Thermal machines at the single trajectory level & stochastic excursions

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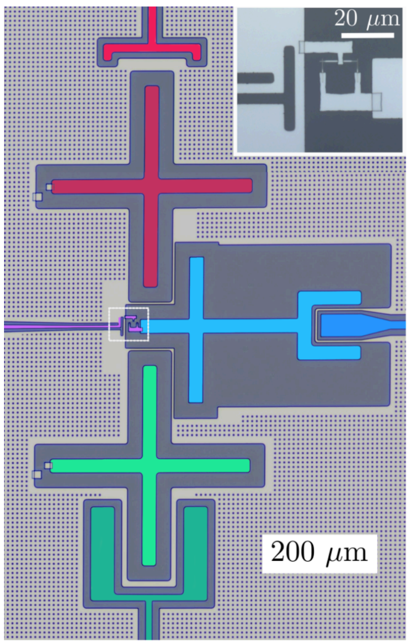
Absorption refrigeration

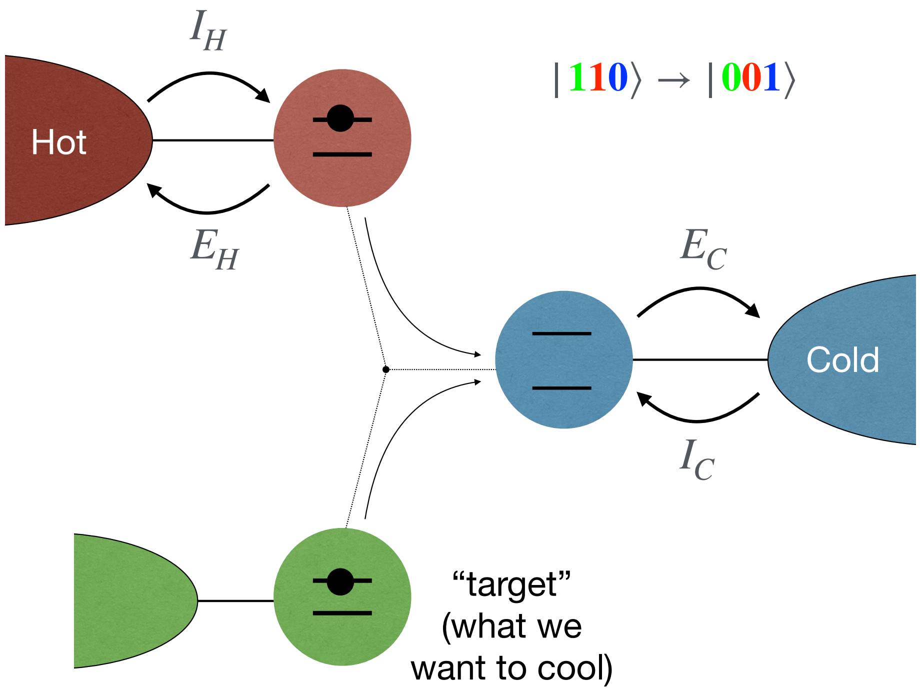
3-body resonant interaction

$$\omega_h + \omega_t = \omega_c$$

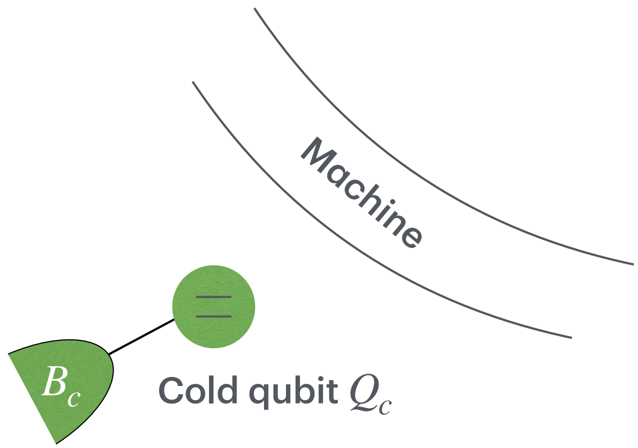
Generates excitation transfer

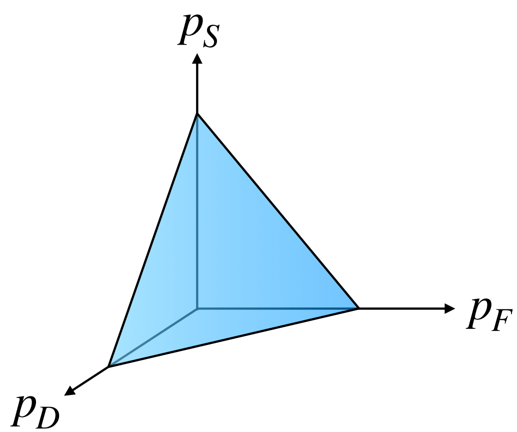
$$|110\rangle \rightarrow |001\rangle$$





Aamir, M. A. et al. "**Thermally driven quantum refrigerator autonomously resets superconducting qubit**" arXiv.2305.16710 (2023).





$$p_S + p_F + p_D = 1$$

Stochastic events:

- $B_c \to Q_c \to M$ SUCCESS
- $B_c \to Q_c \to B_c$ FAILURE (BOUNCE)
- $M \to Q_c \to M$ FAILURE (BOUNCE
- $M \to Q_c \to B_c$ DISASTER!

Each event takes a different amount of time.

Steady-state picture: currents J_H and J_C

On-demand picture: If an excitation suddenly appears in green, extract it as fast as possible.

Changes the questions.

Key question: what is the right question?

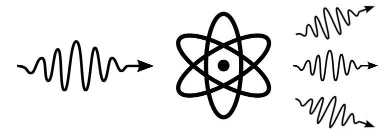
- How long does it take to cool?
- How many things can go wrong before it works?
- What is the entropy production of a single cooling event?

The toolbox of quantum measurement & thermodynamics, We cannot see quantum systems...

All we see is data ...1110000100010011100111101100...

