Memory effects in Gaussian Collisional Models

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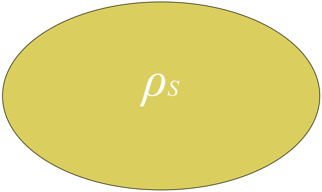
Motivation

Closed quantum systems

The state of the system in quantum mechanics is described by the density matrix ρ_s:

- Hermicity: $\rho_s = \rho_s^\dagger$,
- Positivity: $\rho_s \geq 0$,
- Normalization: $\mathrm{Tr}(\rho_s) = 1$.



We often assumed that the system is isolated, evolving under von Neumman's equation:

$$\frac{d\rho_s(t)}{dt} = -i[H, \rho_s(t)].$$

Breuer & Petruccione (2002). The theory of open quantum systems. Oxford University Press.

Open quantum systems

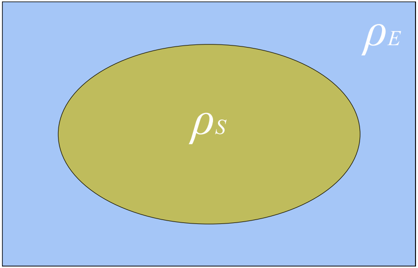
Generally, the system ρ_S is interacting with an environment ρ_E.
Still, the whole bipartite ρ_{SE} is closed.

$$\frac{d\rho_{SE}(t)}{dt} = -i[H_{SE}, \rho_{SE}(t)],$$

with solution:

$$\rho_{SE}(t) = U(t)\,\rho_{SE}(0)\,U^\dagger(t),$$

where $U(t) = e^{-iH_{SE}t}$.



How do we obtain the evolution for the system only?

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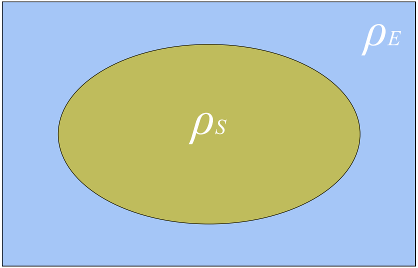


How do we obtain the evolution for the system only?

Markovian vs Non-Markovian Evolution

We can get an analytic evolution when the interaction is weak enough that information translated from the system to the environment never comes back to the system[1]:

$$\frac{d\rho_S}{dt} = -i[H, \rho_S] + \sum_k g_k \left(L_k \rho_S L_k^\dagger - \frac{1}{2} \left\{ L_k^\dagger L_k, \rho_S \right\} \right). \quad (1)$$

More generally, the dynamics can be written as:

$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \int_0^t \mathcal{K}_{t-t'}[\rho_S(t')]\, dt', \quad (2)$$

where $\mathcal{K}_{t-t'}$ is a linear superoperator called the memory kernel.

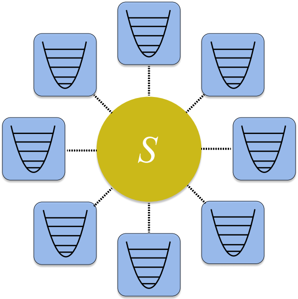
[1] Lindblad, G. (1976). On the generators of quantum dynamical semigroups. Communications in Mathematical Physics, 48(2), 119-130.

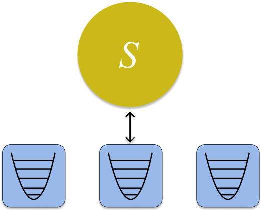
Importance of Non-Markovianity

- Realistic quantum systems are open quantum systems evolving under non-unitary evolutions.
- Strong system-environment coupling, finite reservoirs, low temperatures, large initial system-environment correlations, among others.
- Applications of quantum memory: quantum Brownian motion in optomechanical systems, chaotic systems, continuous variable quantum key distribution, quantum metrology, time-invariant quantum discord.

Collisional Model

An alternative description of open quantum systems is through collisional models.

