Non-Abelian Quantum Transport and Thermosqueezing Effects

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Marshak Lectureship award
APS March Meeting, Chicago, USA
March 18th, 2022
Overview

• Classical Onsager theory of transport
• Non-Abelian transport
• Collision models
• Linear response theory
• Application: Thermosqueezing

PRX QUANTUM 3, 010304 (2022)

Non-Abelian Quantum Transport and Thermosqueezing Effects

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Onsager theory

Entropy production rate

\[ \dot{\Sigma} = \sum_k \delta \lambda_k J_k = \sum_{k \ell} L_{k \ell} \delta \lambda_k \delta \lambda_\ell \]  
(fluxes \times forces)

Onsager’s main results:

- \( L \) is symmetric: Peltier & Seebeck are equal.
- \( L \) is positive semi-definite: \( \dot{\Sigma} \geq 0 \)

- Fluxes: \( J_k = d\langle Q_k \rangle/dt \). Generated by gradients of affinities.
  \( \delta \beta = \beta_L - \beta_R \) and \( -\delta \beta = -\beta_L + \beta_R \mu \).

- Linear response: if the gradients are small
  \( \begin{pmatrix} J_E \\ J_N \end{pmatrix} = \begin{pmatrix} L_{EE} & L_{EN} \\ L_{NE} & L_{NN} \end{pmatrix} \begin{pmatrix} \delta \beta \\ -\delta \beta \mu \end{pmatrix} \)

\( L_{NN} \): Fick’s law of diffusion
Particles flow due to gradient of concentration.

\( L_{EE} \): Fourier’s law
Heat flows due to gradient of temperature.

\( L_{NE} \): Seebeck effect
Gradient of temperature generates a flow of particles/electrons.

\( L_{EN} \): Peltier effect
Gradient of concentration generates heat flow.

Thermoelectric plates in our laptops.
Thermocouples
Onsager theory in the quantum regime

Non-Abelian (non-commuting) charges

- In the quantum domain we can also have transport of charges that do not commute.

\[ \rho = \frac{1}{Z} \exp\left\{ - \sum_k \lambda_k Q_k \right\} \quad [Q_k, Q_\ell] \neq 0 \]

(non-Abelian thermal states - NATS)

- Ex: spin transport \( \rho = \frac{1}{Z} \exp\left\{ - \lambda_x \sigma_x - \lambda_y \sigma_y - \lambda_z \sigma_z \right\} \)

- Ex: Energy & radiation squeezing.
Collision model approach

- We study non-Abelian transport in a collision model approach.
- Sequence of individual collisions between small ancillas of each bath.
- Two systems, A and B, each prepared in states

\[ \rho_{\lambda_k}^X = \frac{1}{Z_x} \exp\left\{ - \sum_k \lambda_k^X Q_k^X \right\}, \quad x = A, B \]

with \( \lambda_k^A \neq \lambda_k^B \)

- Interaction map: \( \rho_{AB}' = U \left( \rho_{\lambda_A}^A \otimes \rho_{\lambda_B}^B \right) U^\dagger \)

When can we talk about transport?

- Transport means the thing leaving one system must equal that entering the other.
- Condition for strict charge conservation (SCC):

\[ [U, Q_k^A + Q_k^B] = 0, \quad \forall k \]

- Define unique current operator

\[ \mathcal{J}_k = U^\dagger Q_k^{(A)} U - Q_k^{(A)} = -U^\dagger Q_k^{(B)} U + Q_k^{(B)} \]

- Average current: \( J_k = \text{tr} \left( \mathcal{J}_k (\pi_A \otimes \pi_B) \right) \)
Entropy production

- Entropy production can be written in a fully information-theoretic way as

\[ \Sigma = I'(A:B) + D(\rho'_A \mid \mid \rho_A) + D(\rho'_B \mid \mid \rho_B) \geq 0 \]

- where

\[ I'(A : B) = S(\rho'_A) + S(\rho'_B) - S(\rho'_{AB}) \]
\[ D(\rho \mid \mid \sigma) = \text{tr}\left\{ \rho \ln \rho - \rho \ln \sigma \right\} \]

- Fully operational: irreversibility due to loss of AB correlations + irreversible local changes in A and B.

- **NATS**: entropy production reduces to Onsager’s result: \( \dot{\Sigma} = \sum_k \delta \lambda_k J_k \).


