Master's project
Instituto de Física da Universidade de São Paulo

Information transport in quench dynamics of quantum spin chains

Artur Machado Lacerda
Supervisor: Gabriel T. Landi

IFUSP

Project summary

The dynamics of quantum spin chains after quantum quenches is known to lead to an incredibly rich set of physical effects, related to the notions of integrability and effective thermalization. Particularly interesting in this scenario is the dynamics of quantum entanglement. The excitations created by the quench will propagate through the chain and consequently carry information from one part to the other, hence entangling the different spins. A more thorough understanding of this process is of general fundamental interest, as it contributes to strengthen the deep connection between non-equilibrium physics and quantum information. In this project we propose a numerical study of information transport in quantum spin chains using tensor network algorithms, including time-dependent density matrix renormalization group and the Trotter decomposition. Our main focus will be on characterizing the dynamics using the concept of conditional mutual information (CMI), which can be used to quantify the ability of one part of the chain to share information with another via disconnected parts. As our main focus, we will study the XXZ spin chain and understand how the gap in the model affects the transmission of information. In addition, we will also consider the XXZ chain under the presence of staggered magnetic fields, a feature which breaks integrability.

1 Introduction

One-dimensional quantum spin chains have been a part of the statistical physics community for more than 50 years, dating at least back to the seminal works of Lieb, Schultz and Mattis on the XY model [1]. Initially, much of the effort was dedicated to equilibrium properties. However, more recently, this shift has focused towards non-equilibrium dynamics. And of all types of non-equilibrium scenarios, the most popular is that of *quantum quenches*. The typical scenario is a system which is prepared in a certain ground-state $|\psi(\lambda_i)\rangle$ of a Hamiltonian $H(\lambda_i)$, which depends on a certain parameter λ initially set to the value λ_i. Then, at $t = 0$ the parameter is suddenly changed to λ_f. After this quench, the state $|\psi(\lambda_i)\rangle$ will no longer be an eigenstate of $H(\lambda_f)$ and will therefore start to evolve in times. The properties of the system during this time evolution turn out to be extremely rich.

A significant progress in this field was given by Calabrese and Cardy in 2005 [2–5], who first put forth a study of the dynamics of quantum entanglement after a quench. The typical scenario is illustrated in Fig. 1(a). The quench creates excitations in the system, which tend to propagate (ballistically) in both directions. However, these excitations also carry information from one part to the other. Hence, they serve to entangle the different parts of the system. In the simplest configuration one considers an infinite chain divided in two parts, A and B, with A representing a certain finite region (as in Fig. 1(a)) and B representing the remainder of the chain. One then asks what is the entanglement, as a function of time, between A and B. The main result of Calabrese and Cardy was that the entanglement was simply proportional to the *number* of quasiparticles that have left A at any given time.

This result offers an incredibly fertile contact between non-equilibrium physics (the dynamics of excitations) and quantum information (the dynamics of entanglement). And in the last decades the physics of quantum quenches in integrable models has advanced considerably, both in our theoretical understanding [6–15] as well as in experimental realizations using ultra-cold atoms in optical lattices [16–23].

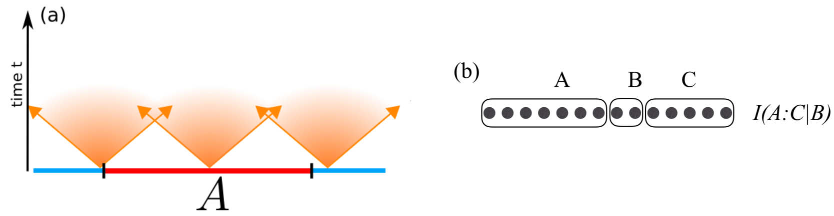

Figure 1: (a) Illustration of the entanglement dynamics in quantum spin chains. Taken from Ref. [15]. (b) Illustration of a tripartition ABC of a quantum spin chain and the conditional mutual information $\mathcal{I}(A : C|B)$.

