Informational steady-states in continuously monitored quantum systems

Gabriel T. Landi
Universidade de São Paulo, Brazil

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The interwebs.

www.fmt.if.usp.br/~gtlandi

In collaboration with

- Mauro Paternostro, Alessio Belenchia, Luca Mancino (Belfast).
- Massimiliano Rossi, Albert Schliesser (Copenhagen).

Entropy Production in Continuously Measured Quantum Systems

Alessio Belenchia,[1] Luca Mancino,[1] Gabriel T. Landi,[2] and Mauro Paternostro[1]

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Editors' Suggestion

Experimental Assessment of Entropy Production in a Continuously Measured Mechanical Resonator

Massimiliano Rossi,[1,2] Luca Mancino,[3] Gabriel T. Landi,[4] Mauro Paternostro,[3] Albert Schliesser,[1,2] and Alessio Belenchia[3,*]

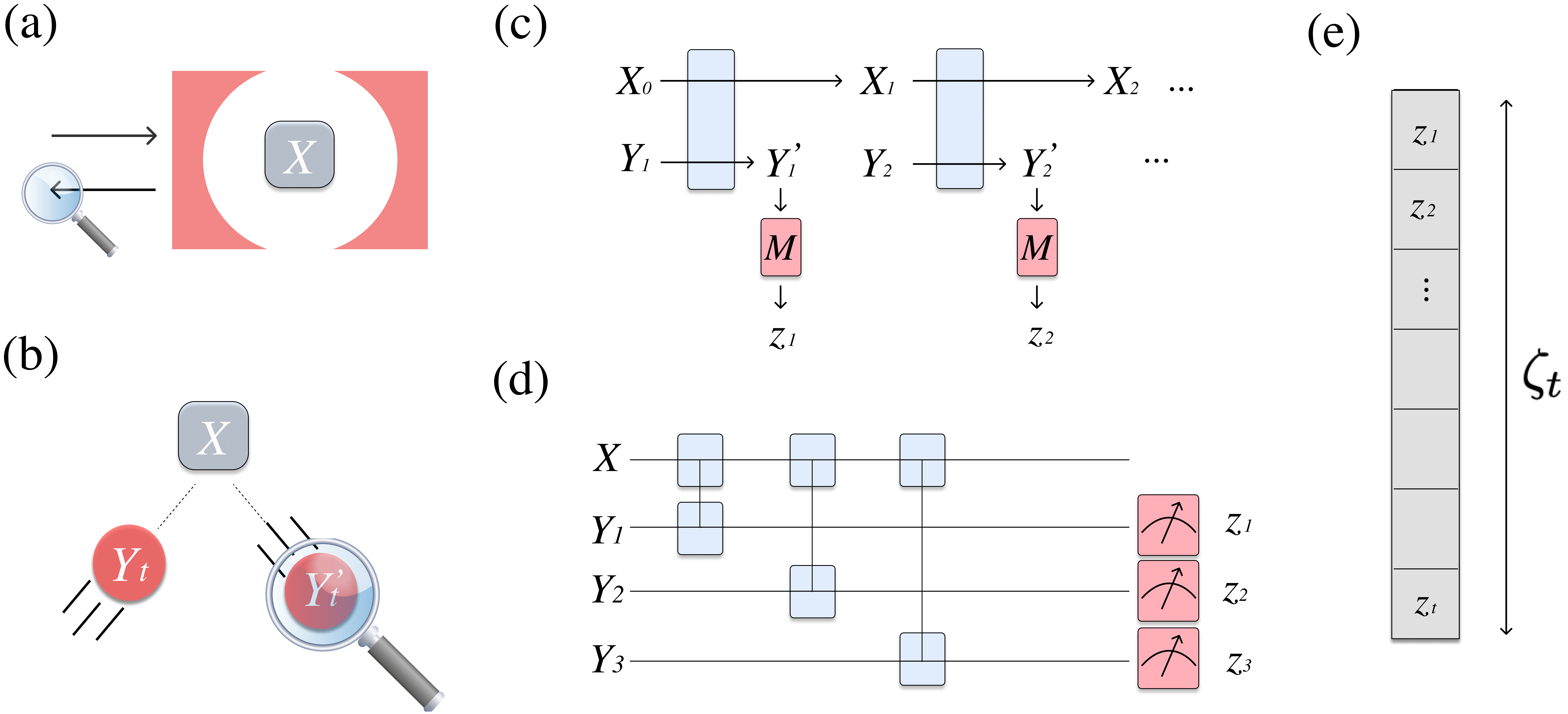
arXiv:2005.03429

Informational steady-states and conditional entropy production in continuously monitored systems

