Informational steady-states in continuously monitored quantum systems

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In collaboration with

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Entropy Production in Continuously Measured Quantum Systems

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Editors' Suggestion

Experimental Assessment of Entropy Production in a Continuously Measured Mechanical Resonator

Massimiliano Rossi,[1,2] Luca Mancino,[3] Gabriel T. Landi,[4] Mauro Paternostro,[3] Albert Schliesser,[1,2] and Alessio Belenchia[3,*]

arXiv:2005.03429

Informational steady-states and conditional entropy production in continuously monitored systems

Gabriel T. Landi,[1,*] Mauro Paternostro,[2] and Alessio Belenchia[2]

In preparation

2nd law at the quantum level

$$\rho'_{XY} = U(\rho_X \otimes \rho_Y)U^\dagger$$

- The degree of irreversibility of this process is quantified by the entropy production:

$$\Sigma = I'(X:Y) + S(\rho'_Y || \rho_Y)$$

$$= S(X') - S(X) + \Phi$$

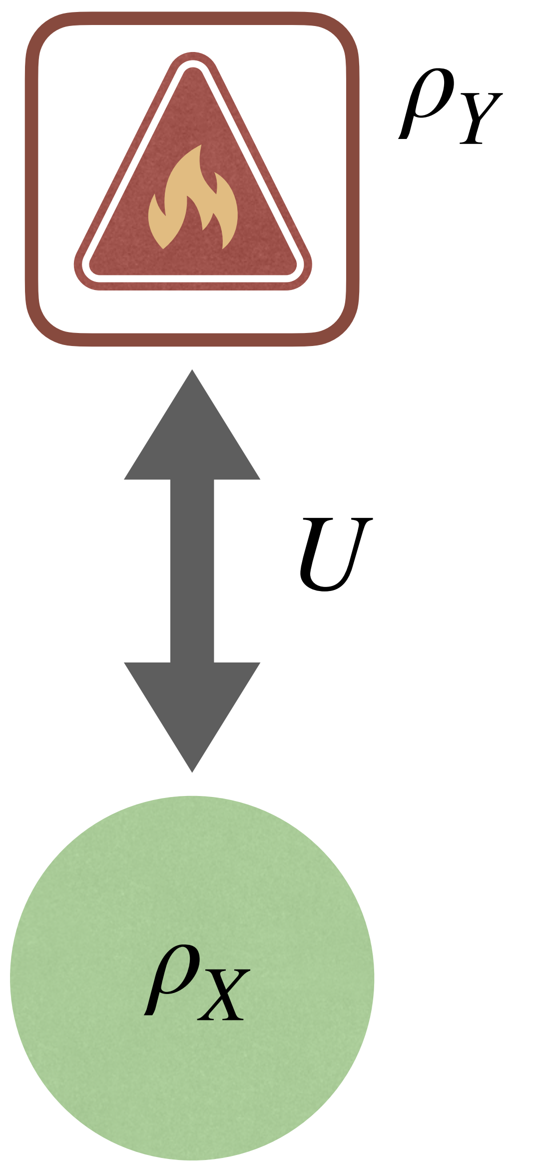
where

$$\Phi = \mathrm{tr}_Y\Big\{(\rho_Y - \rho'_Y)\ln\rho_Y\Big\}$$

is called the **entropy flux**.

$$I'(X:Y) = S(\rho'_X) + S(\rho'_Y) - S(\rho'_{XY})$$

$$S(\rho'_Y || \rho_Y) = \mathrm{tr}\big(\rho'_Y \ln \rho'_Y - \rho'_Y \ln \rho_Y\big)$$



- Φ depends only on Y. Measures change in the "thermodynamic potential" $\ln \rho_Y$
  - If $\rho_Y = e^{-\beta H_Y}/Z_Y$ we get $\Phi = -\beta Q$.

Describes an enormous variety of processes! (maybe a complicated U)

M. Esposito, K. Lindenberg, C. Van den Broeck, "Entropy production as correlation between system and reservoir". New Journal of Physics, **12**, 013013 (2010).

🛂 Conditional entropy production

- Part of the irreversibility stems from our ignorance about the environment.
- Suppose we measure Y after it interacted with X.

$$\rho'_{XY} \to \rho'_{XY|z} = (1 \otimes M_z)\rho'_{XY}(1 \otimes M_z^\dagger)$$

$$p_z = \mathrm{tr}_Y\big(M_z^\dagger M_z \rho'_Y\big)$$

- $\{M_z\}$ = generalized measurement operators acting on Y:

This is a conditional state: It is the state of XY, conditioned on the measurement outcome being z.