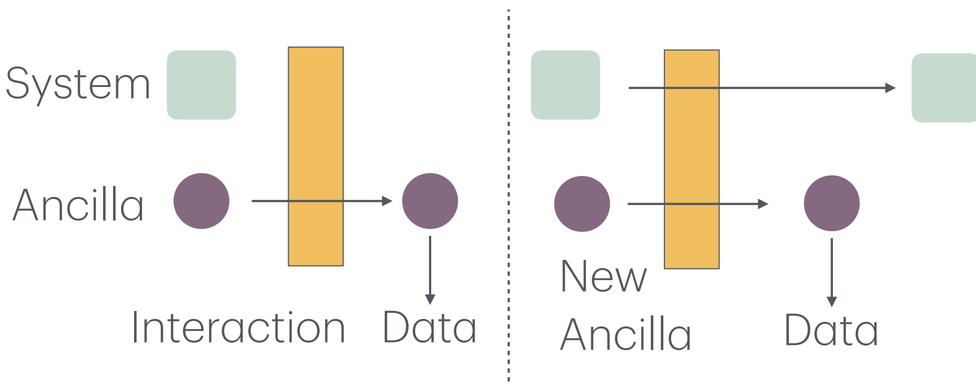
How can we study the stochastic thermodynamics of quantum devices?

- To measure a system we must send in a **probe** (or **ancilla**).
  - S+A interaction encodes information about S on A.
  - Extract information by measuring A.



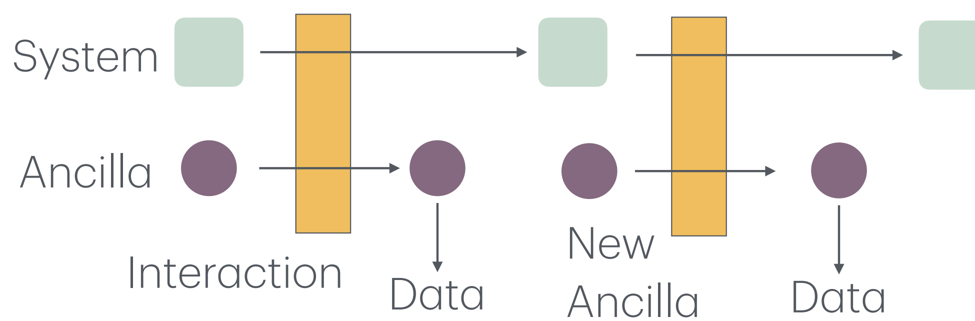
- **Information-back action trade-off:** the more information we want, the more we disturb the system.

Prepare & measure



iid outcomes

Sequential measurements



Correlated outcomes

A simple example

- Qubit: apply unitary U then measure in the computational basis $P_x = |x\rangle\langle x|$ where $x = 0,1$.
- Start in $|\psi_0\rangle$.

  1. Sample first outcome x_1 from $p(x_1) = |\langle x_1 | U | \psi_0 \rangle|^2$.
     Update state to $|\psi_1\rangle = |x_1\rangle$.
  2. Sample second outcome x_2 from $p(x_2 | x_1) = |\langle x_2 | U | x_1 \rangle|^2$.
     Update state to $|\psi_2\rangle = |x_2\rangle$.

- Generates a **bitstring of emitted symbols** $x_{1:n} = (x_1, \ldots, x_n)$.
- Probability of a sequence forms a Markov chain: $P(x_1, \ldots, x_n) = p(x_n | x_{n-1}) \ldots p(x_2 | x_1) p(x_1)$.

Non-projective measurements lead to long memory

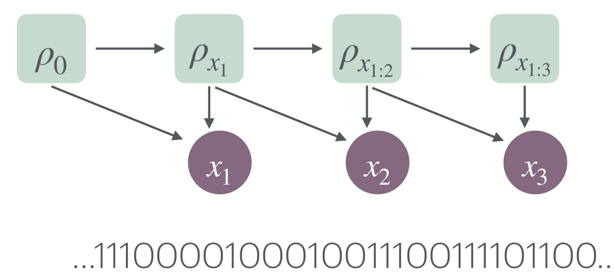
- Apply a set of Kraus operators $\sum_x F_x^\dagger F_x = 1$. Starting at ρ_0:

1. Sample first outcome x_1 from $p(x_1) = \mathrm{tr}\{F_{x_1}\rho_0 F_{x_1}^\dagger\}$. Update state to $\rho_{x_1} = \dfrac{F_{x_1}\rho_0 F_{x_1}^\dagger}{p(x_1)}$.

2. Sample second outcome x_2 from $p(x_2 | x_1) = \mathrm{tr}\{F_{x_2}\rho_{x_1} F_{x_2}^\dagger\}$. Update state to $\rho_{x_{1:2}} = \dfrac{F_{x_2}\rho_{x_1} F_{x_2}^\dagger}{p(x_2 | x_1)}$.

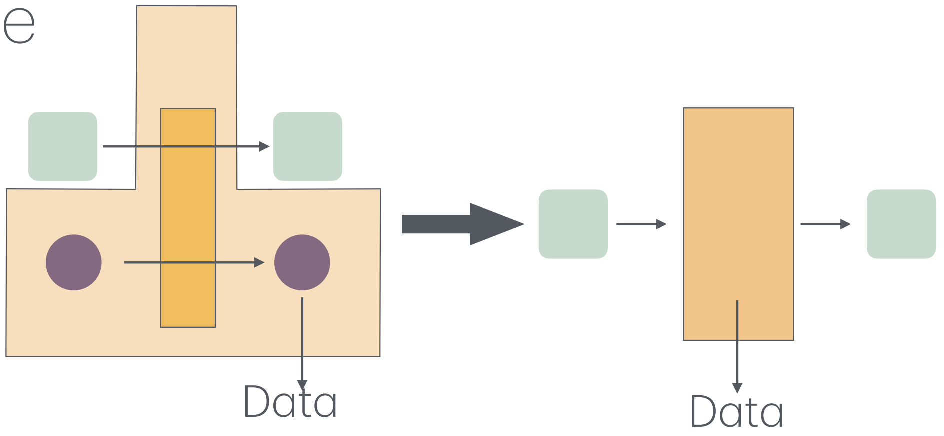
$$p(x_{n+1} | x_{1:n}) = \mathrm{tr}\left\{F_{x_{n+1}}\rho_{x_{1:n}} F_{x_{n+1}}^\dagger\right\} \quad \text{and} \quad \rho_{x_{1:n+1}} = \frac{F_{x_{n+1}}\rho_{x_{1:n}} F_{x_{n+1}}^\dagger}{p(x_{n+1} | x_{1:n})}$$

- String probability is now $P(x_{1:n}) = p(x_n | x_{1:n-1})p(x_{n-1} | x_{1:n-2})\ldots p(x_2 | x_1)p(x_1)$ which is highly non-Markovian.
  - *Evolution of the system is Markovian. But output data is not.*
- Looks like a Hidden Markov Model (HMM):
  - Quantum system is hidden.
  - Measurement outcomes (what we see) = **emitted symbols**



GTL "**Patterns in the jump-channel statistics of open quantum systems**," arXiv 2305.07957

Instruments: simplify and generalize



- Instruments = superoperators:

$$M_x\rho = F_x \rho F_x^\dagger$$

- Update rules become:

$$p(x_{n+1} | x_{1:n}) = \mathrm{tr}\{M_{x_{n+1}} \rho_{x_{1:n}}\}$$

and

$$\rho_{x_{1:n+1}} = \frac{M_{x_{n+1}} \rho_{x_{1:n}}}{p(x_{n+1} | x_{1:n})}$$

Prob. of a string:

$$P(x_{1:n}) = \mathrm{tr}\{M_{x_N} \ldots M_{x_1} \rho_0\}$$

Conditional state

$$\rho_{x_{1:n}} = M_{x_N} \ldots M_{x_1} \rho_0 / P(x_{1:n})$$

Instruments represent the most general kind of measurement possible.