Gabriel T. Landi,[1,*] Mauro Paternostro,[2] and Alessio Belenchia[3,2]

arXiv:2103.06247

CM²: Continuously measured collisional models



Unconditional vs. conditional dynamics

- The unconditional dynamics is governed by the stroboscopic map

$$\rho_{X_t} = \mathscr{E}(\rho_{X_{t-1}}) = \mathrm{tr}_{Y_t}\left\{ U_t\left(\rho_{X_{t-1}} \otimes \rho_{Y_t}\right) U_t^\dagger \right\}$$

- The conditional dynamics, given outcomes

$$\zeta_t = (z_1, z_2, \ldots, z_t)$$

is given by

$$\rho_{X_t|\zeta_t} = \frac{1}{P(\zeta_t)} \mathrm{tr}_{Y_1,\ldots,Y_t}\left\{ M_{z_t} U_t \ldots M_{z_1} U_1 \left(\rho_{X_0} \otimes \rho_{Y_1} \otimes \ldots \otimes \rho_{Y_t}\right) U_1^\dagger M_{z_1}^\dagger \ldots U_t^\dagger M_t^\dagger \right\}$$

where $P(\zeta_t)$ is the normalization constant and also the prob. of the trajectory ζ_t.

- Since the measurements are only in the bath, there is no ***unconditional backaction:***

$$\sum_{\zeta_t} P(\zeta_t)\ \rho_{X_t|\zeta_t} = \rho_{X_t}$$

Unconditional vs. conditional dynamics

- An awkward thing about the map below is that it is not Markovian:

$$\rho_{X_t|\zeta_t} = \frac{1}{P(\zeta_t)} \mathrm{tr}_{Y_1,\ldots,Y_t}\left\{ M_{z_t} U_t \ldots M_{z_1} U_1 \left(\rho_{X_0} \otimes \rho_{Y_1} \otimes \ldots \otimes \rho_{Y_t} \right) U_1^\dagger M_{z_1}^\dagger \ldots U_t^\dagger M_t^\dagger \right\}$$

- But we can define a **trace non-preserving map**

$$\mathscr{E}_z(\rho_X) := \mathrm{tr}_Y \left\{ M_z U (\rho_X \otimes \rho_Y) U^\dagger M_z^\dagger \right\}$$

such that

$$\varrho_{X_t|\zeta_t} = \mathscr{E}_{z_t}(\varrho_{X_{t-1}|\zeta_{t-1}})$$

- This state is not normalized. But we can normalized it at the end. Moreover, what is cool is that

$$\mathrm{tr}\varrho_{X_t|\zeta_t} = P(\zeta_t)$$

Information-theoretic quantities

Unconditional dynamics:

- Information content of ρ_{X_t} is characterized by the von Neumann entropy

$$S(X_t) = -\,\mathrm{tr}\left\{\rho_{X_t} \ln \rho_{X_t}\right\}$$

Conditional dynamics:

- Information content of $\rho_{X_t|\zeta_t}$ is characterized by the quantum-classical conditional entropy

$$S(X_t\,|\,\zeta_t) = \sum_{\zeta_t} P(\zeta_t)\; S(\rho_{X_t|\zeta_t})$$

- This is not the quantum conditional entropy, which can be negative. Here conditioning is only on classical outcomes.

Their difference is the Holevo information:

$$I(X_t : \zeta_t) = S(X_t) - S(X_t\,|\,\zeta_t) = \sum_{\zeta_t} P(\zeta_t) D\left(\rho_{X_t|\zeta_t} \,||\, \rho_{X_t}\right) \geqslant 0$$

Information gain and loss rate.

- The Holevo information $I(X_t : \zeta_t) = S(X_t) - S(X_t | \zeta_t)$ reflects the ***integrated information***, acquired about the system, up to time t.

- The ***differential information gain*** associated only with the information from the last outcome z_t, is

$$G_t = I_c(X_t : z_t | \zeta_{t-1}) = I(X_t : \zeta_t) - I(X_t : \zeta_{t-1})$$

- We also define the ***information rate*** as

$$\Delta I_t = I(X_t : \zeta_t) - I(X_{t-1} : \zeta_{t-1})$$

- We may now write

$$\Delta I_t = G_t - L_t, \qquad L_t = I(X_{t-1} : \zeta_{t-1}) - I(X_t : \zeta_t) \geqslant 0$$

which defines the ***loss rate*** L_t.

Informational steady-state

$\Delta I_{ss} = 0$ **but** $G_{ss} = I_{ss} \neq 0$

Competition between acquisition of information and noise.

2nd law

Unconditional

- The degree of irreversibility of this process is quantified by the entropy production:

$$\Delta\Sigma_t^u = I'(X_t : Y_t') + D(\rho_{Y_t'} || \rho_{Y_t})$$

$$= S(X_t) - S(X_{t-1}) + \Delta\Phi_t^u$$

where

$$\Delta\Phi_t^u = S(Y_t') - S(Y_t) + D(Y_t' || Y_t)$$

$$= \mathrm{tr}_{Y_t}\left\{(\rho_{Y_t} - \rho_{Y_t'})\ln\rho_{Y_t}\right\}$$

is called the **entropy flux**. Depends only on Y_t. Measures change in the “thermodynamic potential” $\ln\rho_{Y_t}$.