In this project we propose to contribute to this emerging field of research, by addressing the question of how to quantify the correlations between *disconnected* parts of a quantum chain. The basic idea is illustrated in Fig. 1(b): a quantum chain of arbitrary size is split into three regions, A, B and C. One then asks how much information is shared between A and C. The relevance of this question lines in the fact that A and C can only communicate through B, which therefore functions as a noisy information channel. It is then natural to ask what is the role of this noisiness of B in transmitting information and how this is affected by the multiple parameters in a model. As is know well established by the Quantum Information community, a faithful quantifier of this type of information transport is the conditional mutual information [24–35], a more general quantifier of tripartite correlations. Prof. Landi recently used this type of quantifier in the context of Lindblad master equations [36]. The approach in this paper can be viewed as a unitary extension of this type of study.

We shall approach this problem by studying the quench dynamics numerically using Tensor Network techniques [37–40]. More specifically the techniques known as time-dependent density matrix renormalization group (tDMRG) and the Trotter decomposition method, which shall be implemented in the `iTensor` library [41]. Our focus will be on the XXZ model, which is known to have an energy gap for a well defined value of the asymmetry parameter. The first part of the project will be to investigate the role of this energy gap in the transmission of information. Next we will consider the effects of adding a staggered magnetic field, which is known to break the integrability of the model [42].

2 Methodology

We now move on to describe the specific goals of this project and the steps that need to be taken to accomplish them.

1. **Familiarize with the basics of one-dimensional quantum chains and quench dynamics:** In this review part of the project the student will focus on some standard references, such as Refs. [1] for the equilibrium properties of spin chains and Refs. [8, 11–13] for quench dynamics.

2. **Familiarize with the basics of the `iTensor` library and Tensor Network algorithms:** this part of the study will be based on Refs. [37–40]. It will also count with the support of other members in our group which are already working with the `iTensor` library, such as Mr. Heitor Peres Casagrande and Rafael Magaldi, two master students, as well as Prof. Luis Gregório, a colleague in my department.

3. **Implement time-evolution algorithms for the dynamics of the XXZ chain:** in view of the fact that the XXZ chain is a widely popular model in the literature, this implementation is already fairly well documented and should not pose a problem for the student.

4. **Implement an algorithm for computing the entanglement entropy of disconnected parts of a quantum chain:** one of the biggest advantages of the matrix product state (MPS) formulation is that it allows one to easily compute the entanglement entropy for a left-right bipartition of the chain. However, the same is not true for more complicated bipartitions or tripartitions. Thus, in this case some effort is still needed. One of the goals for the student will be to implement functions that can extract the von Neumann entropy of arbitrary parts of the chain.

5. **Study the dynamics of the conditional mutual information after quantum quenches:** for this part the student can be guided by the results presented in Ref. [13] for the entanglement entropy of the same model, also computed using Tensor Network algorithms.

6. **Study the role of a staggered magnetic field in the information transport:** as a final (and, to a great extent, optional) part of the project, we propose for the student to investigate the role of an integrability breaking ingredient in the transport of information [42].

3 Execution, time-table and expected results

The project is planned for one and a half years. This is because the student changed supervisors and therefore already concluded all the courses in the masters. Topics 1-4 of Sec. 2 should be concluded within the six months, leaving the remaining year for a thorough investigation of topics 5 and 6. Of course, topics 1 and 2 are continuous and will remain an important part of the student's activities throughout the course of the masters. Throughout the project the student will count with the full support of other members in our department interested in Tensor Networks, as well as possible collaborators in Brazil and abroad. The physics institute already contains all the infrastructure necessary for the execution of this project. Computational resources may become necessary by the end of the project. In this case we shall use the `USP Cloud` computer, which will be paid for with research grants from the São Paulo Funding agency FAPESP. The project has the potential for yielding 2 scientific papers in high-quality international journals.

Finally, we mention that the knowledge that will be acquired by the student in this project extends far beyond this specific area of expertise. The topic here proposed is of interest as it lies in the frontier between statistical mechanics, condensed matter and quantum information. Moreover, Tensor Network algorithms are a blooming field of research which is severely under-represented in Brazil. For this reason, the expertise acquired by the student in this project will help to project him to a privileged position within the Brazilian physics community.

References

[1] E. H. Lieb, T. Schultz, and D. Mattis, “Two soluble models of an antiferromagnetic chain,” *Annals of Physics*, vol. 16, pp. 407–466, 1961.

[2] P. Calabrese and J. Cardy, “Entanglement entropy and quantum field theory,” *Journal of Statistical Mechanics: Theory . . .* , vol. 06002, pp. 1–26, 2005.

