Quantum systems and stochastic excursions

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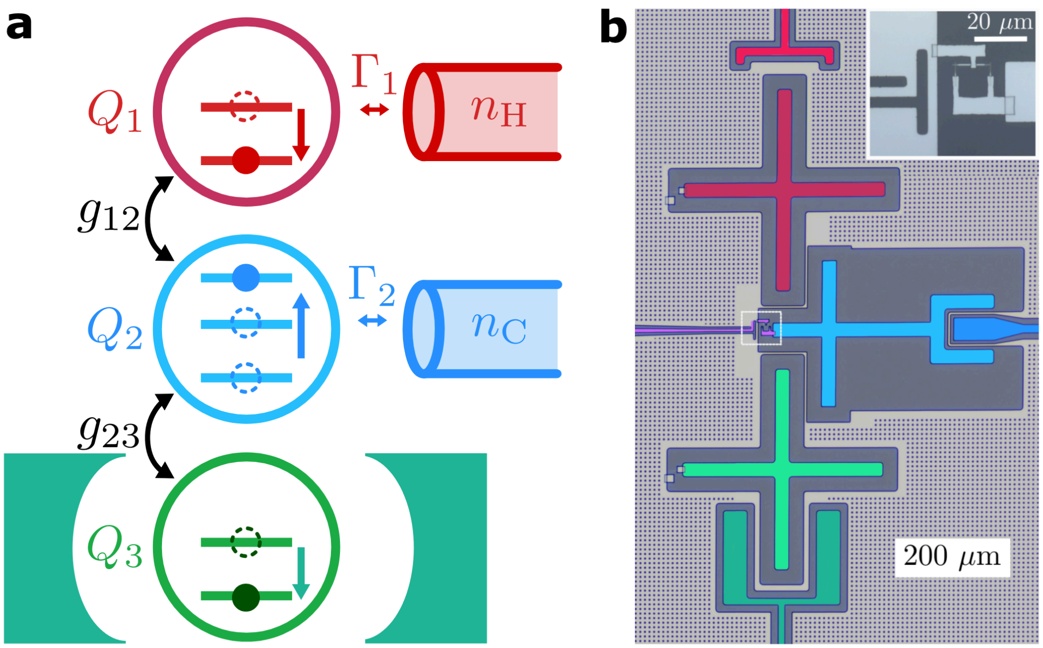
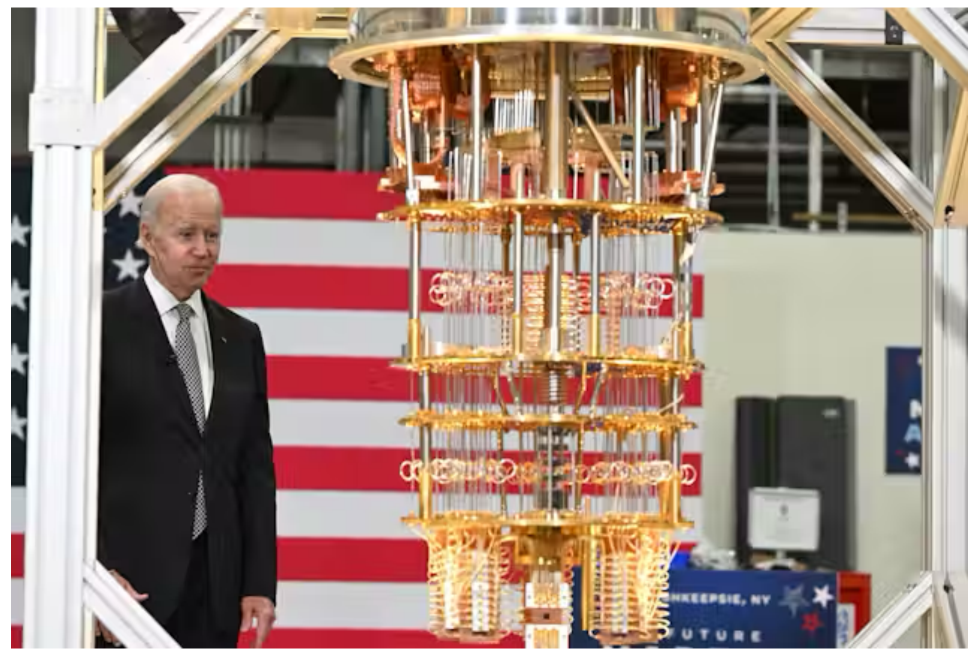
University of Rochester

