Locality in collider tests of quantum mechanics with top quark pairs

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Tests of quantum properties of fundamental particles in high energy colliders are starting to appear. Entanglement and Bell inequality violation in top and antitop quark system is of particular interest since top quarks are unstable particles that undergo a cascade decay. We argue for criteria for the spacelike separation between top and antitop quarks at their different decay stages. We considered causal separation at three different instances: at the top quark decay, at the W boson decay, and at the lepton/jet contact with the macroscopic apparatus. We showed that the spacelike fraction of events is the smallest, when requiring that both top quarks and W bosons decay within the spacelike interval. For high invariant masses, typically required for the Bell inequality violation, this is almost identical to just the top quark decay requirement. We also include an option for the angular correlation of the b quarks from top quark decay to be used for the spin correlation measurement. We require that both top quark and b hadron decays are spacelike separated. Again, we find that at high invariant masses it is almost identical to just the requirement of spacelike separation between top and antitop quarks. We provide numerical values for our proposed criteria. If such a criterion is satisfied, the system is guaranteed to not be in a causal connection.

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I. INTRODUCTION

The ATLAS and CMS Collaborations at the LHC recently reported [1,2] observations of entanglement between the spin degrees of freedom of top and antitop (t and $\bar{t}$) quarks. The experiments found a level of spin correlations in the $t\bar{t}$ pair that exceeded the Peres-Horodecki bound [3–5] which all nonentangled systems must respect. These results prove that the foundations of quantum mechanics (QM) can be successfully investigated at the TeV scale and also in systems with unstable particles. Many new measurements were suggested to test entanglement in different high energy systems [6–11].

In addition to observing entanglement [5], LHC experiments will also seek to observe Bell inequality violations (BIV) [12–14], in $t\bar{t}$ pairs and other high-energy two-particle systems [15–23]. BIVs provide proof of the nonclassicality of quantum theory, by ruling out its compatibility with local hidden variable theories. In Ref. [24], it was argued that, since in collider experiments the measured quantities are not spins but rather particles' momenta, which are commuting variables, a local hidden variable theory can always be constructed that reproduces the observed correlation. Thus, an implicit assumption must be made that the spin direction can be inferred from the angular correlation of the particle's decay products. This means that we assume that we are dealing with a quantum state and characterize the degree of entanglement of this state. Since not all entangled states violate a Bell inequality, BIVs are considered a stronger form of correlations.

A number of loopholes has been identified in connection with an experimental demonstration of BIVs [25–27]. In this work, we investigate the possibility of closing the so-called locality loophole [28,29]. It has been proven that a system of two parties, obeying the laws of classical physics, can exactly reproduce the outcome of a quantum measurement on a maximally entangled state with the exchange of just a few bits of classical information [30]. Therefore, we should ask ourselves what constitutes a convincing observation of nonclassical properties in a collider setting, where the absence of communication is not guaranteed.

Ideally, experimenters should restrict their study to events, where the QM measurements are spacelike separated, to enforce that no classical information can be exchanged between the particles under consideration. However, in a collider setting, this cannot be accomplished on an event-by-event basis for measurements involving the spin degree of freedom, since they are necessarily statistical in nature. In this work, we argue for criteria to enforce such a condition on average in an ensemble of $t\bar{t}$ events considered for a particular analysis. Of course, we have

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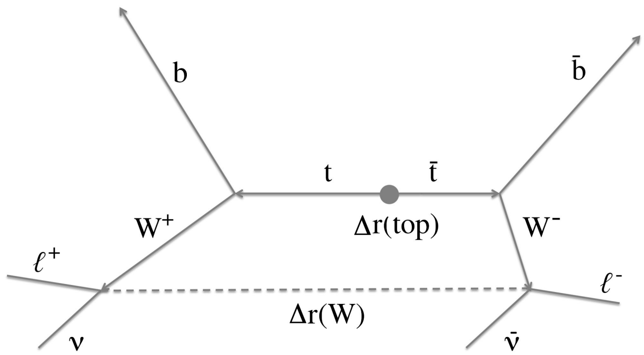

FIG. 1. Schematics of $t\bar{t}$ production with subsequent decay to W bosons and b quarks. Each W boson then decays to a charged lepton and a neutrino. The distance between the tops' decay is denoted $\Delta r(\mathrm{top})$, while the distance between W's decay is denoted $\Delta r(\mathrm{W})$.

no reason to believe that top quarks, or any other pair of fundamental particles, actually possess the ability to exchange information during their lifetime, nor the ability to conspire with their partner to fake the presence of entanglement or Bell violations, when none is actually present; our argument simply attempts to close a loophole that is present in principle.

