Landauer’s principle at zero temperature

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The second law

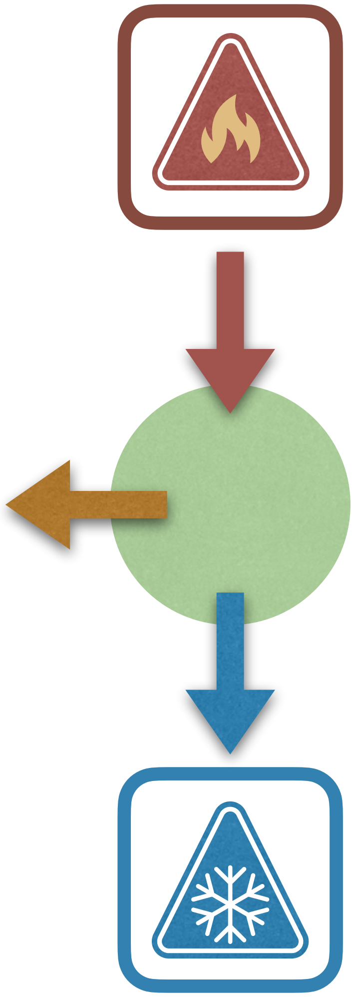
- The 1st law puts heat and work on similar footing and says that, in principle, one can be interconverted into the other.

- For a system coupled to two baths, for instance, we have:

$$\frac{dU}{dt} = \dot{Q}_h + \dot{Q}_c + \dot{W}$$

- Not all such processes are possible, however:

    - This is the purpose of the 2nd law.



- The 2nd law deals with entropy.
    - *Entropy, however, does not satisfy a continuity equation.*
- There can be a flow of entropy from the system to the environment, which is given by the famous Clausius expression $\dot{Q}/T$.
- But, in addition, there can also be some entropy which is spontaneously *produced* in the process. The entropy balance equation thus reads

$$\frac{dS}{dt} = \dot{\Sigma} + \frac{\dot{Q}_h}{T_h} + \frac{\dot{Q}_c}{T_c}$$

- The quantity $\dot{\Sigma}$ is called the **entropy production rate.**
- The second law can now be formulated mathematically as:

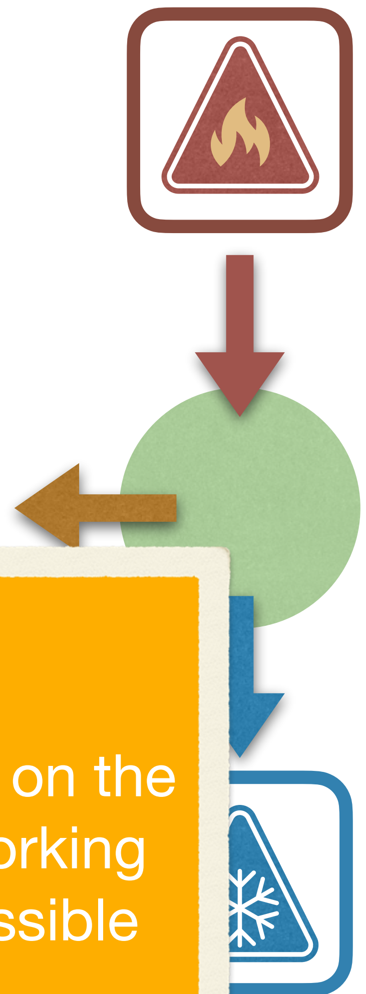
$$\dot{\Sigma} \geq 0$$

Why entropy production matters

- 1st and 2nd laws for a system coupled to two baths:

$$\frac{dU}{dt} = \dot{Q}_h + \dot{Q}_c + \dot{W} = 0$$

$$\frac{dS}{dt} = \dot{\Sigma} + \frac{\dot{Q}_h}{T_h} + \frac{\dot{Q}_c}{T_c} = 0$$



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Carnot's statement of the 2nd law

"The efficiency of a quasi-static or reversible Carnot cycle depends only on the temperatures of the two heat reservoirs, and is the same, whatever the working substance. A Carnot engine operated in this way is the most efficient possible heat engine using those two temperatures."

- *Entropy production is therefore the reason the efficiency is smaller than Carnot:*

$$\eta = \eta_C - \frac{T_c}{\dot{Q}_h}\dot{\Sigma}$$

Entropy and information

- In information theory, *entropy* acquires a different interpretation.
  - S = amount of ignorance (lack of information) one has about the system.
- If the system is described by a set of states $n = 1,2,3,\ldots$, each with probability p_n, the Shannon entropy is defined as

$$S = -\sum_n p_n \ln p_n$$

- The quantity $-\ln p_n$ is called the "surprise" of the state:
  - It measures how surprised we are to observe the system in state n.
  - If $p_n \sim 1 \rightarrow -\ln p_n \sim 0$ (no surprise at all)
  - If $p_n \sim 0 \rightarrow -\ln p_n \gg 1$ (huge surprise)
- Entropy is thus the "average surprise". 🙃

- To have a concrete example, consider a particle which can be found in 1 of 2 sides of a box:

- Suppose we have some *ignorance* about the system:
    - We do not know which side the particle is.
- Let (p_R, p_L) be the probabilities of finding it on the left or on the right.
- Consequently, the "state" of the particle (i.e., left or right) has some entropy associated to it:

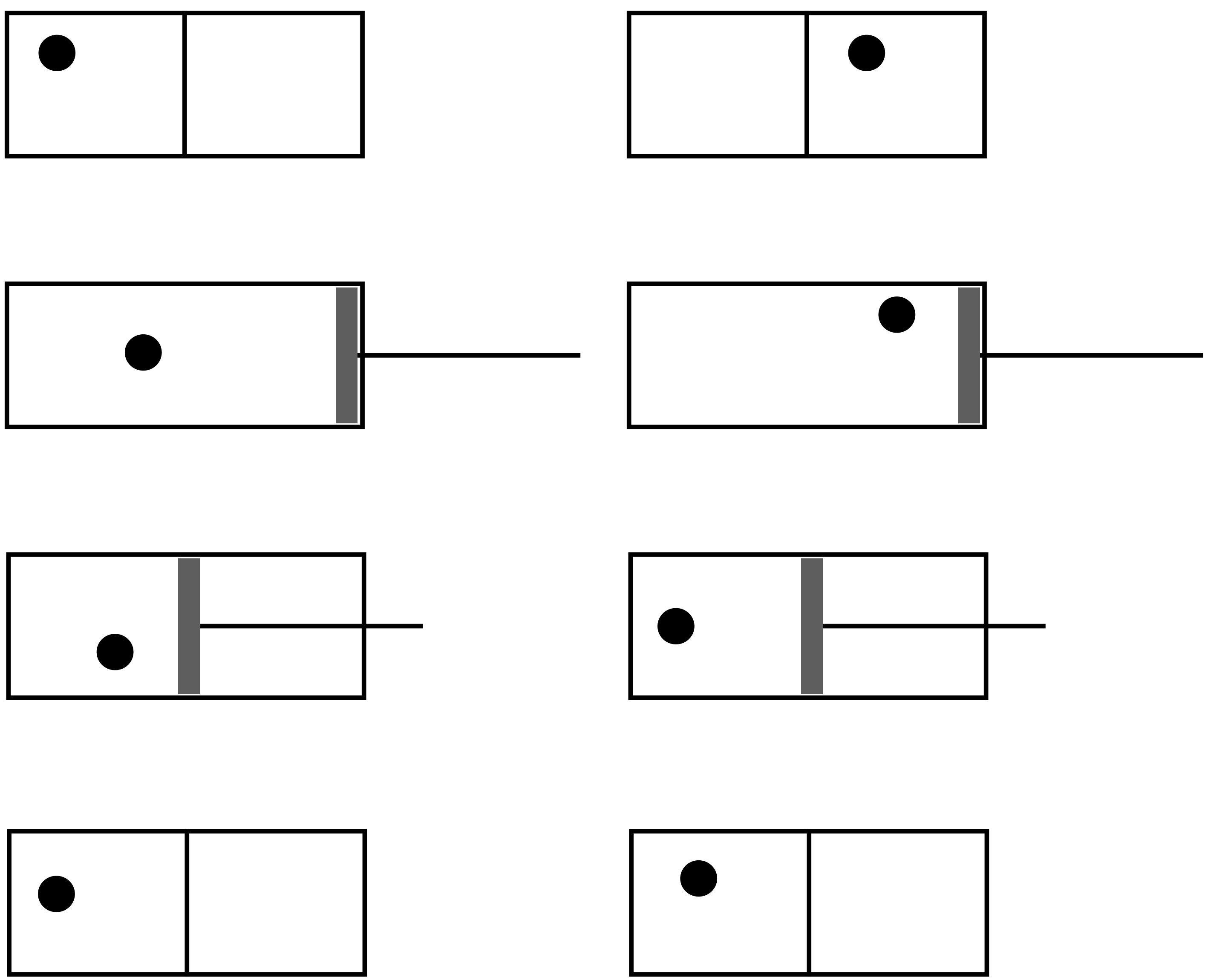
$$S = -\, p_R \ln p_R - p_L \ln p_L$$

- e.g. maximum ignorance: $p_R = p_L = 1/2$ and thus

$$S = \ln 2$$

Information erasure

- Consider now the following procedure:



- Irrespective of the initial state, the final state is always “left”:

$$(p_R, p_L) = (0,1)$$

- Hence the final entropy is zero:

$$S = 0$$

- We call this *information erasure* because any initial information about where the particle was is now forever lost.

Landauer’s principle

- Landauer’s principle states that there is a **fundamental heat cost associated with information erasure**.
    - To erase information one must pay an energy bill:

$$\Delta Q_E \geq -T\Delta S_S$$

    - ΔS_S is the change in entropy of the system
    - ΔQ_E is the amount of heat flowing to the *environment* ($\Delta Q_E > 0$ when energy leaves the system).
- **Landauer’s bound**: minimum heat cost for erasing information.
- For instance, if $\Delta S_S = 0 - \ln 2$, we find

$$\Delta Q_E \geq T \ln 2$$

- Landauer worked for IBM. In computing terms, this is the energy cost required to erase one bit of information.

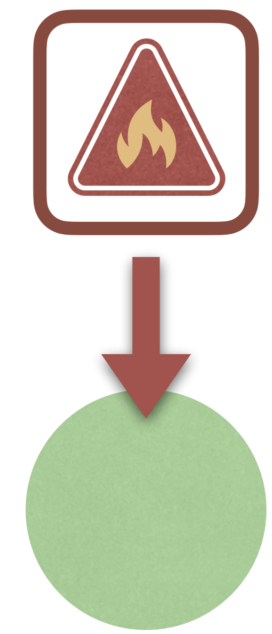
Information is physical

- Landauer's principle is often used to argue that "information is physical".
  - Information is physical because information is *stored* in physical systems and *communicated* using physical systems.
- Information theory is thus not purely mathematical.
- Landauer's principle also highlights a fundamental irreversibility of physical processes:
  - If $\Delta S_S \geq 0$ then $\Delta Q_E \geq -T\Delta S_S$ does not impose any restrictions.
    - There is no energy cost to acquire information.
  - But there is an energy cost to erase it.