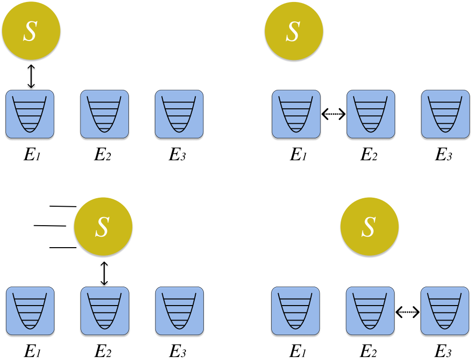




Non-Markovian Collisional Models

We can introduce non-Markovianity in two main ways:

- Ancillas start correlated.
- Environmental collisions.

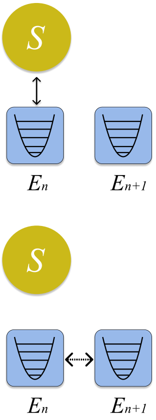


Dynamics

- The interaction between system and ancilla is given by the unitary U_n.
- The interaction between ancilla and ancilla is given by the unitary $V_{n,n+1}$.
- The stroboscopic dynamics generated is:

$$\rho^n = V_{n,n+1} U_n \, \rho^{n-1} \, U_n^\dagger \, V_{n,n+1}^\dagger, \qquad (3)$$

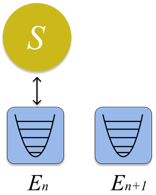
where ρ^n is the global state of $SE_1E_2\dots$ at time n.



Dynamics

- The system S and the ancillas E_n, E_{n+1} are the only involved dynamically with S and E_n in the correlated state $\rho^{n-1}_{SE_n}$.
- Thus, the process can be written as:

$$\rho^n_{SE_nE_{n+1}} = V_{n,n+1} U_n \big(\rho^{n-1}_{SE_n} \otimes \rho_{E_{n+1}}\big) U_n^\dagger \, V^\dagger_{n,n+1}.$$

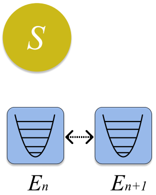


- Tracing out the environment E_n:

$$\rho^n_{SE_{n+1}} = \mathrm{tr}_{E_n}(V_{n,n+1} U_n (\rho^{n-1}_{SE_n} \otimes \rho_{E_{n+1}}) U_n^\dagger V^\dagger_{n,n+1}).$$

- This defines a time-local and CP map:

$$\rho^n_{SE_{n+1}} := \Phi(\rho^{n-1}_{SE_n}). \qquad (4)$$

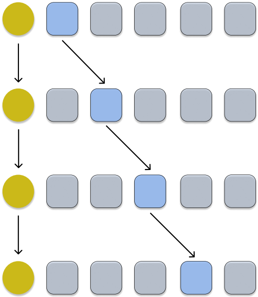


Markovian embedding

- Basic structure of the Markovian embedding $\rho^n_{SE_{n+1}} = \Phi(\rho^{n-1}_{SE_n})$ which is a map from the Hilbert space of SE_n to that of SE_{n+1}.
- We can define $\mathcal{E}_n$ taking ρ^0_S to ρ^n_S:

$$\rho^n_S = \mathcal{E}_n(\rho^0_S) = \mathrm{tr}_{E_{n+1}} \Phi^n(\rho^0_S \otimes \rho_{E_1}),$$

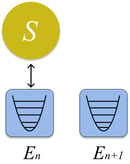
which is CP. But the map $\mathcal{E}_{m\to n}$ taking ρ^m_S to ρ^n_S is generally not.



Gaussianity

We describe the system S by bosonic annihilation operator a and quadratures $Q = (a + a^\dagger)/\sqrt{2}$ and $P = i(a^\dagger - a)/\sqrt{2}$, and the ancillas by bosonic operators $b_1, b_2, \dots$ with quadratures q_n, p_n.

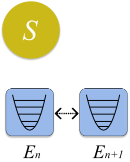
- System ancilla: $U_n = e^{\lambda_S (a^\dagger b_n - b_n^\dagger a)}$.
- Ancilla-ancilla: $V_{n,n+1} = e^{\lambda_e (b_n^\dagger b_{n+1} - b_{n+1}^\dagger b_n)}$.
- Ancilla-ancilla: $\tilde{V}_{n,n+1} = e^{\nu_e (b_n^\dagger b_{n+1}^\dagger - b_n b_{n+1})}$.