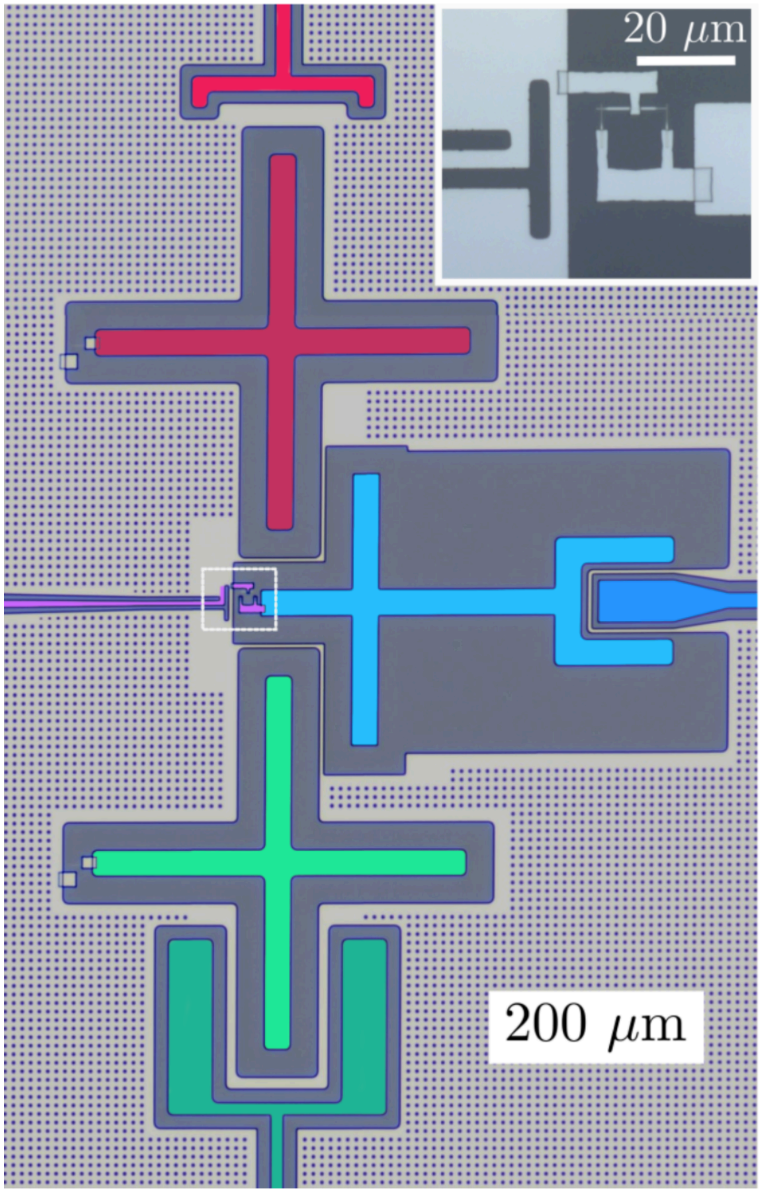
Nov 1st.
Complex systems and information theory seminar