Landauer’s principle is a consequence of the 2nd law

Landauer from the 2nd law

$\Delta Q_E \geq -T\Delta S_S$



- Recall the 2nd law for a system coupled to a single bath:

$$\frac{dS_S}{dt} = \dot{\Sigma} - \frac{\dot{Q}_E}{T}, \qquad \dot{\Sigma} \geq 0$$

- I changed the sign of $\dot{Q}/T$ here because $\dot{Q}_S = -\dot{Q}_E$

- We integrate over some interval of time, leading to

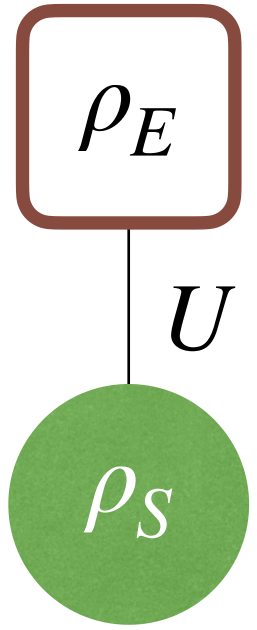
$$\Sigma = \Delta S_S + \frac{\Delta Q_E}{T} \geq 0$$

- This looks exactly like Landauer's principle:
  - It is a direct consequence of the 2nd law $\Sigma \geq 0$.

Microscopic formulation of Landauer's principle

- But this is tricky because:
  - Landauer's bound is defined for the Shannon entropy and for systems of arbitrary size, such as a single bit.
  - The second law uses the thermodynamic entropy and holds only for macroscopic bodies.
- To connect the two universes we must construct a microscopic theory of thermodynamics that is capable of extending the 2nd law to the microscopic domain.
- In this talk I will focus on the quantum version, as it encompasses the classical theory as a particular case.
  - This is the theory we call Quantum Thermodynamics.

Entropy production in quantum systems



- All information about a quantum system is contained in its density matrix ρ.

- Entropy is now quantified by the von Neumann entropy:

$$S(\rho) = -\operatorname{tr}(\rho \ln \rho)$$

- We consider two quantum systems, *S* and *E* (the "environment") in arbitrary states ρ_S and ρ_E.
  - *S* and *E* can have any size: generalization of the bath concept.

- The two then interact with a unitary U, leading to

$$\rho'_{SE} = U\big(\rho_S \otimes \rho_E\big)U^\dagger$$

- This is the quantum version of a system interacting with a bath.

- What is the entropy production?

- The dynamics is unitary and so in principle one could say it is reversible.
  - Indeed, if all of ρ'_{SE} is accessible, everything would be reversible.
- "Irreversibility" depends on which degrees of freedom become inaccessible after the interaction.
- There are many possibilities:
  - System-bath correlations become, in practice, inaccessible.
  - Any changes we make in the bath may also not be recoverable.
  - If measurements are done in the system, quantum coherence may also be lost.
  - etc.
- Each of these features can be gauged using a certain information-theoretic quantifier.

M. Esposito, K. Lindenberg, C. Van den Broeck, *"Entropy production as correlation between system and reservoir"*. New Journal of Physics, **12**, 013013 (2010).

G. Manzano, J. M. Horowitz, J. M. R. Parrondo, *"Quantum fluctuation theorems for arbitrary environments: adiabatic and non-adiabatic entropy production"*,Physical Review X, **8**, 031037 (2018).

- The choice of entropy production which is closest to the classical formulation is:

$$\Sigma = I'(S:E) + D(\rho'_E || \rho_E)$$

where

$$I'(S:E) = S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE}) \text{ = SE correlations}$$

$$D(\rho'_E || \rho_E) = \mathrm{tr}(\rho'_E \ln \rho'_E - \rho'_E \ln \rho_E) \quad \text{= change in the bath}$$

- This formula can be taken as a general *definition* of entropy production for an arbitrary system+bath interaction process.
  - The system and bath can have any size.
  - Their states are arbitrary, except that they start in any (product) state.
  - They interact with an arbitrary unitary *U*.
- Assuming that the bath is in a thermal state, one finds:

$$\Sigma = I'(S:E) + D(\rho'_E || \rho_E) = \Delta S_S + \beta \Delta Q_E \geq 0$$

- In the last 5 years there have been several papers which generalized/improved Landauer's original result:
  - J. Goold, M. Paternostro and K. Modi, *Phys. Rev. Lett*. **114**, 060602 (2015).
  - G. Guarnieri, S. Campbell, J. Goold, S. Pigeon, B Vacchini and M. Paternostro, *NJP*, **19**, 103038 (2017).
  - S. Lorenzo, R. McCloskey, F. Ciccarello, M. Paternostro and G. M. Palma, *Phys. Rev. Lett.,* **115**, 120403 (2015).
  - P. Strasberg, G. Schaller, T. Brandes and M. Esposito, *Phys. Rev. X.*, **7**, 021003 (2017).
  - S. Campbell, G. Guarnieri, M. Paternostro and B. Vacchini, *Phys. Rev. A.*, **96**, 042109 (2017).