Also encompass inefficient measurements

$$M_x \rho = \sum_{k \in x} F_k \rho F_k^\dagger$$

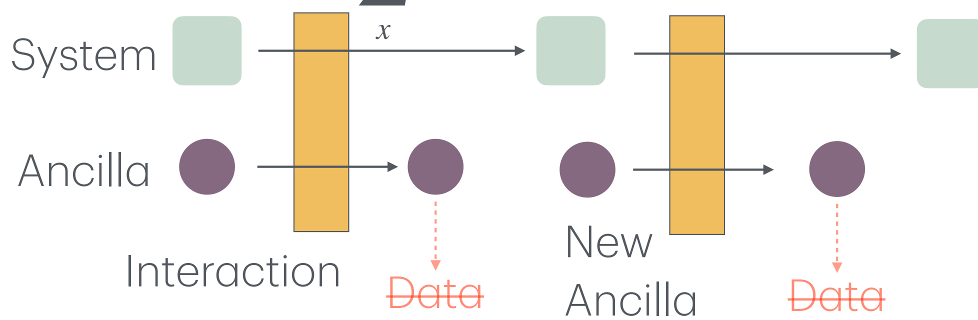
Wiseman, H. M. & Milburn, G. J. **Quantum Measurement and Control.** (Cambridge University Press, New York, 2009)

Unconditional dynamics

- If we measure but don't record the outcome the state of the system still changes (measurement back action)

- Ex: collision model or master equation.

$$\rho' = \sum_x p_x \rho'_x = \sum_x M_x \rho = \mathscr{M}\rho \qquad \text{where} \qquad \mathscr{M} = \sum_x M_x$$

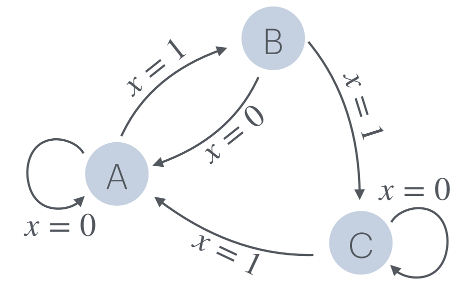


- $\mathscr{M}$ is a quantum channel.

- After n steps: $\rho_n = \mathscr{M}^n \rho_0$.

- Describes the average impact that the interaction with the ancilla causes in the system.

Connection to Hidden Markov Models



- $P(x, \sigma | \sigma')$ = prob. that system goes from $\sigma' \to \sigma$ while emitting a symbol x.
  - If HMM state is $\pi(\sigma')$ the prob. that we observe symbol x is

$$p(x) = \sum_{\sigma,\sigma'} P(x, \sigma | \sigma')\pi(\sigma')$$

- If outcome was x, bayesian update the state of the hidden layer:

$$\pi(\sigma | x) = \frac{P(x, \sigma)}{p(x)} = \frac{\sum_{\sigma'} P(x, \sigma | \sigma')\pi(\sigma')}{p(x)}$$

- Define substochastic matrices: $(M_x)_{\sigma,\sigma'} = P(x, \sigma | \sigma')$ and $\langle 1 | = (1,\ldots,1)$. Then

$$p(x) = \langle 1 | M_x | \pi \rangle \qquad \text{and} \qquad | \pi_x \rangle = \frac{M_x | \pi \rangle}{p(x)}$$

Compare with

$$p(x) = \mathrm{tr}\{M_x \rho\}$$

and

$$\rho_x = \frac{M_x \rho}{p(x)}$$

Milz, S. & Modi, K. "**Quantum Stochastic Processes and Quantum non-Markovian Phenomena**". PRX Quantum 2, 030201 (2021)

Prediction

- **Mixed state representation & unifilar models:** if we know $\rho_{x_{1:n}}$ and we observe x_{n+1} we know with certainty that the system evolved to $\rho_{x_{1:n+1}}$.

- Usefulness: data compression

$$p(x_{n+1} \,|\, x_{1:n}) = p(x_{n+1} \,|\, \rho_{x_{1:n}})$$

If we can know the internal state, we can make statistical predictions of future outcomes.

- Example: figuring out the internal state of a large language model.

F. Binder, J. Thompson, M. Gu, "**Practical unitary simulator for non-Markovian complex processes**," *Phys. Rev. Lett.* **120** 240502 (2018).

Quantum jumps

Mark Mitchison

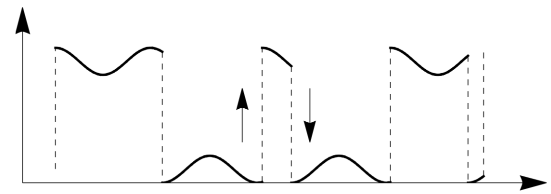
Michael Kewming

Patrick Potts

GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts **"Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics,"** PRX Quantum 5, 020201 (2024)

GTL "**Patterns in the jump-channel statistics of open quantum systems**," arXiv 2305.07957

- Consider a quantum master equation



$$\frac{d\rho}{dt} = \mathscr{L}\rho = -i[H,\rho] + \sum_{x=1}^{r} L_x\rho L_x^\dagger - \frac{1}{2}\{L_x^\dagger L_x, \rho\}$$

- The infinitesimal evolution can be written as a set of instruments:

$$\rho_{t+dt} = e^{\mathscr{L}dt}\rho_t = \sum_x M_x\rho_t$$

(jump) $M_x\rho = dt\ L_x\rho L_x^\dagger = dt\ \mathscr{J}_x\rho$ for $x = 1,2,\ldots,r$

(no jump) $M_0\rho = \rho + dt\mathscr{L}_0\rho$ where $\mathscr{L}_0\rho = -i[H,\rho] - \frac{1}{2}\sum_{x=1}^{r}\{L_x^\dagger L_x, \rho\}$

- $p_x = \mathrm{tr}\{M_x\rho\} = dt\,\mathrm{tr}\{L_x^\dagger L_x\rho\}$ is infinitesimal: most of the time the system evolves with no jump.