We formulate the criteria in such a way that if satisfied the system is guaranteed to avoid the causal connection at every "decision point" in its evolution. We take the approach of the original paper by John Bell [12] and first consider the classical explanation of the entanglement, that is by the exchange of information via the "hidden variables." The schematics of the $t\bar{t}$ production and decay is shown in Fig. 1. As was pointed out in [31] the entanglement is not destroyed at the point of particles' decay. For this reason, it is important to guarantee that there is no causal connection at any instance of interaction, of which there are several. First, the top and antitop quarks are produced via a strong interaction, which could be either a gluon fusion (by far the likely process in the LHC) or quark and antiquark annihilation. At this point, the two particles are necessarily in causal contact, and their quantum state in terms of the level of spin correlation is determined. If the system is in a pure triplet/singlet state, the spins of top and antitop quarks are parallel/antiparallel [16,22,23]. Either one is in a mixed state with spin pointing up or down with respect to some chosen axis, but once one of the particles is detected to have a particular spin orientation, the other one must have the corresponding one. The two particles move apart, and each one decays via a weak process to a W boson and a b quark. If the two decays happen within a causality cone, one might argue that the angular correlation of the decay products could be "agreed upon" between the top and antitop quarks through the interaction via hidden variables. $W^{\pm}$ bosons then decay into the particles that are actually observed experimentally—leptons and light quark jets. (From now on for brevity, we will refer to these decay products as leptons, though the arguments apply to the light quark jets just as well.) Spin correlations, and therefore entanglement and BIV, are measured via angular correlations of the leptons [32]. For this reason, if the decays of the two W bosons are in a causal contact, again an argument could be made that the exchange of hidden variables conspired to mimic an entangled state. There is also a possibility that spacelike separated top quark decays produce W bosons that become timelike separated at the moment of their decay. To exclude this possibility, we also consider a requirement that both types of decays are spacelike separated. Finally, the leptons come into contact with macroscopic apparatus, and the entanglement is destroyed [33]. If up to this point the leptons were in causal connection, again the entangled state can be arranged by the hidden variables. In this paper, we limit our discussion to the event kinematics without considering the specifics of the apparatus and construct a critical criterion for entanglement/BIV as follows. If at any of the discussed decision points (top quark decays, W boson decays, and contacts of leptons with the macroscopic apparatus) the system was in a causal connection, the value of the criterion is set to its mathematically allowed maximum. If there is no possibility of a causal connection at any of these decision points, the system must be separable according to the classical explanation. The criterion is then set to the maximal value allowed for a separable system. Should the observed value of the entanglement/BIV level exceed this critical value of the criterion, it cannot be explained by the classical communication.

A couple of remarks on the assumptions made in the presented analysis are prudent. First, we treat short-lived top quarks and W bosons as particles, not as fields. Second, we assume that the location of the decay can be inferred from the kinematics of the particle, while, strictly speaking, the Heisenberg uncertainty principle should be applied. It should be noted, though, that both counter arguments are based on a quantum mechanical treatment of the system, while we are trying to refute the classical explanation of entanglement. Third, the fact that the particles are spacelike separated at the decay point does not exclude a possibility that their light cones overlapped in the past. This is always true for particles moving at subluminous speeds, which were in interaction at some point. Despite being in contact, the system remains in a mixed state at least until one of the top quarks decays; hence, the points of particles decays must be spacelike separated to exclude causal contact.

This work is organized as follows. After a brief introduction to top spin correlations and entanglement, in Sec. II, we propose criteria to exclude an explanation of entanglement/BIV based on the exchange of the hidden variables. These criteria are based on f, the fraction of events where the decays of top and antitop quarks, together with their decay products, are spacelike separated. Next, in Sec. III, we evaluate f, in the definitions described above.

We use Monte Carlo simulation of the top pair production and decay kinematics. Since this is a straightforward task, we propose that analyzers use their own best MC simulations to determine the relevant spacelike fractions. This way, detector efficiency and resolution will also be taken into account. For the top quark decays, we also show analytical calculation of f. Out of these criteria, we recommend the most stringent one. We also discuss the same strategy applied to b jets from $t\bar{t}$ decay. We conclude in Sec. IV.