This defines two types of evolutions:

- Beam splitter evolution:

$$\rho^n = V_{n,n+1} U_n \, \rho^{n-1} U_n^\dagger \, V_{n,n+1}^\dagger \qquad (5)$$



- Two-mode Squeezing evolution:

$$\rho^n = \tilde{V}_{n,n+1} U_n \, \rho^{n-1} U_n^\dagger \, \tilde{V}_{n,n+1}^\dagger \qquad (6)$$

Continuous variable

The evolution of the expectation value of any observable is:

$$\frac{d}{dt}\langle\mathcal{O}\rangle = i\langle[H,\mathcal{O}]\rangle.$$

The Gaussian dynamics is fully characterized by the evolution of the first moments $\vec{y} = (\langle Q\rangle, \langle P\rangle, \langle q_1\rangle, \langle p_1\rangle \dots)$, and the covariance matrix $\sigma = \frac{1}{2}\langle\{Y_i, Y_j\}\rangle - \langle Y_i\rangle\langle Y_j\rangle$.

$$\frac{d\vec{y}}{dt} = M\vec{y}, \quad \frac{d}{dt}\sigma = M\sigma + \sigma M^{\mathsf{T}},$$

with solution

$$\vec{y}(t) = S\,\vec{y}(0), \quad \sigma(t) = S\,\sigma(0)\,S^{\mathsf{T}}, \quad S = e^{Mt}.$$

Serafini, A. (2017). Quantum continuous variables: a primer of theoretical methods. CRC press.

Equivalence

- Initial state:

$$\rho^0 = \rho_S^0 \otimes \rho_E \otimes \rho_E \otimes \cdots \longrightarrow \sigma^0 = \mathrm{diag}(\theta^0, \epsilon, \epsilon, \dots)$$

- Interactions:

$$U_n, V_{n,n+1}, \tilde{V}_{n,n+1} \longrightarrow S_n, S_{n,n+1}, \tilde{S}_{n,n+1}$$

- Dynamics:

$$\frac{d\rho}{dt} = i[H,\rho] \longrightarrow \frac{d\sigma}{dt} = M\sigma + \sigma M^{\mathsf{T}}$$

- Evolution:

$$\rho^n = V_{n,n+1} U_n \rho^{n-1} U_n^\dagger V_{n,n+1}^\dagger \longrightarrow \sigma^n = S_{n,n+1} S_n \sigma^{n-1} S_n^{\mathsf{T}} S_{n,n+1}^{\mathsf{T}}$$

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Sympletic Matrices

$$U_n \longrightarrow S_n = \begin{pmatrix} x & 0 & y & 0 \\ 0 & \mathbb{I} & 0 & 0 \\ -y & 0 & x & 0 \\ 0 & 0 & 0 & \mathbb{I} \end{pmatrix},$$

$$V_{n,n+1} \longrightarrow S_{n,n+1} = \begin{pmatrix} \mathbb{I} & 0 & 0 & 0 \\ 0 & z & w & 0 \\ 0 & -w & z & 0 \\ 0 & 0 & 0 & \mathbb{I} \end{pmatrix},$$

$$\tilde{V}_{n,n+1} \longrightarrow \tilde{S}_{n,n+1} = \begin{pmatrix} \mathbb{I} & 0 & 0 & 0 \\ 0 & \tilde{z} & \tilde{w}\sigma_z & 0 \\ 0 & \tilde{w}\sigma_z & \tilde{z} & 0 \\ 0 & 0 & 0 & \mathbb{I} \end{pmatrix}.$$

where $x = \cos(\lambda_s),\ y = \sin(\lambda_s),\ z = \cos(\lambda_e),\ w = \sin(\lambda_e),$
$\tilde{z} = \cosh(\nu_e),\ \tilde{w} = \sinh(\nu_e).$