[3] P. Calabrese and J. Cardy, “Evolution of entanglement entropy in one-dimensional systems,” *Journal of Statistical Mechanics: Theory and Experiment*, no. 4, pp. 15–38, 2005.

[4] P. Calabrese and J. Cardy, “Time dependence of correlation functions following a quantum quench,” *Physical Review Letters*, vol. 96, no. 13, p. 136801, 2006.

[5] P. Calabrese and J. Cardy, “Quantum quenches in extended systems,” *Journal of Statistical Mechanics: Theory and Experiment*, no. 6, 2007.

[6] V. Eisler, D. Karevski, T. Platini, and I. Peschel, “Entanglement evolution after connecting finite to infinite quantum chains,” *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2008, p. P01023, 2008.

[7] J. Eisert, M. Cramer, and M. B. Plenio, “Colloquium: Area laws for the entanglement entropy,” *Reviews of Modern Physics*, vol. 82, no. 1, pp. 277–306, 2010.

[8] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, “Colloquium: Nonequilibrium dynamics of closed interacting quantum systems,” *Reviews of Modern Physics*, vol. 83, no. 3, pp. 863–883, 2011.

[9] A. Coser, E. Tonni, and P. Calabrese, “Entanglement negativity after a global quantum quench,” *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2014, no. 12, 2014.

[10] J. S. Cotler, M. P. Hertzberg, M. Mezei, and M. T. Mueller, “Entanglement growth after a global quench in free scalar field theory,” *Journal of High Energy Physics*, vol. 2016, no. 11, pp. 1–32, 2016.

[11] P. Calabrese, F. H. Essler, and G. Mussardo, “Introduction to ’Quantum Integrability in out of Equilibrium Systems’,” *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2016, no. 6, 2016.

[12] F. H. Essler and M. Fagotti, “Quench dynamics and relaxation in isolated integrable quantum spin chains,” *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2016, no. 6, 2016.

[13] V. Alba and P. Calabrese, “Entanglement dynamics after quantum quenches in generic integrable systems,” pp. 1–43, 2017.

[14] V. Alba and P. Calabrese, “Entanglement and thermodynamics after a quantum quench in integrable systems,” *Proceedings of the National Academy of Sciences*, vol. 114, no. 30, pp. 7947–7951, 2017.

[15] V. Alba and P. Calabrese, “Quantum information dynamics in multipartite integrable systems,” vol. 1, no. 2, pp. 1–8, 2018.

[16] S. Hofferberth, I. Lesanovsky, B. Fischer, T. Schumm, and J. Schmiedmayer, “Non-equilibrium coherence dynamics in one-dimensional Bose gases,” *Nature*, vol. 449, no. 7160, pp. 324–327, 2007.

[17] M. Cheneau, P. Barmettler, D. Poletti, M. Endres, P. Schauß, T. Fukuhara, C. Gross, I. Bloch, C. Kollath, and S. Kuhr, “Light-cone-like spreading of correlations in a quantum many-body system,” *Nature*, vol. 481, no. 7382, pp. 484–487, 2012.

[18] S. Trotzky, Y. A. Chen, A. Flesch, I. P. McCulloch, U. Schollwöck, J. Eisert, and I. Bloch, “Probing the relaxation towards equilibrium in an isolated strongly correlated one-dimensional Bose gas,” *Nature Physics*, vol. 8, no. 4, pp. 325–330, 2012.

[19] M. Gring, M. Kuhnert, T. Langen, T. Kitagawa, B. Rauer, M. Schreitl, I. Mazets, D. A. Smith, E. Demler, and J. Schmiedmayer, “Relaxation and Prethermalization in an Isolated Quantum System,” *Science*, vol. 337, no. September, pp. 1318–1323, 2012.

[20] T. Langen, R. Geiger, M. Kuhnert, B. Rauer, and J. Schmiedmayer, “Local emergence of thermal correlations in an isolated quantum many-body system,” *Nature Physics*, vol. 9, no. 10, pp. 640–643, 2013.

[21] T. Fukuhara, P. Schauß, M. Endres, S. Hild, M. Cheneau, I. Bloch, and C. Gross, “Microscopic observation of magnon bound states and their dynamics,” *Nature*, vol. 502, no. 7469, pp. 76–79, 2013.

