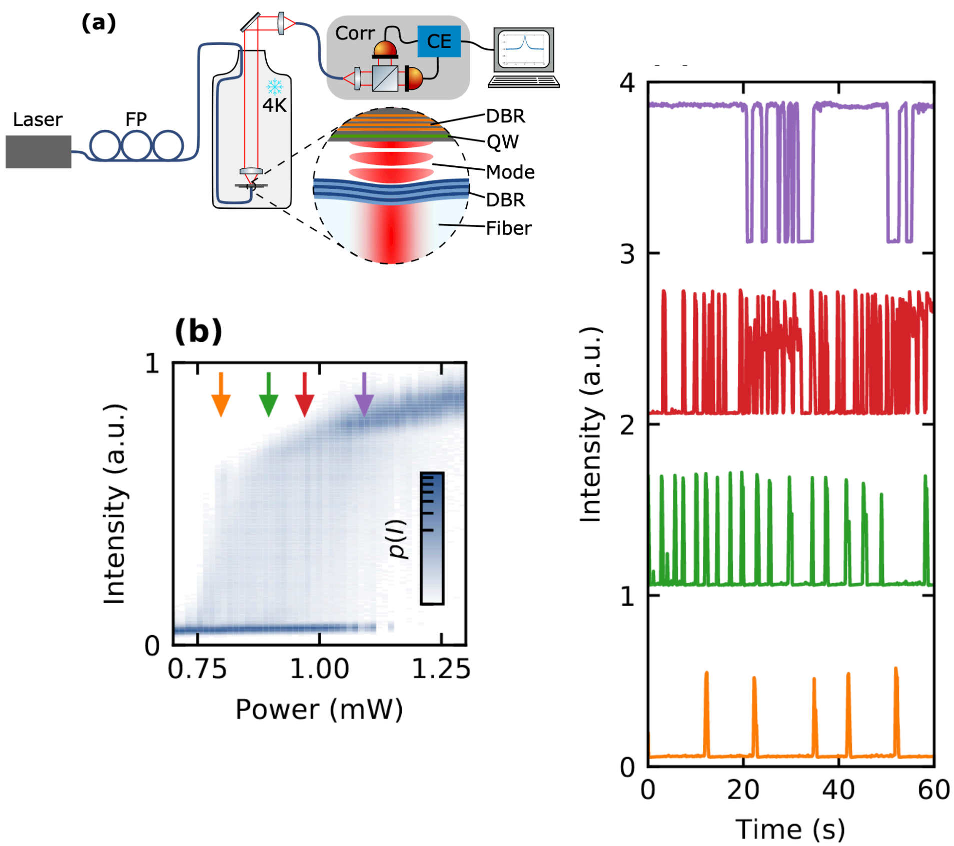
$$\rho_{t+dt} = e^{\mathscr{L}dt}\rho_t = \sum_x M_x\rho_t$$

(jump) $\quad M_x\rho = dt\ L_x\rho L_x^\dagger = dt\ \mathscr{J}_x\rho \quad$ for $\quad x = 1,2,\ldots,r$

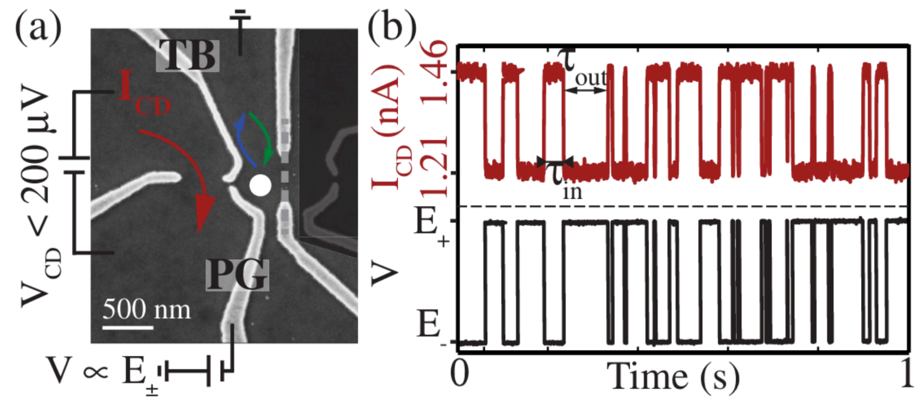
(no jump) $\quad M_0\rho = \rho + dt\mathscr{L}_0\rho \quad$ where $\quad \mathscr{L}_0\rho = -i[H,\rho] - \dfrac{1}{2}\sum_{x=1}^{r}\{L_x^\dagger L_x, \rho\}$

- $p_x = \mathrm{tr}\{M_x\rho\} = dt\,\mathrm{tr}\{L_x^\dagger L_x\rho\}$ is infinitesimal: most of the time the system evolves with no jump.

GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics**," PRX Quantum 5, 020201 (2024)



Fink *et. al.*, "**Signatures of a dissipative phase transition in photon correlation measurements**"
Nature Physics **14** 365-369 (2018)



Hofmann, *et. al.* "**Measuring the Degeneracy of Discrete Energy Levels Using a GaAs / AlGaAs Quantum Dot**,"
Phys Rev. Lett **117**, 206803 (2016)

Jumps with multiple channels

- Each jump operator L_x is a “channel"

$$\frac{d\rho}{dt} = \mathscr{L}\rho = -i[H,\rho] + \sum_{x=1}^{r} L_x \rho L_x^\dagger - \frac{1}{2}\{L_x^\dagger L_x, \rho\}$$

- t-ensemble: final time is fixed, total number of jumps is a random variable.
- N-ensemble: total number of jumps is fixed, final time is a random variable.

- Jumps occur over random times and over random channels.

- Quantum trajectory = list of channels and their corresponding time-tags:

$$(x_1, \tau_1), (x_2, \tau_2), \ldots, (x_N, \tau_N) \qquad \tau_j = t_j - t_{j-1}$$

One-jump instrument

$$M_{x\tau} = \mathscr{J}_x e^{\mathscr{L}_0 \tau}$$

Quantum jumps without time-tags:

$$M_x = -\mathscr{J}_x \mathscr{L}_0^{-1}$$

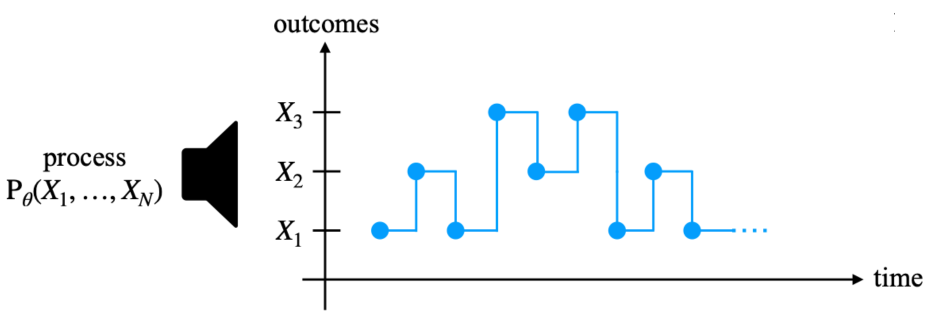
GTL "**Patterns in the jump-channel statistics of open quantum systems**," arXiv 2305.07957

Parameter estimation with stochastic processes

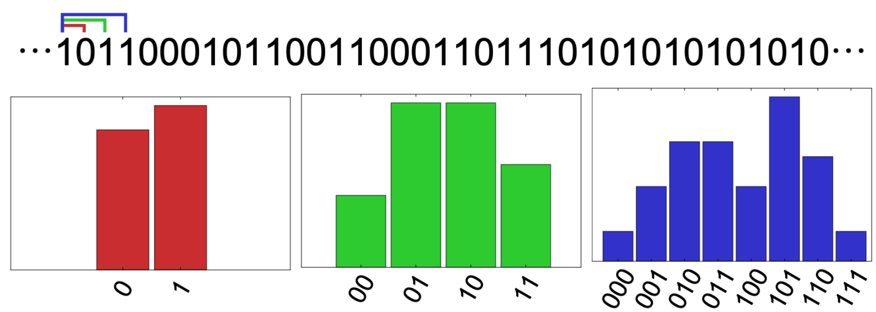
Fisher information in stochastic processes

- For a stochastic process with fixed Markov order M

$$F(X_{1:N}) = F(X_{1:M}) + (N-M)F(X_{M+1}\,|\,X_{1:M}) \simeq NF(X_{M+1}\,|\,X_{1:M})$$



Data compression: count sequences



Radaelli, M., Landi, G. T., Modi, K. & Binder, F. C. **Fisher information of correlated stochastic processes**. New J. Phys. 25, 053037 (2023).
Smiga, J. A., Radaelli, M., Binder, F. C. & Landi, G. T. **Stochastic metrology and the empirical distribution**. Phys. Rev. Research 5, 033150 (2023)
Radaelli, M., Smiga, J. A., Landi, G. T. & Binder, F. C. **Parameter estimation for quantum jump unraveling**. arXiv 2402.06556 (2024)