Sympletic evolution

The step from σ^{n-1} to σ^n involves only S, E_n and E_{n+1}:

$$\sigma^{n-1}_{SE_nE_{n+1}} = \begin{pmatrix} \theta^{n-1} & \xi^{n-1}_n & 0 \\ \xi^{n-1,\mathsf{T}}_n & \epsilon^{n-1}_n & 0 \\ 0 & 0 & \epsilon \end{pmatrix}. \tag{7}$$

We then apply to the evolution:

$$\sigma^{n}_{SE_nE_{n+1}} = S_{n,n+1} S_n \left(\sigma^{n-1}_{SE_nE_{n+1}}\right) S_n^{\mathsf{T}} S_{n,n+1}^{\mathsf{T}}. \tag{8}$$

Only three entries are needed for the dynamics: the system θ^n, the ancilla ϵ^n_{n+1} and their correlations ξ^n_{n+1}.

Beam splitter evolution

Let us analyze the beam splitter case:

$$\theta^n = x^2\theta^{n-1} + y^2\epsilon_n^{n-1} + xy(\xi_n^{n-1} + \xi_n^{n-1,\mathsf{T}}),$$

$$\epsilon_{n+1}^n = z^2\epsilon + w^2\Big[x^2\epsilon_n^{n-1} + y^2\theta^{n-1} - xy(\xi_n^{n-1} + \xi_n^{n-1,\mathsf{T}})\Big],$$

$$\xi_{n+1}^n = w\Big[xy(\theta^{n-1} - \epsilon_n^{n-1}) + y^2\xi_n^{n-1,\mathsf{T}} - x^2\xi_n^{n-1}\Big].$$

These equations can be recast in terms of the Markovian embedding:

$$\gamma^{n+1} = X\gamma^n X^{\mathsf{T}} + Y,$$

where

$$\gamma^n = \begin{pmatrix} \theta^n & \xi_{n+1}^n \\ \xi_{n+1}^{n,\mathsf{T}} & \epsilon_{n+1}^n \end{pmatrix},\ X = \begin{pmatrix} x & y \\ yw & -wx \end{pmatrix},\ Y = \begin{pmatrix} 0 & 0 \\ 0 & z^2\epsilon \end{pmatrix}.$$

Two-mode squeezing evolution

For the two-mode squeezing, we get:

$$\gamma^{n+1} = X\gamma^{n}X^{\mathsf{T}} + Y,$$

where

$$\gamma^{n} = \begin{pmatrix} \theta^{n} & \xi^{n}_{n+1} \\ \xi^{n,\mathsf{T}}_{n+1} & \epsilon^{n}_{n+1} \end{pmatrix}, \; X = \begin{pmatrix} x & y \\ -y\tilde{w}\sigma_z & \tilde{w}x\sigma_z \end{pmatrix}, \; Y = \begin{pmatrix} 0 & 0 \\ 0 & \tilde{z}^{2}\epsilon \end{pmatrix}.$$

System's Evolution

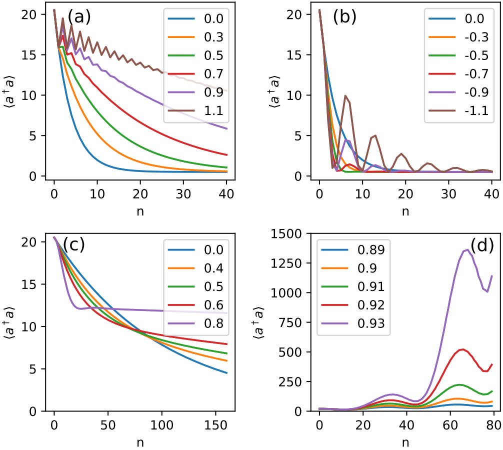

Figure 1: Number of excitations in the system as a function of time. (a,b) BS dynamics with $\lambda_s = 0.5$ and different values of λ_e (with $\lambda_e > 0$ in (a) and $\lambda_e < 0$ in (b)). (c,d) Same, but for the TMS with $\lambda_s = 0.1$ and different values of ν_e (with $\nu_e < \nu_e^{\text{crit}}$ in (a) $\nu_e \geqslant \nu_e^{\text{crit}}$ in (b), where $\nu_e^{\text{crit}} = \sinh^{-1}(1) \simeq 0.8813$). The ancillas are assumed to start in the vacuum, and the system in a thermal state with $\langle a^\dagger a\rangle^0 = 20$.