II. TOP SPIN CORRELATIONS, ENTANGLEMENT, AND BELL VIOLATIONS

Let us give a quick overview of the strategy used to measure spin correlations and the corresponding conditions for observing entanglement and Bell inequality violations. The correlation between the spin orientations of top and antitop quarks is described by the spin correlation matrix C_{ij}, which directly enters the top/antitop spin density matrix,

$$\rho = \frac{\mathbf{1}\otimes\mathbf{1} + B_{1i}\sigma_i\otimes\mathbf{1} + B_{2j}\mathbf{1}\otimes\sigma_j + C_{ij}\sigma_i\otimes\sigma_j}{4}, \quad (1)$$

where $i, j = 1, 2, 3$ and σ are the Pauli matrices. The vectors B_{1i} and B_{2j} represent the spin polarization of the top and antitop quarks, while the C_{ij} matrix parametrizes their spin correlations. The angular distribution of the top and antitop quark decay products is used to evaluate C_{ij} experimentally, and therefore extract spin correlations. In some regions of phase space, correlations are so strong that they can only be explained by entanglement. According to the Peres-Horodecki criterion, the system is entangled if [32]

$$\Delta_E = C_{nn} + |C_{rr} + C_{kk}| > 1, \quad (2)$$

where C_{nn}, C_{rr}, and C_{kk} are the diagonals of C_{ij} in the helicity basis $\{k, r, n\}$. An even stronger quantum connection is manifested in the violation of Bell inequality, which in our case using Clauser, Horne, Shimony, and Holt (CHSH) formulation [13] reduces to [16]

$$B_{\pm} = |C_{rr} \pm C_{nn}| > \sqrt{2}, \quad (3)$$

where either condition is sufficient to establish a violation.

In [34], a proposal was made to extend the condition in Eq. (2) to ensure its value cannot be explained via classical communication. The locality loophole is more of an interest in the context of the BIV. Here, we continue the same argument to $B_{\pm}$, the Bell markers of Eq. (3), proceeding as follows: the timelike separated events are assumed to have maximum correlations, and therefore the maximum mathematically allowed values of $\Delta_E^{\max} = 3$ and $B_{\pm}^{\max} = 2$, and the spacelike separated events are assumed to have the maximum allowed "classical" values, i.e., those that do not require entanglement or BIV, $\Delta_E^{\text{class}} = 1$ and $B_{\pm}^{\text{class}} = \sqrt{2}$. Then, with f being the fraction of events where the decays of top and antitop quarks, together with their decay products, are spacelike separated, the largest values that could be explained by classical communication are

$$\Delta_E^{\star} = f\Delta_E^{\text{class}} + (1-f)\Delta_E^{\max} = 3 - 2f \quad (4)$$

for entanglement and

$$B_{\pm}^{\star} = fB_{\pm}^{\text{class}} + (1-f)B_{\pm}^{\max} = \sqrt{2}f + 2(1-f) \quad (5)$$

for Bell inequality violation.

III. SPACELIKE FRACTION

All that is left, at this point, is evaluating the probability f that an event is spacelike. Let $\tau_1 = (ct_1, x_1, y_1, z_1)$ and $\tau_2 = (ct_2, x_2, y_2, z_2)$ be the four-dimensional coordinates of the two points in question. To guarantee that the connection between the top and antitop quarks is of quantum nature, we need to make sure that these points are separated by a spacelike interval,

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 - (ct_1 - ct_2)^2 > 0. \quad (6)$$

We will evaluate the value of the interval between the decays of the top quarks, W bosons, or points of lepton contact with the apparatus. It is important to remember that experimentally we do not measure the top quark or the W boson decay length. Thus, in the cases of top quark and W boson decays, we can only determine f on statistical basis. This is not an issue, however, since the entanglement and BIV markers are also determined as averages on an ensemble of events. In the lepton contact definition, the directions of leptons and the location of their production vertex are known event by event; hence, the interval between first contacts with the apparatus can in principle be extracted for each event. The same is true if the angular correlations between the b jets are used to evaluate the spin correlation between the top and antitop quarks.