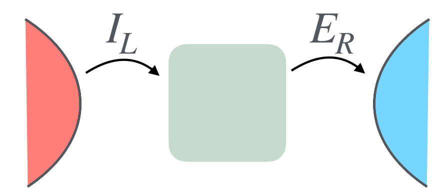
GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts **"Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics,"** PRX Quantum 5, 020201 (2024)

The t and the N ensembles

- **t-ensemble:** t_f fixed. $\hat{N}$ is random.
  - Instruments: $M_0\rho = (1 + dt\mathscr{L}_0)\rho$ and $M_x\rho = dt\ L_x\rho L_x^\dagger$ for $x = 1,2,\ldots r$
  - Trajectory: $00000x_1 00000000x_2 00000\ldots$
- **N-ensemble:** N is fixed. $\hat{t}_f$ is random.
  - Instruments: $M_{x,\tau}\rho = \mathscr{J}_x e^{\mathscr{L}_0\tau}\rho$
  - Trajectory: $(x_1, \tau_1), (x_2, \tau_2), \ldots, (x_N, \tau_N)$ $\quad \tau_j = t_j - t_{j-1}$
- Quantum jumps without time tags: we know a jump happened, but do not know when
  - Instruments: $M_x = -\mathscr{J}_x\mathscr{L}_0^{-1}$.
  - Trajectory: $x_1, x_2, \ldots$

A. A. Budini, R. M. Turner, and J. P. Garrahan. "**Fluctuating Observation Time Ensembles in the Thermodynamics of Trajectories**." Journal of Statistical Mechanics: Theory and Experiment 2014 (3): P03012 (2014)

Quantum jumps without time tags



- Lattice with L sites, each of which can have 0 or 1 particles.
  - excitations can be injected on the left (I_L)
  - or extracted on the right (E_R).
  - And they can tunnel back and forth through the chain: not monitorable.

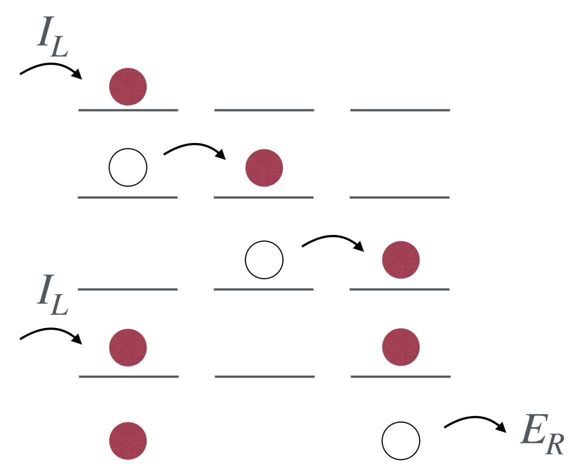
- All we would observe are symbols: $I_L I_L E_R$.

Prob. of a string:

$$P(x_{1:n}) = \mathrm{tr}\left\{M_{x_N}\ldots M_{x_1}\rho_0\right\}$$

Conditional state

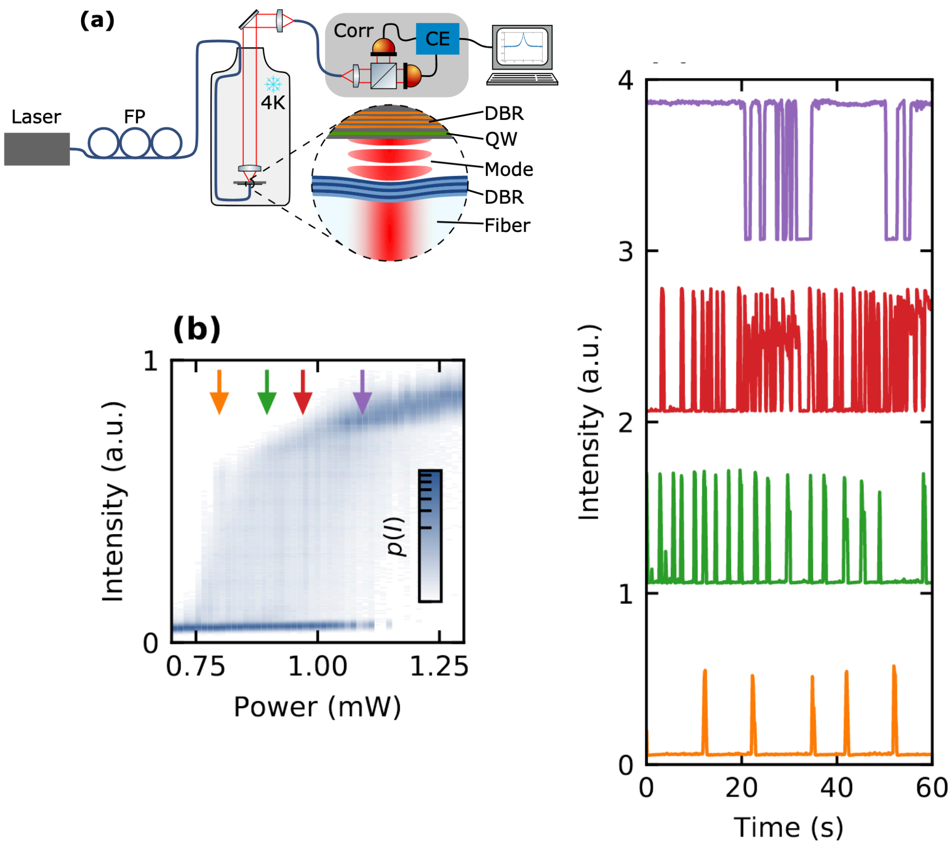
$$\rho_{x_{1:n}} = M_{x_N}\ldots M_{x_1}\rho_0 / P(x_{1:n})$$



GTL "**Patterns in the jump-channel statistics of open quantum systems**," arXiv 2305.07957