Memory effects in Collisional Models

Quantum non-Markovianity

Classically, a process is non-Markovian if the conditional probability of the future states depends on the precedent events.

- Information flow: The backflow of information quantifies the ability of the dynamics to communicate past information to the future.
- Map divisibility: The map $\mathcal{E}_n$ is CP by construction. However, the intermediate map $\mathcal{E}_{m\to n}$ in general is not CP. Conversely, Markovian maps are always CP.

Rivas, A., Huelga, S. F., & Plenio, M. B. (2014). Quantum non-Markovianity: characterization, quantification and detection. Reports on Progress in Physics, 77(9), 094001.

Mutual Information

Memory effects must be related to correlations that develop between system and bath.

- In the collisional model, the relevant correlations are between S and ancilla E_{n+1} at time n before its explicit interaction.
- A useful measure of correlations is the mutual information[2]:

$$\mathcal{I}^n(SE_{n+1}) = S(\rho_S^n) + S(\rho_{E_{n+1}}^n) - S(\rho^n),$$

  where S is the von Neumann entropy.
- We can compute the MI in terms of the eigenvalues of γ^n.

[2] Nielsen, M. A., & Chuang, I. (2002). Quantum computation and quantum information.

Mutual Information

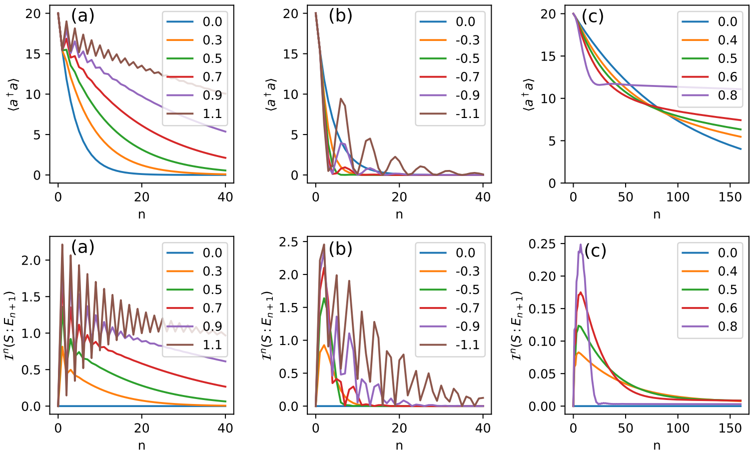

Figure 2: Mutual Information for the BS (a,b) and TMS (c) dynamics. (a,b) BS with $\lambda_s = 0.5$ and different values of λ_e (with $\lambda_e > 0$ in (a) and $\lambda_e < 0$ in (b)). (c) TMS with $\lambda_s = 0.1$ and different values of ν_e (with $\nu_e < \nu_e^{\text{crit}}$ in (c) where $\nu_e^{\text{crit}} = \sinh^{-1}(1) \simeq 0.8813$). The ancillas are assumed to start in the vacuum, and the system in a thermal state with $\langle a^\dagger a\rangle^0 = 20$.

Memory Kernel

A much older measure is the memory kernel $\mathcal{K}_{t-t'}$[3,4]:

$$\frac{d\rho_S}{dt} = -i[H_S, \rho_S] + \int_0^t \mathcal{K}_{t-t'}[\rho(t')]\, dt'$$

- The collisional model analog will act on the system's CM:

$$\theta^{n+1} = x^2\theta^n + \sum_{r=0}^{n-1} \mathcal{K}_{n-r-1}(\theta^r) + G_n, \tag{9}$$

where G_n is a contribution coming from the ancilla initial state, and the memory kernel $\mathcal{K}_n$ on the X matrix with:

$$\mathcal{K}_n(\theta) = \sum_{ij} \kappa_{ij}^n M_i \theta M_j^{\mathsf{T}}. \tag{10}$$

where M_i are a complete set of matrices $\{\mathbb{I}_2, \sigma_z, \sigma_+, \sigma_-\}$.