- What is the entropy production and flux, conditioned on these outcomes?

$$\Sigma_c = S(X'|z) - S(X) + \Phi_c \qquad \text{where} \qquad S(X'|z) = \sum_z p_z S(\rho'_{X|z})$$

is the quantum-classical conditional entropy

- How to define Σ_c and Φ_c?
- Natural generalization of the flux:

$$\Phi_c = \sum_z p_z \mathrm{tr}\Big\{(\rho_Y - \rho'_{Y|z})\ln \rho_Y\Big\}$$

$$= \mathrm{tr}\Big\{(\rho_Y - \tilde{\rho}_Y)\ln \rho_Y\Big\}$$

where $\tilde{\rho}_Y = \sum_z p_z \rho'_{Y|z}$.

- But very often $\mathrm{tr}(\tilde{\rho}_Y \ln \rho_Y) = \mathrm{tr}(\rho'_Y \ln \rho_Y)$, so

$$\Phi_c = \Phi$$

Flux is physical; no subjective component associated to information acquired.

- The unconditional and conditional $\Sigma's$ are thus

$$\Sigma_u = S(X') - S(X) + \Phi$$

$$\Sigma_c = S(X'|z) - S(X) + \Phi$$

- Whence,

$$\Sigma_c = \Sigma_u - I$$

where

$$I = S(X') - S(X'|z) = \sum_z p_z S(\rho'_{X|z}||\rho'_X)$$

is the Holevo χ quantity ✅.

- One may show that

$$0 \leqslant \Sigma_c \leqslant \Sigma_u$$

- Thus, the conditional entropy production still satisfies a 2nd law ($\Sigma_c \geqslant 0$).

- But it is also smaller than the unconditional one:

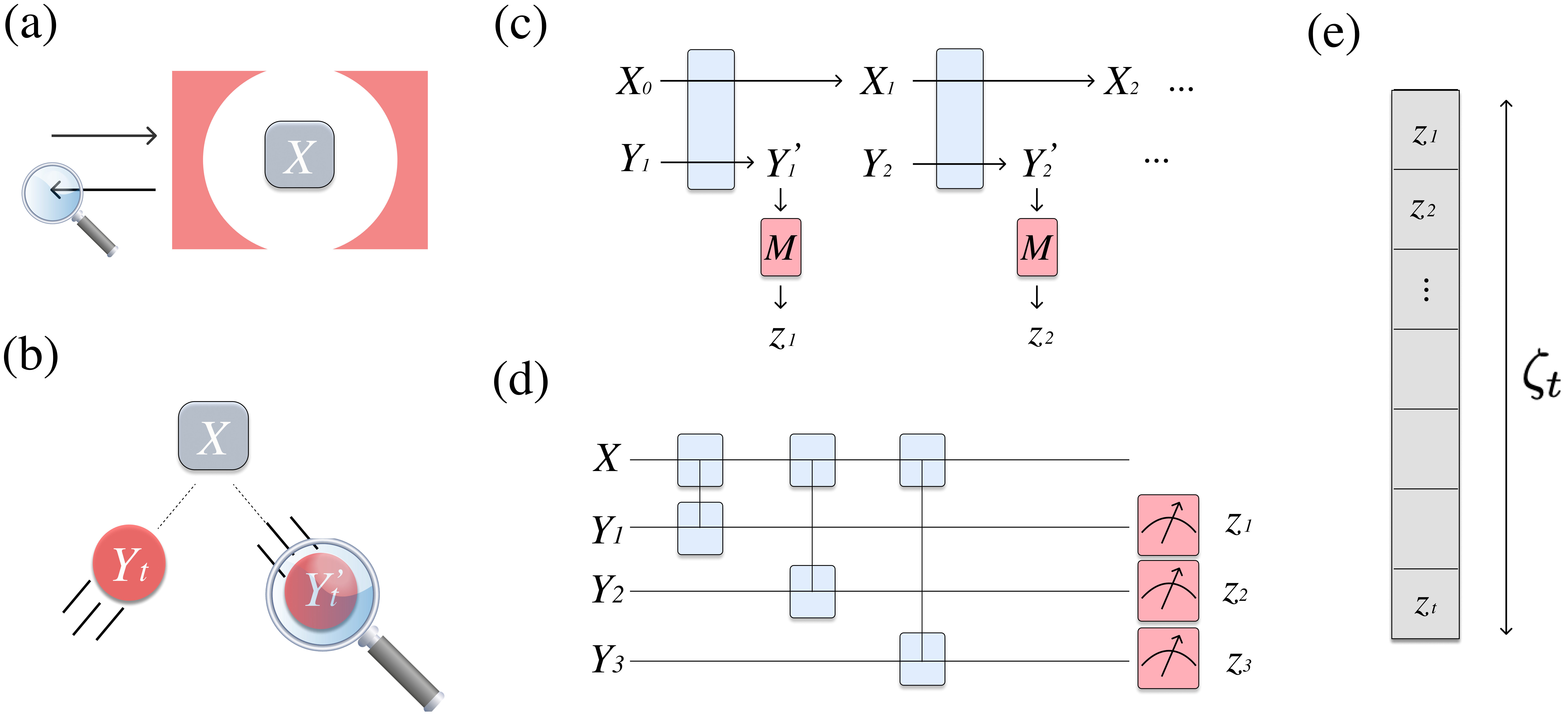
  - Conditioning makes the process more reversible.