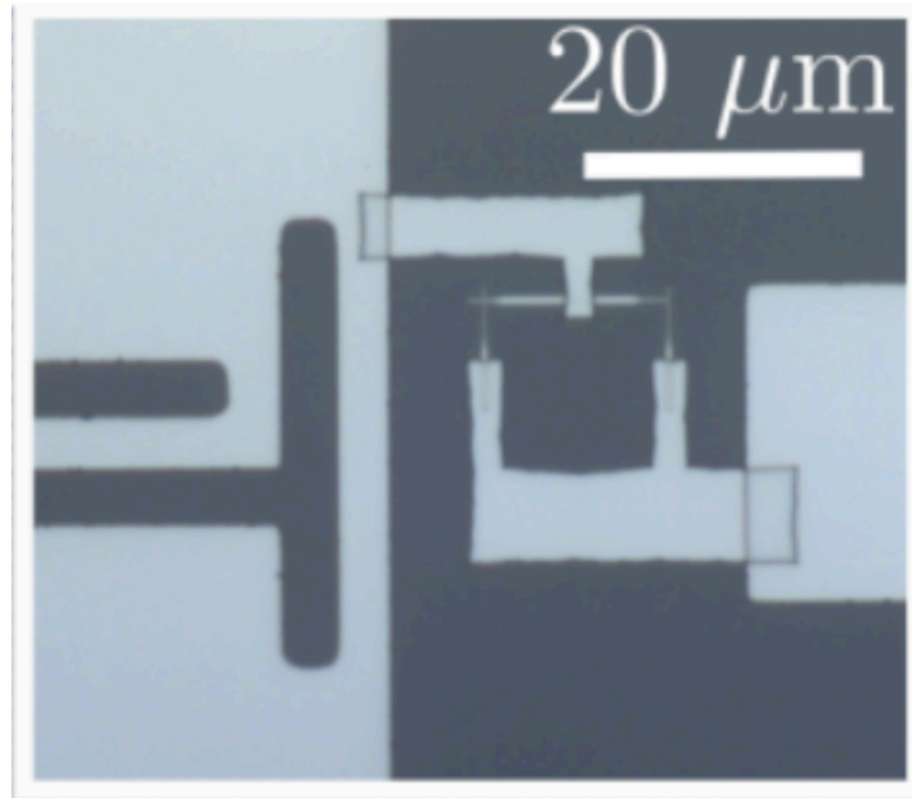
https://www.pas.rochester.edu/~gtlandi

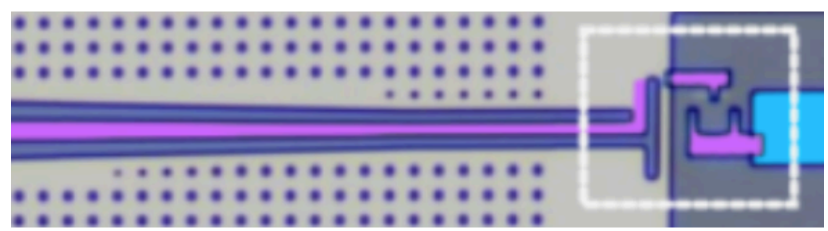




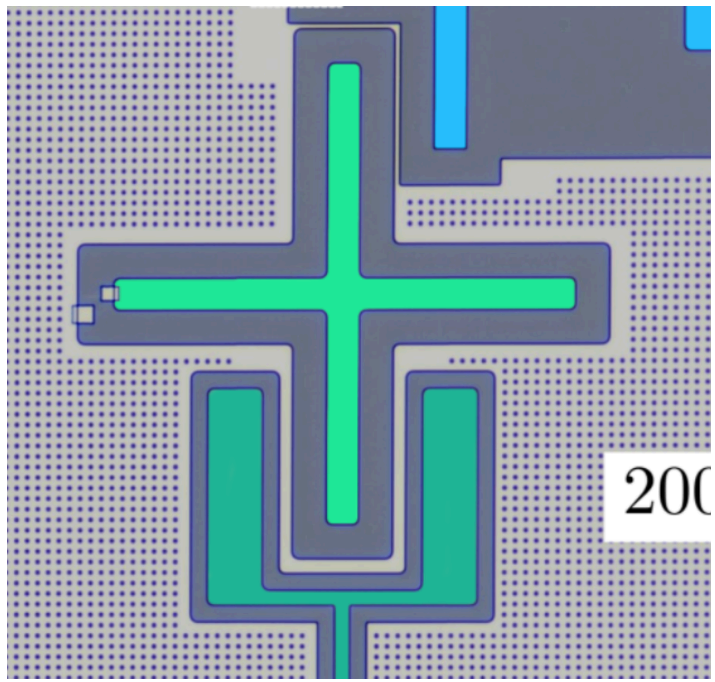
Aamir, M. A. et al. "**Thermally driven quantum refrigerator autonomously resets superconducting qubit**" arXiv.2305.16710 (2023). To appear in Nature Physics.


20 μm
200 μm

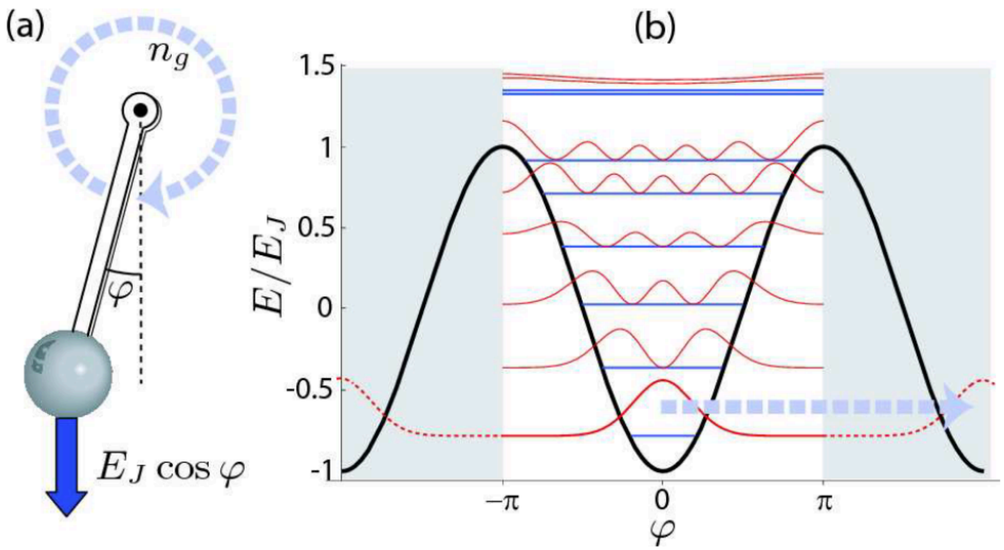

20 μm



Readout from overlap



Single transmon: quantum states are like harmonics of a vibrating string



$$H = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\varphi}$$

Mechanical analogy

$$H_{\text{pend}} = \frac{\hat{L}_z^2}{2m\ell^2} - mg\ell\cos\hat{\varphi}$$

Koch, J. et al. "**Charge insensitive qubit design derived from the Cooper pair box**", Phys. Rev. A 76, 042319 (2007)