A. Top quark decay

Let us consider the center of mass of the top and antitop quark system. In this reference frame, the velocities of the two particles are equal in magnitude and opposite in direction $\vec{\beta_t} = -\vec{\beta}_{\bar{t}}$. For brevity, we shall refer to the magnitude of these vectors as β, which is related to the invariant mass of the $t\bar{t}$ system, $M_{t\bar{t}}$, through the simple relation

$$\beta = \sqrt{1 - (2m_t/M_{t\bar{t}})^2}, \quad (7)$$

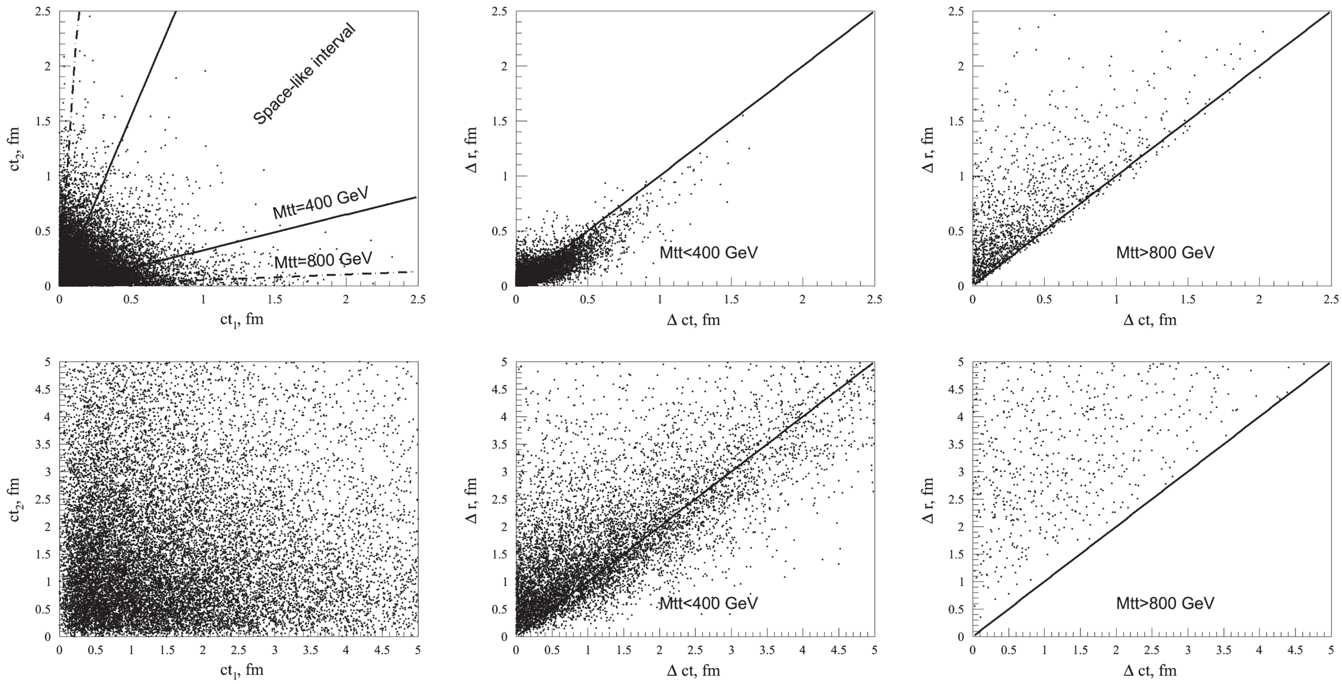

FIG. 2. Top row: top quark decays. Bottom row: W boson decays. Left: time from production to decay of particle vs that of antiparticle. Lines show the limits of spacelike interval for $M_{t\bar{t}} = 400$ GeV (solid) and $M_{t\bar{t}} = 800$ GeV (dashed). Center and right: distance vs time difference between particle decays for $M_{t\bar{t}} < 400$ GeV (center) and $M_{t\bar{t}} > 800$ GeV (right). Events below the diagonal are timelike separated, while the ones above the diagonal are spacelike separated.

where m_t is the top quark mass. The corresponding relativistic factor is $\gamma = 1/\sqrt{1-\beta^2}$.