Conditional

- We want to define $\Delta\Sigma_t^c, \Delta\Phi_t^c$ such that:

$$\Delta\Sigma_t^c = S(X_t | \zeta_t) - S(X_{t-1} | \zeta_{t-1}) + \Delta\Phi_t^c$$

- A natural choice is:

$$\Delta\Phi_t^c = S(Y_t' | z_t) - S(Y_t) + \sum_{z_t} P(z_t)\ D(\rho_{Y_t'|z_t} || \rho_{Y_t})$$

$$= \mathrm{tr}\left\{(\rho_{Y_t} - \tilde{\rho}_{Y_t})\ln\rho_{Y_t}\right\}$$

where $\tilde{\rho}_{Y_t} = \sum_{z_t} M_{z_t}\rho'_{Y_t}M_{z_t}^\dagger$.

M. Esposito, K. Lindenberg, C. Van den Broeck, “Entropy production as correlation between system and reservoir”. New Journal of Physics, **12**, 013013 (2010).

2nd law

Conditional

- We want to define $\Delta\Sigma_t^c, \Delta\Phi_t^c$ such that:

$$\Delta\Sigma_t^c = S(X_t|\zeta_t) - S(X_{t-1}|\zeta_{t-1}) + \Delta\Phi_t^c$$

- A natural choice is:

$$\Delta\Phi_t^c = S(Y_t'|z_t) - S(Y_t) + \sum_{z_t} P(z_t)\ D(\rho_{Y_t'|z_t}||\rho_{Y_t})$$

$$= \mathrm{tr}\Big\{(\rho_{Y_t} - \tilde{\rho}_{Y_t})\ln\rho_{Y_t}\Big\}$$

where $\tilde{\rho}_{Y_t} = \sum_{z_t} M_{z_t}\rho_{Y_t}' M_{z_t}^\dagger$.

- We normally have $\tilde{\rho}_{Y_t} \neq \rho_{Y_t'}$.
  - But many measurement strategies lead to

$$\mathrm{tr}\Big\{\tilde{\rho}_{Y_t}\ln\rho_{Y_t}\Big\} = \mathrm{tr}\Big\{\rho_{Y_t'}\ln\rho_{Y_t}\Big\}$$

- As a consequence, the conditional and unconditional fluxes become equal:

$$\Delta\Phi_t^c = \Delta\Phi_t^u$$

- Makes sense: flux is physical; should not depend on subjective information from the experimenter.

2nd law

Summarizing

$$\Delta\Sigma_t^u = S(X_t) - S(X_{t-1}) + \Delta\Phi_t^u$$

$$\Delta\Sigma_t^c = S(X_t\,|\,\zeta_t) - S(X_{t-1}\,|\,\zeta_{t-1}) + \Delta\Phi_t^u$$

$$\therefore \qquad \Delta\Sigma_t^c = \Delta\Sigma_t^u - \Delta I_t \geqslant 0$$

The two entropy productions are related by the Holevo information rate.

Integrated quantities

$$\Sigma_t^\alpha = \sum_{\tau=1}^{t} \Delta\Sigma_\tau^\alpha, \qquad \alpha = u, c$$