Basic rules

Superposition	Mixture
In many states at the same time	Maybe many states. Ignorance.
Very quantum-y	Classical
Lost if left alone (“decoherence”)	

When left alone:

- E_0 = ground-state = state with the smallest energy.
- If left alone, systems will tend to decay to the ground-state.
- Thermal fluctuations can populate excited states.
- $p_n/p_0 = e^{-(E_n-E_0)/k_BT}$: populations decrease exponentially with energy separation.
- Any coherences are eventually lost after some time due to noise (decoherence)

When driven:

- External pulses can excite system.
- External pulses can create superposition.

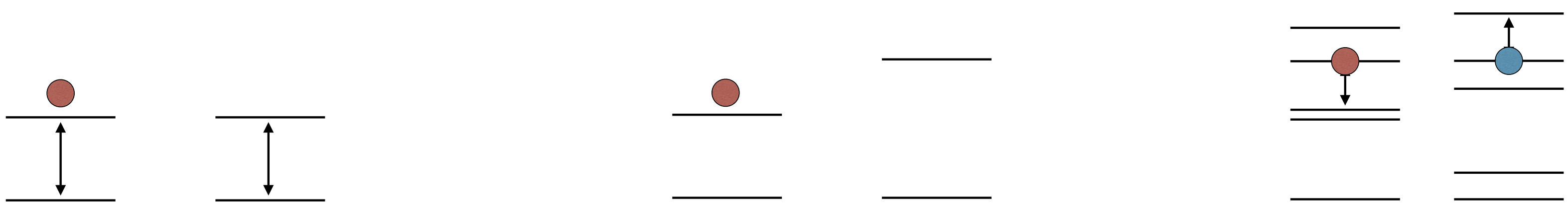
Quantum systems do not live in real space.

- They live in Hilbert space.
- System @ energy levels = classical-ish.
- Wobbly-wobbly = quantum-ish.
- Quantum-ish to classical-ish due to decoherence.

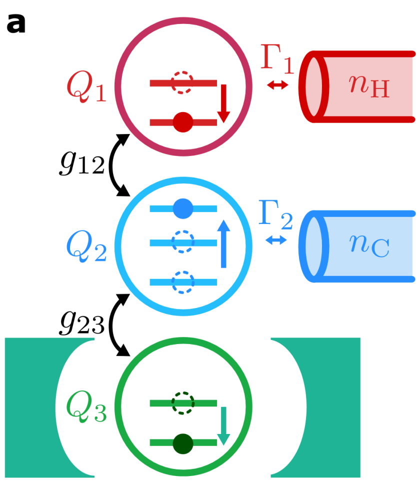
Superposition	Mixture
In many states at the same time	Maybe many states. Ignorance.
Very quantum-y	Classical
Lost if left alone (“decoherence”)	

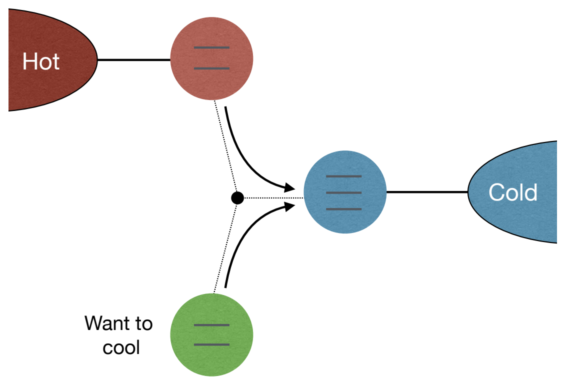
Interactions between quantum systems:

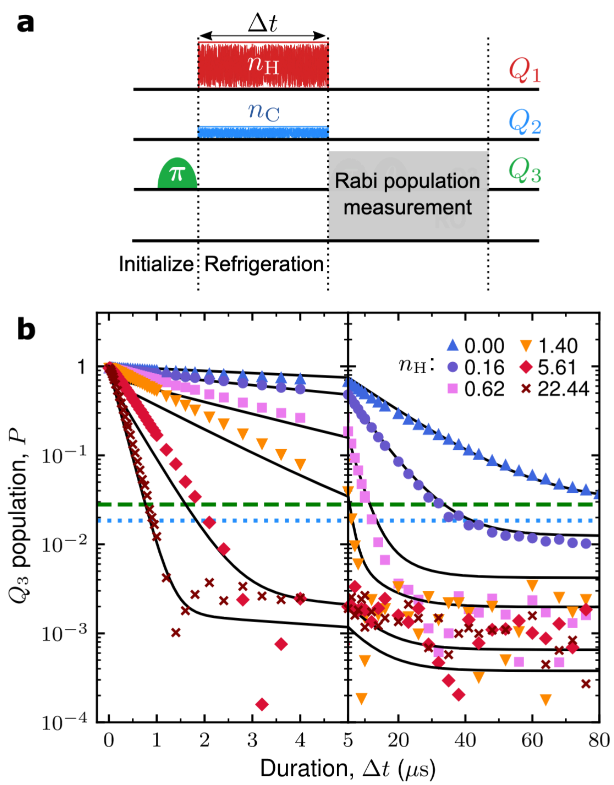
- Weak coupling → stuff only happens when there are resonances.



