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INVITED TALK

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Thermodynamics of continuously measured quantum systems

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Thermodynamics of continuously measured quantum systems

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QTD, October 20th, 2020.

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In collaboration with

- Mauro Paternostro, Alessio Belenchia, Luca Mancino (Belfast).
- Massimiliano Rossi, Albert Schliesser (Copenhagen).

Entropy Production in Continuously Measured Quantum Systems

Alessio Belenchia,[1] Luca Mancino,[1] Gabriel T. Landi,[2] and Mauro Paternostro[1]

arXiv:1908.09382 (accepted in NPJQI)

PHYSICAL REVIEW LETTERS **125,** 080601 (2020)

Editors' Suggestion

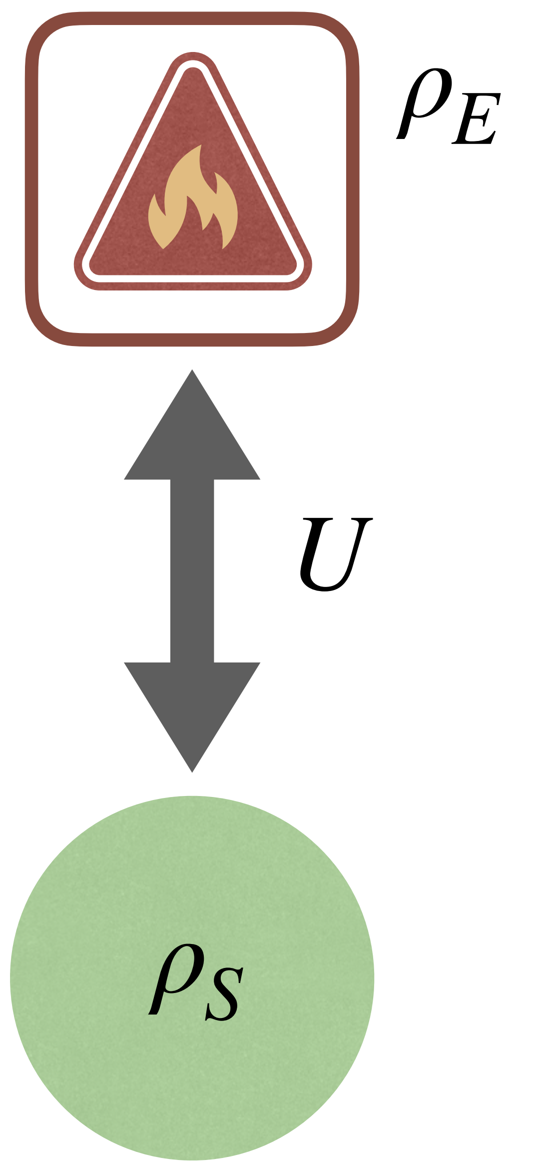
Experimental Assessment of Entropy Production in a Continuously Measured Mechanical Resonator

Massimiliano Rossi,[1,2] Luca Mancino,[3] Gabriel T. Landi,[4] Mauro Paternostro,[3] Albert Schliesser,[1,2] and Alessio Belenchia[3,*]

arXiv:2005.03429

2nd law at the quantum level

$$\rho'_{SE} = U(\rho_S \otimes \rho_E)U^\dagger$$



- The degree of irreversibility of this process is quantified by the entropy production:

$$\Sigma = I'(S:E) + S(\rho'_E || \rho_E)$$

$$= \Delta S_S + \Phi$$

where

$$\Phi = \mathrm{tr}_E\left\{(\rho_E - \rho'_E)\ln \rho_E\right\}$$

is called the **entropy flux**.

$$I'(S:E) = S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE})$$

$$S(\rho'_E || \rho_E) = \mathrm{tr}\left(\rho'_E \ln \rho'_E - \rho'_E \ln \rho_E\right)$$

- Φ depends only on E. Measures change in the “thermodynamic potential” $\ln \rho_E$
  - If $\rho_E = e^{-\beta H_E}/Z_E$ we get $\Phi = -\beta Q_E$.

Describes an enormous variety of processes! (maybe a complicated U)

M. Esposito, K. Lindenberg, C. Van den Broeck, “Entropy production as correlation between system and reservoir”. New Journal of Physics, **12**, 013013 (2010).