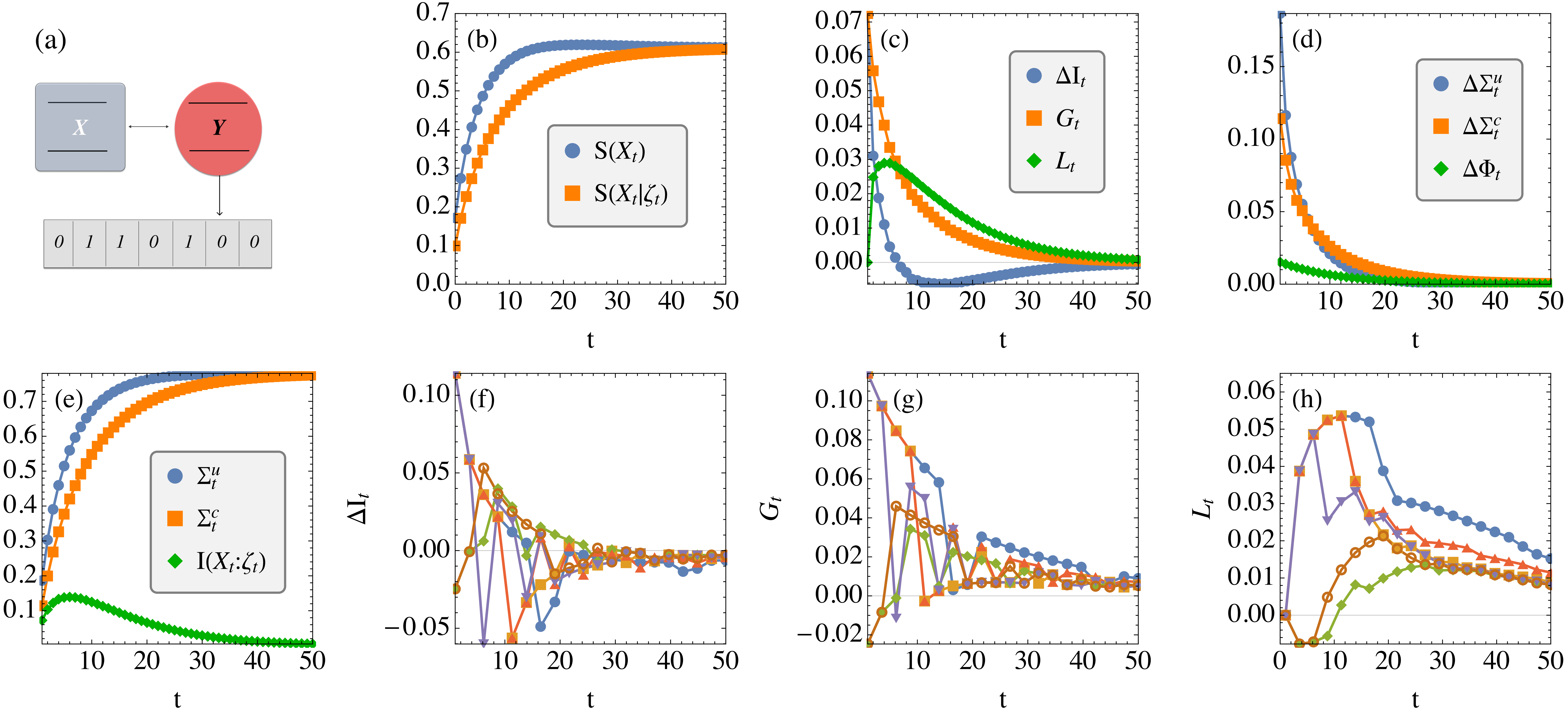
satisfy

$$\Sigma_t^u - \Sigma_t^c \geqslant \sum_{\tau=1}^{t} G_\tau \geqslant 0$$

Conditioning always makes the process more reversible.

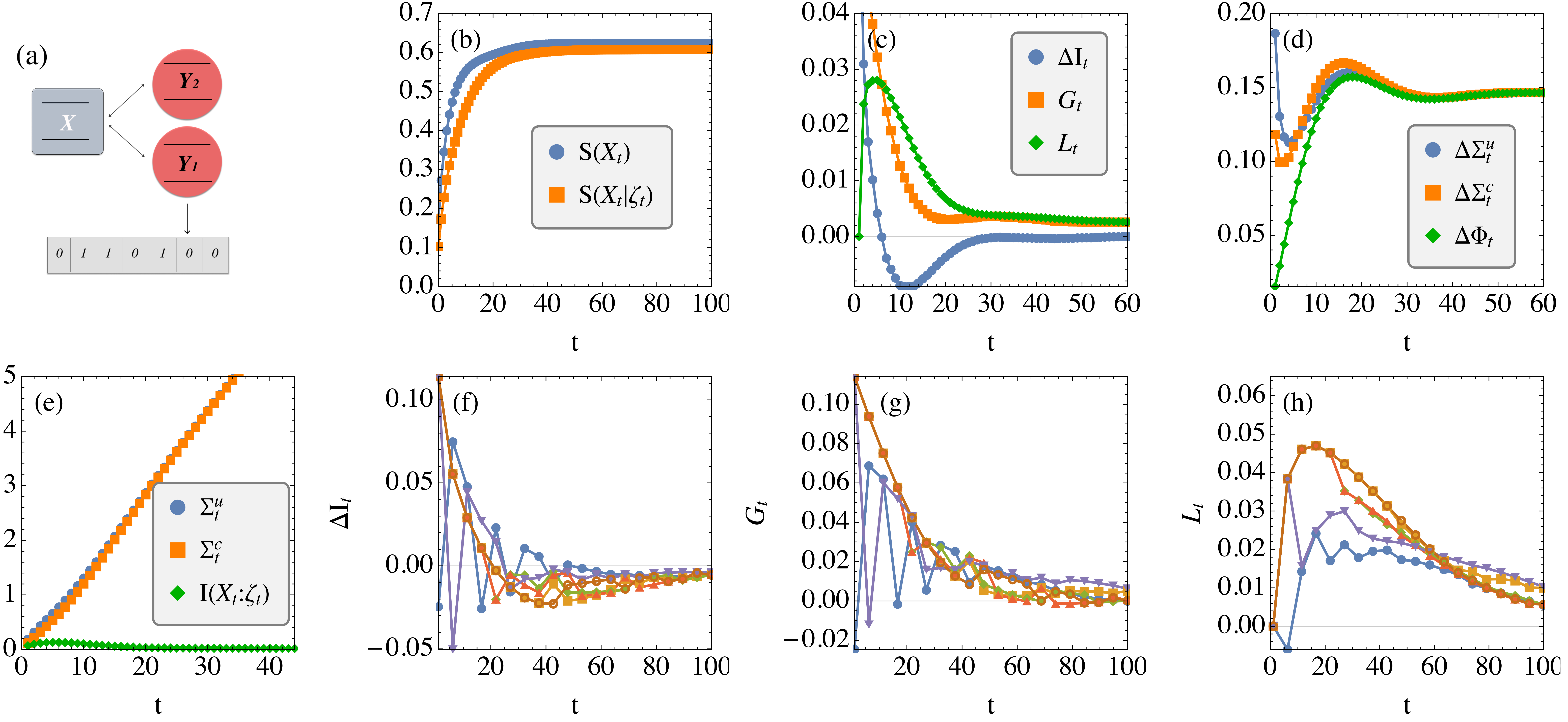
Minimal qubit models - Single-qubit ancilla