LETTER

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Experimental verification of Landauer's principle linking information and thermodynamics

Antoine Bérut[1], Artak Arakelyan[1], Artyom Petrosyan[1], Sergio Ciliberto[1], Raoul Dillenschneider[2] & Eric Lutz[3]†

nature physics

LETTERS

https://doi.org/10.1038/s41567-018-0070-7

Corrected: Publisher Correction

Quantum Landauer erasure with a molecular nanomagnet

R. Gaudenzi[1]*, E. Burzurí[1], S. Maegawa[2], H. S. J. van der Zant[1] and F. Luis[3]

MATERIALS SCIENCE

Experimental test of Landauer's principle in single-bit operations on nanomagnetic memory bits

Jeongmin Hong,[1] Brian Lambson,[2] Scott Dhuey,[3] Jeffrey Bokor[1]*

PRL 113, 190601 (2014) PHYSICAL REVIEW LETTERS week ending 7 NOVEMBER 2014

High-Precision Test of Landauer's Principle in a Feedback Trap

Yonggun Jun,* Momčilo Gavrilov, and John Bechhoefer†

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Experimental demonstration of information to energy conversion in a quantum system at the Landauer limit

J. P. S. Peterson[1], R. S. Sarthour[1], A. M. Souza[1], I. S. Oliveira[1], J. Goold[2], K. Modi[3], D. O. Soares-Pinto[4] and L. C. Céleri[5]

PHYSICAL REVIEW LETTERS **120,** 210601 (2018)

Editors' Suggestion Featured in Physics

Single-Atom Demonstration of the Quantum Landauer Principle

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Trouble at $T \to 0$

Trouble at $T \to 0$

- Landauer's bound becomes trivial in the limit $T \to 0$:

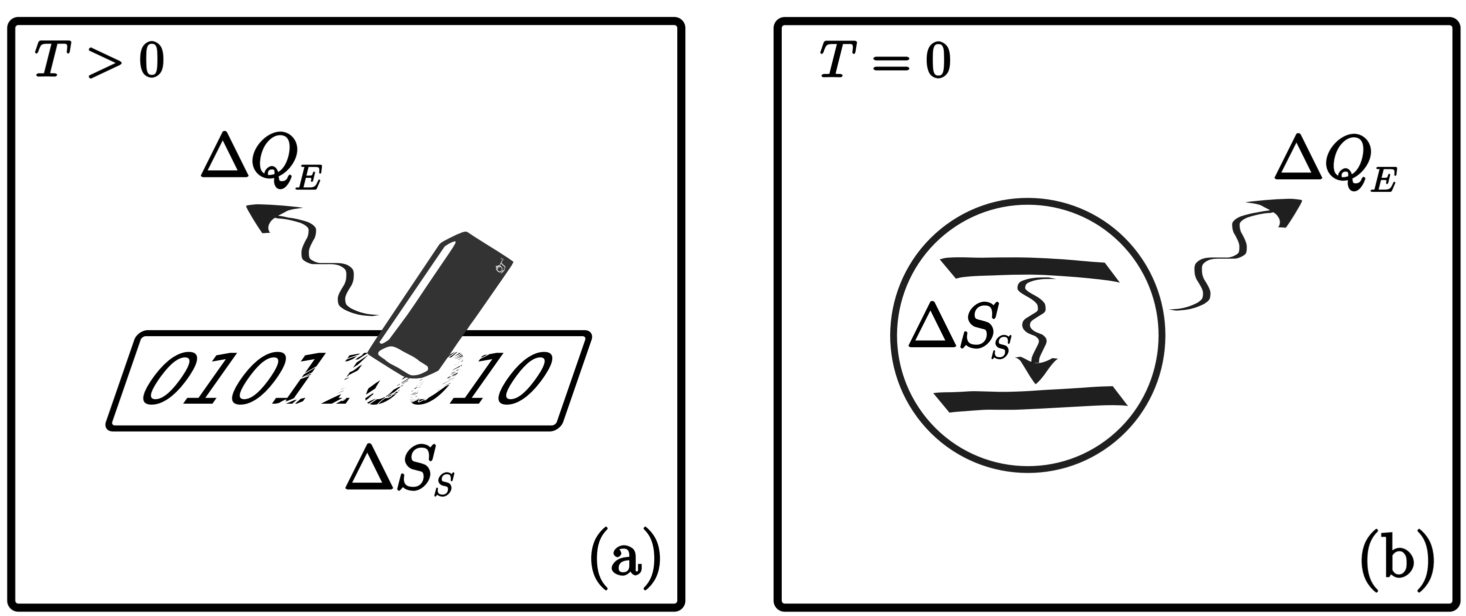
$$\Delta Q_E \geq -T\Delta S_S \quad \to \quad \Delta Q_E \geq 0$$

- All it says is that the heat cost for erasure is non-negative. But otherwise, it is *independent* of the amount of erasure.

- Take, as an example, spontaneous emission:

  - To erase information about an atom, we must emit a photon.

  - Energy therefore *was* emitted. But the bound does not capture it.