🛂 Conditional entropy production

- Part of the irreversibility stems from our ignorance about the environment.

- Suppose we measure E after it interacted with S.

$$\rho'_{SE} \to \rho'_{SE|k} = (1 \otimes M_k)\rho'_{SE}(1 \otimes M_k^\dagger)$$

$$p_k = \mathrm{tr}_E\big(M_k^\dagger M_k \rho'_E\big)$$

$\{M_k\}$ = generalized measurement operators acting on E:

This is a conditional state: It is the state of SE, conditioned on the measurement outcome being k.

- What is the entropy production and flux, conditioned on these outcomes?

$$\Sigma_c = \sum_k p_k S(\rho'_{S|k}) - S(\rho_S) + \Phi_c$$

How to define Σ_c and Φ_c?

- Natural generalization of the flux:

$$\Phi_c = \sum_k p_k \mathrm{tr}\Big\{(\rho_E - \rho'_{E|k})\ln \rho_E\Big\}$$

- If measurement is non-disturbing,

$$\sum_k p_k \rho'_{E|k} = \rho'_E.$$

- Whence:

$$\Phi_c = \Phi$$

Flux is physical; no subjective component associated to information acquired.

- The unconditional and conditional $\Sigma's$ are thus

$$\Sigma = S(\rho'_S) - S(\rho_S) + \Phi$$

$$\Sigma_c = \sum_k p_k S(\rho'_{S|k}) - S(\rho_S) + \Phi$$

- Whence,

$$\Sigma_c = \Sigma - \chi_M(\rho'_S)$$

where

$$\chi_M(\rho'_S) = S(\rho'_S) - \sum_k p_k S(\rho'_{S|k}) = \sum_k p_k S(\rho'_{S|k} || \rho'_S)$$

is the Holevo quantity ✅.

- One may show that

$$0 \leqslant \Sigma_c \leqslant \Sigma$$

- Thus, the conditional entropy production still satisfies a 2nd law ($\Sigma_c \geqslant 0$).

- But it is also smaller than the unconditional one:

  - Conditioning makes the process more reversible.

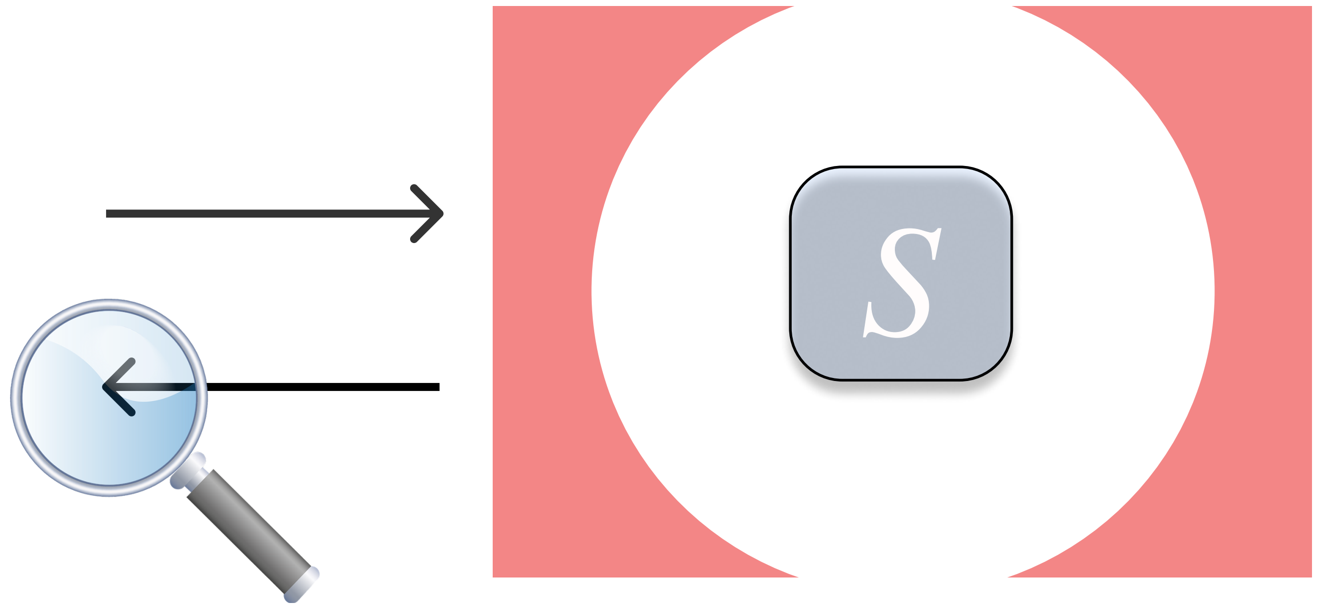
K. Funo, Y. Watanabe and M. Ueda, "Integral quantum fluctuation theorems under measurement and feedback control". PRE, **88**, 052121 (2013).