Thermal ancilla qubit + partial SWAP.

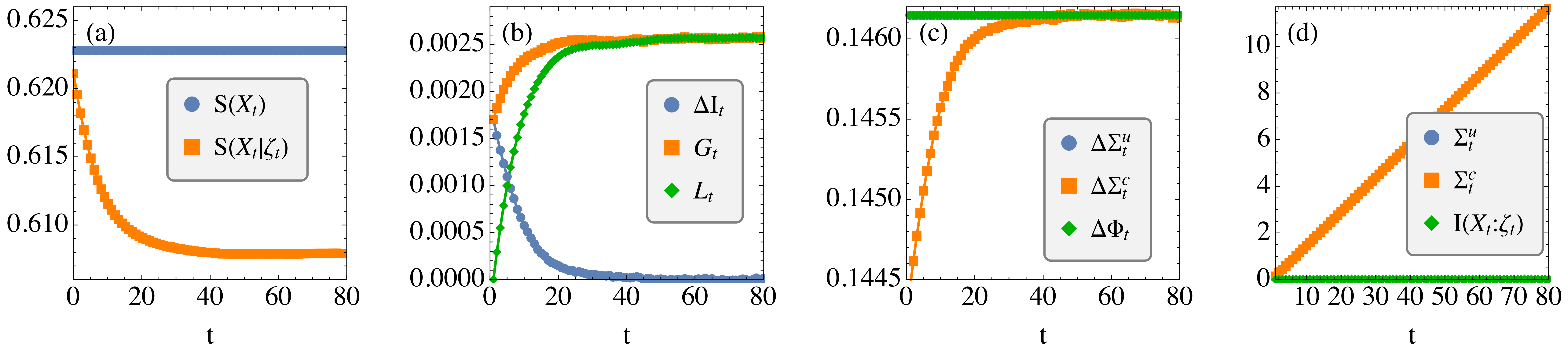


Minimal qubit models - Two-qubit ancilla

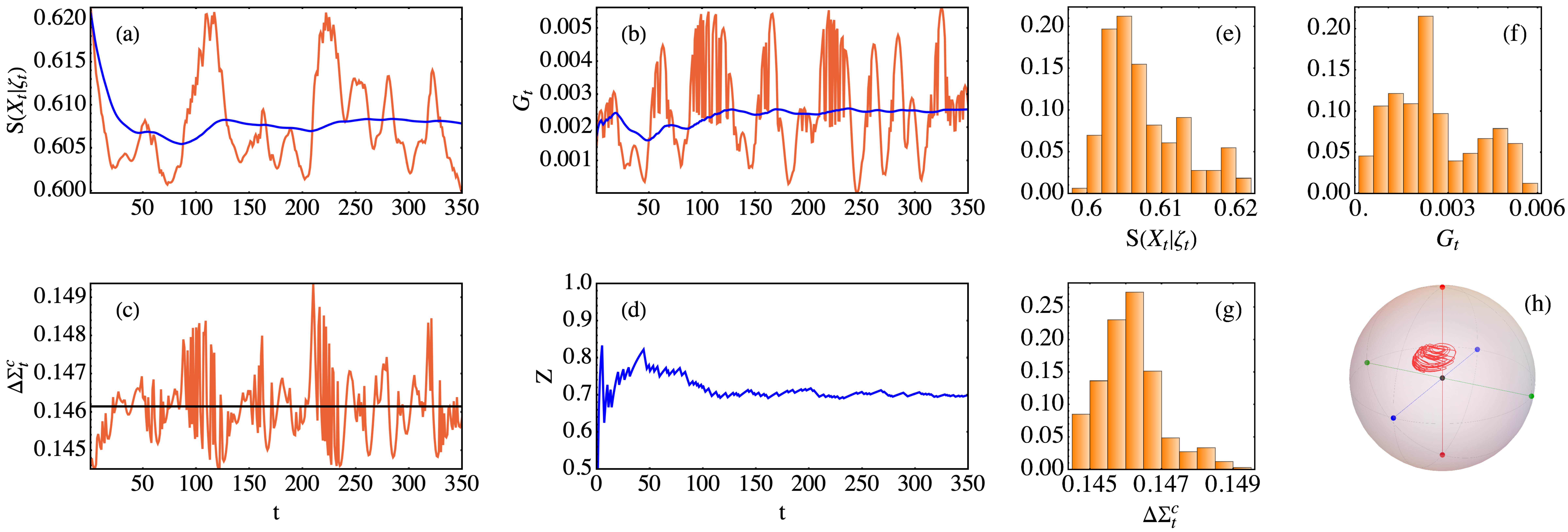
One ancilla thermal. The other prepared in $|+\rangle$
Sequential partial SWAPs