Cooling colder than the coldest bath







Transmon 3 is part of a quantum computer. We want to keep it cool.

Coldest bath, determined by the dilution fridge.

We can cool below the coldest bath using an **absorption refrigerator**.

Idea: only resonant transition is

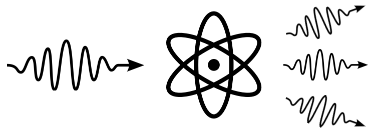
$\omega_1 + \omega_3 =$ double gap of 2

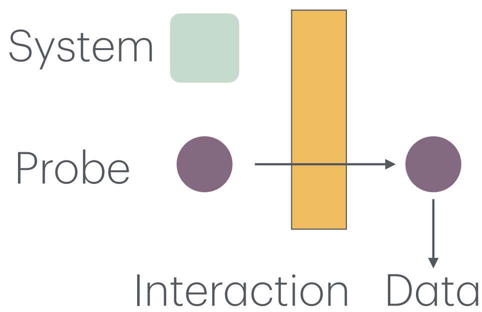
- Excitation arrives in 3.
- Combines with excitation in 1 (hot; plenty).
- Transferred to cold.

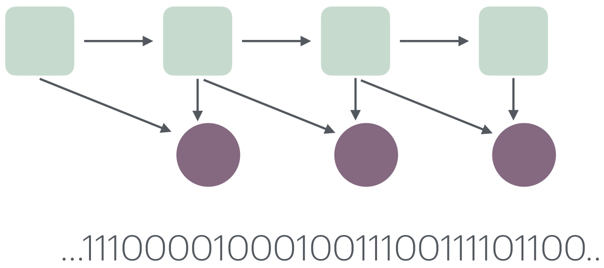
$$1_{Q_3} 1_{Q_1} \rightarrow 2_{Q_2}$$

Quantum complications

- We cannot directly “see” quantum systems.
    - To see = to measure = to send in a probe to interact with it.
    - Probe has a **back-action**: perturbs the system.
- Information is always indirect. If we try to see it, we perturb it.
- Similarity to Hidden Markov Models (HMM):
    - Quantum system is hidden.
    - Measurement outcomes (what we see) = data.
- All we can do is Bayesian updates:
    - What is our best guess for the state of the system given the data?

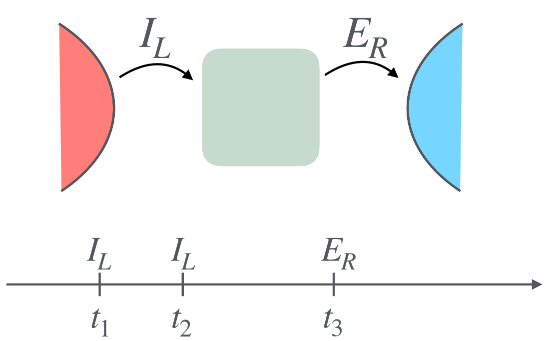






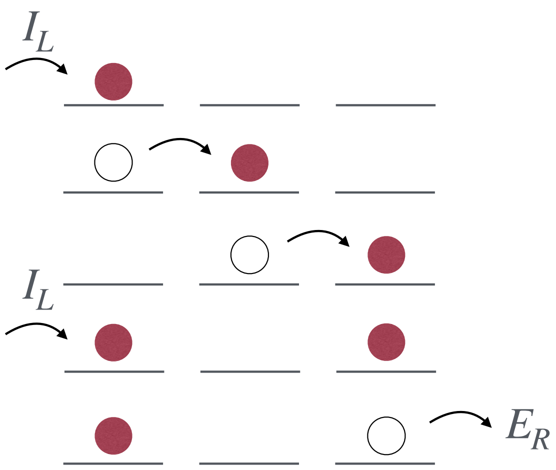
Ex: Injection/extraction on a lattice

- Lattice with L sites that can be empty or occupied.
  - Particles can be injected on the left (I_L)
  - or extracted on the right (E_R).
  - And they can jump back and forth through the chain: *not monitorable.*