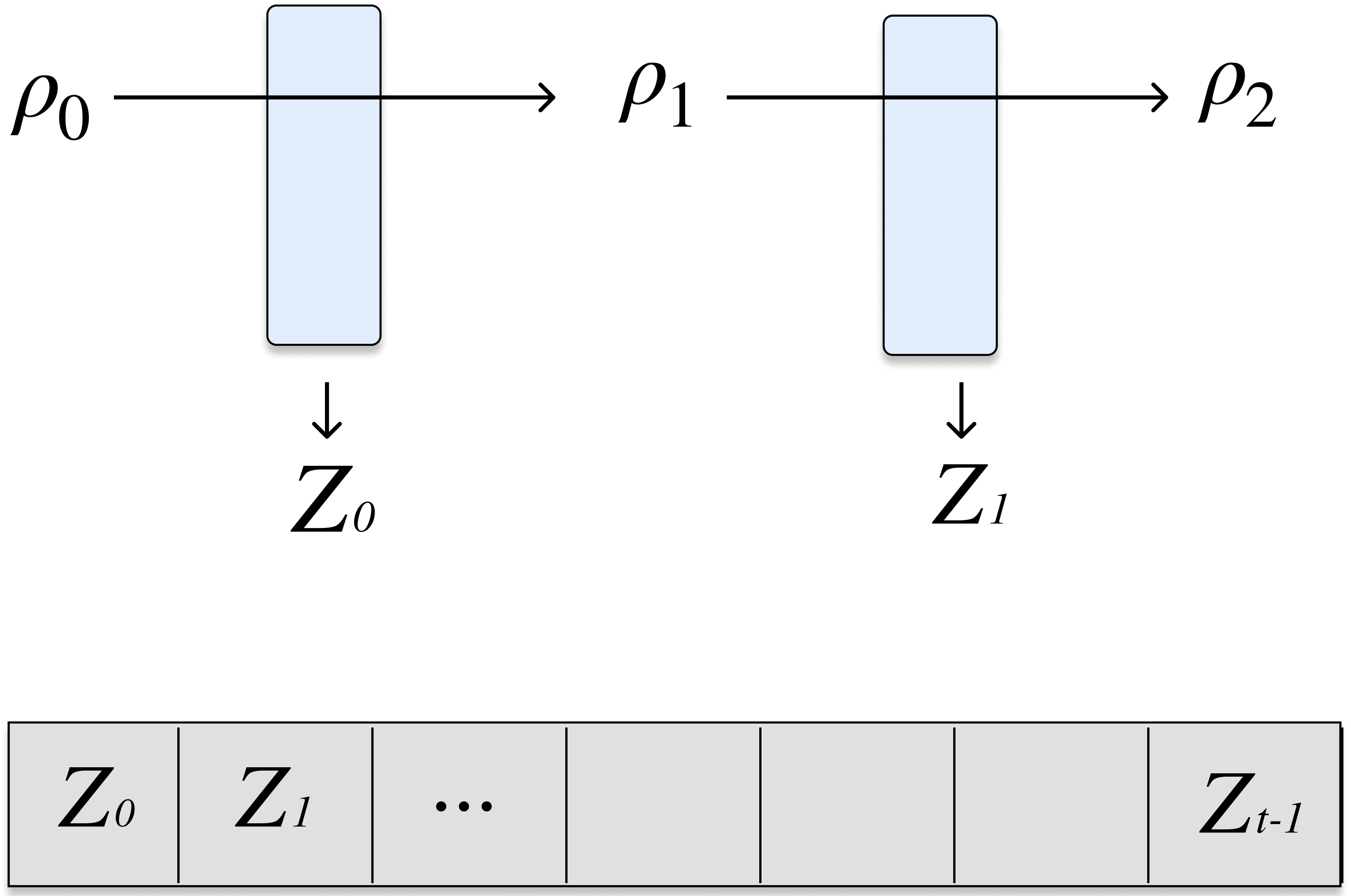
GTL and M. Paternostro, "Irreversible entropy production, from quantum to classical", arXiv:2009.07668

M. Naghiloo, J. J. Alonso, A. Romito, E. Lutz, K. Murch, "Information Gain and Loss for a Quantum Maxwell's Demon". PRL **121**, 030604 (2018).

