Master's project
Instituto de Física da Universidade de São Paulo

Non-Markovianity in correlated Gaussian collisions

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Project summary

The theory of open quantum systems has advanced considerably in the last decade. However, several extremely important aspects still remain poorly understood. One, in particular, concerns the notion of non-Markovianity in quantum systems. That is, how to quantify the amount of memory that an open quantum evolution has about the past. Unlike in classical stochastic processes, where non-Markovianity is by now well characterized, in the quantum realm deep questions still remain. This is particularly important in view of the growing interest in quantum information processing, as non-Markovianity is a manifestation of information backflow; i.e., a flow of information from the past to the future. In this project we propose a theoretical study of how non-Markovianity is induced when a system interacts sequentially with a series of ancillas, a method known as repeated interactions. Moreover, we shall focus on the case of Gaussian preserving maps, for which substantial analytical progress can be achieved. The main goal will be to implement and characterize two protocols for non-Markovian evolution. In the first the ancillas are assumed to be prepared in a multipartite entangled state while in the second the ancillas are allowed to interact with each other in addition to interacting with the system. Both scenarios provide mechanisms for information backflow and can thus be used as platforms to investigate the manifestations of non-Markovianity. Thus, we hope that this study will help to shed light on some of the possible mechanisms behind information flows in open quantum systems, which could have a potential impact for several quantum information processing applications.

1 Introduction

The recent surge of interest in quantum information processing has highlighted the need for furthering our knowledge on the concept of *information flow*. Unlike classical systems, in the quantum realm information leaks are much more efficient, so that when a system interacts with an environment, information about the former is inevitably transferred to the latter. When the environment is sufficiently large, this information may never return, in which case the dynamics is called Markovian. But when the environment is finite, there may eventually be a *backflow of information* [1–3], which characterizes a non-Markovian evolution. From the point of view of causality, information backflow can be interpreted as the ability of the past to communicate information to the future, through the present [4]. Having a more thorough understanding of this important problem is considered an essential step in the quest for developing new information processing applications.

In this project we shall not be specifically concerned with the question of how to quantify non-Markovianity, a problem which has been the subject of considerable debate in recent years (for a recent review, see [5]). Instead, we shall be concerned with the physical implementation of non-Markovianity in controlled environments. More specifically, we propose to study non-Markovianity in the context of repeated interactions [6–10]. The basic idea is depicted in Fig. 1. A system S is put to interact with a series of ancillas. Each interaction lasts for a finite amount of time, after which that ancilla is discarded and a fresh new one is introduced. This type of discrete-time dynamics can be used to model a vast number of interesting scenarios, as it offers great control over the system's evolution.

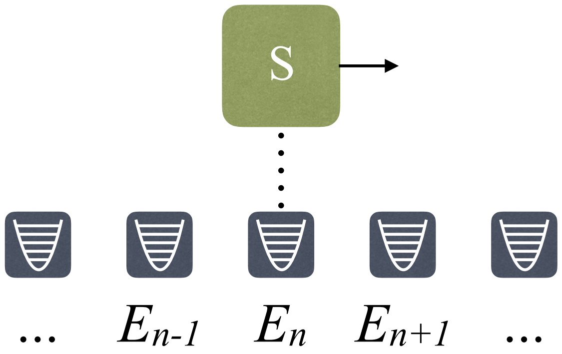

Figure 1: Schematics of the repeated interactions method. A system S is allowed to interact sequentially with a series of environmental ancillas E_1, E_2, etc. After each interaction, which lasts for a finite time, the corresponding ancilla is discarded.

The method of repeated interactions depicted in Fig. 1 is Markovian by construction, since the ancillas are discarded after each interaction, so that no information of the system can ever backflow. Conversely, one may introduce non-Markovianity by one of two methods. The first is to assume that the ancillas are prepared in a globally entangled state. In that way, the interactions at different times will no longer be independent and the dynamics will be effectively non-Markovian. The second is to assume that after the interaction between S and E_n, the ancilla E_n interacts with some other future ancilla E_m $(m > n)$. That way, information of the system in the past will be transported into the future, so that when the time comes for E_m to interact with S, it will already contain a certain amount of information about it.

The natural appeal of this type of physical construct has led to a surge of studies in recent years [11–19]. However, all of these studies rely on qubits as the basic units for the dynamics. This makes the Hilbert space prohibitively large as the number of environmental ancillas increase. Instead, we propose in this project to approach this problem from the perspective of continuous variable *Gaussian preserving maps*. Gaussian systems are fully characterized by their $2N \times 2N$ covariance matrix (where N is the total number of modes present). Moreover, if the dynamics is such as to preserve Gaussianity, then the open-system evolution can be entirely described in terms of a map for the covariance matrix, with no need to ever work with density matrices. This provides an enormous simplification to the problem, since it allows for the dimension of the "effective" Hilbert space to grow linearly, instead of exponentially.