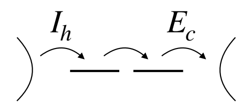
Stochastic operation of thermal machines

Patrick Potts

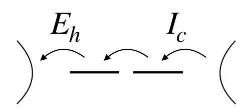
Abhaya Hegde

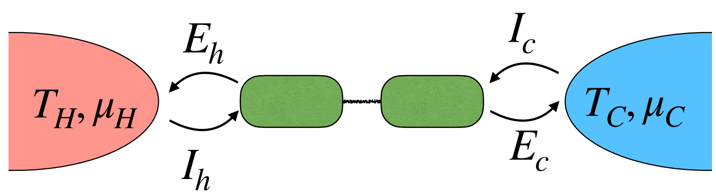
Abhaya S. Hegde, Patrick P. Potts, GTL, "**Time-resolved Stochastic Dynamics of Quantum Thermal Machines**," arXiv:2408.00694

- Double quantum dot
  - Engine process: uses thermal gradient to extract chemical work .

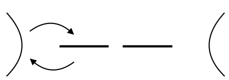


  - Refrigerator process: uses chemical work to make heat flow from cold to hot.

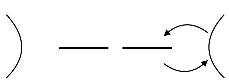




- There can also be “idle cycles”
  - “Idle hot”



  - “Idle cold”



Can we identify individual cycles from a bitstring?

$I_hE_cI_cI_hE_hE_cI_hI_cE_hI_c$

Impossible in general, if excitations are indistinguishable

$$I_cI_hE_hE_c = \begin{cases} I_cI_hE_hE_c \\ I_cI_hE_hE_c \end{cases}$$

Manzano, Gonzalo, and Roberta Zambrini “**Quantum Thermodynamics under Continuous Monitoring: A General Framework**,” AVS Quantum Science 4 (2): 025302 (2022).

Single excitation assumption

$$\frac{d\rho}{dt} = \underbrace{-i[H,\rho]}_{\text{Unitary work}} + \underbrace{\sum_n D[K_n]\rho}_{\text{Work reservoirs}} + \sum_{\alpha\in\{h,c\}} \sum_j \underbrace{\gamma^-_{\alpha j} D[L_{\alpha j}]\rho}_{\text{Extraction to bath } \boldsymbol{\alpha}} + \underbrace{\gamma^+_{\alpha j} D[L^\dagger_{\alpha j}]\rho}_{\text{Injection from bath } \boldsymbol{\alpha}}$$

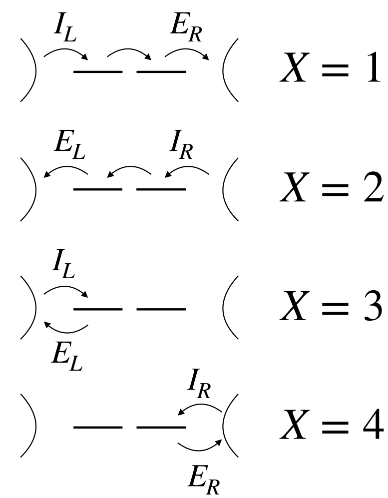
- Result: for cycles to be identifiable the string must always have injections followed by extractions.

$$\ldots I.E.I.E.I.E.I.E.I.E.\ldots$$

- Condition: Hilbert space must be split in 2.
  - $L^\dagger_{\alpha j}$ injects → post-injection subspace.
  - $L_{\alpha j}$ extracts → post-extraction subspace.

$$\text{Hilbert space} = \left(\begin{array}{c:c} \text{Post injection} & \\ \hdashline & \text{Post extraction} \end{array}\right)$$

Bitstrings of jumps → bitstrings of cycles



$$\ldots I_\bullet E_\bullet I_\bullet E_\bullet I_\bullet E_\bullet I_\bullet E_\bullet \ldots = \ldots X_\bullet X_\bullet X_\bullet X_\bullet \ldots$$

- We can use this to answer the following questions:
  - What is the probability that the next cycle is of type X and takes a time $\boldsymbol{\tau}$?
  - How are cycles correlated with each other?
  - What is the average time required to complete each cycle?
  - How many idle cycles happen between two useful cycles?
- Define instruments

$$M_{X\tau} = \int_0^\tau dt\ \mathcal{J}_{E_X} e^{\mathcal{L}_0(\tau - t)} \mathcal{J}_{I_X} e^{\mathcal{L}_0 t}$$

with 2 emitted symbols: $X = 1,2,3,4$ and cycle duration $\boldsymbol{\tau}$

Cycle probabilities

π_E = Jump Steady-State

Correct state to get long-time statistics

- Then prob. a cycle is of type X and takes a time τ: $p_{X,\tau} = \mathrm{tr}\{M_{X\tau}\pi_E\}$.