🔍 Continuous weak measurements

- What about systems that are continuously monitored by a weak probe?
- Things become more interesting because now we have the entire **measurement record** to take into account.
  - For instance, there will be both integral and differential information gains.





H. M. Wiseman and G. J. Milburn, "Quantum Measurement and Control".
K. Jacobs, "Quantum Measurement Theory".

Gaussian continuous weak measurements

- Experimentally relevant and easier to handle.
    - Quadrature operators: $x = (q_1, p_1, q_2, p_2, \ldots)$
        - Average: $\bar{x} = \langle x \rangle$
        - Covariance matrix (CM): $\sigma_{ij} = \frac{1}{2}\langle \{x_i, x_j\} \rangle - \langle x_i \rangle \langle x_j \rangle$.
- We must track both the conditional and unconditional dynamics.
    - Unconditional means we monitor (there is still backaction) but we don't care about the results. Described by a Lindblad MEq.
    - Conditional dynamics is stochastic because we condition on random outcomes. Described by a stochastic MEq.

A. Serafini, "Quantum Continuous Variables: A Primer of Theoretical Method".

M. G. Genoni, L. Lami, and A. Serafini, "Conditional and unconditional Gaussian quantum dynamics", Contemp. Phys. **57**, 331 (2016).

- Unconditional variables evolve as:

$$\frac{d\bar{x}_u}{dt} = A\bar{x}_u + b$$

where A, b depend on both unitary and dissipative dynamics.

- Similarly, the CM evolves according to the Lyapunov equation:

$$\frac{d\sigma_u}{dt} = A\sigma_u + \sigma_u A^T + D$$

where D is called the diffusion matrix.

- The continuous measurement will cause the mean $\bar{x}_c$ to evolve stochastically according to the Langevin equation:

$$\frac{d\bar{x}_c}{dt} = (A\bar{x}_c + b) + (\sigma_c C^{\mathsf{T}} + \Gamma^{\mathsf{T}})\xi(t)$$

where C, Γ are matrices and $\xi(t)$ is a vector of white noises.

- The CM, on the other hand, evolves deterministically:

$$\frac{d\sigma_c}{dt} = A\sigma_c + \sigma_c A^{\mathsf{T}} + D - \chi(\sigma_c)$$

where

$$\chi(\sigma) = (\sigma_c C^{\mathsf{T}} + \Gamma^{\mathsf{T}})(C\sigma + \Gamma) \geqslant 0$$

(update matrix) describes the information gained due to the measurement.

M. G. Genoni, L. Lami, and A. Serafini, “Conditional and unconditional Gaussian quantum dynamics“, Contemp. Phys. **57**, 331 (2016).

🔁 Thermodynamics of Gaussian CMs

- In the case of continuous measurements, the relevant quantity is the entropy production **rate**.

- We formulate the thermodynamics of this model using a semi-classical representation in terms of the Wigner function $W(x)$ (standard approach does not work).

  - The Wigner function, conditioned on a given outcome for the average, is $W_c(x|\bar{x})$.

  - The variable $\bar{x}$ is classical, with probability distribution $p(\bar{x})$.

  - The conditional and unconditional Wigner functions are thus associated by a Kalman filter:

$$W_u(x) = \int W_c(x|\bar{x})p(\bar{x})d\bar{x}$$

A. Belenchia, L. Mancino, GTL and M. Paternostro, "Entropy Production in Continuously Measured Quantum Systems", arXiv:1908.09382. Accepted in npj Quantum Information.