All we observe are symbols

$$I_L I_L E_R$$



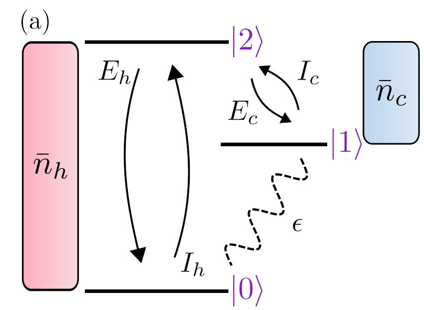
A simpler quantum machine: 3-level maser

Patrick Potts

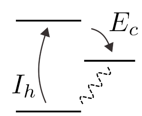
Abhaya Hegde

Abhaya S. Hegde, Patrick P. Potts, GTL, “**Time-resolved Stochastic Dynamics of Quantum Thermal Machines**,” arXiv:2408.00694

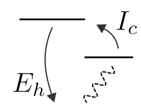
A simpler quantum machine: 3-level maser



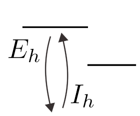
- A quantum system with 3 levels can be used to generate coherent microwave radiation (maser).
- Clockwise cycle: emit a photon.

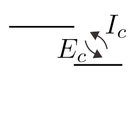


- Anti-clockwise cycle: absorbs a photon.
  - Undesirable but can happen.



- Idle cycles (bounces): machine failed; nothing happened.





Questions:

- What is the probability that the next cycle is of a given type.
- What is the probability a cycle takes a certain time?
- How are cycles correlated with each other?
- What is the average time required to complete each cycle?
- How many idle cycles happen between two useful cycles?

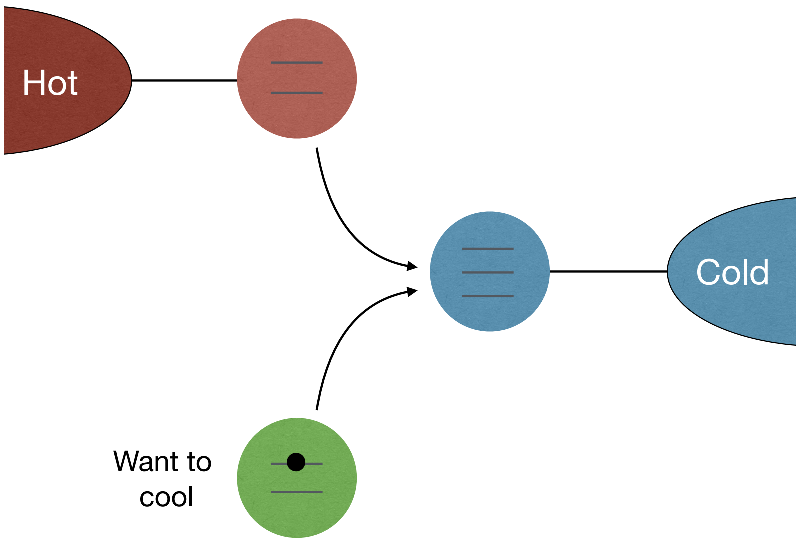
Scovil, H. E. D. & Schulz-DuBois, E. O. **Three-level masers as heat engines**. Physical Review Letters 2, 262 (1959).

Stochastic excursions

Guilherme Fiusa

Abhaya Hegde

Motivation: cooling on demand



Excursion:
- Starts when an excitation arrives at the green qubit.
- Ends the first time the excitation leaves the green qubit.

Many things can happen in between.

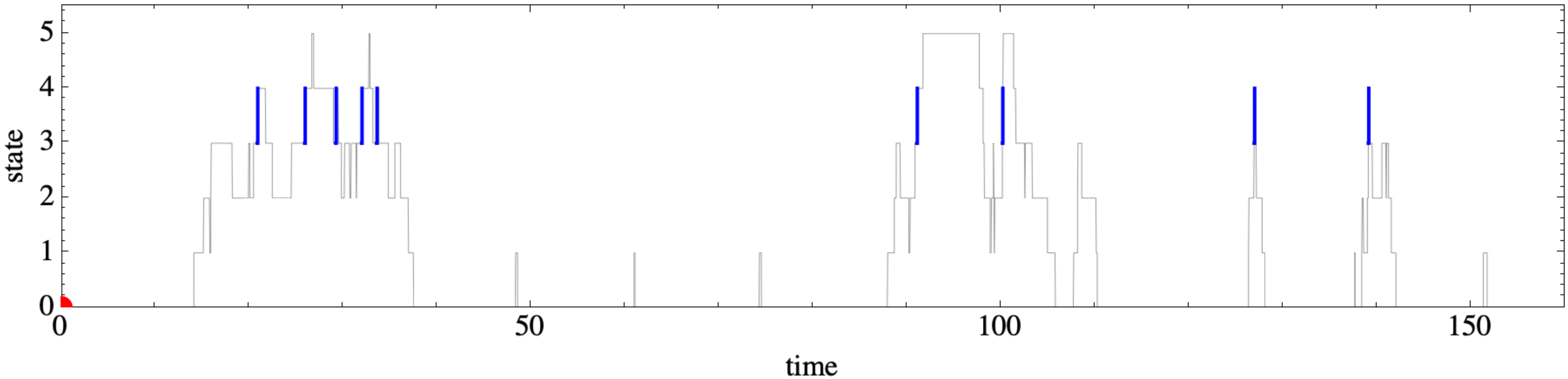
There are many ways that the cooling process can fail before it succeeds.