K. Funo, Y. Watanabe and M. Ueda, "Integral quantum fluctuation theorems under measurement and feedback control". PRE, **88**, 052121 (2013).

GTL and M. Paternostro, "Irreversible entropy production, from quantum to classical", arXiv:2009.07668

M. Naghiloo, J. J. Alonso, A. Romito, E. Lutz, K. Murch, "Information Gain and Loss for a Quantum Maxwell's Demon". PRL **121**, 030604 (2018).

CM²: Continuously measured collisional models



Information-theoretic quantities

- The unconditional dynamics is governed by the stroboscopic map

$$\rho_{X_t} = \mathscr{E}(\rho_{X_{t-1}}) = \mathrm{tr}_{Y_t}\left\{ U_t\left(\rho_{X_{t-1}} \otimes \rho_{Y_t}\right) U_t^\dagger \right\}$$

- And its information content is thus summarized by the von Neumann entropy

$$S(X_t) = -\,\mathrm{tr}\left\{\rho_{X_t} \ln \rho_{X_t}\right\}$$

- The conditional dynamics, on the other hand, is governed by (up to a normalization)

$$\rho_{X_t|\zeta_t} = \mathscr{E}_{z_t}(\rho_{X_{t-1}|\zeta_{t-1}}) = \mathrm{tr}_{Y_t}\left\{ M_{z_t} U_t\left(\rho_{X_{t-1}} \otimes \rho_{Y_t}\right) U_t^\dagger M_{z_t}^\dagger \right\}$$

- And its information content is thus summarized by the quantum-classical conditional entropy

$$S(X_t\,|\,\zeta_t) = \sum_{\zeta_t} P(\zeta_t)\; S(\rho_{X_t|\zeta_t})$$

Their difference is the Holevo information:

$$I(X_t : \zeta_t) = S(X_t) - S(X_t\,|\,\zeta_t) = \sum_{\zeta_t} P(\zeta_t) D\left(\rho_{X_t|\zeta_t} \,||\, \rho_{X_t}\right) \geqslant 0$$

Gain rate/Loss rate - ISS

- The change in Holevo information can have any sign:

$$\Delta I_t = I(X_t : \zeta_t) - I(X_{t-1} : \zeta_{t-1})$$

- But we can split it into a Gain rate and a Loss rate

$$\Delta I_t = G_t - L_t$$

$$G_t = I(X_t : z_t | \zeta_{t-1}) = I(X_t : \zeta_t) - I(X_t : \zeta_{t-1}) \geqslant 0$$

$$L_t = I(X_{t-1} : \zeta_{t-1}) - I(X_t : \zeta_{t-1}) \geqslant 0$$

Informational steady-state:

$$\Delta I_{ISS} = 0$$

but

$$G_{SS} = L_{SS} \neq 0.$$

Thermodynamics

- The entropy flux/production is now the same as before:
  - Unconditional:

$$\Delta\Sigma_t^u = S(X_t) - S(X_{t-1}) + \Delta\Phi_t$$

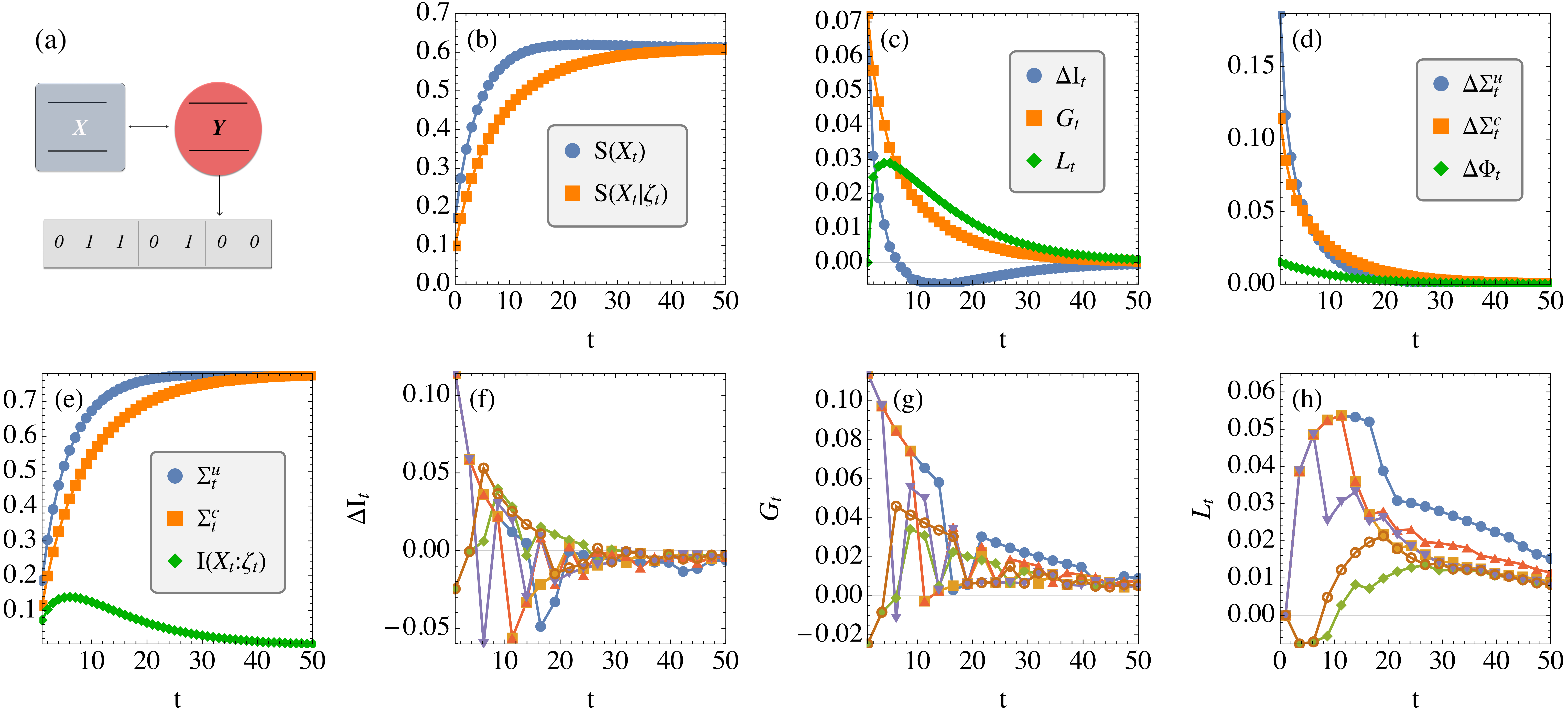
  - Conditional:

$$\Delta\Sigma_t^c = S(X_t | \zeta_t) - S(X_{t-1} | \zeta_{t-1}) + \Delta\Phi_t$$
$$= \Delta\Sigma_t^u - \Delta I_t$$

  - Flux is again the same in both.
- In an ISS $\Delta I_{ISS} = 0$ so $\Delta\Sigma_{ISS}^c = \Delta\Sigma_{ISS}^u$.

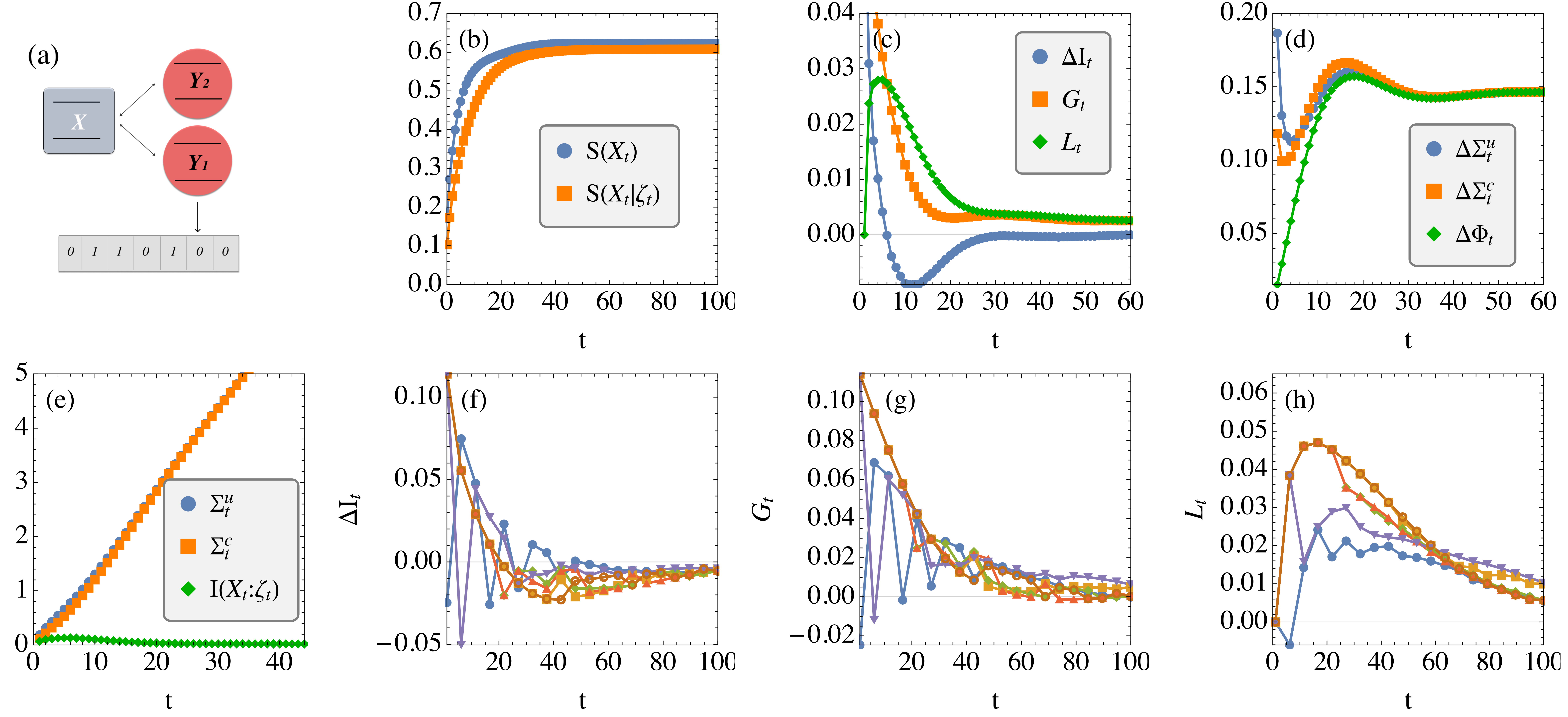
Minimal qubit models - Single-qubit ancilla