- As an alternative representation of entropy, we use

$$S_u = -\int W_u(x) \ln W_u(x) dx$$

and

$$S_c = -\int p(\bar{x}) d\bar{x} \int W_c(x \,|\, \bar{x}) \ln W_c(x \,|\, \bar{x}) dx$$

- Their difference represents the net amount of information acquired by the measurement record:

$$I = S_u - S_c \geqslant 0$$

- This is the phase-space analog of the Holevo quantity. Exactly the same idea ✅.

$$\left(\chi_M(\rho'_S) = S(\rho'_S) - \sum_k p_k S(\rho'_{S|k}) \right)$$

G. Adesso, D. Girolami, A. Serafini, “Measuring gaussian quantum information and correlations using the Rényi entropy of order 2”. PRL **109**, 190502 (2012).

🅿️ Unconditional production/flux

- The unconditional Wigner function evolves according to a Fokker-Planck equation:

$$\frac{\partial W_u}{\partial t} = \mathrm{div} J(W_u)$$

where

$$J(W_u) = (Ax + b)W_u - \frac{D}{2}\nabla W_u$$

is a quasi-probability current.

- The entropy production and flux rates are

$$\Pi_u = 2\int \frac{dx}{W_u} J^T D^{-1} J \geqslant 0$$

$$\Phi_u = -2\int J^T D^{-1} A \, dx$$

- The stochastic MEq is translated into a stochastic Fokker-Planck equation:

$$\frac{\partial W_c}{\partial t} = \mathrm{div}\left[J + J_{\mathrm{sto}}\right]$$

where

$$J_{\mathrm{sto}} = W_c(\sigma_c C^T + \Gamma^T)\xi(t)$$

- One can show that the flux does not change:

$$\Phi_c = \Phi_u$$

as intuitively expected.

J. P. Santos, GTL, M. Paternostro, "The Wigner entropy production rate", PRL **118**, 220601 (2017).

A. Belenchia, L. Mancino, GTL and M. Paternostro, "Entropy Production in Continuously Measured Quantum Systems", arXiv:1908.09382. Accepted in npj Quantum Information.

- Hence, as before, we will have

$$\Pi_u = \dot{S}_u + \Phi_u$$
$$\Pi_c = \dot{S}_c + \Phi_u$$
$$\therefore \qquad \Pi_c = \Pi_u - \dot{I}$$

- In particular, the net rate of information gain can be shown to be

$$\dot{I} = \frac{1}{2}\mathrm{tr}\left[D(\sigma_c^{-1} - \sigma_u^{-1})\right] - \frac{1}{2}\mathrm{tr}\left[\chi(\sigma_c)\sigma_c^{-1}\right] := \dot{L} - \dot{G}$$

✓ The 1st term is the information loss rate due to the dissipation ($\propto D$).

✓ The 2nd term is the information gain rate, due to the update matrix $\chi(\sigma_c)$

- In the steady-state $\dot{I} = 0$. But this does not mean we are no longer acquiring information.
  - What it means is that $\dot{G} = \dot{L}$: the information acquired is balanced by the information dissipated.

GTL, M. Paternostro, A. Belenchia,
“Thermodynamics and information in continuously monitored collisional models”,
In preparation.

Informational steady-state

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Experimental Assessment of Entropy Production in a Continuously Measured Mechanical Resonator

Massimiliano Rossi,[1,2] Luca Mancino,[3] Gabriel T. Landi,[4] Mauro Paternostro,[3]
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arXiv:2005.03429