Traditional questions: excursion times (First Passage times, Travel times, sojourn times,...)

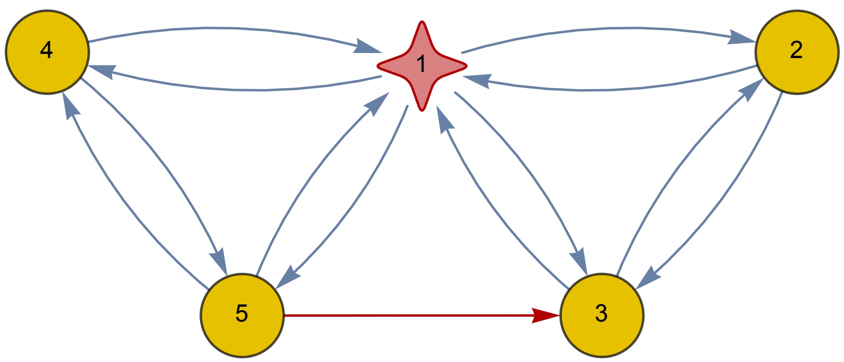
What we are interested: statistics of **counting observables.**

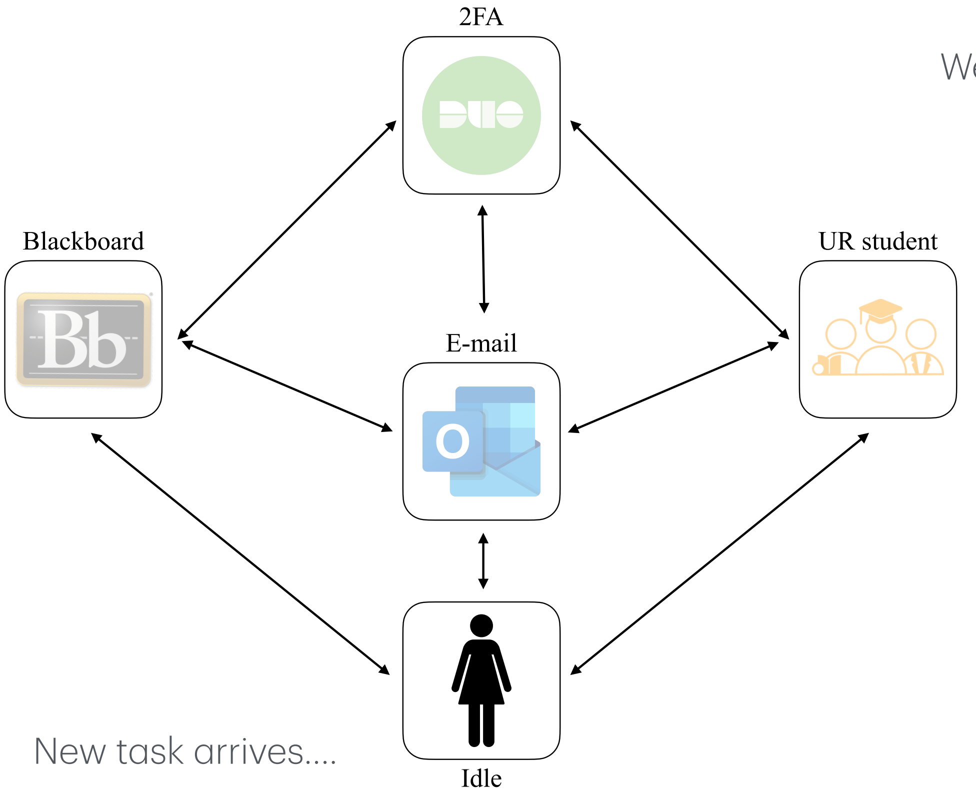
So far: fully classical.

Example 1



Example 2





New task arrives....

We can attribute contextual meaning to any transition

During an excursion…

- ☑ How many 2-factor authentications?
- ☑ How many e-mails were sent?
- ☑ How many faculty did she have speak to?

Our results

Classical rate equation: $\frac{dp_x}{dt} = \sum_y W_{xy} p_y - \Gamma_x p_x$

Excursion time $\hat{\mathcal{T}}$

Counting variables $\hat{N}_{xy}(\hat{\mathcal{T}})$

Linear counting observable: $\hat{Q}(\hat{\mathcal{T}}) = \sum_{x,y} \nu_{xy} \hat{N}_{xy}(\hat{\mathcal{T}})$.