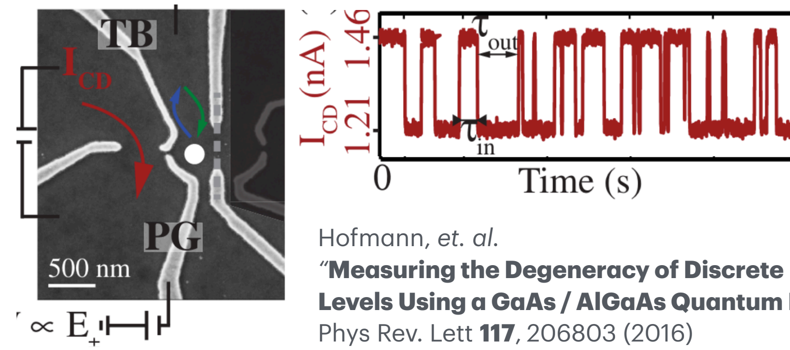
Direct and indirect observation of quantum jumps

Quantum jumps = observable clicks in the environment



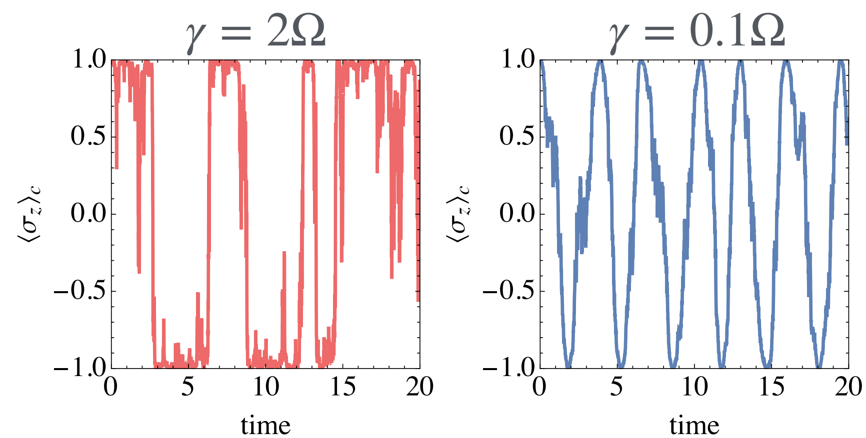
Fink *et. al.*, "**Signatures of a dissipative phase transition in photon correlation measurements**"
Nature Physics **14** 365-369 (2018)

Quantum jumps observed indirectly through continuous measurements of the system



Hofmann, *et. al.*
"**Measuring the Degeneracy of Discrete Energy Levels Using a GaAs / AlGaAs Quantum Dot**,"
Phys Rev. Lett **117**, 206803 (2016)

Driven qubit



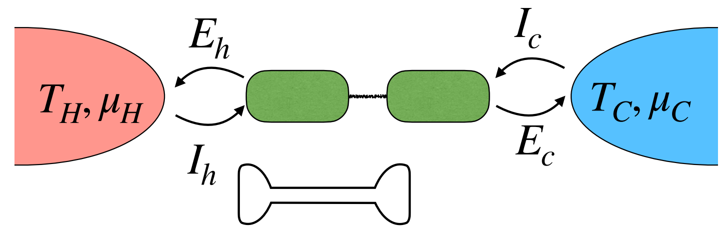
$$\frac{d\rho}{dt} = -i[\Omega\sigma_x, \rho] + \gamma D[\sigma_z]\rho$$

Double quantum dot - 3-level system

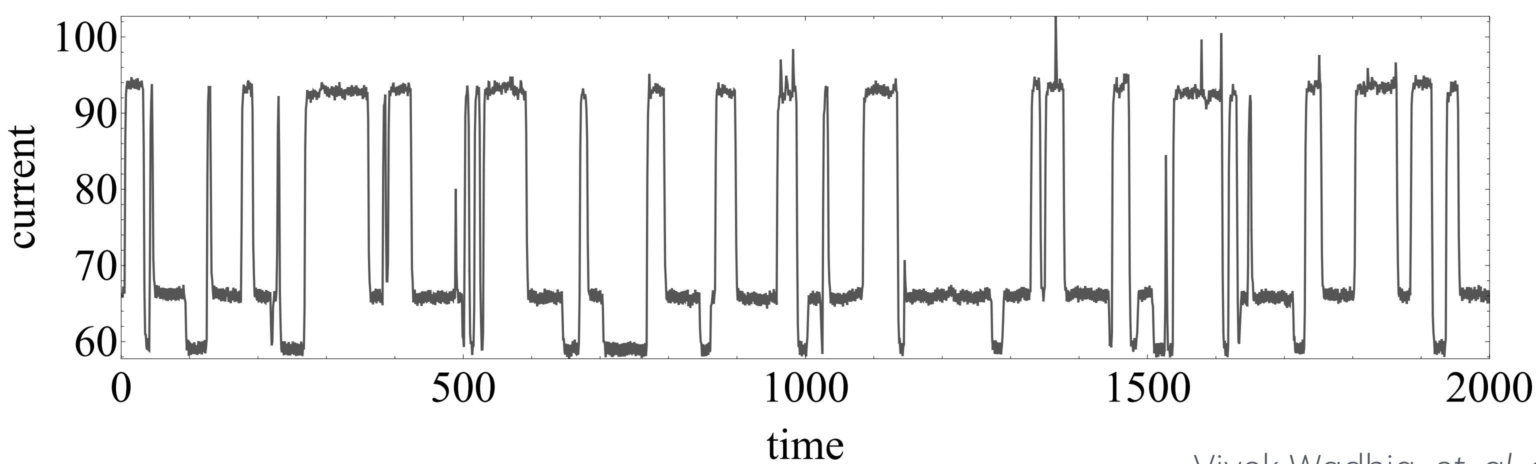
- Two dots + Coulomb blockade
  $\rightarrow$ 3 levels only: $|0\rangle, |L\rangle, |R\rangle$

- One-to-one mapping between system transition and jump channel

  - e.g. $|0\rangle \overset{I_H}{\rightarrow} |L\rangle$ or $|R\rangle \overset{E_C}{\rightarrow} |0\rangle$, etc.



QPC asymmetrically placed



Vivek Wadhia, *et. al.* arXiv 2502.00096

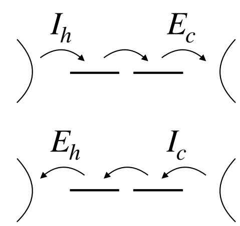
Stochastic operation of thermal machines

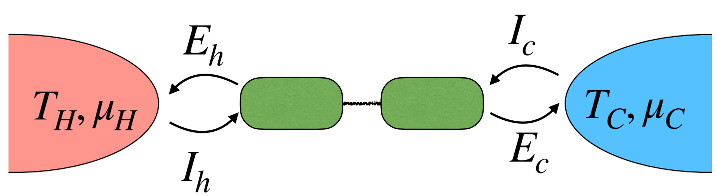
Patrick Potts

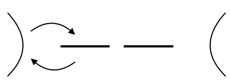
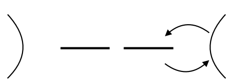
Abhaya Hegde

Abhaya S. Hegde, Patrick P. Potts, GTL, “**Time-resolved Stochastic Dynamics of Quantum Thermal Machines**,” arXiv:2408.00694

- Double quantum dot
  - Engine process: uses thermal gradient to extract chemical work .
  - Refrigerator process: uses chemical work to make heat flow from cold to hot.