🇩🇰 Copenhagen setup

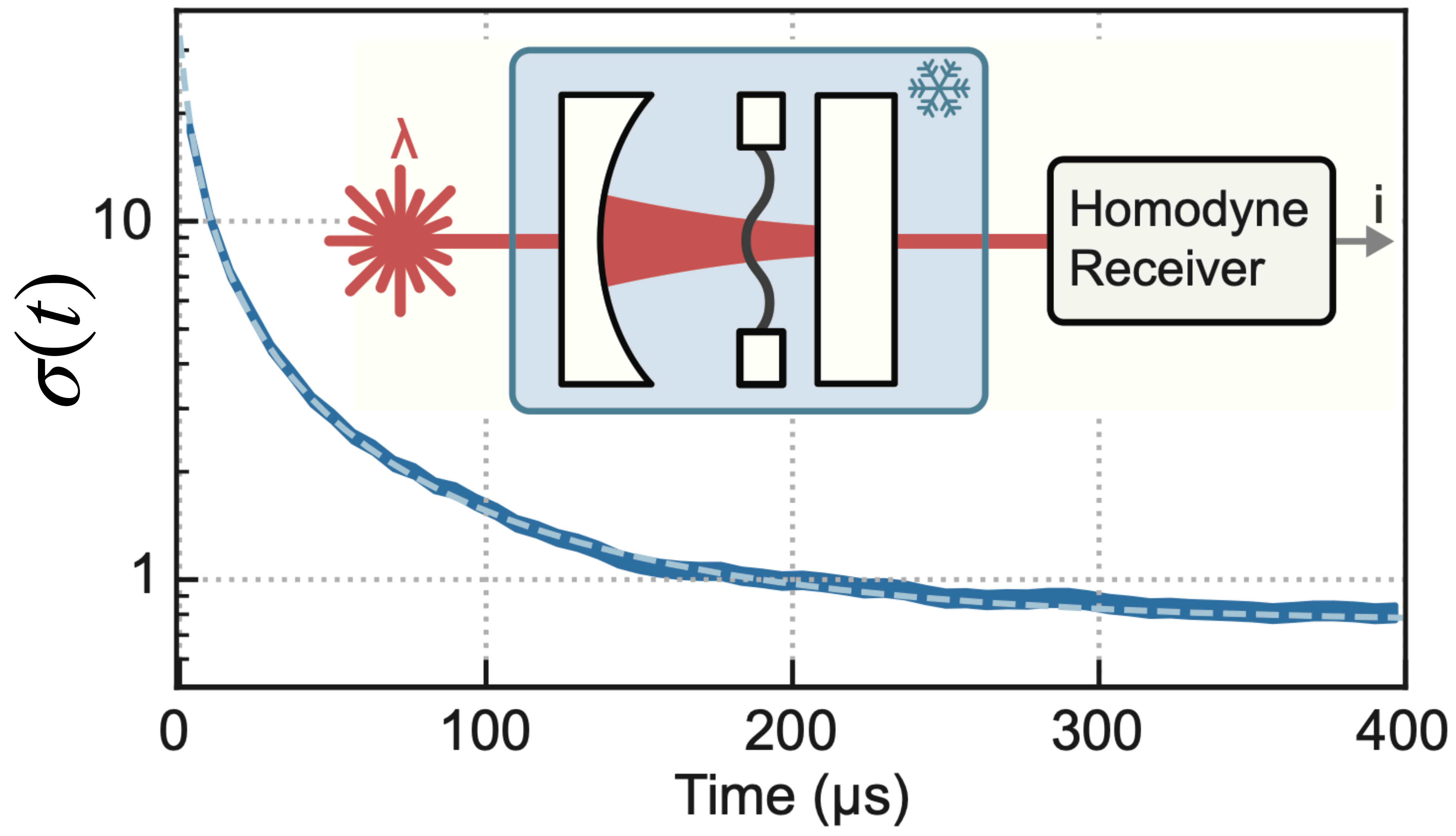
- Optomechanical system continuously monitored by an optical field.
- Competition: Thermal bath vs. Measurement.
- Quadratures of the mechanical mode: $x = (q, p)$
- CM $\sigma \propto$ 𝟙
- Unconditional dynamics tends to $\bar{x}_u = 0$

$$\sigma_u = \bar{n} + 1/2 + \Gamma_{qba}/\Gamma_m$$

- Conditional dynamics evolves instead to

$$\frac{dx}{dt} = -\frac{\Gamma_m}{2}x + \sqrt{4\eta\Gamma_{\mathtt{qba}}}\sigma_c(t)\xi(t)$$

$$\frac{d\sigma_c}{dt} = \Gamma_m(\sigma_u - \sigma_c) - 4\eta\Gamma_{\mathtt{qba}}\sigma_c^2$$



Informational steady-state:

Conditional dynamics relaxes to a colder state, $\sigma_c < \sigma_u$, which can only be maintained by continuously monitoring S.