Thermal ancilla qubit + partial SWAP.

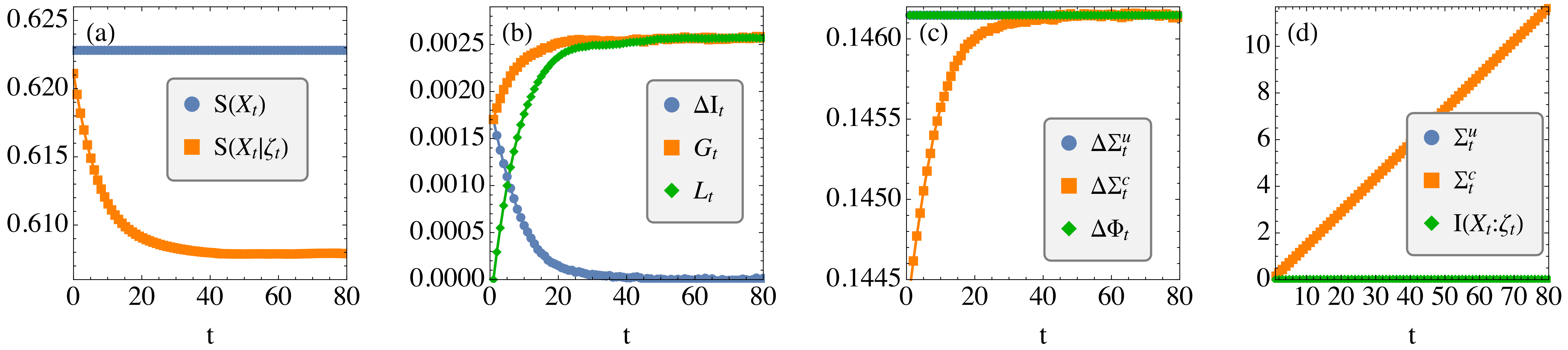


Minimal qubit models - Two-qubit ancilla

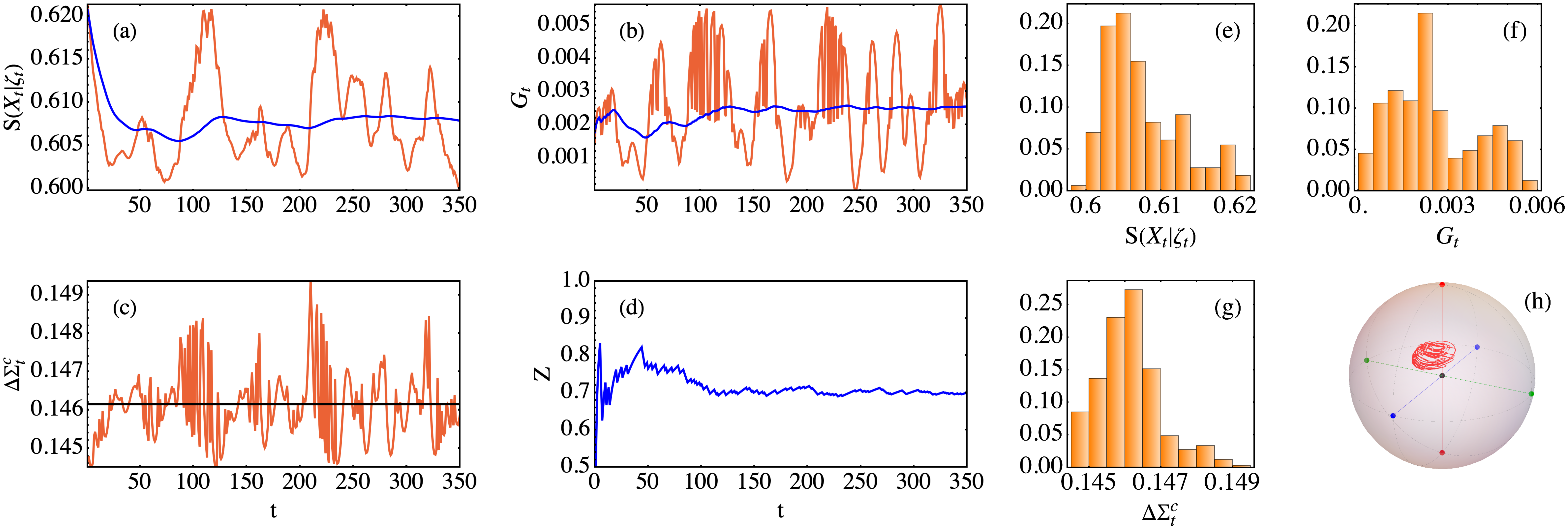
One ancilla thermal. The other prepared in $|+\rangle$
Sequential partial SWAPs