- If we don't care about how long a cycle takes, we just need to marginalize the instrument:

$$M_X = \int_0^\infty d\tau\ M_{X\tau}$$

Relation to steady-state currents:

$$I = \frac{p_1 - p_2}{E(\tau)}$$

- Prob. of obtaining each cycle type

$$p_X = \mathrm{tr}\{M_X \pi_E\}$$

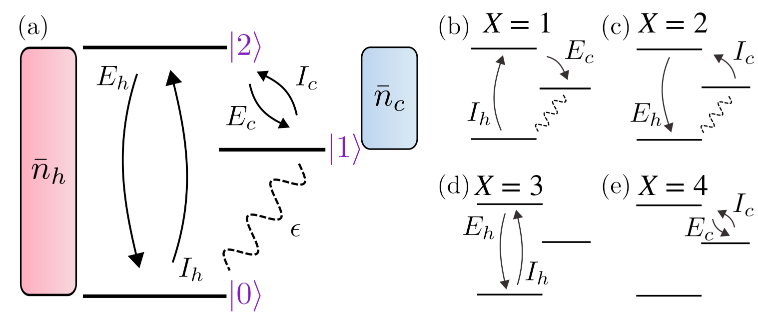
- Conditional cycle times: if cycle is of type X, how long it takes?

$$E(\tau|X) = \int_0^\infty d\tau\ \tau \frac{p_{X,\tau}}{p_X}$$

Correlations between cycles:

$$P(X_1, \tau_1, \ldots, X_n, \tau_n) = \mathrm{tr}\{M_{X_n\tau_n} \ldots M_{X_1\tau_1}\pi_E\}$$

Results for the 3-level maser



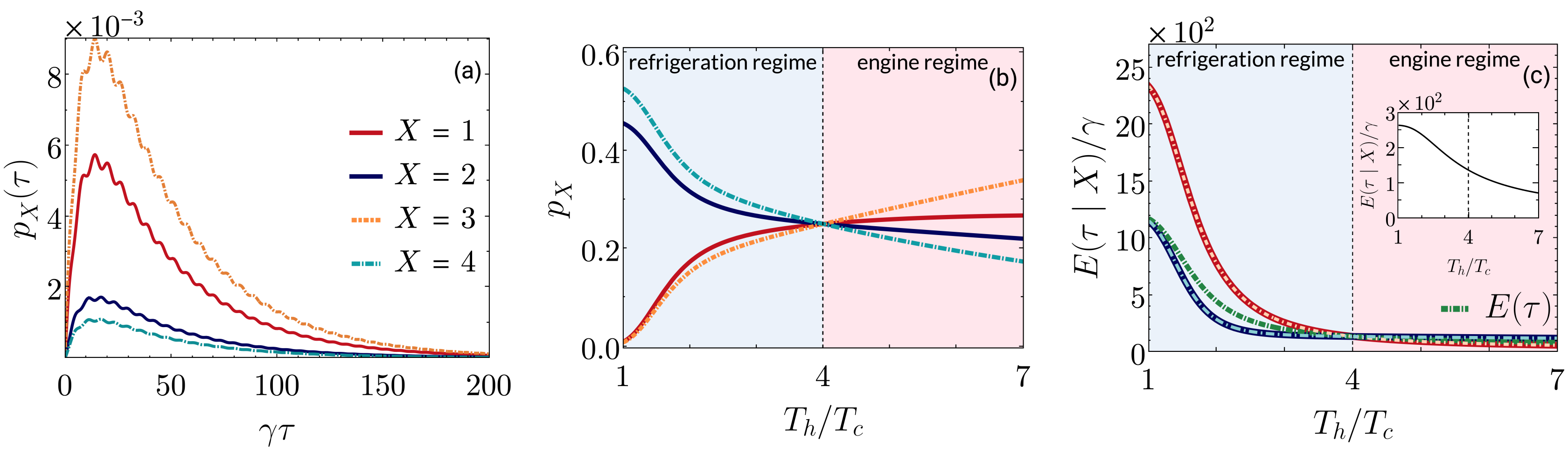
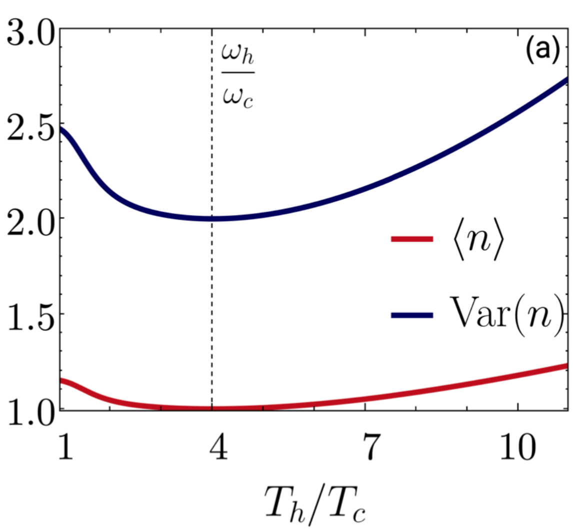