This project will count with the collaboration of Prof. Gerardo Adesso, from the University of Nottingham, and a collaborator in the context of the FAPESP project 2017/07973-5. It will also count with the collaboration of Profs. Marcelo França (UFRJ) and Nadja Bernardes (UFPE). Finally, it will count with the assistance of Dr. Jader Pereira dos Santos, a post-doc in our group.

2 Methodology

We now a point-by-point list of goals for this project, together with the methodology we shall employ to accomplish them:

1. **Familiarize the student with the tools of Gaussian quantum information [20–23]**. Gaussian systems are one of the expertises of Prof. Landi, which has extensive experience in a wide range of techniques. This familiarization process will also count with the assistance of other students in the group, as well as Dr. Jader Pereira dos Santos, a post-doc.

2. **Familiarize the student with the method of repeated interactions [6–10]**. The method of repeated interactions has recently seen a revival of interest from the open quantum systems community. It also corresponds to an expertise of Prof. Landi, which has recently given an important contribution to this field by reconciling the method with thermodynamics [10]. Thus, in this part of the project the student will also count with the support from all members of the group.

3. **Review the tools for quantifying non-Markovianity in general [1, 5, 24] and of Gaussian systems in particular [25].** This part of the project will count with the collaboration of Prof. Nadja Bernardes, which is one of the authors of Ref. [25].

4. **Implement a framework for working with Gaussian maps in the context of the repeated interactions method, motivated by the results in [14, 17].** This corresponds to the first novel contribution expected from the student. It will be based on the results of Prof. Landi published in Ref. [10], but extended to the more general multi-ancilla scenario.

5. **Study general properties of the repeated interactions method to infer under which conditions is it possible to obtain analytical results.** In this part of the project the student will focus on exploring the different types of interactions between system and ancilla and, under which conditions, they can lead to closed formulas for the covariance matrix of the system.

6. **Implement the dynamics assuming a general entangled state for the ancillas.** This is the second original contribution expected from the student. It corresponds to a natural step and will be based on Refs. [11–19].

7. **Implement the dynamics assuming a general ancilla-ancilla interaction scheme.** This is similar to the previous point and the third main contribution of the student.

8. **Study Hamiltonian graph states [26, 27] as a possible tool for generating interesting environmental states.** This part of the project will make use of other techniques being studied in the group of Prof. Landi for dealing with multipartite entangled states. The generation of Hamiltonian graph states is already well established and so the main question the student should be able to address is whether these states offer interesting prospects when applied to the study of non-Markovianity. The student will also focus on multipartite states for which analytical calculations are in principle possible.

9. **Investigate the role of quantum vs. classical correlations in information backflow [28].** This part of the project is to be considered optional and will only be investigated if the student progresses considerably on the previous points. The goal is to use the techniques of Ref. [28] to study whether quantum discord (a measure of quantum correlations beyond entanglement) plays a role in information backflow.

10. **Investigate whether it is possible to use the concept of conditional mutual information as a tool for quantifying non-Markovianity [29–31].** Similar to the previous point, this part will be optional. It is motivated in particular by Ref. [31] which show, in a classical scenario, how the conditional mutual information (a measure of tripartite quantum correlations) can be used to quantify the degree of non-Markovianity. Extending these ideas to the method of repeated interactions is not trivial. However, if possible, it would give a genuinely novel approach to characterizing non-Markovianity in open quantum systems.

3 Execution, time-table and expected results

The project is planned for two years. Topics 1-3 of Sec. 2 should be concluded within the first year. Topics 4 and 5 should be concluded at most by the end of the 3rd semester. Finally, topics 6 and 7 should, ideally, be concluded after 21 months, leaving 3 months for the optional problems and for writing the dissertation. Throughout the project the student will count with the full support of the entire group of Prof. Landi as well as possible collaborators in Brazil and abroad. The physics institute already contains all the infrastructure necessary for the execution of this project. The project involves only analytical calculations or low-cost numerical routines, so that no sophisticated computational resources will be necessary. The project has the potential for yielding 2 scientific papers in high-quality international journals.

Finally, we mention that the knowledge that will be acquired by the student in this project extends far beyond this specific area of expertise. In fact, the main motivation for this problem was precisely to combine three fields of research which have all seen a surge of interest in the last decade. The tools of Gaussian quantum information have applications across quantum information sciences, from quantum communications to quantum computing. The method of repeated interactions is a versatile tool which can be used to model open system dynamics in several important quantum platforms, from trapped ions to superconducting qubits. Finally, the topic of non-Markovianity touches deeply on the foundations of quantum information and therefore will help put the student in contact with the frontiers of this exciting field.

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