Starting from the ISS:



Single-shot scenario



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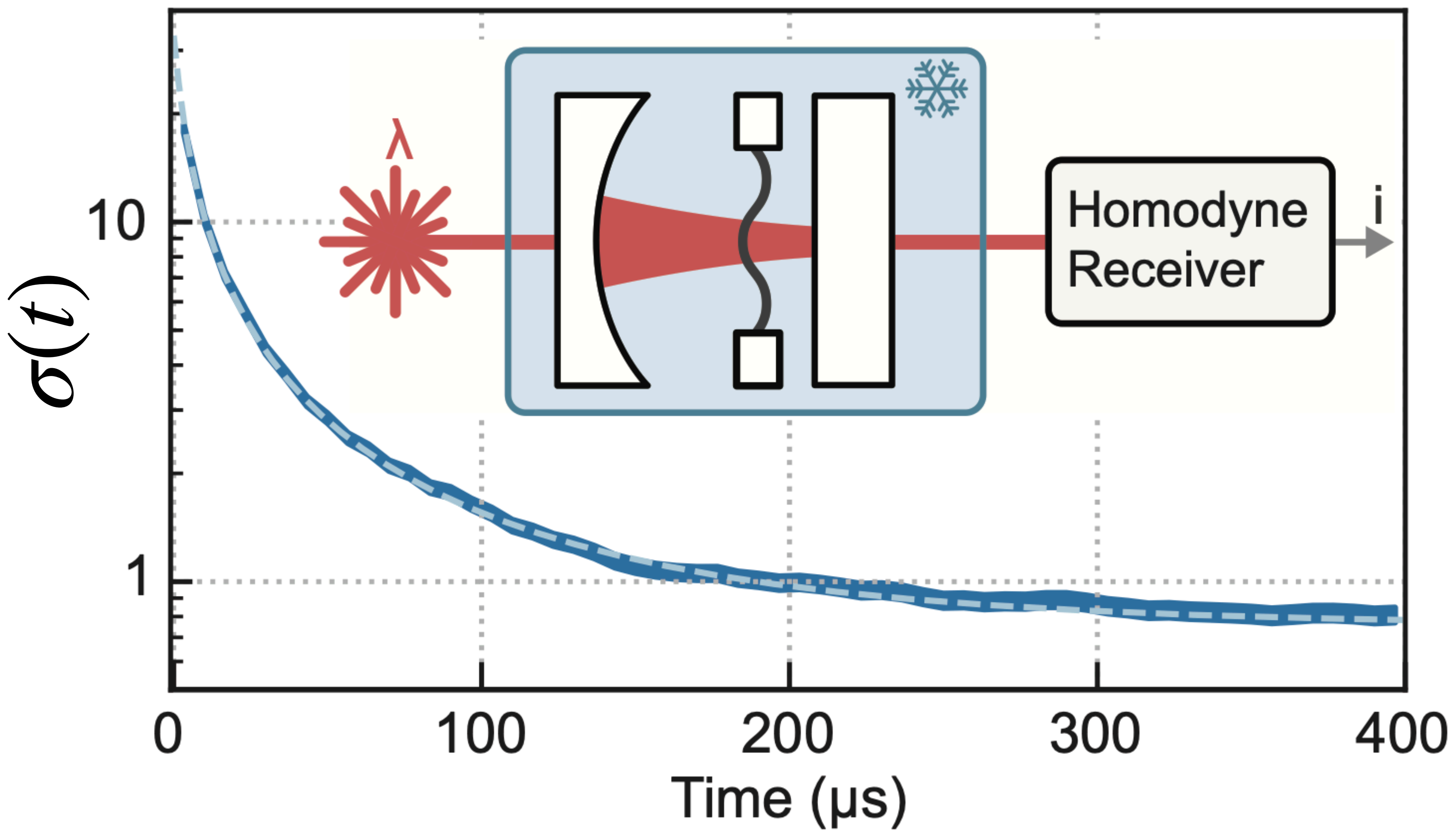
🇩🇰 Copenhagen setup