A tighter Landauer bound

- Now I would like to show how it is possible to derive a modified bound, which:
  - Is always tighter than the original.
  - Tends to it at high temperatures.
  - But yields non-trivial information when $T \to 0$.
- Landauer's bound stems from the positivity of:

$$\Sigma = I'(S:E) + D(\rho_E' || \rho_E) \geq 0$$

- Instead, we focus only on the positivity of:

$$I'(S:E) \geq 0$$

Derivation

- The initial bath state is thermal.
    - But its final state $\rho'_E = \text{tr}_S(\rho'_{SE})$ is not.
- Define a reference thermal state $\rho_E(T')$ which is thermal, but at a temperature T' such that

$$\text{tr}\left\{H_E \rho_E(T')\right\} = \text{tr}\left\{H_E \rho'_E\right\} := E_E(T')$$

- From the MaxEnt principle $S(\rho_E(T')) \geq S(\rho'_E)$ so that

$$\Delta S_S + \Delta S_E^{\text{th}} \geq \Delta S_S + \Delta S_E$$

$$= I'(S:E)$$

$$\geq 0$$

- Thus

$$\Delta S_S + \Delta S_E^{\text{th}} \geq 0$$

- This is the 1st result we will need.

- Next define the functions

$$\mathcal{Q}(T') = \Delta Q_E = \int_T^{T'} C_E(\tau) d\tau \qquad \text{and} \qquad \mathcal{S}(T') = \Delta S_E^{\text{th}} = \int_T^{T'} \frac{C_E(\tau)}{\tau} d\tau$$

- Here $C_E(T)$ is the equilibrium heat capacity of the bath.
- We may then write

$$\Delta S_E^{\text{th}} = \mathcal{S}(T') = \mathcal{S}(\mathcal{Q}^{-1}(\Delta Q_E))$$

- Plugging this in $\Delta S_S + \Delta S_E^{\text{th}} \geq 0$ we then finally get

$$\mathcal{S}(\mathcal{Q}^{-1}(\Delta Q_E)) \geq -\Delta S_S$$

- Finally, inverting $\mathcal{S}$ and $\mathcal{Q}$, we get:

$$\Delta Q_E \geq \mathcal{Q}(\mathcal{S}^{-1}(-\Delta S_S))$$

About this modified bound

- Our new bound is:

$$\Delta Q_E \geq \mathcal{Q}(\mathcal{S}^{-1}(-\Delta S_S))$$

- It is identical in spirit to Landauer's original bound (same assumptions).
    - Landauer's is universal because it assumes only 1 thing: that the bath is thermal and has a temperature T.
- Compared to Landauer, we require only one additional piece of information: the bath equilibrium heat capacity $C_E(T)$.
    - However, our bound is always always tighter.
        - Requires a bit more info, but is also always better.

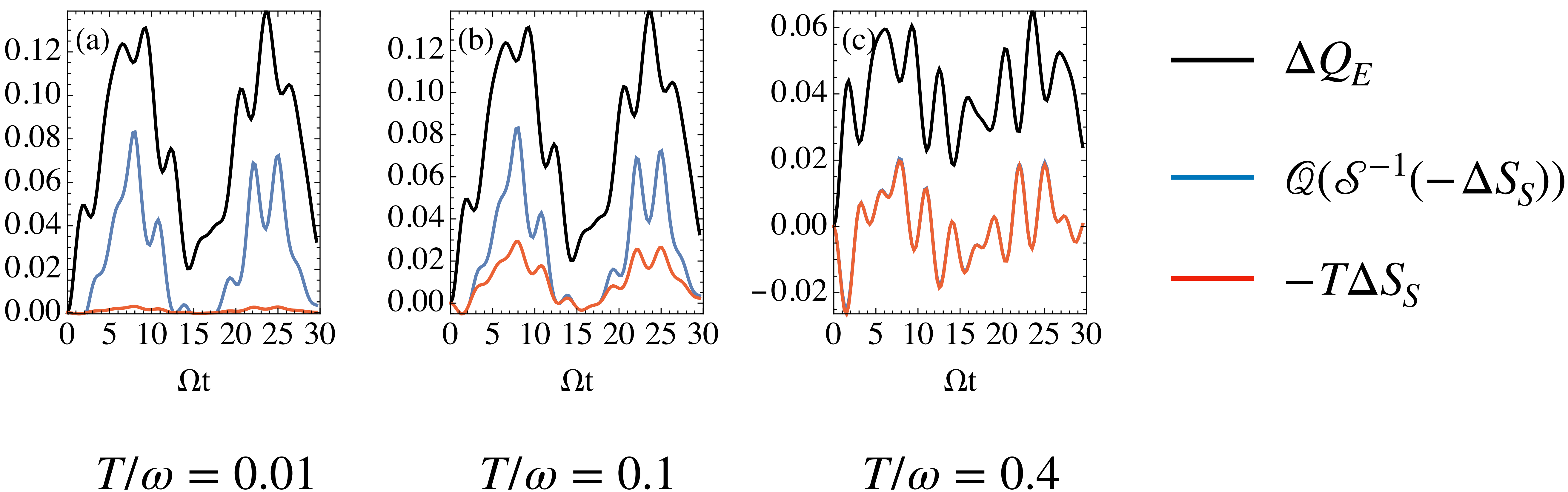
Applications

Cavity QED

- Consider a 2-level atom interacting with an optical cavity field via the Rabi model:

$$H = \hbar\omega a^\dagger a + \frac{\hbar\omega}{2}\sigma_z + \hbar g(a + a^\dagger)\sigma_x$$

- The atom is the system and the cavity is the bath.



Waveguide QED

- Next consider the emission of the atom into a 1D waveguide of length L.