Production and flux at the trajectory level

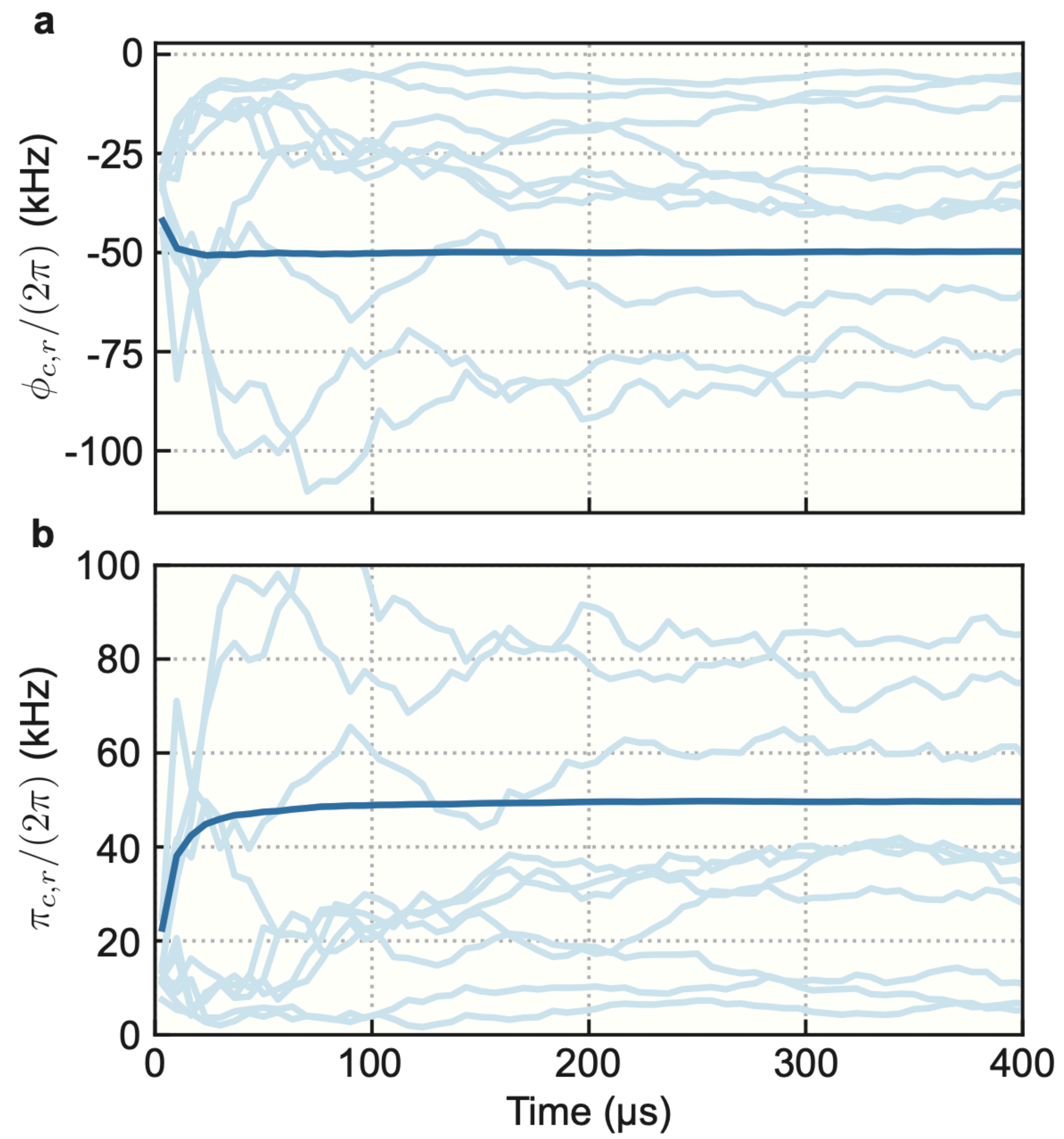


FIG. 2. **Stochastic entropy flux and production rates.** **a,** The stochastic entropy flux rates (light blue) for a sample of 10 trajectories. The dark blue line is the ensemble average over all the trajectories. **b,** The stochastic entropy production rates (light blue) and the ensemble average (dark blue), for the same sample of trajectories.