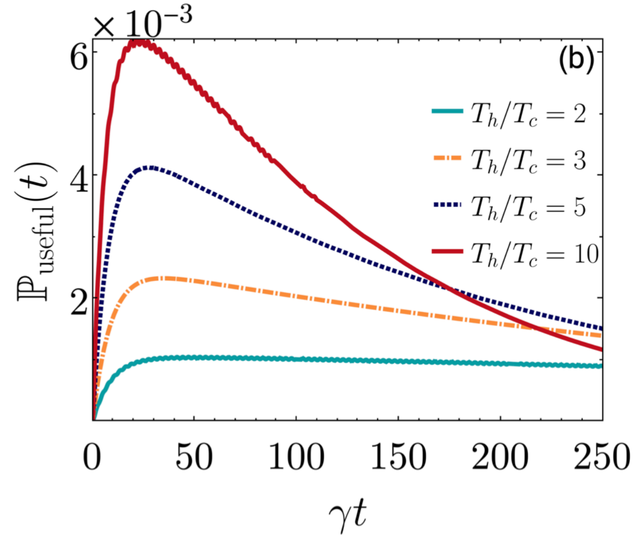
FIG. 3. **(a)** Probability of observing a cycle X within a duration τ [Eq. (9)] at resonance $\Delta = 0$ and $T_h/T_c = 10$. **(b)** Total probability of observing a cycle X [Eq. (10)] and **(c)** expectation values for cycle duration [Eqs. (11), (12)] as a function of the ratio of bath temperatures. A vertical line at $T_h/T_c = \omega_h/\omega_c$ separates the refrigerator and engine regimes. The inset shows all expectation values nearly converge at resonance. The parameters are fixed (in units of $T_c = 1$) at $\gamma_h = \gamma_c \equiv \gamma = 0.05$, $\omega_h = 8$, $\omega_c = 2$, $\omega_d = 4$, $\epsilon = 0.5$ unless mentioned otherwise.

Machine intermittency

Number of idle cycles between two useful cycles



Time between two useful cycles



$M_u = M_{X=1} + M_{X=2}$ and $M_{\text{id}} = M_{X=3} + M_{X=4}$

$$\mathbb{P}_u(n) = \frac{\mathrm{tr}\{M_u M_{\text{id}}^n M_u \pi_E\}}{\mathrm{tr}\{M_u \pi_E\}}$$

$\mathbb{P}_u(t)$ = similar, but a bit more complicated.