Linear response theory
**Symmetric logarithmic derivative**

The proof of our result uses concepts from quantum parameter estimation.

We define the SLD for each charge/affinity pair:

\[
\Lambda_k \rho_\lambda + \rho_\lambda \Lambda_k = 2 \frac{d \rho_\lambda}{d \lambda_k}
\]

For commuting charges \( \Lambda_k = \langle Q_k \rangle - Q_k \)

The Onsager matrix can then be written as

\[
L_{k\ell} = -\frac{1}{2} \langle \{ \mathcal{J}_k, \Lambda_\ell \} \rangle
\]

Onsager reciprocity follows from time-reversal invariance.

**Main result**

If the charges \( Q_k \) and the dynamics are time-reversal invariant, then the Onsager matrix can be written as

\[
L_{k\ell} = \frac{1}{2} \int_0^1 dy \ \text{cov}_y(\mathcal{J}_k, \mathcal{J}_\ell)
\]

where \( \mathcal{J}_k = U^\dagger Q_k^{(A)} U - Q_k^{(A)} \) and

\[
\text{cov}_y(A, B) = \text{tr}(A \rho^y B \rho^{1-y}) - \text{tr}(A \rho) \text{tr}(B \rho)
\]

is the \( y \)-covariance, with \( \rho = \rho^A_\lambda \otimes \rho^B_\lambda \) being the equilibrium state.

For commuting charges we recover the *Kubo formula*

\[
L_{k\ell} = \text{cov}(\mathcal{J}_k, \mathcal{J}_\ell)
\]
The entropy production can be written as

\[ \Sigma = \frac{1}{2} \int_{0}^{1} dy \text{cov}_y(D, D), \quad D = \sum_k \delta \lambda_k \mathcal{J}_k \]

This can be further split as

\[ \Sigma = \Sigma_{\text{comm}} - I \]

where \( I \) is the Wigner-Yanase-Dyson skew information (a quantifier of coherence)

\[ I(\pi, D) = \frac{1}{2} \int_{0}^{1} dy \ \text{tr} \left( [\pi^y, D][\pi^{1-y}, D] \right) \geq 0 \]

Reduction in the entropy production due to quantum coherence.


Note that \( D \) is the operator associated to the entropy production:

\[ \Sigma = \langle D \rangle \]

In the commuting case, we would have the Fluctuation-Dissipation relation

\[ \langle D \rangle_{AB} = \frac{1}{2} \text{Var}(D)_{\text{eq}} \]

Non-commutativity breaks the FDR:

\[ \langle D \rangle_{AB} = \frac{1}{2} \text{Var}(D)_{\text{eq}} - I \]

is the \( y \)-covariance, with \( \rho = \rho_A^\lambda \otimes \rho_B^\lambda \) being the equilibrium state.

For commuting charges we recover the Kubo formula

\[ L_{k\ell} = \text{cov}(\mathcal{J}_k, \mathcal{J}_\ell) \]
Thermosqueezing
Squeezed Thermal Reservoirs as a Resource for a Nanomechanical Engine beyond the Carnot Limit

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(Received 25 April 2017; revised manuscript received 25 July 2017; published 13 September 2017)

LETTER

Efficiency of heat engines coupled to nonequilibrium reservoirs
Obinna Abah and Eric Lutz
Published 2 May 2014 • Copyright © EPLA, 2014
EPL (Europhysics Letters), Volume 106, Number 2
Citation Obinna Abah and Eric Lutz 2014 EPL 106 20001

Entropy production and thermodynamic power of the squeezed thermal reservoir
Gonzalo Manzano, Fernando Galve, Roberta Zambrini, and Juan M. R. Parrondo
Phys. Rev. E 93, 052120 – Published 10 May 2016

Squeezed Thermal Reservoirs as a Resource for a Nanomechanical Engine beyond the Carnot Limit
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Phys. Rev. X 7, 031044 – Published 13 September 2017
Thermosqueezing

• Single QHO:

\[ \rho = \frac{1}{Z} \exp\left\{ -\beta H - \beta \mu A \right\}, \quad H = \frac{\omega}{2}(p^2 + x^2), \quad A = \frac{\omega}{2}(p^2 - x^2) \]

• Two charges, \( H \) (energy) and \( A \) (asymmetry). Satisfy SU(1,1) algebra.