[3] Nakajima, S. (1958). On quantum theory of transport phenomena. Progress of Theoretical Physics

[4] Zwanzig, R. (1960). Ensemble method in the theory of irreversibility. The Journal of Chemical Physics, 33(5).

Memory Kernel

We start with the dynamics difference equation:

$$\gamma^{n+1} = X\gamma^n X^{\mathsf{T}} + Y. \tag{11}$$

Vectorizing the difference equation, we get:

$$\vec{\gamma}^{n+1} = (X \otimes X)\vec{\gamma}^n + \vec{Y}. \tag{12}$$

We introduce projection matrices on the subspaces:

$$P_S = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & 0 \end{pmatrix}, \quad P_E = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{I} \end{pmatrix}.$$

We introduce the Nakajima-Zwanzig projection operators $P = P_S \otimes P_S$ and $Q = 1 - P$:

$$P\vec{\gamma}^{n+1} = P(X \otimes X)P\vec{\gamma}^n + \sum_{r=0}^{n-1} \hat{K}_{n-r-1} P\vec{\gamma}^r + \vec{\mathcal{G}}_n. \tag{13}$$

Beam splitter MK

$$\mathcal{K}_n(\theta) = \sum \kappa_{ij}^n M_i \theta M_j^{\mathsf{T}}, \quad M_i = \mathbb{I}_2, \sigma_z, \sigma_+, \sigma_-$$

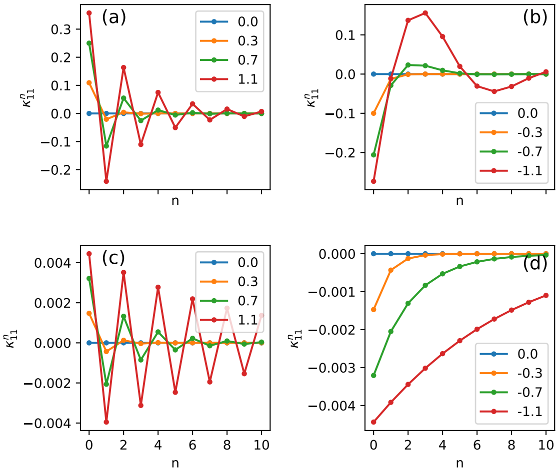

Figure 3: The memory Kernel for the BS dynamics. The only non-zero entry is κ_{11}^n, proportional to the identity. The plots are for $\lambda_s = 0.5$ (upper panel) and $\lambda_s = 0.05$ (lower panel), with $\lambda_e > 0$ (left) and $\lambda_e < 0$ (right).

Two-mode Squeezing MK

$$\mathcal{K}_n(\theta) = \sum \kappa_{ij}^n M_i \theta M_j^{\mathsf{T}}, \quad M_i = \mathbb{I}_2, \sigma_z, \sigma_+, \sigma_-$$

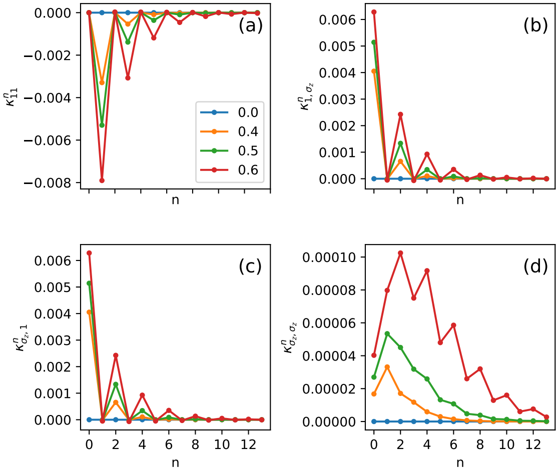

Figure 4: The memory Kernel for the (stable) TMS dynamics, with $\lambda_s = 0.1$ and different values of λ_e. Each curve corresponds to a different entry of the memory kernel; namely, κ_{11}^n, κ_{1,σ_z}^n, $\kappa_{\sigma_z,1}^n$ and $\kappa_{\sigma_z,\sigma_z}^n$.