Starting from the ISS:



Single-shot scenario



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🇩🇰 Copenhagen setup

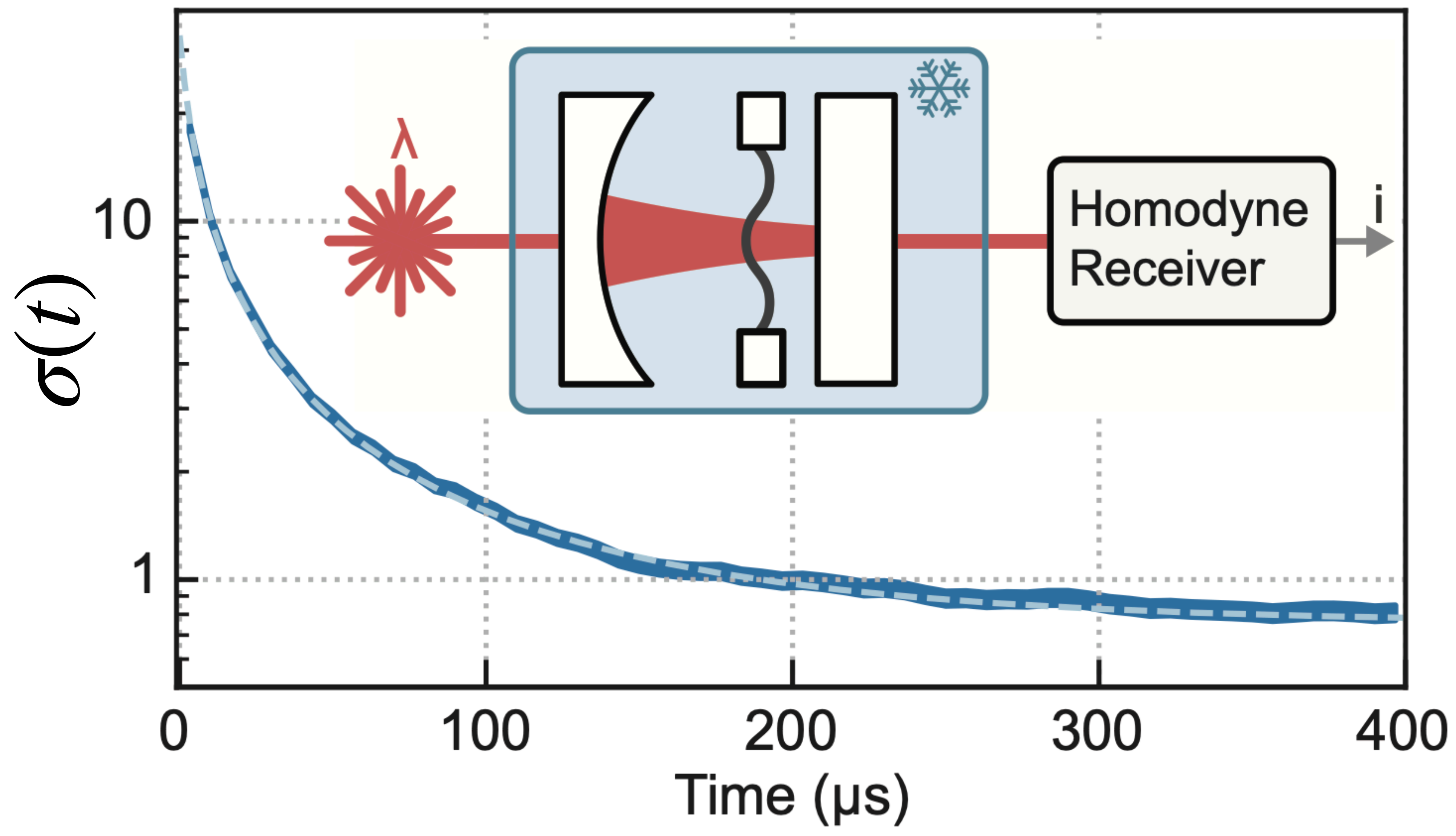
- Optomechanical system continuously monitored by an optical field.
- Competition: Thermal bath vs. Measurement.
- Quadratures of the mechanical mode: $x = (q, p)$
- Unconditional dynamics tends to $\bar{x}_u = 0$