• Onsager coefficients:

\[ L_{QQ} = L_{QA} = \beta \delta_{\mu} \]

\[ L_{AQ} = L_{AA} = L_{Q\bar{A}} = \beta \delta_{\mu} \]

\[ \kappa = -\beta^2 L_{QQ} \]

\[ G = -\beta L_{AA} \]

\[ \dot{Q}_{\text{diss}} = \frac{\Sigma}{\beta} = \kappa \delta \frac{T^2}{T} + J_A G \]

New Joule-like heating term due to squeezing.

Charge preserving Gaussian unitary

Unitary which preserves both energy and squeezing:

\[ U = \exp\{-g\tau(a_1^\dagger a_2^\dagger - a_2^\dagger a_1)\} \]

Actually the only one which is also Gaussian (quadratic).
FIG. 2. (a)–(c) Thermosqueezing Onsager coefficients $L_{11}, L_{12}, L_{22}$ on the log scale, computed from Eqs. (19), in units of $(\hbar \omega)^2 \sin^2(g \tau)$, as a function of the inverse temperature $\beta$ (in units of $\hbar \omega/k_B$) and the adimensional squeezing parameter $r$. 
Entropy reduction

Recall that

\[ \Sigma = \frac{1}{2} \text{var}(D) - \frac{1}{2} \int_0^1 dy \ 2I_y(\pi, D) \]

Define the entropy reduction due to non-commutativity

\[ \mathcal{R} = \frac{1}{2\Sigma} \int_0^1 dy I_y(\pi, D) \]

Classical case corresponds to \( \mathcal{R} = 0 \).
Cross coefficients

Thermopower, or Squeezing-Seebeck (Squeebeck) coefficient

\[ S = \frac{1}{T} \frac{L_{AQ}}{L_{AA}} \]

(flow of squeezing due to gradient of temperature)

Squeezing-Peltier (Squeetier (?)) coefficient:

\[ \Pi = \frac{L_{QA}}{L_{AA}} \]

(flow of heat due to gradient in squeezing)

The two are related by \( \Pi = TS \)
Spin $S$ Heisenberg dynamics
• Spin $S$ operators:

$$S_z |m\rangle = m |m\rangle , m = S, S-1, \ldots, -S$$

• Two spins in NATS:

$$\rho_{\lambda_A} = \frac{1}{Z}$$

• Interact with Heisenberg unitary

$$U = \exp\{-it S_A \cdot S_B\} = \exp\{-it (S_A x S_B x + S_A y S_B y + S_A z S_B z)\}$$
Conclusions & outlook

• Quantum mechanics opens up the way for performing transport of non-commuting charges.

• We put forth a framework suitable for describing this in the linear response regime.

Perspectives:

• If the charges do not commute, how can we actually measure them?

• Current fluctuations and Thermodynamic Uncertainty Relations.

• Concrete applications of thermosqueezing.

THANK YOU!

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Extra slides
SU(1,1) algebra

The charges $Q_1, Q_2, Q_3$ form a non-Abelian group:

$$[Q_1, Q_2] = 2iQ_3$$
$$[Q_3, Q_1] = 2iQ_2$$
$$[Q_2, Q_3] = -2iQ_1$$

Onsager matrix

Unitary which preserves both heat ($J_Q = J_H - \mu J_A$) and squeezing:

$$J_Q = L_{QQ} \delta_\beta - L_{QA} \beta \delta_\mu$$
$$J_A = L_{AQ} \delta_\beta - L_{AA} \beta \delta_\mu$$

with

$$L_{QQ} = f_\tau (1 - \mu^2) \bar{n}(\bar{n} + 1)$$
$$L_{QA} = L_{AQ} = f_\tau \mu \bar{n}(\bar{n} + 1)$$
$$L_{AA} = f_\tau (1 - \mu^2)^{-1} \left[ \mu \bar{n}(\bar{n} + 1) + \frac{\tanh \alpha}{\alpha} (\bar{n}^2 + \bar{n}/2 + 1/2) \right]$$

where

$$\bar{n} = (e^{\beta \omega} - 1)^{-1}, \quad f_\tau = \omega^2 \sin^2 (g \tau), \quad \alpha = \beta \omega \sqrt{1 - \mu^2}$$