Beam splitter MK

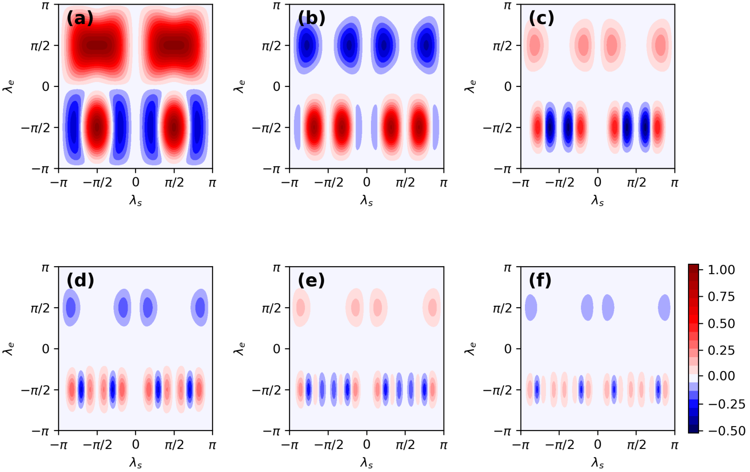

Figure 5: Diagrams for the MK of the BS dynamics. Each plot shows κ^n_{11} in the (λ_s, λ_e) plane for a different value of n, from $n = 0$ to $n = 5$.

CP-Divisibility

Let us return to map divisibility. Given that the inverse map $\mathcal{E}^{-1}$ exists for all times $t > 0$, we can define the intermediate maps:

$$\mathcal{E}_{m \to n} = \mathcal{E}_n \circ \mathcal{E}_m^{-1}.$$

Even though $\mathcal{E}_n$ and $\mathcal{E}_m$ are CP by construction, the intermediate map $\mathcal{E}_{m \to n}$ will not necessarily be. Hence, by measuring how much the intermediate map $\mathcal{E}_{m \to n}$ departs from the CP map, we are measuring the degree of non-Markovianity of the time evolution.

CP-Divisibility

- At the level of CM, any gaussian CPTP map have the form $\theta \to \mathcal{X}\theta\mathcal{X}^{\mathsf{T}} + \mathcal{Y}$, where $\mathcal{X}$ and $\mathcal{Y}$ are matrices satisfying[5]:

$$\mathcal{M}[\mathcal{X}, \mathcal{Y}] := 2\mathcal{Y} + i\Omega - i\mathcal{X}\Omega\mathcal{X}^{\mathsf{T}} \geq 0$$

- We come back to the difference equations and solve them:

$$\gamma^n = X^n \gamma^0 (X^{\mathsf{T}})^n + \sum_{r=0}^{n-1} X^{n-r-1} Y (X^{\mathsf{T}})^{n-r-1}.$$

- The evolution of the system's CM from 0 to n is:

$$\theta^n = \mathcal{X}_n \theta^0 \mathcal{X}_n^{\mathsf{T}} + \mathcal{Y}_n$$

where the matrix $\mathcal{X}_n = (X^n)_{11}$ and the other matrix $\mathcal{Y}_n = (X^n)_{12}\epsilon(X^{n\mathsf{T}})_{12} + \sum_{r=0}^{n-1} \left[X^{n-r-1} Y (X^{\mathsf{T}})^{n-r-1}\right]_{11}.$

[5] Lindblad, G. (2000). Cloning the quantum oscillator. Journal of Physics A: Mathematical and General, 33(28).

CP-Divisibility

- To probe whether the dynamics is divisible, we consider the map taking the system from n to $m > n$:

$$\theta^m = \mathcal{X}_{mn}\theta^n\mathcal{X}_{mn}^\mathsf{T} + \mathcal{Y}_{mn},$$

where $\mathcal{X}_{mn} = \mathcal{X}_m\mathcal{X}_n^{-1}$, $\mathcal{Y}_{mn} = \mathcal{Y}_m - \mathcal{X}_{mn}\mathcal{Y}_n\mathcal{X}_{mn}^\mathsf{T}$.