$$\sigma_u = \bar{n} + 1/2 + \Gamma_{qba}/\Gamma_m$$

- Conditional dynamics evolves instead to

$$\frac{dx}{dt} = -\frac{\Gamma_m}{2}x + \sqrt{4\eta\Gamma_{\mathtt{qba}}}\sigma_c(t)\xi(t)$$

$$\frac{d\sigma_c}{dt} = \Gamma_m(\sigma_u - \sigma_c) - 4\eta\Gamma_{\mathtt{qba}}\sigma_c^2$$



Informational steady-state:

Conditional dynamics relaxes to a colder state, $\sigma_c < \sigma_u$, which can only be maintained by continuously monitoring S.

Production and flux at the trajectory level

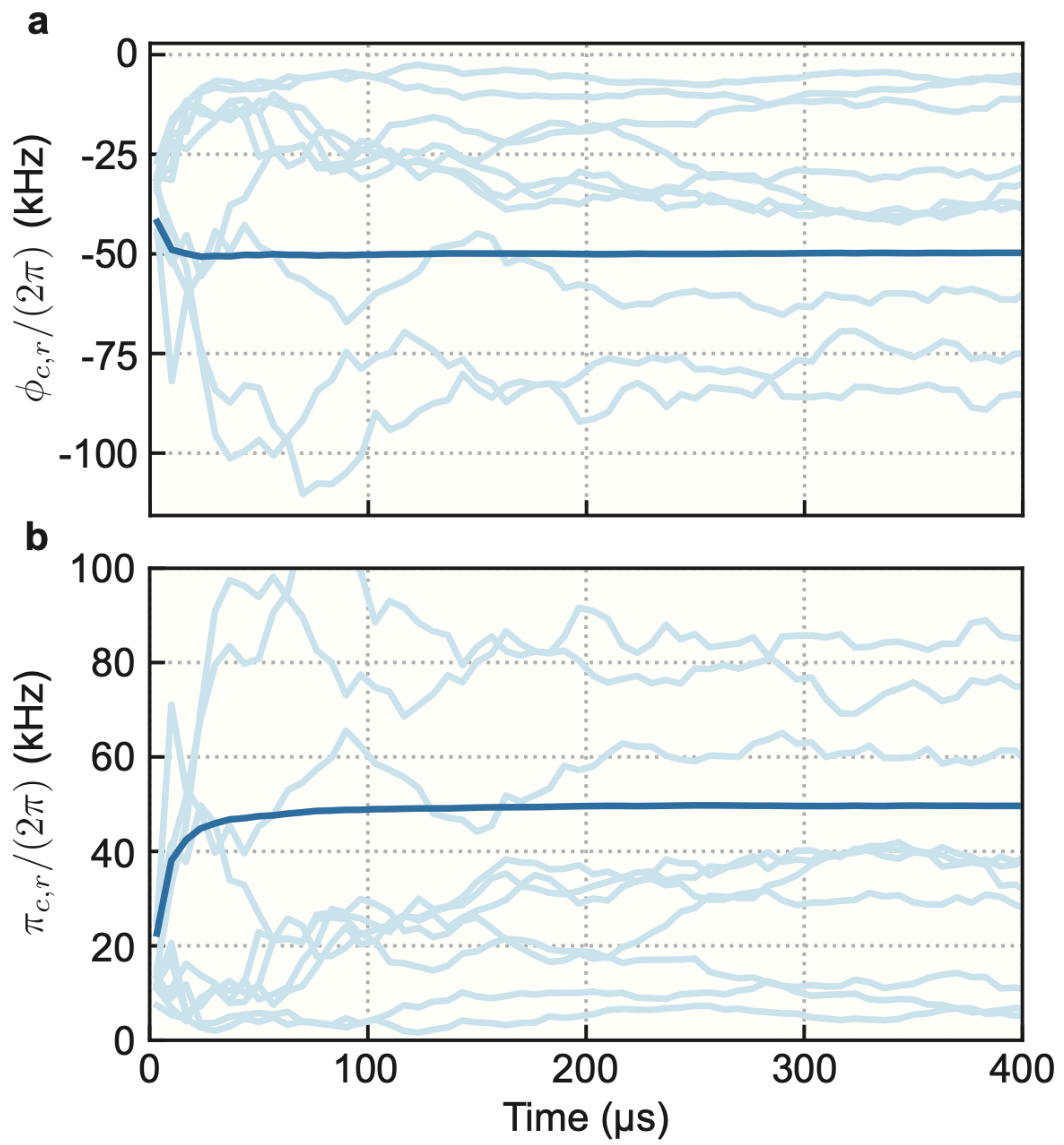


FIG. 2. **Stochastic entropy flux and production rates. a,** The stochastic entropy flux rates (light blue) for a sample of 10 trajectories. The dark blue line is the ensemble average over all the trajectories. **b,** The stochastic entropy production rates (light blue) and the ensemble average (dark blue), for the same sample of trajectories.