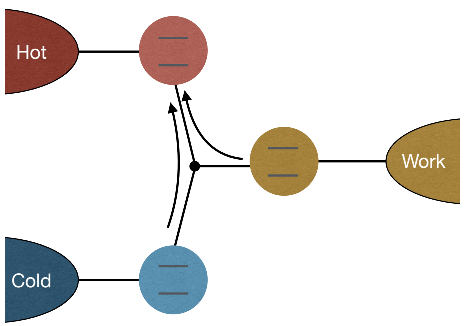
Cooling on demand

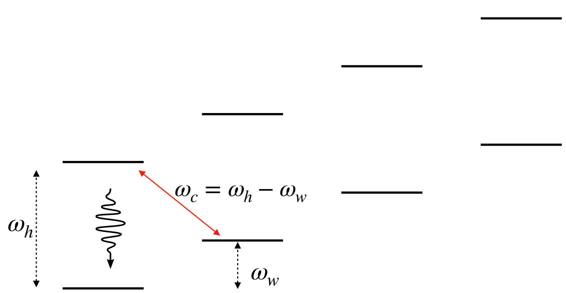
Guilherme Fiusa

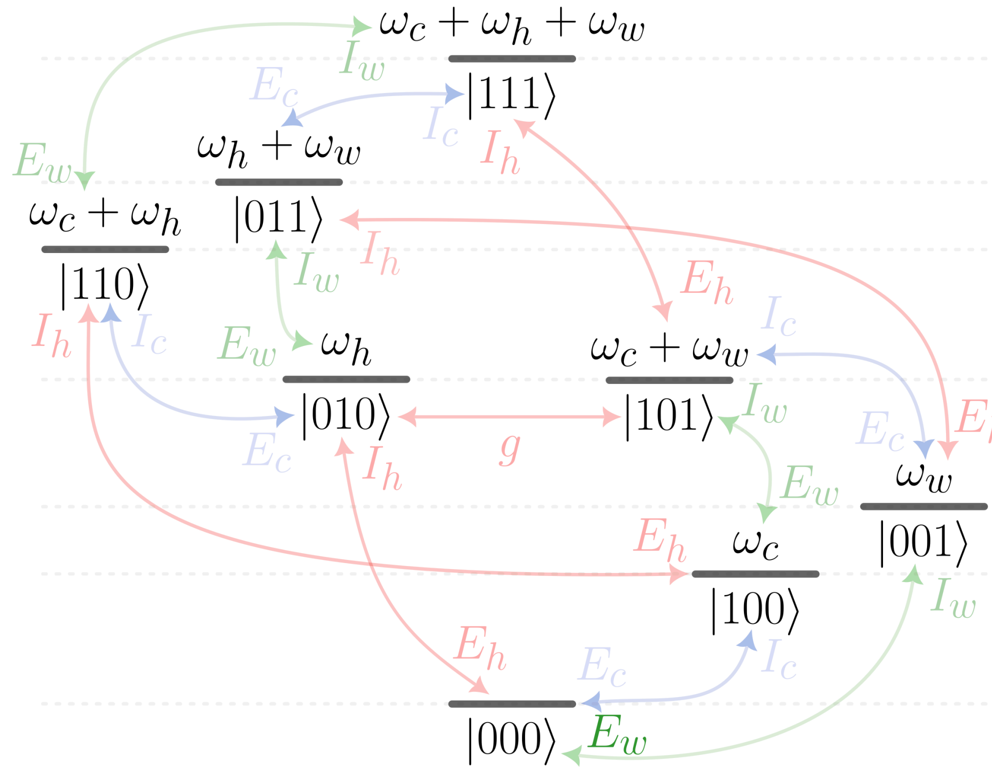
Abhaya Hegde

Quantum absorption refrigerator

- Like sideband laser cooling, but with autonomous heat baths.





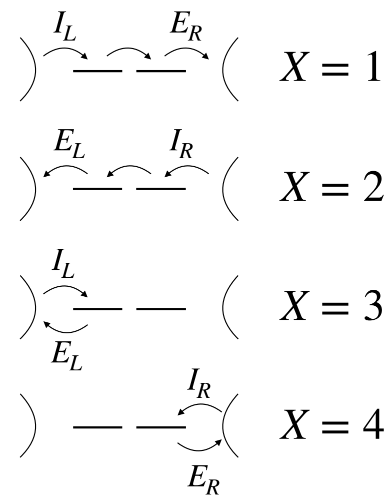


Aamir *et al* "**Thermally driven quantum refrigerator autonomously resets superconducting qubit,**" arXiv:2305.16710

Conclusions

Thank you!

- Sequential quantum measurements → **time-series** of correlated stochastic outcomes.
  - Bayesian inference of the quantum state, given outcomes.
  - Unveiling the thermodynamics from measurement data.
    - Stochastic operation of a thermal machine.
- Open question: machine intermittency vs. current fluctuations?



GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics**," PRX Quantum 5, 020201 (2024)

GTL "**Patterns in the jump-channel statistics of open quantum systems**," arXiv 2305.07957

Abhaya S. Hegde, Patrick P. Potts, GTL, "**Time-resolved Stochastic Dynamics of Quantum Thermal Machines**," arXiv:2408.00694