[22] T. Langen, S. Erne, R. Geiger, B. Rauer, T. Schweigler, M. Kuhnert, W. Rohringer, I. E. Mazets, T. Gasenzer, and J. Schmiedmayer, “Experimental observation of a generalised Gibbs ensemble,” *Science*, vol. 348, no. 6231, p. 207, 2015.

[23] T. Fukuhara, A. Kantian, M. Endres, M. Cheneau, P. Schauß, S. Hild, D. Bellem, U. Schollwöck, T. Giamarchi, C. Gross, I. Bloch, and S. Kuhr, “Quantum dynamics of a mobile spin impurity,” *Nature Physics*, vol. 9, no. 4, pp. 235–241, 2013.

[24] A. D. Wyner, “A definition of conditional mutual information for arbitrary ensembles,” *Information and Control*, vol. 38, no. 1, pp. 51–59, 1978.

[25] J. P. Crutchfield and K. Young, “Inferring statistical complexity,” *Physical Review Letters*, vol. 63, no. 2, pp. 105–108, 1989.

[26] R. Renner and U. Maurer, “About the mutual (conditional) information,” *IEEE International Symposium on Information Theory*, p. 364, 2002.

[27] P. Hayden, R. Jozsa, D. Petz, and A. Winter, “Structure of States Which Satisfy Strong Subadditivity of Quantum Entropy with Equality,” *Communications in Mathematical Physics*, vol. 246, no. 2, pp. 359–374, 2004.

[28] B. Ibinson, N. Linden, and A. Winter, “Robustness of quantum Markov chains,” *Communications in Mathematical Physics*, vol. 277, no. 2, pp. 289–304, 2008.

[29] M. Gu, K. Wiesner, E. Rieper, and V. Vedral, “Quantum mechanics can reduce the complexity of classical models,” *Nature Communications*, vol. 3, pp. 762–765, 2012.

[30] W. Brown and D. Poulin, “Quantum Markov Networks and Commuting Hamiltonians,” *arXiv*, no. Theorem 4, 2012.

[31] M. Papapetrou and D. Kugiumtzis, “Markov chain order estimation with conditional mutual information,” *Physica A: Statistical Mechanics and its Applications*, vol. 392, no. 7, pp. 1593–1601, 2013.

[32] O. Fawzi and R. Renner, “Quantum Conditional Mutual Information and Approximate Markov Chains,” *Communications in Mathematical Physics*, vol. 340, no. 2, pp. 575–611, 2015.

[33] K. Kato and F. G. S. L. Brandão, “Quantum Approximate Markov Chains are Thermal,” no. 4, 2016.

[34] M. Berta, F. G. S. L. Brandão, C. Majenz, and M. M. Wilde, “Conditional Decoupling of Quantum Information,” pp. 1–6, 2018.

[35] F. C. Binder, J. Thompson, and M. Gu, “Practical Unitary Simulator for Non-Markovian Complex Processes,” *Physical Review Letters*, vol. 120, no. 24, p. 240502, 2018.

[36] W. T. B. Malouf, J. Goold, G. Adesso, and G. T. Landi, “Information transport in quantum non-equilibrium steady-states,” no. Cmi, pp. 39–42, 2018.

[37] S. R. White, “Density matrix formulation for quantum renormalization groups,” *Physical Review Letters*, vol. 69, no. 19, pp. 2863–2866, 1992.

[38] G. De Chiara, M. Rizzi, D. Rossini, and S. Montangero, “Density Matrix Renormalization Group for Dummies,” pp. 1–26, 2006.

[39] I. P. McCulloch, “From density-matrix renormalization group to matrix product states,” *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2007, no. 10, pp. P10014–P10014, 2007.

[40] U. Schollwöck, “The density-matrix renormalization group in the age of matrix product states,” *Annals of Physics*, vol. 326, no. 1, pp. 96–192, 2011.

[41] “ITensor. http://itensor.org.”

[42] J. J. Mendoza-Arenas, S. R. Clark, and D. Jaksch, “Coexistence of energy diffusion and local thermalization in nonequilibrium X X Z spin chains with integrability breaking,” *Physical Review E*, vol. 91, p. 042129, 2015.