- Optomechanical system continuously monitored by an optical field.
- Competition: Thermal bath vs. Measurement.
- Quadratures of the mechanical mode: $x = (q, p)$
- Unconditional dynamics tends to $\bar{x}_u = 0$

$$\sigma_u = \bar{n} + 1/2 + \Gamma_{qba}/\Gamma_m$$

- Conditional dynamics evolves instead to

$$\frac{dx}{dt} = -\frac{\Gamma_m}{2}x + \sqrt{4\eta\Gamma_{\mathtt{qba}}}\sigma_c(t)\xi(t)$$
$$\frac{d\sigma_c}{dt} = \Gamma_m(\sigma_u - \sigma_c) - 4\eta\Gamma_{\mathtt{qba}}\sigma_c^2$$



Informational steady-state:

Conditional dynamics relaxes to a colder state, $\sigma_c < \sigma_u$, which can only be maintained by continuously monitoring S.

Production and flux at the trajectory level

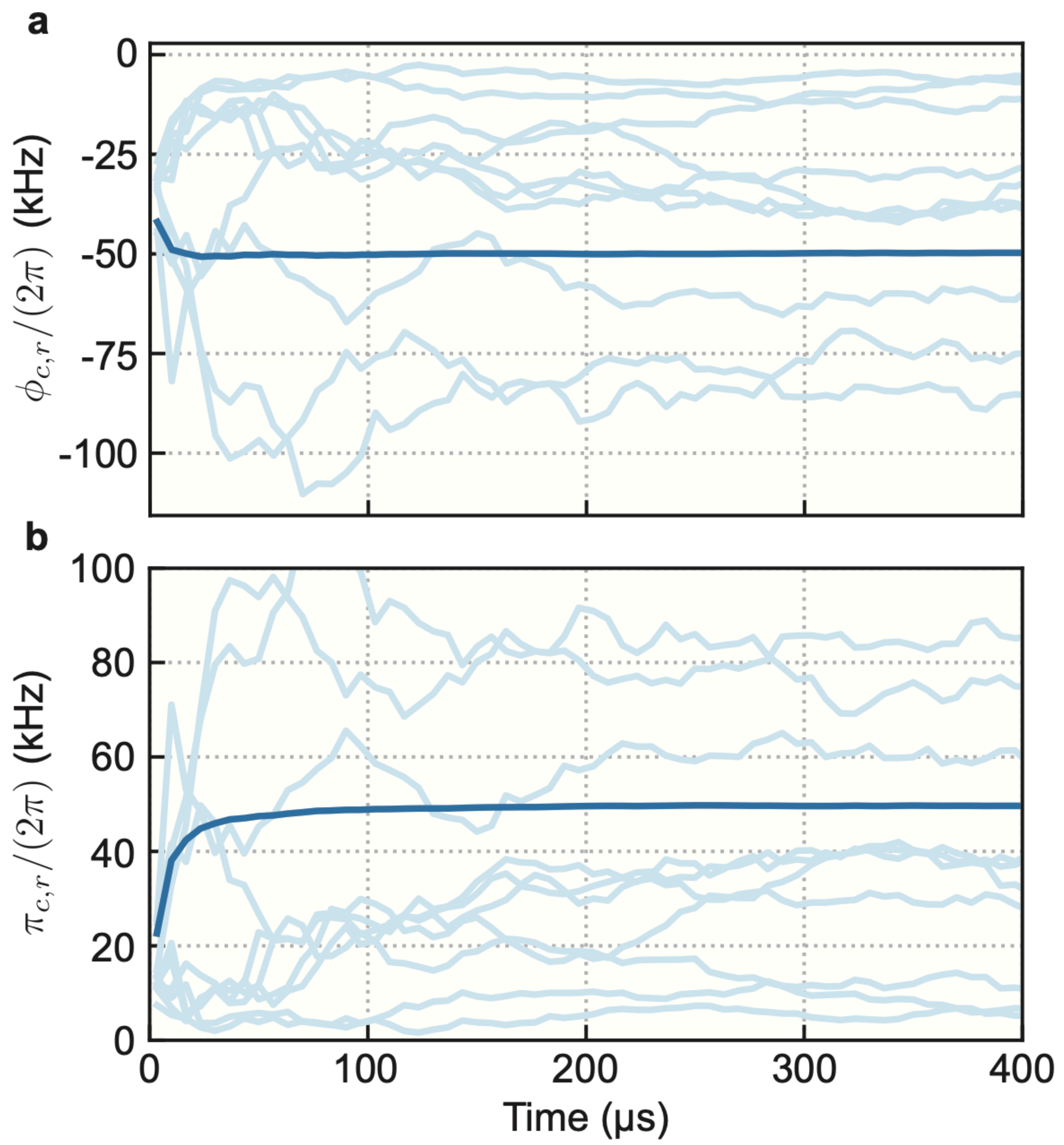


FIG. 2. **Stochastic entropy flux and production rates. a,** The stochastic entropy flux rates (light blue) for a sample of 10 trajectories. The dark blue line is the ensemble average over all the trajectories. **b,** The stochastic entropy production rates (light blue) and the ensemble average (dark blue), for the same sample of trajectories.