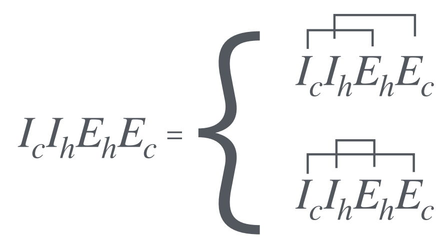


- There can also be "idle cycles" (bounces)
  - "Hot bounce" 
  - "Cold bounce" 

Can we identify individual cycles solely from a bitstring?

$I_h E_c I_c I_h E_h E_c I_h I_c E_h I_c$

Impossible in general, if excitations are indistinguishable



Manzano, Gonzalo, and Roberta Zambrini "**Quantum Thermodynamics under Continuous Monitoring: A General Framework**," AVS Quantum Science 4 (2): 025302 (2022).

Single excitation assumption

$$\frac{d\rho}{dt} = -\underbrace{i[H,\rho]}_{\text{Unitary work}} + \underbrace{\sum_n D[K_n]\rho}_{\text{Work reservoirs}} + \sum_{\alpha\in\{h,c\}} \sum_j \underbrace{\gamma^-_{\alpha j} D[L_{\alpha j}]\rho}_{\text{Extraction to bath } \alpha} + \underbrace{\gamma^+_{\alpha j} D[L^\dagger_{\alpha j}]\rho}_{\text{Injection from bath } \alpha}$$

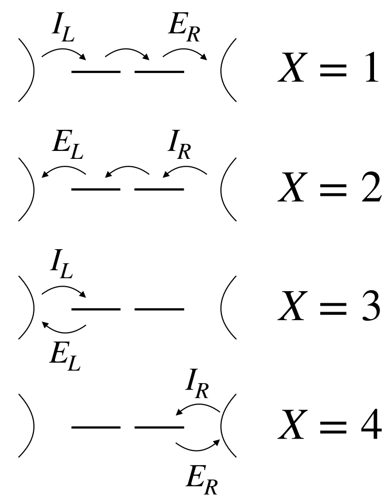
- Result: for cycles to be identifiable the string must always have injections followed by extractions.

$$\ldots I.E.I.E.I.E.I.E.I.E.\ldots$$

- Condition: Hilbert space must be split in 2.
  - $L^\dagger_{\alpha j}$ injects → post-injection subspace.
  - $L_{\alpha j}$ extracts → post-extraction subspace.

Hilbert space =

Bitstrings of jumps → bitstrings of cycles



$$\ldots I_\bullet E_\bullet I_\bullet E_\bullet I_\bullet E_\bullet I_\bullet E_\bullet \ldots = \ldots X_\bullet X_\bullet X_\bullet X_\bullet \ldots$$

- We can use this to answer the following questions:
  - What is the probability that the next cycle is of type X and takes a time $\boldsymbol{\tau}$?
  - How are cycles correlated with each other?
  - What is the average time required to complete each cycle?
  - How many idle cycles happen between two useful cycles?
- Define instruments

$$M_{X\tau} = \int_0^\tau dt \; \mathcal{J}_{E_X} e^{\mathcal{L}_0(\tau - t)} \mathcal{J}_{I_X} e^{\mathcal{L}_0 t}$$

with 2 emitted symbols: $X = 1,2,3,4$ and cycle duration $\boldsymbol{\tau}$

Cycle probabilities

π_E = Jump Steady-State

Correct state to get long-time statistics

- Then prob. a cycle is of type X and takes a time τ: $p_{X,\tau} = \mathrm{tr}\{M_{X\tau}\pi_E\}$.

- If we don't care about how long a cycle takes, we just need to marginalize the instrument:

$$M_X = \int_0^\infty d\tau \; M_{X\tau}$$

- Prob. of obtaining each cycle type

$$p_X = \mathrm{tr}\{M_X \pi_E\}$$

Relation to steady-state currents:

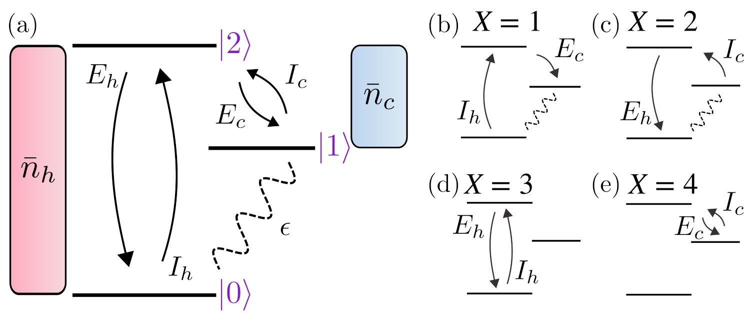
$$I = \frac{p_1 - p_2}{E(\tau)}$$

- Conditional cycle times: if cycle is of type X, how long it takes?

$$E(\tau | X) = \int_0^\infty d\tau \; \tau \frac{p_{X,\tau}}{p_X}$$

Correlations between cycles:

$$P(X_1, \tau_1, \ldots, X_n, \tau_n) = \mathrm{tr}\{M_{X_n\tau_n} \ldots M_{X_1\tau_1}\pi_E\}$$



Double quantum dot or 3-level system

$$\langle n \rangle = \frac{p_{\text{id}}}{p_u} = \frac{p_3 + p_4}{p_1 + p_2}, \qquad T = \frac{p_3 E(\tau\,|\,3) + p_4 E(\tau\,|\,4)}{p_1 E(\tau\,|\,1) + p_2 E(\tau\,|\,2)},$$