- The dynamics is considered divisible when the intermediate maps are CPTP Gaussian map $\mathcal{M}[\mathcal{X}_{mn}, \mathcal{Y}_{mn}] \geq 0$.
- This can also be used as a figure of merit[6]:

$$\mathcal{N}_{mn} = \sum_k \frac{|m_k| - m_k}{2}, \quad \{m_k\} = \mathrm{eigs}\Big(\mathcal{M}[\mathcal{X}_{mn}, \mathcal{Y}_{mn}]\Big).$$

[6] Torre, G., Roga, W., & Illuminati, F. (2015). Non-markovianity of gaussian channels. PRL, 115.

Beam splitter CP-Divisibility

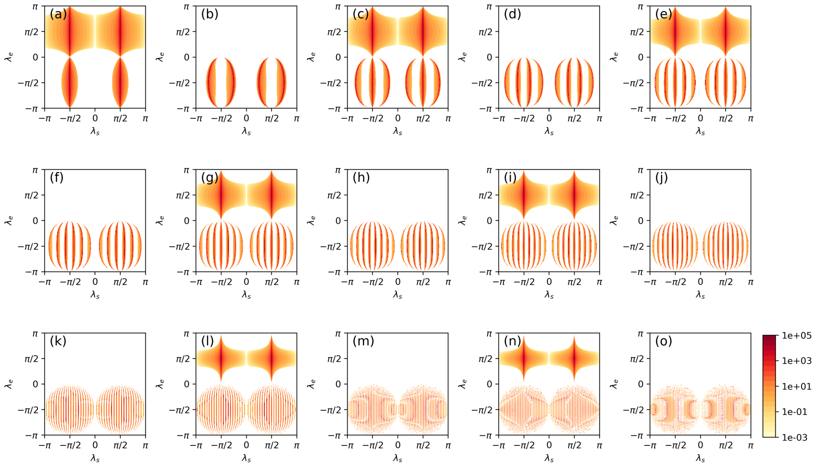

Figure 6: CP-divisibility measure $\mathcal{N}_{n+1,n}$ in the (λ_s, λ_e) plane for the BS dynamics. Each plot corresponds to a different values of n: in the first 2 lines, n ranges from 1 to 10 in steps of 1. In the 3rd line, $n = 20, 21, 30, 31, 40$.

Two-mode squeezing CP-Divisibility

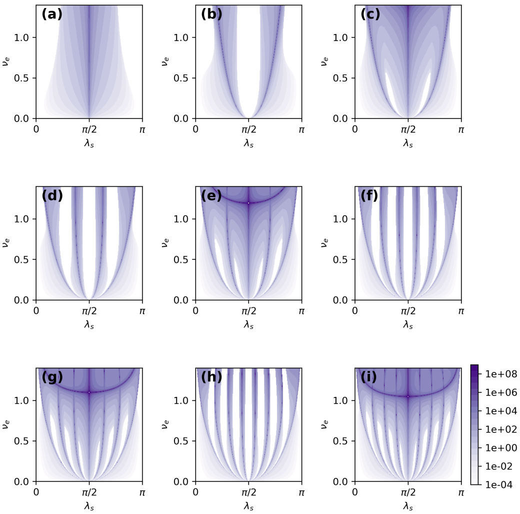

Figure 7: CP-divisibility measure $\mathcal{N}_{n+1,n}$ in the (λ_s, ν_e) plane for the TMS dynamics. Each plot corresponds to a different values of n, from 1 to 9 in steps of 1.

Conclusions

Conclusions

- We presented a robust framework for studying non-Markovianity in collisional models from multiple perspectives.
- We showed that the dynamics can be cast in terms of a Markovian embedding of the covariance matrix.
- This yields closed expressions for the mutual information, the memory kernel, and the divisibility monotone.
- We analyzed in detail two types of interactions, a beam splitter and a two-mode squeezing. Yet the results can be easily generalized to other Gaussian interactions.

Conclusions

Results of this work were reported in the preprint:

- Camasca, R.R. and Landi, G.T., 2020. Memory kernel and divisibility of Gaussian Collisional Models. arXiv preprint arXiv:2008.00765.
- Python Libraries: https://github.com/gtlandi/gaussianonmark