Information gain/loss rates characterizing the information steady-state

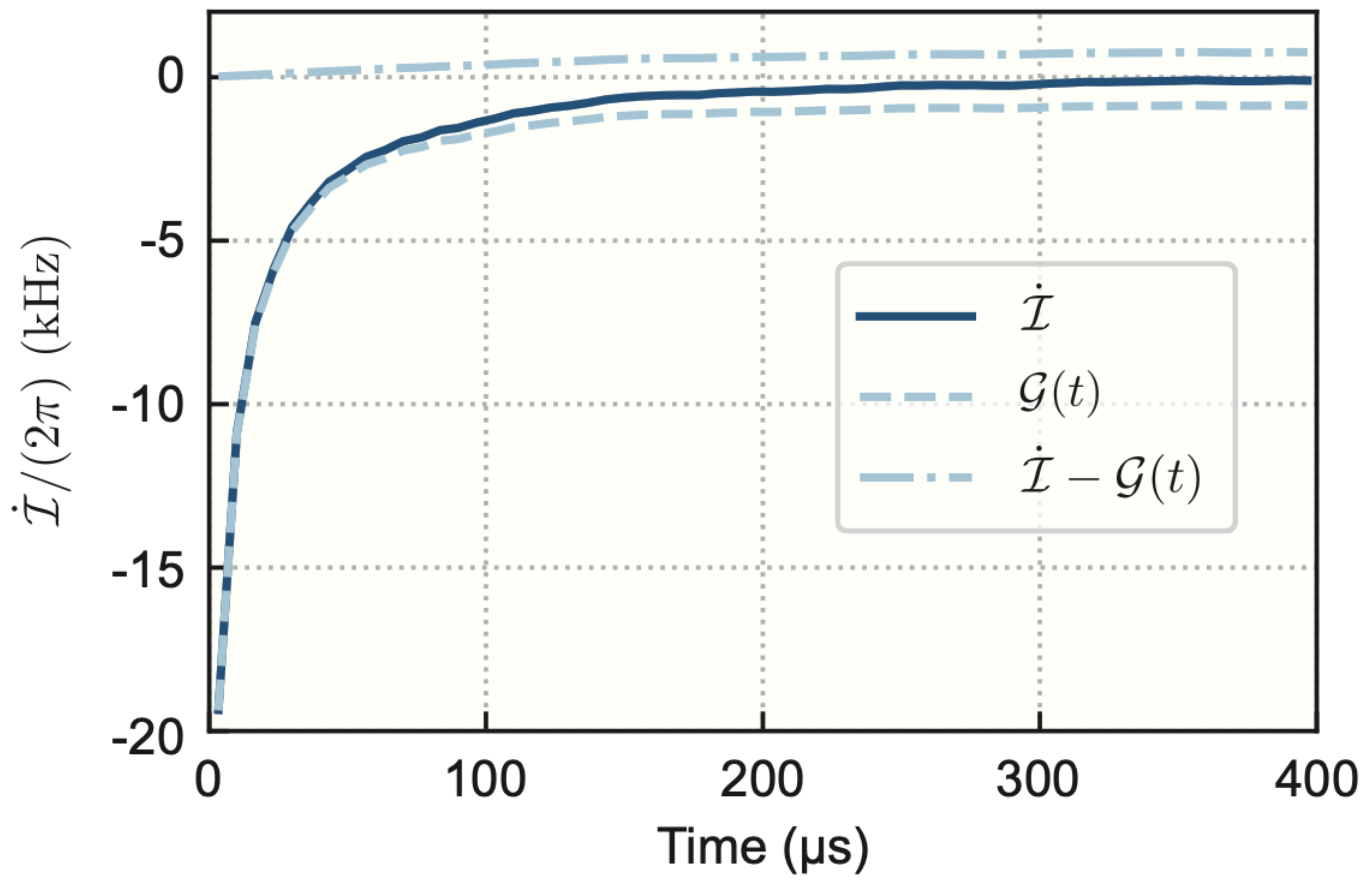


FIG. 3. **Informational contribution to the entropy production rate.** We obtain the informational contribution (dark blue) from the entropy production. The dashed (dot-dashed) line is the differential gain of information due to the measurement (loss of information due to noise input by the phonon bath).

Conclusions

- Knowing something about the bath makes the process less irreversible.
- The **conditional entropy production** quantifies this effect.
- We put forth a framework based on **continuously monitored collisional models** to address this scenario:
  - Clear conditions for identifying **informational steady-states**.
  - We also provide an **experimental assessment** of the entropy production at the level of stochastic trajectories in a quantum optomechanical system.

Thank you! 🙃

Quantum Thermodynamics
QT²
Quantum Transport

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