The weights ν_{xy} can be used to append contextual meaning to each transition.

New result: mathematical tool to efficiently calculate the full distribution $P(\hat{Q}_1, \ldots, \hat{Q}_r, \hat{\mathcal{T}})$ within a single excursion.

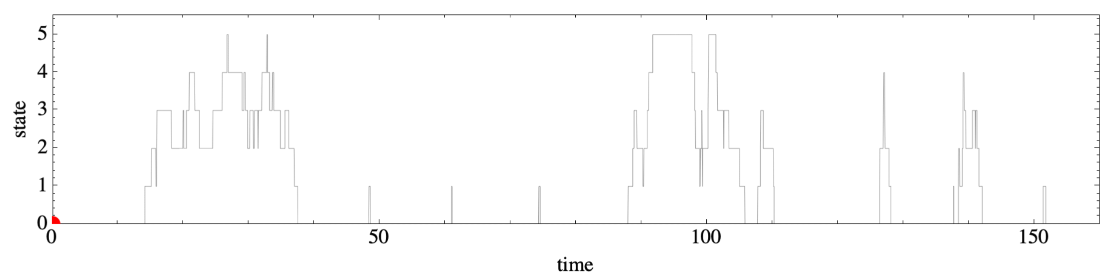
Optimal time-keeping

- Suppose we want to use this stochastic system as a clock.
- Each excursion = 1 tick of the clock.
- How do we build an optimal estimator for time?

$$\hat{\theta} = \sum_{x,y} \nu_{xy} \hat{N}_{xy}$$

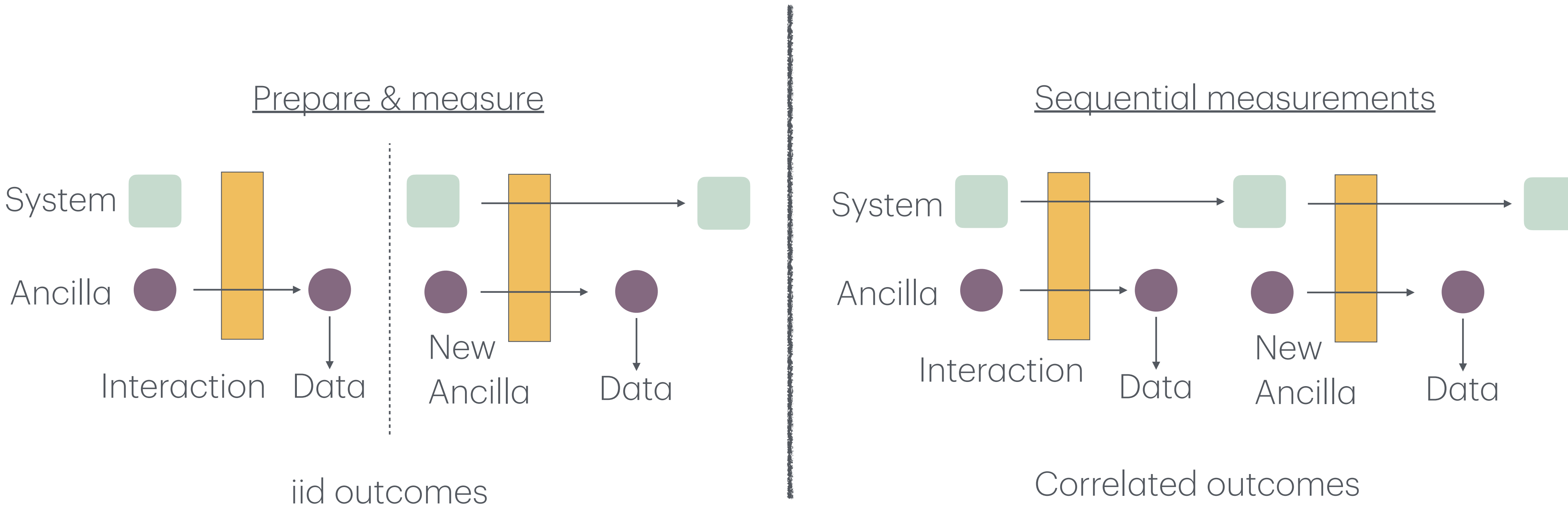
  - Choose ν_{xy} such that $E(\hat{\theta}) = t$.

- Optimal choice is $\hat{\theta} = \sum_{x,y} \frac{\hat{N}_{xy}}{\Gamma_y}$



Prech, K. et al. "Optimal time estimation and the clock uncertainty relation for stochastic processes". arXiv 2406.19450.

Thank you!


Prepare & measure
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Ancilla
Interaction
Data
New
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Data
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Sequential measurements
System
Ancilla
Interaction
Data
New
Ancilla
Data
Correlated outcomes