- The waveguide is characterized by a continuum of modes with dispersion relation $\omega_k = ck$. Hence

$$E_E(T) = \sum_k \frac{\hbar\omega_k}{e^{\beta\omega_k} - 1} = \frac{\pi L}{12\hbar c} T^2$$

- Similarly, the entropy can be found to be

$$S_E(T) = \frac{\pi L}{6\hbar c} T$$

- Thus

$$\Delta Q_E \geq -T\Delta S_S + \frac{3\hbar c}{\pi L} \Delta S_S^2$$

- The first term is the original bound; it vanishes when $T \to 0$
    - But the 2nd remains.

Heat capacity examples

- We can also provide explicit forms for the bound by assuming different scalings for the bath's heat capacity.

- We focus on $T = 0$.

- Phonons: $C_E = aT^3$

$$\therefore \qquad \Delta Q_E \geq \frac{3^{4/3}}{4} \frac{(-\Delta S_S)^{4/3}}{a^{1/3}}$$

- Gapped system (e.g. BCS superconductor): $C_E = be^{-\delta/T}$

$$\therefore \qquad \Delta Q_E \geq \delta \frac{(-\Delta S_S)}{\ln(-b/\Delta S_S)}$$

Summary

- Landauer's bound $\Delta Q_E \geq -T\Delta S_S$ provides a fundamental link between thermodynamics and information.
- The bound is powerful because it is universal.
  - Assumes minimal information about the system and process.
- But it trivializes when $T \to 0$.
- In this talk I showed how one can derived a modified bound which
  - Is always tighter than the original.
  - Tends to it at high temperatures.
  - Yields non-trivial information when $T \to 0$.
- The bound has exactly the same spirit as the original and assumes only 1 additional ingredient: knowledge of the bath's heat capacity.

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Landauer's Principle at Zero Temperature

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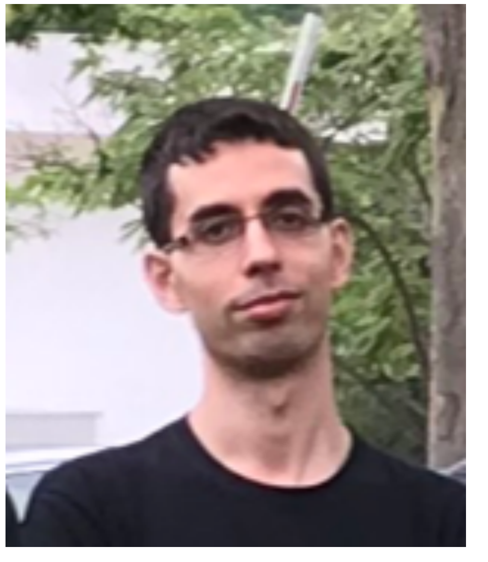

André M. Timpanaro
UFABC

Thank you ❄️🔁🔥🙃

Quantum thermodynamics
QT^2
quantum transport
IFUSP

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Extra slides

Flow of heat

- The 2nd law reads

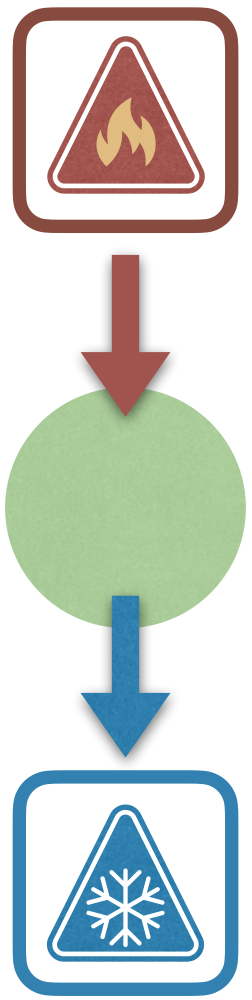
$$\dot{\Sigma} = -\frac{\dot{Q}_h}{T_h} - \frac{\dot{Q}_c}{T_c} \geq 0$$

- But if there is no work involved,

$$\dot{Q}_c = -\dot{Q}_h$$

$$\therefore \qquad \dot{\Sigma} = \left(\frac{1}{T_c} - \frac{1}{T_h}\right)\dot{Q}_h \geq 0$$

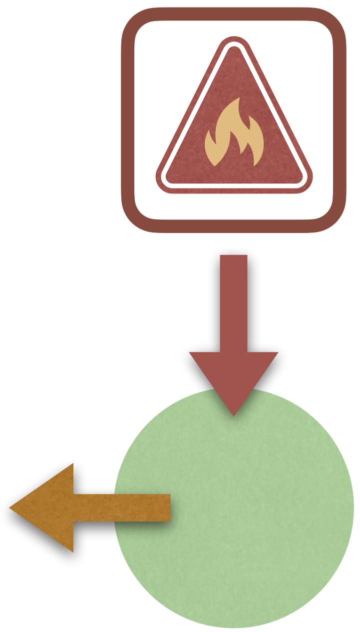
- *Heat flows from hot to cold.*



Clausius' statement of the 2nd law

"Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time."

Work from a single bath



- Finally, suppose there is only one bath present:

$$\dot{W} = -\dot{Q}_h$$

$$\dot{\Sigma} = -\frac{\dot{Q}_h}{T_h} = \frac{\dot{W}}{T_h} \geq 0$$

- Positive work (in my definition) means an external agent is *doing* work on the system.