The top quark is an unstable particle, and its lifetime is exponentially distributed with a time constant corresponding to the top width $\Gamma_t = 1.42$ GeV. We can direct the z axis along the direction of flight of the top quark, so that the four-dimensional coordinates of the top/antitop decay points are $(\gamma ct_1, 0, 0, \beta\gamma ct_1)$ and $(\gamma ct_2, 0, 0, -\beta\gamma ct_2)$. To guarantee that the connection between the top and antitop quarks is of quantum nature, we need to make sure that their decays are separated by a spacelike interval,

$$(\beta\gamma ct_1 + \beta\gamma ct_2)^2 - (\gamma ct_1 - \gamma ct_2)^2 > 0. \tag{8}$$

Thus, the condition for spacelike separation is

$$\frac{1-\beta}{1+\beta} t_1 < t_2 < \frac{1+\beta}{1-\beta} t_1. \tag{9}$$

This is shown by the solid lines corresponding to different values of $M_{t\bar{t}}$ in Fig. 2 (top left), obtained via a Monte Carlo simulation accurate at next-to-leading order in QCD using POWHEG [35]. The events are distributed according to the survival probability of each top quark that decays exponentially but is boosted by the time dilation factor γ. Even though it is impossible to say on an event-by-event basis when the decays are timelike or spacelike separated, we can approach the problem statistically. Integrating the probability density function over the area between the solid lines, we find a simple relation,

$$f = \beta. \tag{10}$$

Note, that this fraction does not depend on the actual decay times. Since β depends on $M_{t\bar{t}}$, so does f, and indeed for $M_{t\bar{t}}$ going to infinity, we find that f approaches 1. To appreciate the dependence of f on $M_{t\bar{t}}$, in Fig. 2, we also split $t\bar{t}$ phase space in the regions $M_{t\bar{t}} < 400$ GeV (top center) and $M_{t\bar{t}} > 800$ GeV (top right). The fraction of the spacelike separated events, above the diagonal, is significantly lower for the events near production threshold compared to those at high invariant mass.

B. *W* boson decay

Next, we consider the W boson decays. The lifetime of the W boson is exponentially distributed with a time constant corresponding to its width, $\Gamma_W = 2.085$ GeV. The two measurements are then separated by the time and distance of the top quark decays plus that of the W boson decays, as shown schematically in Fig. 1. The time elapsed from the $t\bar{t}$ production to decays of the W^+ and W^- bosons, also obtained via a Monte Carlo simulation, is shown in Fig. 2 (bottom left). The distance separation vs time separation between W boson decays is also shown for events with $M_{t\bar{t}} < 400$ GeV (bottom center) and $M_{t\bar{t}} >$ 800 GeV (bottom right). Similar to the top quark decays,

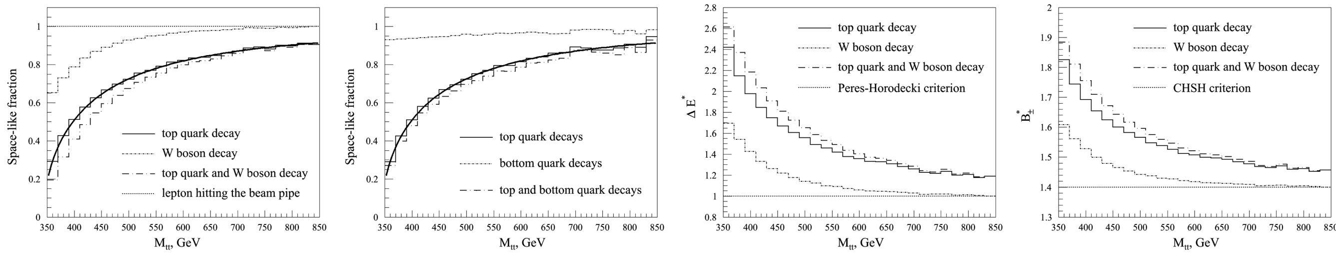

FIG. 3. Legend inline. Far and center left: fraction of events with spacelike separated instances using four different definitions. The solid line shows the result of the analytical calculation, $f = \beta$, for the top quark decays. Center and far and right: critical values of Δ_E (center) and $B_\pm$ (right) as a function of $M_{t\bar{t}}$ based on the considered definitions. The dotted lines show the critical values in the absence of our proposed corrections and coincide with the definition based on lepton contact.

we observe that the probability of spacelike separation is significantly lower for events near $t\bar{t}$ production threshold compared to those at high invariant mass.

It is important to note that in this case more events are above the diagonal, that is, spacelike separated, compared to what we had found for the top quark decay.