Information gain/loss rates characterizing the information steady-state

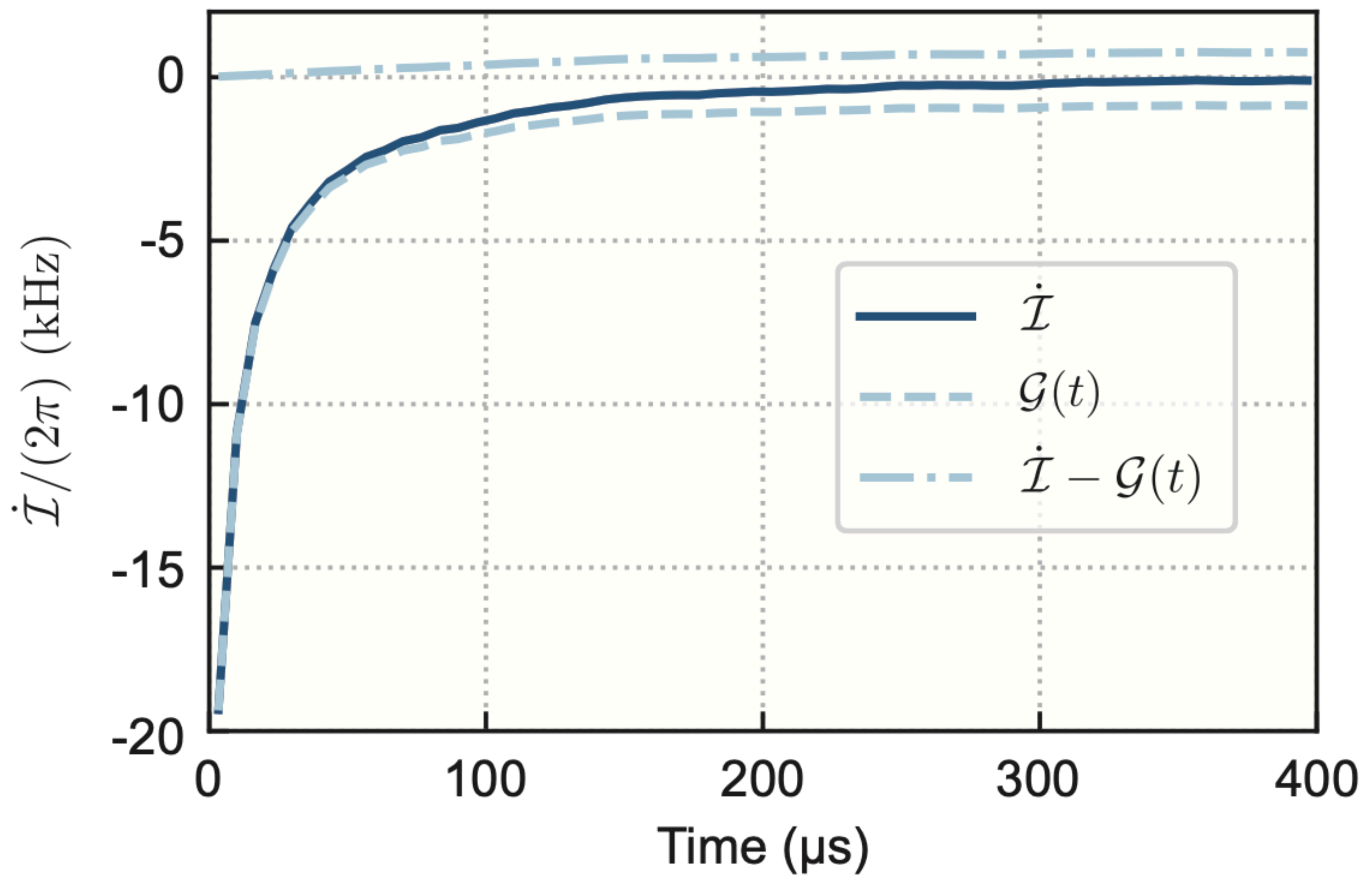


FIG. 3. **Informational contribution to the entropy production rate.** We obtain the informational contribution (dark blue) from the entropy production. The dashed (dot-dashed) line is the differential gain of information due to the measurement (loss of information due to noise input by the phonon bath).

Conclusions

- Knowing something about the bath makes the process less irreversible.
- The **conditional entropy production** quantifies this effect.
- We put forth a framework based on **continuously monitored collisional models** to address this scenario:
  - Clear conditions for identifying **informational steady-states**.
  - We also provide an **experimental assessment** of the entropy production at the level of stochastic trajectories in a quantum optomechanical system.

Thank you! 🙃

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