Kelvin-Planck statement of the 2nd law

"It is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work."

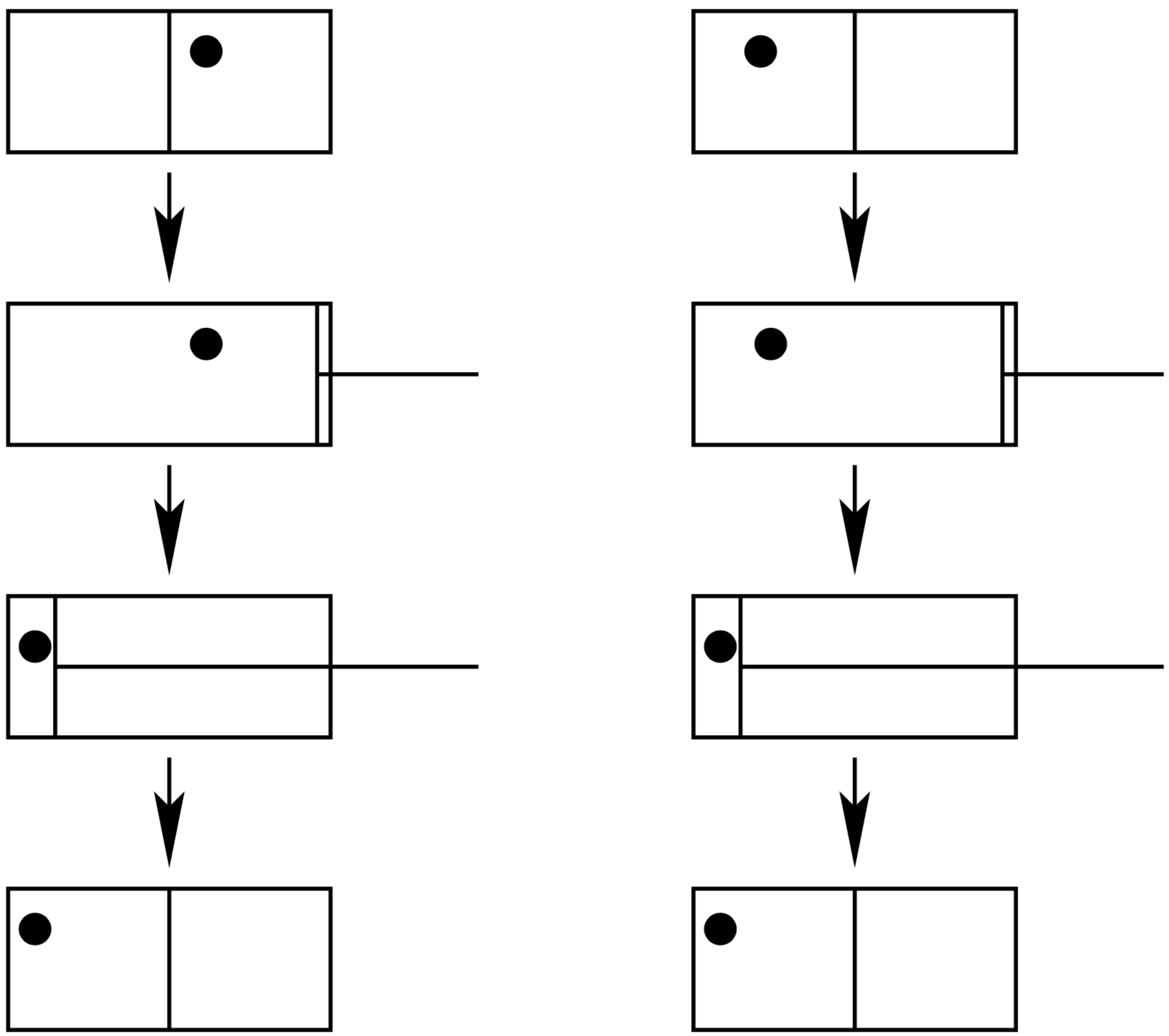
Information content

- Suppose you have an object in your hand, such as a coin or a deck of cards.
    - What is the *information content* of this object?
- Call a friend:
    - The two of you share some background information to make information possible (like establishing what a “coin” is).
    - But your friend does not know the state of the object (e.g. if the coin is Head or Tails).
- Information content = size of the set of instructions that your friend requires to be able to reconstruct the state of the object.

M. B. Plenio, V. Vitelli, “*The physics of forgetting: Landauer’s erasure principle and information theory”,* Contemporary Physics ***42****, 25 - 60 (2001).*

- We can understand Landauer's principle using basic thermodynamics.

- The particle is an ideal gas so $pV = T$



- The work in an isothermal compression is

$$W = -\int_{V_1}^{V_2} p dV = T \ln V_1 / V_2$$

- Minimum work is when one compresses up to the middle of the partition, so $V_1 = 2V_2$