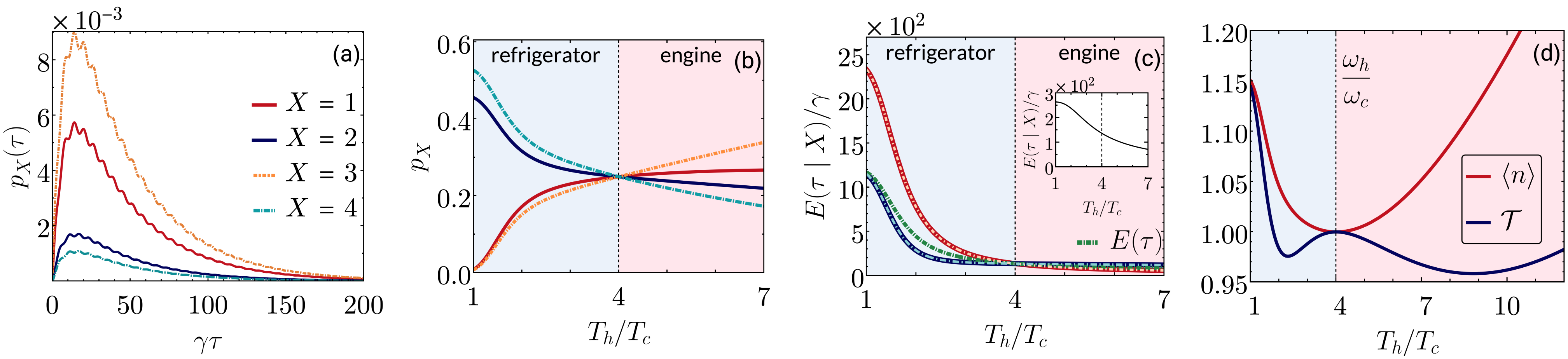


FIG. 3. **(a-d)** Statistics of cycles in three-level maser from Fig. 2. **(a)** Probability of observing a cycle X within a duration τ [Eq. (9)] at resonance $\omega_d = \omega_h - \omega_c$ and $T_h/T_c = 10$. **(b)** Total probability of observing a cycle X [Eq. (10)] and **(c)** expectation values for cycle duration [Eqs. (11), (12)] as a function of the ratio of bath temperatures. A vertical line at $T_h/T_c = \omega_h/\omega_c$ separates the refrigerator and engine regimes. The inset shows all expectation values nearly converge at resonance. **(d)** Mean of intervening idle cycles between useful cycles and ratios of fraction of idle-to-useful times against bath gradient. The parameters are fixed (in units of $T_c = 1$) at $\gamma_h = \gamma_c \equiv \gamma = 0.05$, $\omega_h = 8$, $\omega_c = 2$, $\omega_d = 4$, $\epsilon = 0.5$ unless mentioned otherwise.

Stochastic excursions

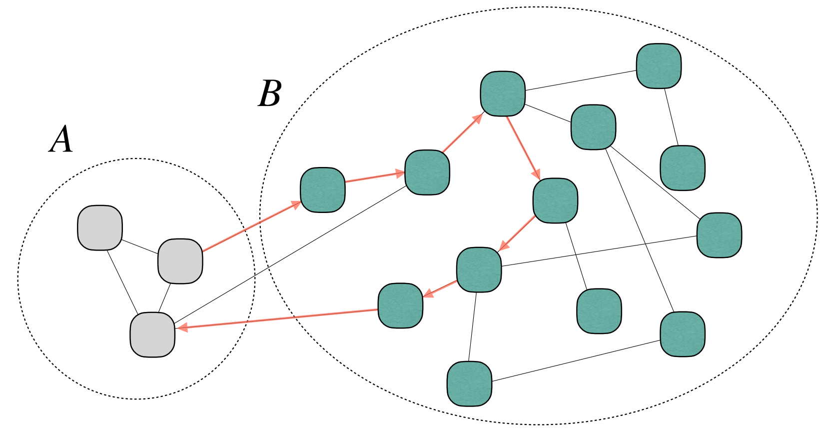

Guilherme Fiusa


Abhaya Hegde


Pedro Harunari

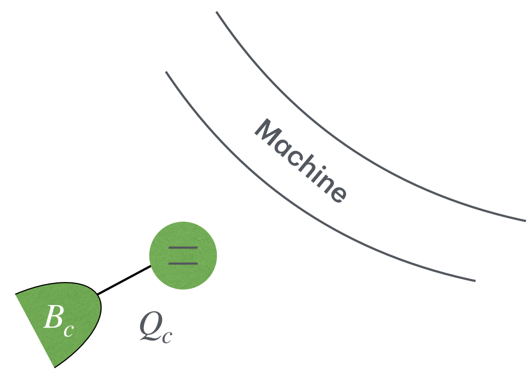
See also poster by Guilherme Fiusa this afternoon!

Stochastic excursions

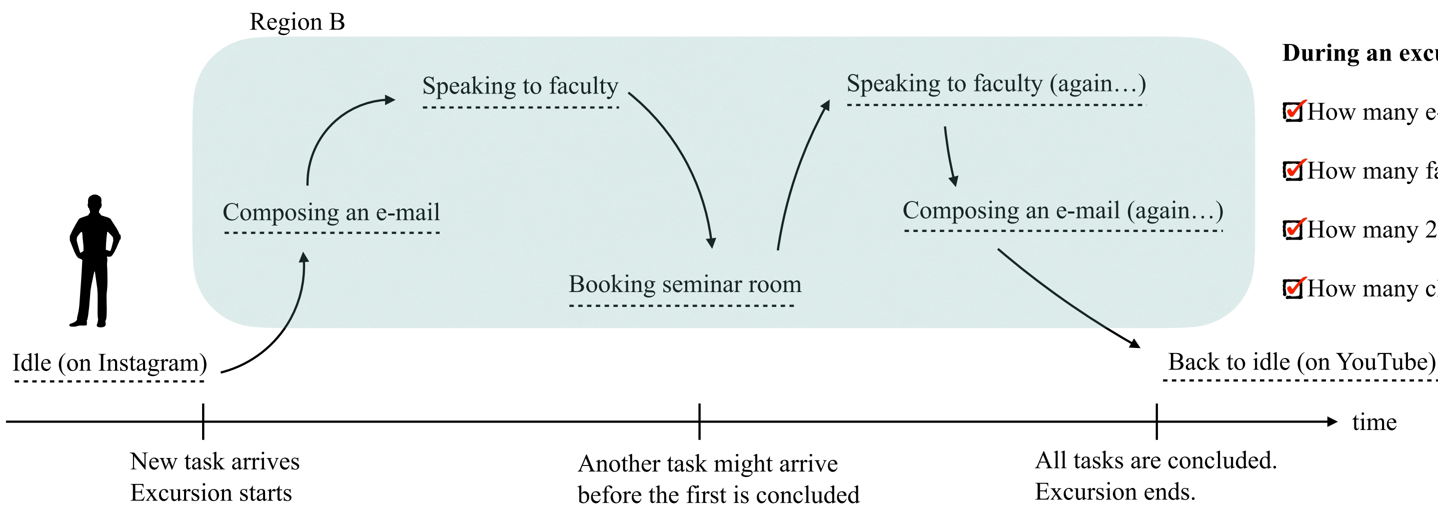


- Starts when system leave A.
- Ends when system first comes back to A.
- Generalizes the notion of cycles.
- Excursion time $\hat{T}$ is random: **first passage time**. Well known and extensively studied.
- Our question: **statistics of counting observables within a single excursion $\hat{Q}(\hat{T})$.**

  - An excursion starts whenever $Q_c = 1$.
  - It ends when $Q_c = 0$.
  - In between, many things can happen because the machine can have many internal states.