C. Leptons contact with macroscopic apparatus

While top quark and W boson decays happen on the scale of a femtometer, 1 fm $= 10^{-15}$ m, contact of leptons (or jets produced by light quarks) with the macroscopic apparatus, presumably the beam pipe, happens on the scale of 1 cm. Since leptons (and hadrons) have negligible mass compared to the LHC collision energy, they can be considered as moving close to the speed of light. A simple calculation shows that in the limit of zero lepton mass and the same origin the instances of lepton's contact with the beam pipe are always spacelike separated. Based on the simulation, where lepton's masses and the difference between their production points are taken into the account, we find that this is also true in almost 100% of the events—the timelike fraction is $O(10^{-5})$. This argument is similar to the one illustrated in Fig. 3 of [36] except the speed of electron or muon from top quark decay is much closer to the speed of light than the speed of τ lepton produced at $\sqrt{s} = 10$ GeV, making the fraction of timelike events negligible.

D. b quark decay

There is also an option to use the angular distribution of the b quarks to evaluate the spin correlation of top and antitop quarks, albeit with a lower spin analyzing power than leptons [37]. b quarks hadronize into long-lived b hadrons, which are identified if tracks of their charged decay products do not point to the primary vertex. In this case, since the position of the b hadron decay vertex can be measured, it is possible to tell on event-by-event basis if these decays vertices are spacelike or timelike separated. For completeness, we include this option in our discussion.

E. Summary

In Fig. 3 (far left), we show the spacelike fraction f as a function of $M_{t\bar{t}}$ based on the top quark and W boson decays and leptons hitting the beam pipe. We also include a more stringent requirement that both types of decays happen in a spacelike interval. For low $M_{t\bar{t}}$, this requirement reduces the fraction of spacelike events, while for $M_{t\bar{t}} > 800$ GeV, it essentially coincides with the requirement based on top quark decay. For comparison, we also show the result of the analytical calculation for top quark decays, which is in excellent agreement with the simulation. The case when instead of leptons b quarks are used for the $t\bar{t}$ spin correlation measurement is demonstrated in Fig. 3 (center left). Again, the requirement of the top quark decays being spacelike is the most stringent. The requirement that b quarks also decay in the spacelike interval does not change f significantly.

Of course, we advocate for selecting the most stringent requirement of top quarks and W bosons both being in the spacelike interval, when evaluating f. Therefore, the critical values for the entanglement marker Δ_E and for the Bell markers $B_\pm$ become a function of $M_{t\bar{t}}$ and are shown in Fig. 3 (center and far right). If the observed value of Δ_E or $B_\pm$ exceeds these critical values with enough significance, we can say that the detection of entanglement or of Bell inequality violations cannot be argued away by assuming classical communication.

In the argument presented above, one critical assumption is that the top quark decay time is not correlated with that of the antitop quark. Should that not be the case, e.g., the events are clustered in the upper and lower wedges in Fig. 2 (top left), while still individually following the exponential distributions, the locality loophole cannot be closed. Since measuring top quark decay length is out of the question, a system consisting of a pair of B-mesons or τ leptons might be better suited for addressing this potential issue.

IV. CONCLUSION

In light of the observation of entanglement in $t\bar{t}$ pairs, of concrete prospects for detecting Bell violations, and in general for a future Quantum Information program at the LHC, it is crucial to carefully analyze our strategies, to ensure that such counterintuitive physical phenomena are detected experimentally in a convincing manner. Part of this endeavor consists of making sure "alternative" explanations for quantum phenomena, however unlikely, are ruled out. We analyzed the locality loophole and proposed criteria to establish that the observed values of Δ_E or $B_{\pm}$ cannot be explained by classical communication. We considered causal separation at three different instances: at the top quark decay, at the W boson decay, and at the lepton/jet contact with the macroscopic apparatus. We showed that the spacelike fraction of events is the smallest, when requiring that both top quarks and W bosons decay within spacelike interval. For high invariant masses, typically required for the Bell inequality violation, this is almost identical to just the top quark decay requirement. We also included an option for the angular correlation of the b quarks from top quark decay to be used for the spin correlation measurement. We require that both top quark and b hadron decays are spacelike separated. Again, we find that at high invariant masses it is almost identical to just the requirement of spacelike separation between top and antitop quarks. We provide numerical values for our proposed criteria.

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