Information gain/loss rates characterizing the information steady-state

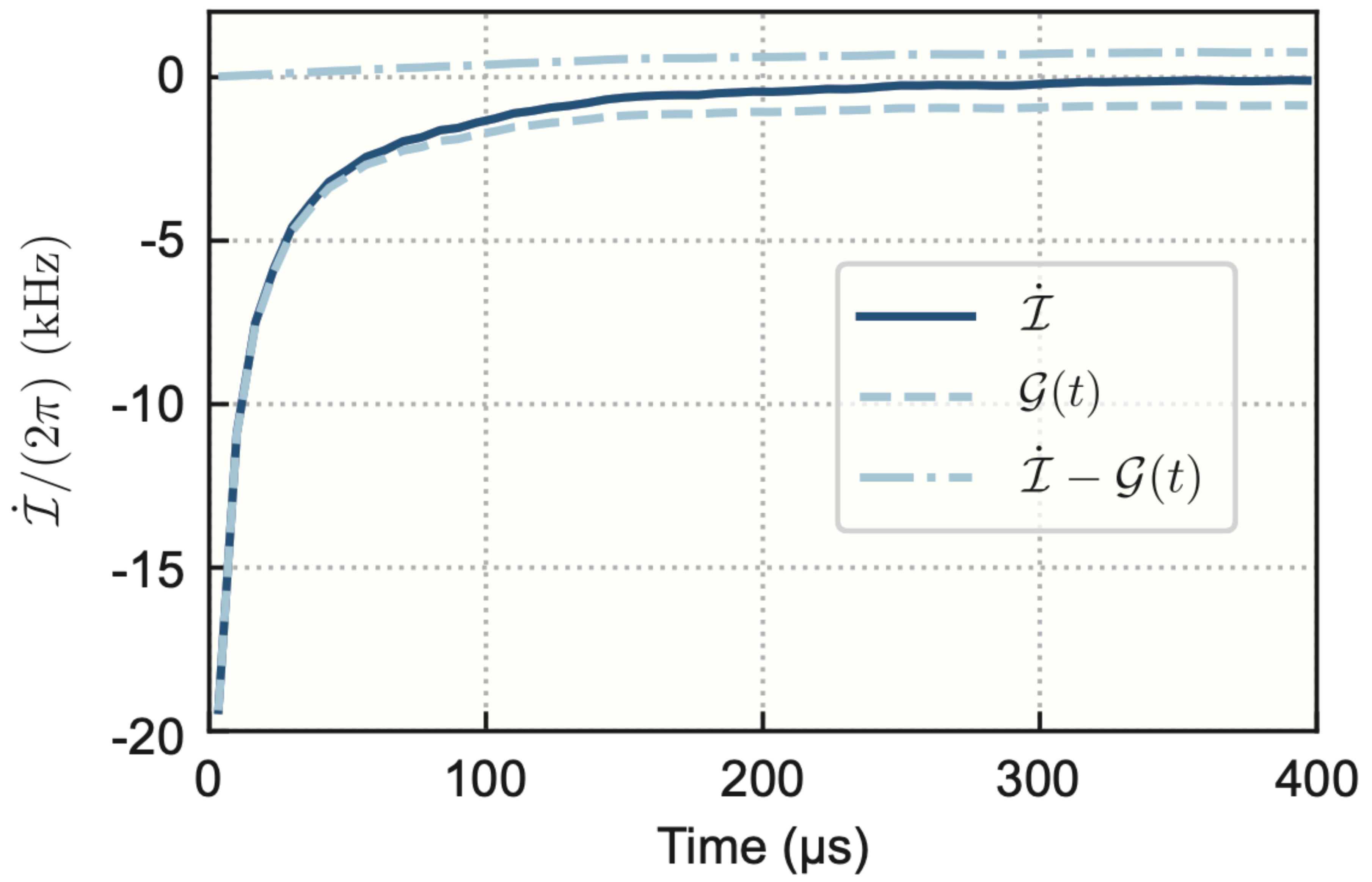


FIG. 3. **Informational contribution to the entropy production rate.** We obtain the informational contribution (dark blue) from the entropy production. The dashed (dot-dashed) line is the differential gain of information due to the measurement (loss of information due to noise input by the phonon bath).

Conclusions

- Knowing something about the bath makes the process less irreversible.
- The conditional entropy production quantifies this effect.
- But quantifying this for continuously monitored quantum systems is not trivial.
  - We put forth a framework for GCV systems.
    - Rich and clear physical interpretation.
  - We also provide an experimental assessment of the entropy production at the level of stochastic trajectories in a quantum optomechanical system.

Thank you! 🙃

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