Stochastic excursions in other contexts



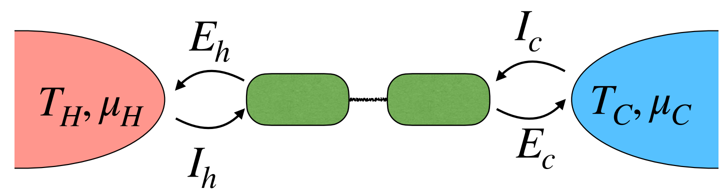
During an excursion…

- ☑How many e-mails were sent?
- ☑How many faculty did they have speak to?
- ☑How many 2-factor authentications?
- ☑How many clueless grad students?

Guilherme Fiusa, Gabriel T. Landi,
"**Queued quantum collision models**", arXiv 2403.19408

Foraging models

Rate equations - incoherent dynamics



- Our approach is based on classical (Pauli) master equati

$$\frac{dp_x}{dt} = \sum_{y \neq x} \left\{ W_{xy} p_y - W_{yx} p_x \right\}$$

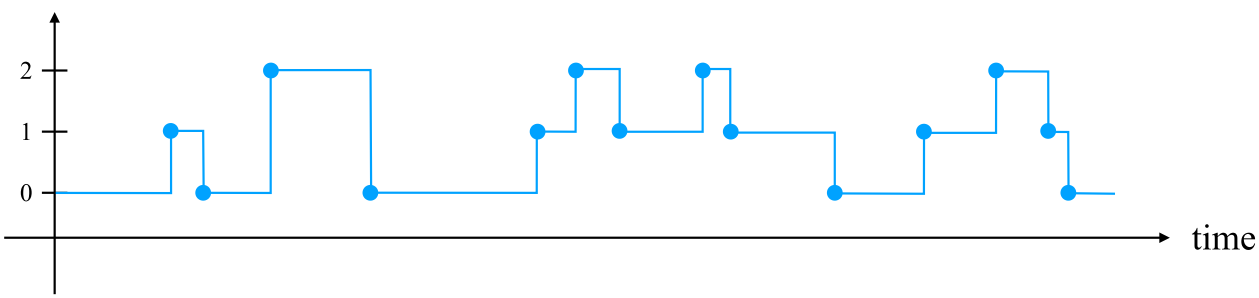
- Can be derived from quantum master equations using perturbation theory, in some regimes.
  - Can offer a very good approximation.

Counting variables $\hat{N}_{xy}(t)$

Build physical currents through counting observables

$$\hat{Q}(t) = \sum_{x,y} \nu_{xy} \hat{N}_{xy}(t)$$

Weights ν_{xy} provide contextual meaning (physics) to each transition



K. Prech, P. Johansson, E. Nyholm ,GTL, C. Verdozzi , P. Samuelsson , P. P. Potts
"**Entanglement and thermokinetic uncertainty relations in coherent mesoscopic transport**". Phys. Rev. Res. 5, 023155 (2023).

Connection with Full Counting Statistics

- FCS deals with the long-time statistics of counting observables.
- Define the tilted matrix (ξ = counting field)

$$\mathbb{W}^{\xi}_{xy} = \begin{cases} W_{xy} e^{i\xi\nu_{xy}} & x \neq y \\ -\sum_z W_{zx} & x = y \end{cases}$$

$$P(q,t) = C \int_{-\infty}^{\infty} \frac{d\xi_1 \dots d\xi_r}{(2\pi)^r} \langle y_A | W_{AB\xi} e^{\mathbb{W}_{B\xi} t} W_{BA\xi} | x_A \rangle e^{-iq\xi},$$

- **Exchange FT within each excursion:** if $\hat{Q} = \hat{\Sigma}$ = entropy production then $P(\Sigma) = e^{-\Sigma} P(-\Sigma)$.
- The first and second moments of any excursion-related quantity can be connected with steady-state results:

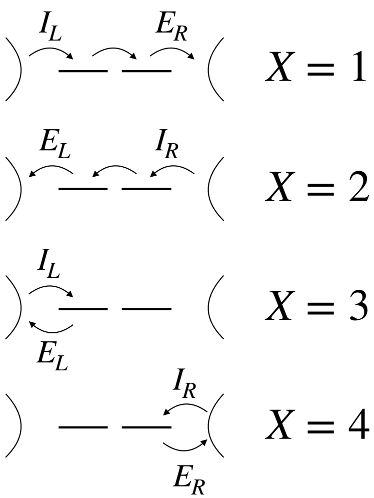
$$J = \frac{E(\hat{Q})}{\mu} \quad \text{where} \quad \mu = E(\hat{T})$$

$$D = \frac{\text{var}(Q)}{\mu} + \frac{E(\hat{Q})\Delta^2}{\mu^3} - \frac{2E(\hat{Q}}{\mu}\text{cov}(\hat{Q},\hat{T}) \quad \text{where} \quad \Delta^2 = \text{var}(\hat{T}).$$

Conclusions

Thank you!

- Sequential quantum measurements = **time-series** of correlated stochastic outcomes.
  - Bayesian inference of the quantum state, given outcomes.
  - Unveiling the thermodynamics from measurement data.
  - Stochastic operation of a thermal machine.
    - Open question: machine intermittency vs. current fluctuations?
  - Stochastic excursions: so far classical, but very exciting.



GTL, Michael J. Kewming, Mark T. Mitchison, Patrick P. Potts "**Current fluctuations in open quantum systems: Bridging the gap between quantum continuous measurements and full counting statistics**," PRX Quantum 5, 020201 (2024)

GTL "**Patterns in the jump-channel statistics of open quantum systems**," arXiv 2305.07957

Abhaya S. Hegde, Patrick P. Potts, GTL, "**Time-resolved Stochastic Dynamics of Quantum Thermal Machines**," arXiv:2408.00694