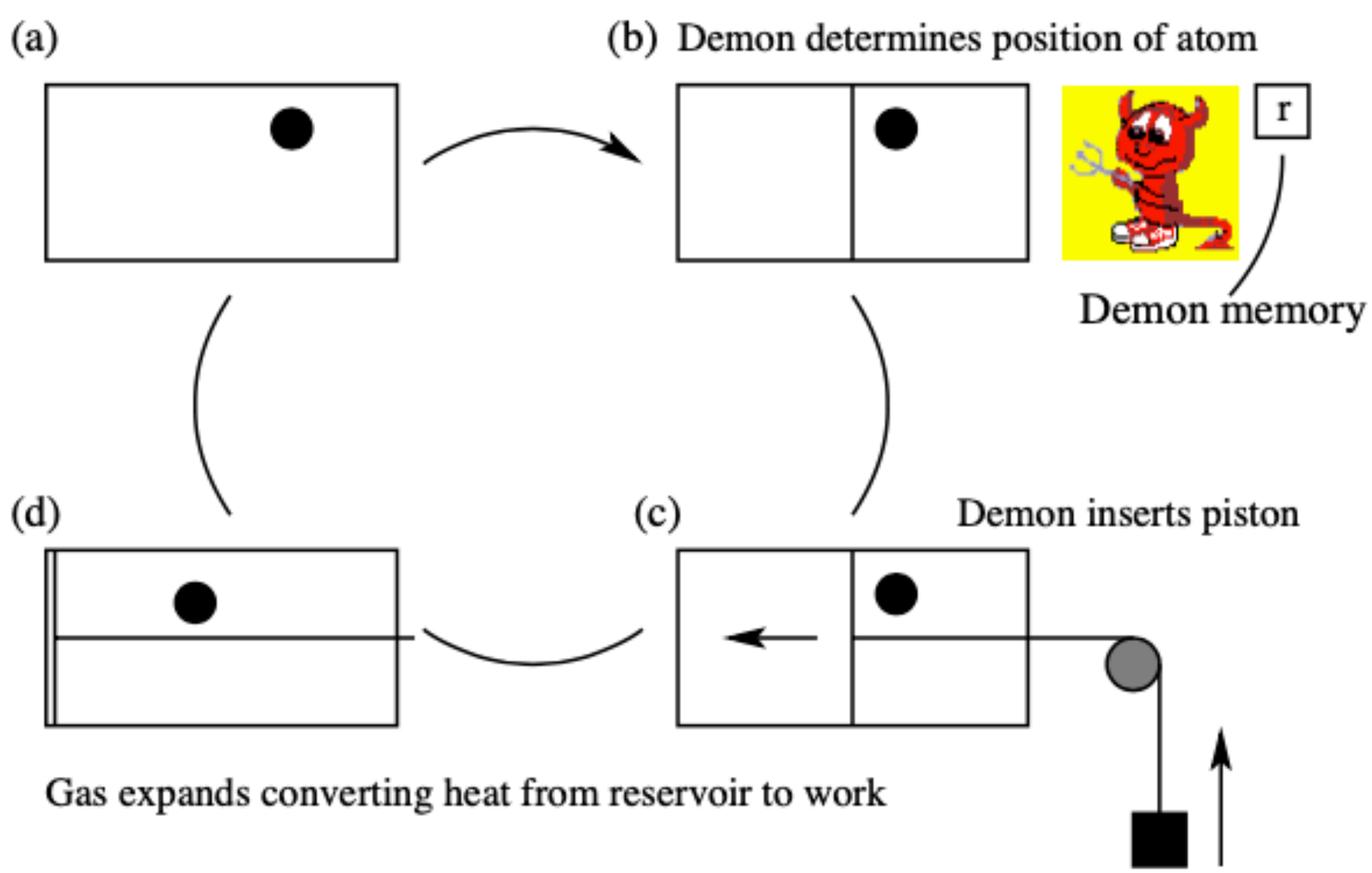
$$W_{\text{min}} = T \ln 2$$

- The energy of the particle $U(T)$ remains constant, so this work must be converted into heat:

$$\Delta Q_E = W_{\text{min}} = T \ln 2$$

Maxwell demon

Seem to violate the 2nd law.



- Bennet used Landauer's principle to solve this paradox:
- There is no energy cost for the demon to acquire information.
    - But there is an energy cost to erase it!

Kelvin-Planck statement of the 2nd law

"It is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work."

Thermal case

- Let us check that we indeed recover the standard expressions when the bath is thermal.
- Since the dynamics is unitary

$$S(\rho'_{SE}) = S(\rho_S \otimes \rho_E) = S(\rho_S) + S(\rho_E)$$

- Whence

$$I'(S:E) = S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE})$$

$$= S(\rho'_S) + S(\rho'_E) - S(\rho_S) - S(\rho_E)$$

$$= \Delta S_S + \Delta S_E$$

- The entropy production will then be

$$\Sigma = \Delta S_S + S(\rho'_E) - S(\rho_E) + D(\rho'_E || \rho_E)$$

- Opening up the last terms:

$$S(\rho_E') - S(\rho_E) + D(\rho_E' || \rho_E) = -\,\mathrm{tr}(\rho_E' \ln \rho_E') + \mathrm{tr}(\rho_E \ln \rho_E)$$

$$+\mathrm{tr}\Big(\rho_E' \ln \rho_E' - \rho_E' \ln \rho_E\Big)$$

$$= \mathrm{tr}\Big\{(\rho_E - \rho_E')\ln \rho_E\Big\}$$

- This result is still general. Now we assume that the bath is in a thermal state

$$\rho_E = \frac{e^{-\beta H_E}}{Z_E}$$

- This is then seen to be the heat entering the bath:

$$\mathrm{tr}\Big\{(\rho_E - \rho_E')\ln \rho_E\Big\} = \beta \Delta Q_E = \beta\Big(\langle H_E \rangle' - \langle H_E \rangle\Big)$$

- Whence:

$$\Sigma = \Delta S_S + \beta \Delta Q_E$$